Circuit analyzer for an induction motor

by Allan K Hammell

A THESIS Submitted to the Graduate faculty in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University

Abstract:
The circuit analyzer for polyphase induction motors is designed for use in the Electrical Engineering laboratory at Montana State College. The circuit analyzer is used to obtain the current, voltage, power input, power output, and efficiency for various motor speeds. The long and laborious calculations of the analytical solution are unnecessary. Tests showed that the circuit analyzer results compared very favorably with those of the analytical solution.

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Allan K Hammell

ABSTRACT

The circuit analyzer for polyphase induction motors is designed for use in the Electrical Engineering laboratory at Montana State College. The circuit analyzer is used to obtain the current, voltage, power input, power output, and efficiency for various motor speeds. The long and laborious calculations of the analytical solution are unnecessary. Tests showed that the circuit analyzer results compared very favorably with those of the analytical solution.

CIRCUIT ANALYZER FOR AN INDUCTION MOTOR

INTRODUCTION

The polyphase induction motor is the most common alternating-current motor in use at the present time for industrial application. The motor is inexpensive, rugged, and has a nearly constant speed. Squirrel-cage motors have no external connections to the rotor and can be totally enclosed for use in adverse conditions where sparking or abrasive dust may be hazardous.

The wide-spread use of the polyphase induction motor makes it feasible to have a circuit analyzer for the study of induction motors.

Circuit analysis has been used in the study of transmission line short-circuit currents by means of a calculating board. The transmission line calculating board is set up with resistances connected to simulate a short circuited transmission network. Direct-current voltages are applied at points where voltage sources exist and the currents at various points in the network are measured to find the behavior of the system.

Since both inductive reactance and resistance are present in an induction motor, power factor is an important item. Consequently, an a-c system must be used in a circuit analyzer for an induction motor. The equivalent circuit of an induction motor, derived on pages 8 to 13, is used for the circuit analyzer designed for use in the Electrical Engineering laboratory at Montana State College.

INDUCTION MOTOR THEORY

Principles of Operation

The essential feature in the theory of a polyphase induction motor is a revolving field produced by the stator. The operation of the induction motor may be explained by representing the revolving field by a rotating field structure NS in Fig. 1. On the same shaft, there is a short circuited rotor R that is free to rotate in any direction.

As the field rotates, a current is induced in the windings of the rotor. With a clockwise rotation of the field, the current in the conductor under the north pole will be out of the paper. The field produced by this current will be in a counter clockwise direction and consequently, a clockwise torque will be exerted to the rotor by the interaction of the two fields. A similar torque is produced in conductors under the south pole since both the conductor current and the field flux are opposite to those of the north pole.

The rotor will not attain the same speed as the field because the existence of torque depends upon the field cutting across the conductors.

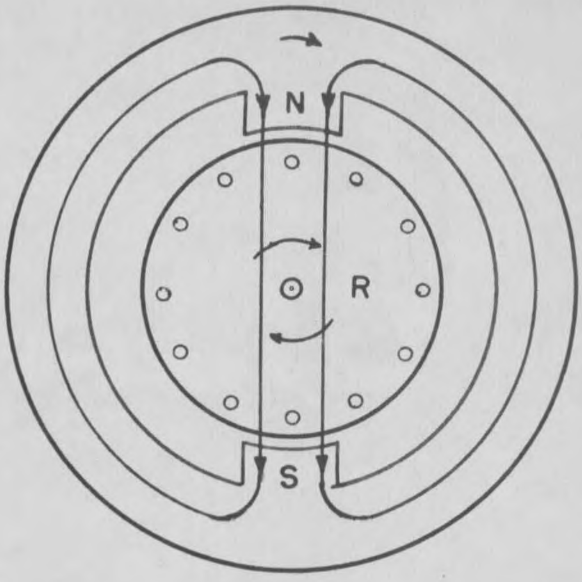

FIG. 1 PRINCIPLE OF INDUCTION MOTOR.

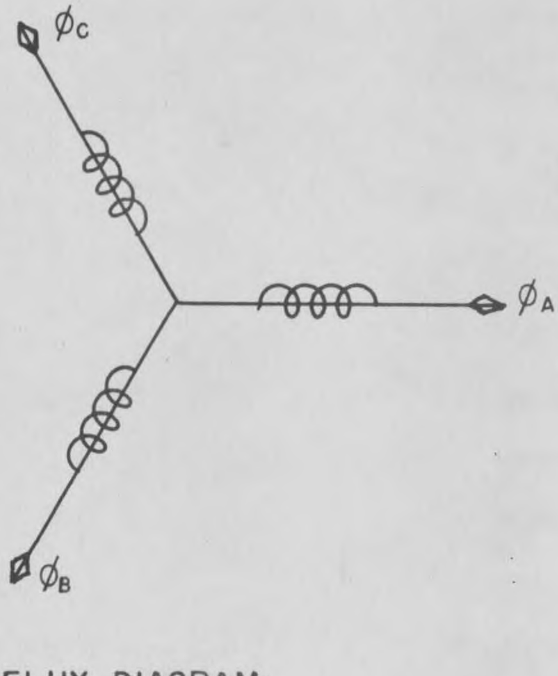

FIG. 2 STATOR FLUX DIAGRAM.

The Rotating Field

The field of an induction motor is shown to be rotating and of constant magnitude by considering the current in the stator windings and the space relationship of the windings. Although the windings are spread over the periphery of the stator, each set of windings has an axis. The axes are equally spaced in the stator with an angle of $\frac{360}{m}$ electrical degrees between them, where m is the number of phases.

The currents in a three-phase system are

$$i_a = I_M \cos \omega t \tag{1}$$

$$i_b = I_M \cos (\omega t - 120°) \tag{2}$$

$$i_c = I_M \cos (\omega t + 120°) \tag{3}$$

Since the flux is in phase with the inducing current, the flux equations for the three-phase stator represented in Fig. 2 may be written as

$$\phi_a = \Phi \cos \omega t = \frac{\Phi}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) \tag{4}$$

$$\phi_b = \Phi \cos (\omega t - 120°) = \frac{\Phi}{2}\left(e^{j(\omega t - 120°)} + e^{-j(\omega t - 120°)}\right) \tag{5}$$

$$\phi_c = \Phi \cos (\omega t + 120°) = \frac{\Phi}{2}\left(e^{j(\omega t + 120°)} + e^{-j(\omega t + 120°)}\right) \tag{6}$$

The resultant flux of the stator is the vector sum of the three fluxes and is expressed by

$$\phi = \phi_a + \phi_b \, e^{j120°} + \phi_c \, e^{-j120°} \tag{7}$$

Then

$$\phi = \frac{\bar{\Phi}}{2}\Big[(e^{j\omega t} + e^{-j\omega t}) + e^{j120°}\left(e^{j(\omega t-120°)} + e^{-j(\omega t-120°)}\right)$$

$$+ e^{-j120°}\left(e^{j(\omega t+120°)} + e^{-j(\omega t+120°)}\right)\Big] \qquad (8)$$

$$\phi = \frac{\bar{\Phi}}{2}\left[3e^{j\omega t} + e^{-j\omega t}\left(1 + e^{j120°} + e^{j240°}\right)\right] \qquad (9)$$

From symmetrical components,

$$1 + e^{j120°} + e^{j240°} = 0 \qquad (10)$$

The resultant flux

$$\phi = \frac{3}{2}\,\bar{\Phi}\,e^{j\omega t} \qquad (11)$$

has a constant magnitude $\frac{3}{2}\bar{\Phi}$ and a rotation of line frequency. It should be noted that the rotation is in terms of electrical degrees. The frequency of rotation is converted into mechanical speed by the equation

$$n = \frac{120° f_1}{p} \qquad (12)$$

where n is the speed of the field in revolutions per minute;

f_1 is the frequency of the supply system in cycles per second and

p is the number of poles in the stator.

<u>Equivalent Circuit Diagram</u>

The polyphase induction motor has many characteristics in common with the transformer. The motor may be considered as a polyphase transformer with the stator represented by the primary and the rotor represented as a secondary that may move with respect to the primary. At standstill, there is no relative motion between the rotor and the stator, and the motor is similar to a transformer with a short-circuited secondary.

As the motor builds up speed, the frequency of the rotating field with respect to the rotor is reduced. The relationship of the rotor frequency to the synchronous frequency is called slip, which is defined as:

$$s = \frac{n_s - n}{n_s} \tag{13}$$

where s is the slip in per-unit values;

n_s is the synchronous speed in revolutions per minute; and

n is the rotor speed in revolutions per minute.

The rotor speed is $(1 - s)\, n_s$ and the frequency of the induced currents in the rotor is sf_1.

When the rotor is at standstill ($s = 1$), the transformer analogy for one phase of the motor is shown by Fig. 3. When the rotor is revolving, the secondary voltage $\dot{E}_{2s}$ is $s\dot{E}_2$ and the reactance X_{2s} is sX_2.[1] Fig. 4 is the equivalent circuit with the rotor revolving. The mechanical power output of the motor may be represented by a load with a voltage $(1 - s)\, \dot{E}_2$ applied to its terminals. Since the mechanical output is represented in electrical units, Fig. 4 may be converted to the equivalent circuit of a transformer with a ficticious load connected to the secondary.

The values of equivalent resistance and reactance representing the ficticious load may be derived from the relation

$$\dot{E}_{2s} = s\dot{E}_2 = \dot{I}_{2s}\, \dot{Z}_{2s} \tag{14}$$

[1]See Appendix A (p. 28) for symbols.