Judging the relative qualities and merits of Galerkins approximate solutions to a dyamically loaded beam system

by Thomas Michael Hanson

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University

Abstract:
Galerkin's method is applied to a beam structure that is forced to nonlinear behavior by a dynamic load. Nonlinearities in the system include a nonlinear stress-strain relation and consideration of geometry changes due to large deflections.

Several trial deflection shapes are assumed as approximate solutions of the problem and these trial shapes are combined in various manners in an effort to produce a better quality solution. All resulting solutions are studied in the light of three criteria that are postulated in an attempt to define the relative merits and qualities of approximate solutions.

It is concluded that although the criteria are good guidelines to finding reasonable solutions, they are not strict in defining a good quality solution.

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Date October 7, 1970

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ABSTRACT

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Several trial deflection shapes are assumed as approximate solutions of the problem and these trial shapes are combined in various manners in an effort to produce a better quality solution. All resulting solutions are studied in the light of three criteria that are postulated in an attempt to define the relative merits and qualities of approximate solutions.

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CHAPTER I: INTRODUCTION

Earlier works concerned with problems of nonlinear deflections have usually only considered single component systems such as beams, plates, or shells. It is of interest to apply techniques used in solving these single component problems to problems involving a system of such components.

Solution techniques very often used on nonlinear problems are finite difference methods [7,8][1] and Galerkin's method.[1,2,4,5] A popular method for both linear and nonlinear structures is the finite element method.[10,17] Comparisons of the finite difference and Galerkin methods have been made [3] but criteria for judging the relative qualities of solutions as given by Galerkin's method have only been hypothesized. Convergence proofs for certain linear problems are sometimes stated, [9] however, little is known about the convergence for nonlinear problems. If one is confronted with two or more approximate solutions to a problem, it is of particular interest to be able to discuss the relative merits and qualities of each solution.

Previous investigations of nonlinear, dynamically loaded systems have included a study of a blast loaded cantilever beam [1] and of a blast loaded plate.[2] Large deflections of thin elastic beams have been studied [3] and Galerkin methods have been applied

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[1] Numbers in brackets refer to literature consulted.

to nonlinear problems of finite elastodynamics.[11] Considerable work has been done utilizing rigid-plastic theory which is valid only for moderately large deflections.[4,5,6]

This paper studies a simple structural system of beams forced to large displacements by a blast load. Galerkin's modified method is used to reduce the nonlinear partial differential equations of motion to ordinary differential equations which can be solved by numerical methods. Nonlinearities in the system arise from geometrical changes of large deflections and consideration of a nonlinear stress-strain relation. The effects of various one-term approximate solutions are studied and the deflection shapes are combined in an attempt to improve the resulting answer. Methods of combining the assumed deflection shapes are investigated and compared. All results are studied in an effort to define the relative qualities of the various trial solutions.

The purpose of this paper will be to test the hypothesis that the quality of an assumed solution to a nonlinear problem solved by Galerkin's technique can be judged in part by a combination of three criteria. These criteria are:

Criterion 1: The quality of an assumed solution can be measured by the magnitude of the absolute area under the equation residual curve. This is a measure of the "closeness" of the equation residual to zero as implied by Finlayson and Scriven in their discussion of weighted residual methods.[9]

Criterion 2: The larger the resulting deflection of the system, the better the assumed solution. This is suggested by Anderson [11] in that an assumed solution to a linear system, other than the exact solution will result in an increased system stiffness and therefore smaller deflections of the system. This is stating that forcing the deflection of a system in a shape other than the true deflection shape will require more energy than forcing the deflection in the exact shape.

Criterion 3: The more uniform the distribution of the equation residual about zero, the better the assumed solution. Koszuta[1] states that a good assumed solution will exhibit a more uniform distribution over a greater portion of the solution interval.

Trial solutions to the beam system are studied and discussed in considerstion of the above criteria.

CHAPTER II: SYSTEM MODEL FORMULATION

2.1 The System

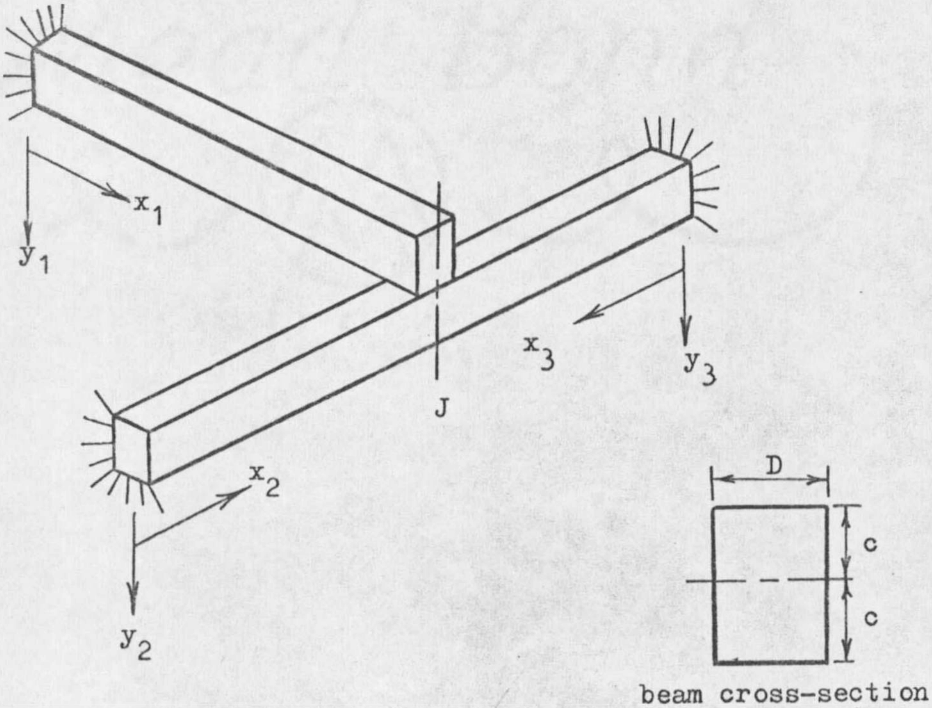

Figure 1. The Beam System

An example problem that will test the postulates given in Chapter I and that will fulfill the requirement that Galerkin's method be applicable to a beam system is shown in Figure 1. It consists of a fixed-simply supported beam resting on the midspan of a fixed-fixed beam. It will be convenient to consider the fixed-fixed beam as two seperate beams joined at J so that the coordinate directions, x_1, y_1, x_2, y_2, and x_3, y_3, each describe a different beam. Beams 1,