Fully-developed turbulent flow in smooth and rough-walled pipe
by John Leonard Gow

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
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Abstract:
FulIy-developed turbulent flow in both smooth and rough-walled pipes was investigated in a Reynolds number range 30,000 - 500,000, Experimental mean velocity profiles, root-mean-square fluctuating velocity profiles, cross-correlations. of the fluctuating velocities, and friction factor data are presented in the thesis for flow in a smooth pipe and two sand-roughened pipes: R/e = 208 and R/e = 26.4. The quantity R/ε is the ratio of the actual pipe radius to the average sand particle size. All velocity data-that were obtained were with hot-wire anemometers employing standard measuring techniques.

The smooth pipe data agree very well with the data of previous investigators and satisfy the Navier-Stokes equations for fully-developed axisymmetric turbulent flow. For Reynolds numbers less than 50,000, the data obtained using the R/e=208 rough pipe exhibit; essentially the same characteristics as the smooth tube data. However, for Reynolds numbers greater than 50,000 in the R/e = 208 rough pipe and for all flow rates investigated in the-R/ε = 26.4 rough pipe, the data differ from smooth pipe data; the difference being a function of roughness size as well as Reynolds number.

The rough pipe experimental data do not agree with emperical data obtained, employing -the Navier-Stokes equations for fully-developed axisymmetric turbulent flow; although the experimental data do agree with limited rough-pipe data of other investigators. Apparently, the axisymmetric, fully-developed flow model prescribed for smooth pipes does not represent the turbulent flow in rough-walled pipes.

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NOMENCLATURE

Symbol	Description
A, A', B, B'	Emperical constants
D	Pipe diameter
e_1, e_2	Linearized voltage signals
$\sqrt{\overline{e_1^2}}, \sqrt{\overline{e_2^2}}$	Root-mean-square linearized voltage signals
f	Friction factors
h	Distance between parallel plates
I	Hot wire current
K	Calibration constant
K_s	Equivalent sand diameter
ℓn	Logarithm to base e
log	Logarithm to base 10
P	Mean pressure at any point in the pipe
r	Radial coordinate
R	Pipe radius
R'	Hot wire resistance
Q	Flow rate
T	Temperature
Re	Reynolds number $= \frac{U_{avg} D}{\nu}$
U	Mean velocity in axial direction

Symbol	Description
U_{avg}	Bulk mean velocity
U_{max}	Centerline mean velocity
U_*	Shear velocity $(\tau_o/\rho)^{1/2}$
$U+$	Dimensionless mean velocity (U/U_*)
u	Instantaneous fluctuation component in axial direction
$\sqrt{\overline{u}^2}$	Root-mean-square of fluctuating velocity in axial direction
V	Mean velocity component in radial coordinate direction
v	Instantaneous value of fluctuating velocity component in radial direction
$\sqrt{\overline{v}^2}$	Root-mean-square of fluctuating velocity in radial coordinate direction
W	Mean velocity component in angular coordinate direction
w	Instantaneous value of fluctuating velocity in angular direction
$\sqrt{\overline{w}^2}$	Root-mean-square of fluctuating velocity in angular coordinate direction
y	radial distance, $y = 0$ is the wall
Y^+	Dimensionless distance parameter (yU_*/ν)
z	Axial coordinate
ε	Average sand grain size
$\varepsilon U_*/\nu$	Roughness parameter

Symbol	Description
e	Air density
ν	Kinematic viscosity
θ	Angular coordinate
τ	Turbulent shear stress
τ_o	Wall shear stress
∇^2	$\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

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Fully-developed turbulent flow in both smooth and rough-walled pipes was investigated in a Reynolds number range 30,000 - 500,000. Experimental mean velocity profiles, root-mean-square fluctuating velocity profiles, cross-correlations of the fluctuating velocities, and friction factor data are presented in the thesis for flow in a smooth pipe and two sand-roughened pipes: $R/\varepsilon = 208$ and $R/\varepsilon = 26.4$. The quantity R/ε is the ratio of the actual pipe radius to the average sand particle size. All velocity data that were obtained were with hot-wire anemometers employing standard measuring techniques.

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I. INTRODUCTION

Fully developed turbulent flow over a smooth boundary has been extensively examined both theoretically and experimentally. Although strictly empirical, the accepted theories for flow in a smooth-walled tube agree well with experimental data and have been successfully applied in areas of heat transfer and hydraulic design. However, smooth tube theory has not been adequately modified to predict velocity profiles, shear stress distributions, or convective heat transfer characteristics of the flow in tubes with rough boundaries.

The rough surface has been shown by Dippery[1] to be a more effective heat transfer surface when compared with the smooth surface in a restricted range of Prandtl and Reynolds numbers. The larger surface area is, of course, a factor; but the marked difference in velocity profiles is also thought to be important. Since neither acceptable theory, nor adequate experimental velocity data has been available, an evaluation of the effect of the velocity profile has not been possible.

Consequently, the purpose of this investigation was to collect information on the properties of flow in sand-roughened pipes that might be eventually employed to more fully explain the heat transfer characteristics of rough-walled pipes.

An examination of the momentum and continuity equations governing turbulent pipe flow shows that the principle quantities of interest are the temporal mean velocity profiles and fluctuating velocity correlations. Root-mean-square values of fluctuating velocity components and friction factors are also presented for completeness.

II. LITERATURE REVIEW

A review of the literature indicates the considerable amount of work undertaken in the investigation of turbulent flow. However, close scrutiny of the textbooks which treat turbulent theory and present experimental data, reveals that only a few individuals have actually been responsible for the advancement and understanding of this complex phenomenon. Presented here is a review of the pertinent references dealing with the theoretical and experimental accomplishments for fully-developed turbulent flow in both smooth and rough surface conduits.

The important theories of turbulent flow can be classed into two categories, empirical and statistical. Unlike laminar flow, turbulent flow theory has never been fully manageable from momentum or energy considerations alone. The empirical theory, historically first, attempted to describe the phenomenon on the basis of mean temporal velocities. The statistical theory is based on a knowledge of the fluctuating velocity components and their correlations.

In the late 1800's, Reynolds modified the Navier-Stokes equations, generally applicable only to laminar flow, to include the fluctuating velocity components and the apparent turbulent shear stresses. This was done in an attempt to illustrate the role played by the fluctuating velocities in the transfer of momentum. The Reynolds transformation is presented in a thorough manner by Corcoran, Opfell and Sage.[2] Employing the concept of turbulent stresses, Prandtl proposed his mixing length theory[3] to explain the high friction factors encountered in turbulent flow. Prandtl reasoned that the high friction factors were due to an exchange of momentum between fluid layers as particles moved transversely to the direction of mean flow. The term "mixing length" was introduced to describe the distance a particle would have to travel in order to attain the same momentum as that of a different layer. Prandtl developed an expression for the turbulent shear stress and introduced his universal velocity distribution equation which has remained one of the most useful for predicting point mean velocities. The equation proposed by Prandtl is

$$U = \frac{U_*}{K} \ln y + C \qquad (2.1)$$

where U_* is the shear velocity, K is a universal empirical constant obtained from experimental data, and C is a constant of integration that must be evaluated from boundary conditions. Evaluation of this integration constant at the pipe centerline yields the velocity deficiency equation

$$\frac{U_{max}-U}{U_*} = \frac{1}{K} \ln \frac{R}{y} \qquad (2.2)$$

Perhaps the best known adaption of this equation is the universal logarithmic velocity equation for large Reynolds number flow in smooth tubes. This equation has the form

$$U^+ = A + B \ln Y^+ \tag{2.3}$$

where A and B are again empirical constants, $U^+ = U/U_*$ and $Y^+ = yU_*/\nu$.

Subsequent investigations by Prandtl revealed that a thin laminar region existed at boundary for which it can be shown from shear stress considerations that

$$U^+ = Y^+ . \tag{2.4}$$

Employing this postulate and Blasius' friction factor equation, Prandtl also introduced the power law for turbulent velocity distribution as

$$\frac{U}{U_{max}} = (y/R)^{1/n} \tag{2.5}$$

where the exponent n varies, between six and ten, as a function of Reynolds number.

Later von Karman introduced his similarity hypothesis[2] in an attempt to determine the dependence of mixing length on space coordinates. He assumed the turbulent fluctuations differ from point to point only by time and length scale factors. von Karman assumed the mixing length satisfied the equation

$$\ell = K \left| \frac{dU/dy}{d^2U/dy^2} \right| \tag{2.6}$$

where K is an empirical constant to be determined from experimental data. The mixing length thus can be considered independent of velocity magnitude and dependent only on the first and second derivation of U with respect to y.

Applying the similarity hypothesis, combined with the expression for turbulent shear stress as proposed by Prandtl, the universal velocity distribution law of von Karman is established as

$$\frac{U_{max}-U}{U_*} = \frac{1}{K} \left[\ln(1-\sqrt{1-y/R}) + \sqrt{1-y/R} \right] \tag{2.7}$$

A graphical comparison of von Karman's and Prandtl's universal velocity distribution equations is presented in Schlichting.[4] It must be noted here, as pointed out by Schlichting, that neither equation is valid at the wall or at the pipe centerline.

Taylor presented a series of papers treating the statistical considerations of isotropic and homogeneous flow in 1935. These papers and others by Dryden, Kolmogoroff, von Karman and Lin are presented by Friedlander and Topper.[5] However, only the works of Taylor have been applied in boundary layer flow to produce a useful equation for predicting velocity profiles. As a result of his theoretical investigations, Taylor proposed the vorticity transport hypothesis.[2] For a fully developed flow he considered the axial-direction equation of motion and introduced the vorticity vector into the equation. After time averaging the equation and employing a modification of the Reynold's shear stress equation as proposed by Prandtl, Taylor developed the same equation for the turbulent shear stress as did von Karman with the similarity hypothesis. However, Taylor evaluated the velocity distribution employing all the boundary conditions for the centerline of the pipe, and thus the resulting equation

$$\frac{U_{max}-U}{U_*} = \frac{\sqrt{2}}{K} \sin^{-1}[\sqrt{y/R} - \sqrt{y/R}\ \sqrt{1-(y/R)}] \qquad (2.8)$$

agrees well with experimental data at the centerline, but yields a negative infinite mean velocity at the wall which is, of course, not valid.

Wang[6] has developed yet another emperical velocity distribution equation which agrees with experimental data better than do the equations of Prandtl and von Karman. Wang produced an expression for the mixing length from experimental velocity profiles and, employing Prandtl's mixing length theory, derived the following equation:

$$\frac{U_{max}-U}{U_*} = 2.5 \ [\ln \frac{1+\sqrt{1-y/R}}{1-\sqrt{1-y/R}} - 2\tan^{-1}\sqrt{1-y/R}$$

$$- 0.572 \ \ln \frac{1-y/R + 1.75\sqrt{1-y/R} + 1.53}{1-y/R - 1.75\sqrt{1-y/R} \pm 1.53}$$

$$+ 1.14 \tan^{-1} \frac{1.75\sqrt{1-y/R}}{1.53 - (1-y/R)}] . \qquad (2.9)$$

However, because of its complexity, the equation has never been employed as a useful tool.

In 1953, Pai presented papers[7,8] treating turbulent flow strictly from a consideration of the Navier-Stokes equations by employing the Reynold's transformation. Pai derived the same equation for the $\overline{uv}$ correlation as did Laufer,[12]

$$\overline{uv} = \nu \frac{dU}{dr} + \frac{r}{R} U_T^2 \ ; \qquad (2.10)$$

however, he integrated this equation to yield an expression for the mean velocity valid for both laminar and turbulent flow. Employing experimental mean velocity data, Pai developed empirical equations for the $\overline{uv}$ correlation and turbulent mean velocity profile. Excellent agreement with experimental smooth tube $\overline{uv}$ correlation data is indicated by Knudsen and Katz.[3]

General acceptance of Pai's theory is not indicated, because references to it are not found in newer textbooks and recent investigations continue to employ the earlier empirical theories derived from mixing length considerations.

Nikuradse has presented a complete set of mean velocity and friction factor data for both smooth and rough tubes.[9] He correlated his data to Prandtl's mixing length theory and evaluated the empirical constants in the logarithmic equation to yield the equation, for smooth tube flow,

$$U^+ = 5.5 + 2.5 \ln Y^+ \tag{2.11}$$

for data in the turbulent core. However, Knudsen and Katz[3] point out that Nikuradse shifted his original data to correlate with the laminar sublayer hypothesis, and the validity of the constants is questioned.

Nikuradse divided the U^+ versus log Y^+ distribution curve into three distinct parts: the laminar sublayer, for which $U^+ = Y^+$ according to Prandtl; a buffer region in which the transition from laminar to turbulent flow results; and the completely turbulent region. von Karman analyzed the existing data and found the extent of the laminar sublayer to be between $Y^+ = 0$ and $Y^+ = 5$. Between $Y^+ = 5$ and $Y^+ = 70$ is the buffer or transition layer and for $Y^+ > 70$ is the turbulent core.

Nikuradse's data, when fitted in these regions, yielded the following:

Laminar $U^+ = Y^+$

transition $U^+ = -3.05 + 3.5 \ \ln Y^+$

turbulent $U^+ = 5.5 + 2.5 \ \ln Y^+$ (2.12)

Nikuradse's friction factor data is compared with Prandtl's universal law of friction for smooth pipes in Schlichting.[4] Excellent agreement is seen.

Deissler[10] has presented data for fully developed flow in smooth tubes and the equation

$$U^+ = 3.8 + 2.78 \ \ln Y^+ \qquad (2.13)$$

best fitted his data for $Y^+ > 26$ in the turbulent core region. For $Y^+ < 26$, he derived the empirical equation

$$Y^+ = \frac{1}{n} \ \frac{1/\sqrt{2\pi} \int_0^{nU^+} e^{-[(nU^+)^2/2]} d(nU^+)}{(1/\sqrt{2\pi}) \ e^{-[(nU^+)^2/2]}} \qquad (2.14)$$

where the value of n was found experimentally.

Laufer conducted investigations in both circular pipes[11] and rectangular tubes[12] with smooth boundaries. He presented experimental data correlated to theoretical equations derived from the Navier-Stokes equations. The work consisted of a determination of mean velocity profiles, fluctuating velocity profiles and $\overline{uv}$ correlation measurements. The treatment of the Navier-Stokes equations yielded two important results: namely an equation for predicting the $\overline{uv}$ correlation from mean velocity profile data and the fact that the $\overline{vw}$ correlation must be zero for all points in the pipe. The values of $\overline{uv}$ calculated by using the mean velocity profile are subject to error only in calculating the slope of the velocity profile from experimental data. Such calculated values have been shown to agree very well with measured $\overline{uv}$ values.

Many other experimental investigations have been conducted in smooth tubes and a list of the mean velocity logarithmic equations for the turbulent core is presented in Table I as taken from Corcoran, Page, Schlinger and Sage.[13] (See page 15)

A recent investigation by Richardson and McQuivery[14] contains fluctuating velocity data for both smooth and rough boundary open channel flow. RMS measurements of the axial fluctuating componetns for smooth boundary flow are compared to Laufer's data and good agreement is indicated. A comparison of their data to that of the present investigation for flow over a rough boundary is presented in Chapter V.

Although theory of turbulent flow in rough tubes is lacking, experimental data has been presented. The most notable is the investigation of Nikuradse.[9] Nikuradse experimented with artificially roughened pipes and determined mean velocity profiles for a number of pipe radius to sand grain height ratios. His roughness was obtained by cementing sand grains of a given average size to the inside of pipes of various diameters.

Nikuradse's data was correlated to smooth tube flow by altering Prandtl's universal logarithmic equation to contain the roughness height ε. The equation is

$$U^+ = 2.5 \ln \frac{Y}{\varepsilon} + C \qquad (2.15)$$

where C was determined experimentally as a function of the roughness parameter $\varepsilon U^+/\nu$. Nikuradse also fitted straight lines to the U^+ versus log Y^+ data in the turbulent core and found the slopes of these lines constant and equal to the slope obtained in smooth tubes. However, the ordinant intercepts of these straight lines decreased with increasing values of the roughness parameter. For values of the roughness parameter less than five, the U^+ versus log Y^+ distribution coincided with the smooth tube curve. Friction factors were also predicted with excellent accuracy from Prandtl's universal law of friction for smooth pipes.[4] For values of the roughness parameter greater than seventy, the friction factors were found to be independent of Reynold's number and dependent only on the actual roughness size ε. Nikuradse thus divided the flow into three regimes:

$\frac{\varepsilon U_*}{\nu} \leq 5$ - Hydraulically smooth

$5 \leq \frac{\varepsilon U_*}{\nu} \leq 70$ - Transition from smooth to rough flow

$\frac{\varepsilon U_*}{\nu} \geq 70$ - Completely rough.

A detailed comparison is made of Nikuradse's data and the data of the present investigation in Chapter VI.

In 1944, Moody[15] presented extensive friction factor data for commercially rough pipes. The Moody Diagram remains today the most useful method for predicting friction head losses for turbulent flow in rough tubes.

Schlichting,[4] attempting to correlate friction factors for various roughness shapes and distributions to those of Nikuradse's sand roughness, introduced the concept of equivalent roughness. Friction factor data can be substituted into von Karman's friction factor law[4] for completely rough flow,

$$f = \frac{1}{(2 \log(R/K_s) + 1.74)^2} \tag{2.16}$$

to calculate the equivalent sand roughness K_s. Similar measurements have been made by Streeter[16] in pipes roughened with machined groves of different forms and distribution.

Townes[17] studied open channel flow over rough boundaries and noted that the mean velocity distribution was in close agreement with the logarithmic universal equation of Prandtl in the fully turbulent section of the boundary layer. Data close to the boundary is inconsistent with Prandtl's laminar sublayer hypothesis. However, it is noted that the flow in the so-called laminar sublayer is quite complicated, more often that not involving velocity components in all three coordinate directions.

The only data available until now for fluctuating velocity correlations in rough pipes has been presented by Logan and Jones.[18] They measured the $\overline{uv}$ correlation and turbulence intensities following an abrupt increase in surface roughness. A comparison of their data is also presented in Chapter V.

TABLE I

Investigator	Equation
Nikuradse	$U^+ = 5.5 + 2.5 \ln Y^+$
Deissler	$U^+ = 3.8 + 2.78 \ln Y^+$
Laufer	$U^+ = 5.5 + 2.91 \ln Y^+$
Wattendorf	$U^+ = 4.0 + 2.68 \ln Y^+$
Skinner	$U^+ = 6.0 + 2.71 \ln Y^+$

III. EXPERIMENTAL SYSTEM

A. Apparatus

The experimental system illustrated schematically on Figure (1) was employed throughout the investigation. Hot-wire anemometry was used for all velocity measurements.

Air was discharged from a centrifugal fan into the entrance section through eighty feet of flexible duct. Flow rates were controlled with a manual damper on the inlet side of the fan. The entrance section, pictorially illustrated in Figure (2), contained a baffle plate and a series of screens to filter the air and dampen turbulence from the fan and return line. The cross-section of the entrance section is reduced in the transition section to match the test pipe diameter. Seventy feet of one-foot diameter aluminum irrigation pipe was used to ensure fully developed flow at the test section. The pipe was suspended with adjustable brackets so near perfect alignment was possible. The test section (Figure (3)) is mounted on the downstream end of the test pipe and consists of a machined section of one-foot diameter and a traversing mechanism for radial positioning of the hot-wire. This mechanism consists of two parallel cylindrical rods which are spanned by the probe support holder. Motion of the holder on the rods is provided by ball-bearing guides. A micrometer screw feed is mounted on the holder to allow hot-wire movements

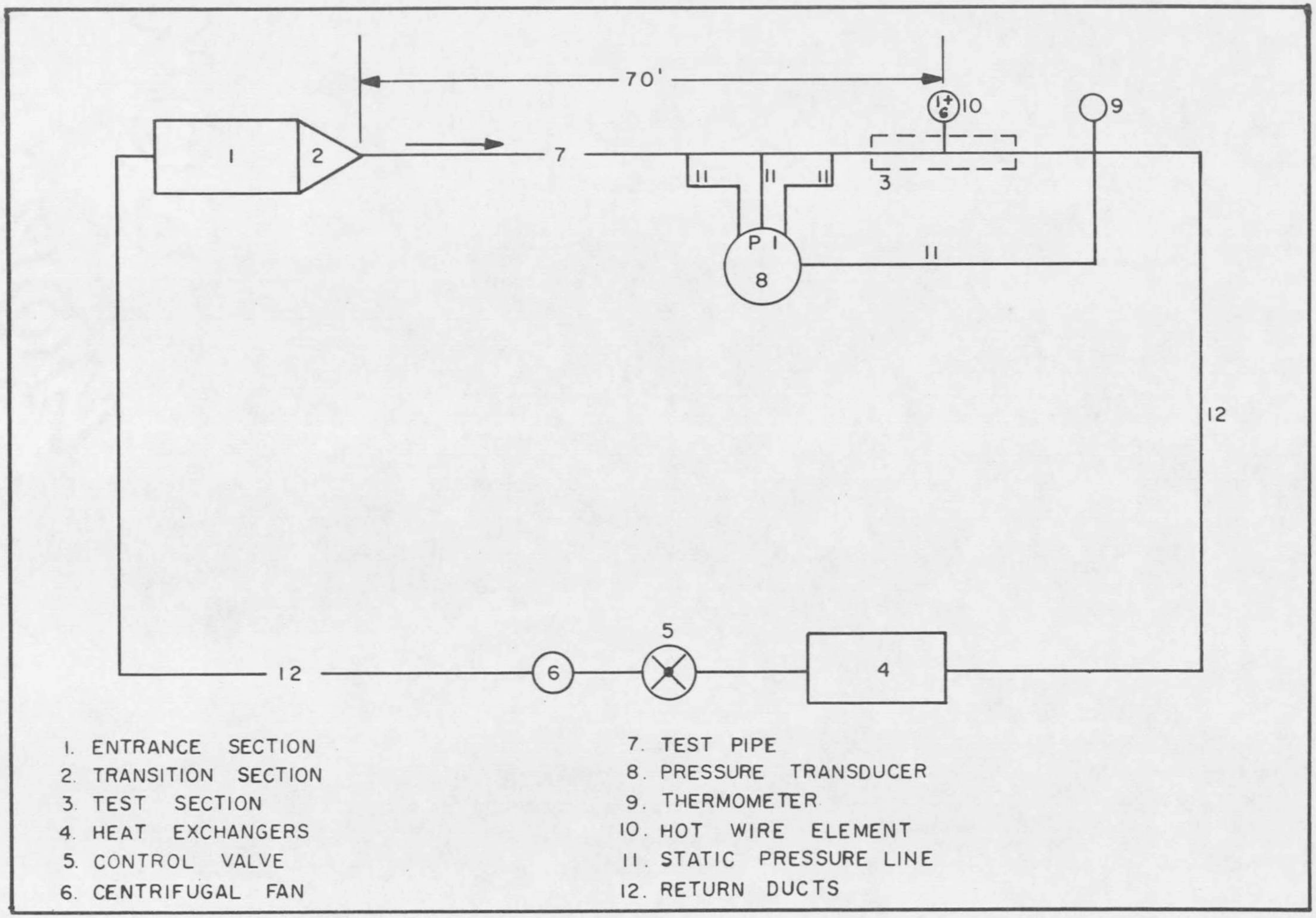

Figure 1. Schematic of Experimental System