A general solution for active and passive earth pressures
by William Smith Hartsog

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Civil Engineering
Montana State University

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The Rankine assumptions were used as a basis to develop equations for active and passive earth pressures within a slope having an infinite extent. The analysis includes: cohesive and noncohesive soils; the angle of internal friction of the soil; seepage forces caused by a ground-water table which is parallel to the ground surface; and the plane on which the stresses are to be found may be at any inclination.

The equations for active and passive pressures were nondimension alized to reduce the number of parameters and aid in programming the expressions for solution on an IBM 1620, Model II, digital computer.

The program systematically varies the parameters in order to develop tables which simplify the solution for a variety of soil properties and slope geometry. The dimensional equations were also programmed for solution for a specific set of soil properties and slope geometry.

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ABSTRACT

The Rankine assumptions were used as a basis to develop equations for active and passive earth pressures within a slope having an infinite extent. The analysis includes: cohesive and noncohesive soils; the angle of internal friction of the soil; seepage forces caused by a groundwater table which is parallel to the ground surface; and the plane on which the stresses are to be found may be at any inclination.

The equations for active and passive pressures were nondimensionalized to reduce the number of parameters and aid in programming the expressions for solution on an IBM 1620, Model II, digital computer. The program systematically varies the parameters in order to develop tables which simplify the solution for a variety of soil properties and slope geometry. The dimensional equations were also programmed for solution for a specific set of soil properties and slope geometry.

A GENERAL SOLUTION FOR ACTIVE AND PASSIVE EARTH PRESSURES

CHAPTER I

INTRODUCTION

In 1860, Rankine presented the original development for lateral earth pressures within a slope of infinite extent. Rankine's development was based on a conjugate stress relationship between vertical stresses and stresses on a vertical plane and considered a dry cohesionless material with either a sloping or a horizontal surface. During the latter part of the nineteenth century, the Mohr circle of stresses was presented for the graphic representation of the state of stress. The analytical work of Rankine was subsequently adapted to a graphical method using the Mohr circle of stresses.

In 1915, Bell extended Rankine's work to include cohesive soils. Bell's graphical procedure made use of Mohr's stress circle, but in an inconvenient manner. Martin (1961) derived an analytical expression to extend Rankine's work to include cohesive soils, but the development includes neither seepage conditions nor a variable plane of investigation.

The development herein extends Martin's analytical development to include stresses on any plane in the slope, and includes seepage stresses in the analysis. It is assumed in the following development that seepage stresses are caused by a water table parallel to the ground surface. A chronological summary of the developments based on Rankine's assumptions is shown in Table I.

The graphical methods presently being used for the determination of

TABLE I

CHRONOLOGICAL SUMMARY OF THE DEVELOPMENT OF INFINITE SLOPE EARTH PRESSURES

	FACTORS CONSIDERED					APPROACH	
INVESTIGATOR	angle of slope	angle of internal friction	unit cohesion of the soil	angle of the plane of investigation	seepage	graphical	analytical
RANKINE	X	X		vertical			X
BELL	X	X	X	vertical		X	
MARTIN	X	X	X	vertical			X
HARTSOG	X	X	X	any	X		X

earth pressures are tedious and time consuming. The analytical expressions developed herein would also be time consuming were it not for the advent of the digital computer.

CHAPTER II

STRESS RELATIONSHIPS ON A PLANE PARALLEL TO THE GROUND SURFACE WITHIN AN INFINITE SLOPE

The term, infinite slope, is defined as a slope of constant inclination of unlimited extent. For practical purposes, a slope that has a length to depth ratio of twenty or greater has been considered an infinite slope.

The classical infinite slope analysis requires that soil properties be constant at any and all vertical sections along the slope. If this requirement is met, the slope can be studied by analyzing the stress conditions within an elemental volume of soil.

Figure 1 shows the basic assumed geometry of the infinite slope. In Figure 1, Z is the depth to the plane of investigation, Z_w is the depth to the water table, and $(Z-Z_w)$ is the vertical distance from the water table to the plane of investigation. The total unit weight of the soil material above the water table is denoted as γ_t, and γ_{sat} is the saturated unit weight of the soil. The angle the ground surface makes with the horizontal is denoted as θ. It is assumed that the planes representing the ground surface and the water table are parallel.

The slope profile with an elemental volume of soil (AA'BB') is shown in Figure 2. If a unit dimension is assumed normal to the plane of Figure 2, and if AA' is assumed to be of unit length, the resulting plane (which shall be referred to as plane A-A') will be of unit area. The force, W, acting on plane A-A', is equal to the weight of the vertical column of soil above plane A-A'.

The equation for the vertical force, W, is found by adding the

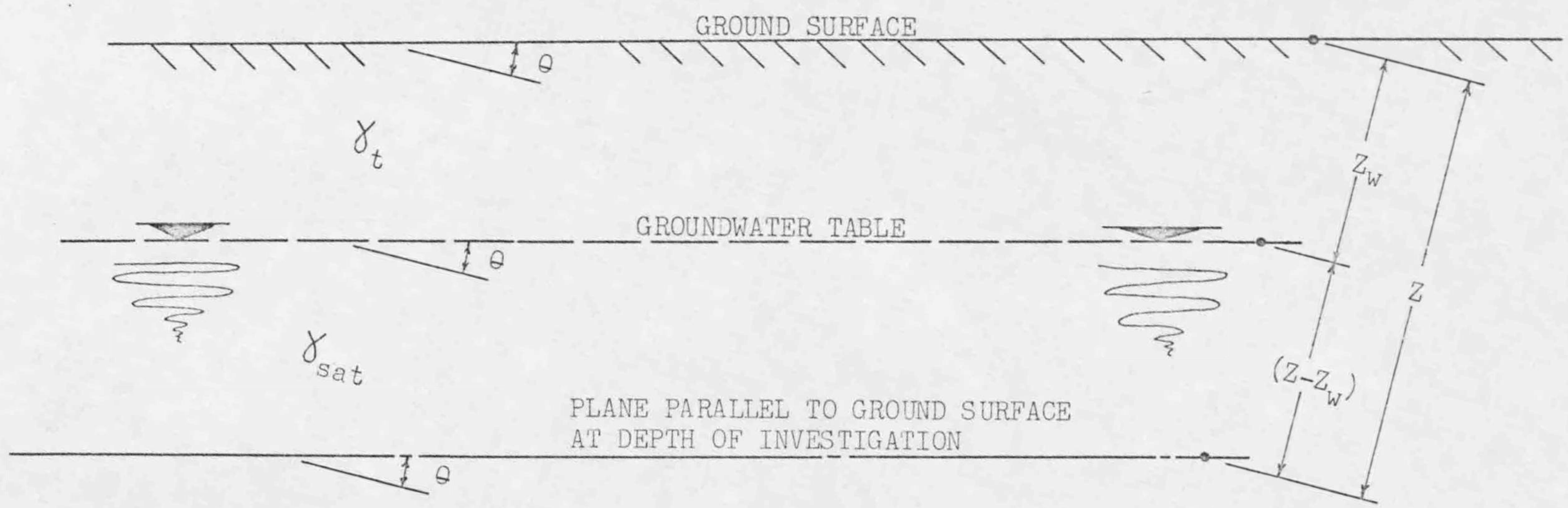

Figure 1. Profile of an infinite slope.