Analysis of hydraulic transients in an injection device for two-phase flow systems
by Gary Allen Hendrix

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Civil Engineering
Montana State University

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The method of characteristics is used to solve the two partial differential equations that describe the phenomenon of waterhammer. The resulting characteristic equations are applied to four basic design cases. These design cases are the basic components of the injection operation. Application of the characteristic equations to the basic design cases yields complete hydraulic information about the flow conditions on either side of all the valves in the system and permits the determination of the gate opening of each valve at any time.

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ABSTRACT

A method is presented for designing the operating rates of seven valves in a system that injects a mixture of solids and water into a high pressure pipeline without inducing hydraulic transients in that pipeline. Hydraulic transients are generated within the injection system itself, however, and the individual valves are operated in a manner that limits the magnitude of the transients.

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CHAPTER I

INTRODUCTION

The hydraulic transport of solids in pipelines has generated great interest in recent years. Mining companies have made extensive use of pipelines in transporting solid materials to processing mills. The Colorado School of Mines in 1963 cited fifty-two examples of such transport in a book (8)* concerning the most current state of the art in transport of solids in steel pipelines. The longest such pipeline in the United States is 108 miles long and is used by the Consolidation Coal Company to transport coal slurries.

The success of pumping solids by mining companies caused pulp mills to consider the possibility of reducing trees to wood chips at harvesting areas and pipelining the chips to the mills in place of transporting the trees by truck or railroad. As a result, the Pulp and Paper Institute of Canada and the Civil Engineering Department of Queen's University in Kinston, Ontario began investigations of the hydraulic feasibility of transporting wood chips in a pipeline.

The hydraulic transport of wood chips offers more problems than those encountered by mining companies in pipelining slurries. Wood chips are large and irregular in size and shape whereas slurries are mixtures of water and finely ground solids. One of the major problems is passing wood chips through pumps. The sizes of the chips rules out the use of positive displacement pumps due to the problems incurred at intake and

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*Numbers in parentheses refer to numbered references in the Literature Cited.

exhaust ports and damage to the chips. The only useable centrifugal pumps commercially available are low head trash pumps which can pass solids, but are rather inefficient in converting input energy to flow energy. Economic spacing of pumping stations on a fully operational pipeline requires that relatively high pressures be developed at each pumping station. The high operating pressure is theoretically attainable if trash pumps are placed in series but the casings and packing in these pumps are not designed for the high pressure operation that some of the pumps would experience.

An alternative to the series of staged pumps is to introduce the solids into the flow without passing them through pumps. A system of pumps and valve was patented by Consolidation Coal Company requiring the chips to pass through only one low head pump (see Figure 1). This system continually injects wood chips into the main pipeline by alternating wood chip flow into the main pipeline from two injection lines. The low pressure pump pumps a wood chip, water mixture into one pipe at low pressure. A high pressure pump, pumping clear water, forces the mixture from a previously charged pipe of the same length into the main pipeline. When the pipe injecting wood chips into the main pipeline becomes completely filled with clear water, switching valves divert the flow such that the pipe, now filled with clear water, feeds the intake of the high pressure pump as wood chips from the low pressure pump fill this line from the opposite end. The pipe, now filled with chips, begins injecting the chips into the main pipeline as clear water from the high pressure

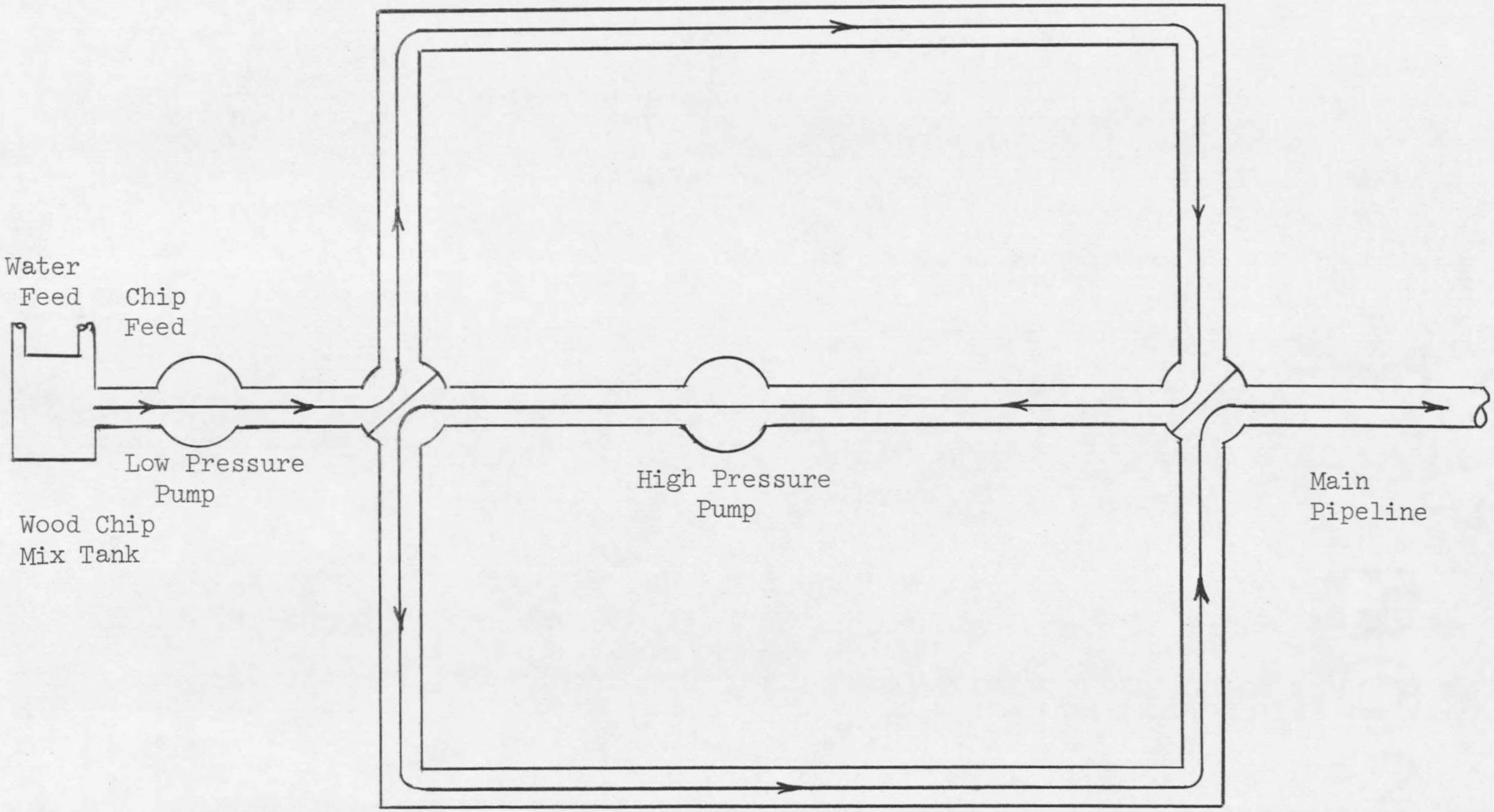

Figure 1. Proposed injection system

pump forces the chips from the line. This system, however, generated severe pressure surges when the valves were operated rapidly and was not used for any commercial operation.

This study was undertaken to determine if this injection system can be operated in a manner which would control the transients developed by the operation of the switching valves.

CHAPTER II

LITERATURE REVIEW

In order to develop a method of controlling the hydraulic transients generated by the injection system described in the preceding chapter, a basic understanding of the problem and methods of analyzing it must be obtained. Hydraulic transients are the time variance of pressure and discharge in a confined piping system and are often referred to as "waterhammer". The waterhammer phenomenon is induced by any external force on a fluid system which would change its initial steady state condition. A valve being opened or closed produces such a force.

As a valve is closed, pressure waves in excess of the operating pressure in the pipe are sent upstream to destroy the momentum of the incoming fluid. These pressure waves travel at a characteristic speed which depends on the physical properties of the fluid and the conduit in which it is flowing. When these positive pressure waves reach the upstream boundary of the pipe, they are reflected back to the valve as negative waves. The magnitude of the negative waves is slightly smaller than that of the positive waves due to frictional damping of the positive waves as they travel up the pipe. As the negative waves return to the valve they interact with positive waves which are still being generated at the valve. The net effect of the negative reflected waves is to modify the pressure increases due to the positive waves generated at the valve. If the valve is closed or nearly closed at the time the first negative wave returns to the valve, there will be no positive waves to interact with the negative wave and the pressure at the valve will drop below operating pressure. In extreme cases the pressure will drop low enough to initiate cavitation. Low pressure waves, lower than operating pressure, are then sent upstream, inverted at the reservoir and returned to the valve. At the valve another series of pressure waves greater than the normal operating pressure will be sent upstream as the reflected negative waves return to the valve. The effect is to produce a sinusoidal pressure fluctuation at every point in the pipe. The frequency of the pressure fluctuation is a/4L where L is the pipe length and a is the wave velocity. The changes in pressure are accompanied by reversals in flow direction and a hammering effect is created in the pipe. The oscillations will persist until friction damps them out. If a valve can be operated slowly enough, the magnitude of the pressure surges are negligible because the pressures are proportional to the rate at which the flow is regulated. However, during emergencies and in special cases such as that encountered in the proposed wood chip injection system, a valve may be required to close very rapidly.

According to Jaeger (3), the differential equations which describe the phenomena of waterhammer were first written in their correct form in the late 1890's by N. Joukowsky. These equations, however, are quite difficult to solve. Lorenzo Allievi is credited with making the first solution to a simplified form of the waterhammer equations (3). Allievi formulated an algebraic solution to the equations which are:

$$V_t = g\ H_x \qquad (1)$$

and

$$\frac{a^2}{g} V_x = H_t \tag{2}$$

a is the wave velocity; g is the gravitational constant; V_x is the partial derivative of velocity with respect to distance; V_t is the partial derivative of velocity with respect to time; H_x is the partial derivative of the head with respect to distance; and H_t is the partial derivative of the head with respect to time. Because the equations in their correct form are:

$$gH_x + VV_x + V_t + \frac{fV|V|}{2D} = 0 \tag{3}$$

and

$$H_t + \frac{a^2}{g} V_x + V \sin \theta + VH_x = 0 \tag{4}$$

θ is the slope of the pipe, Allievi's solution is limited in the precision of its results.

Graphical methods of solving the same forms of the differential equations that Allievi used were developed in the later 1920's (5). Louis Bergeron (1) and John Parmakian (5) have written excellent references on the graphical solutions of the waterhammer equations. The principle difficulties with graphical methods are the accuracy of the answers and the time required to make a solution.

With the advent of the digital computer in the 1950's methods of solving differential equations, which before had been impractical to solve, became practical. In 1960, Mary Lister (4) presented a numerical method for solving hyperbolic differential equations, like the waterhammer equations, by the method of characteristics. The characteristic solution to a set of partial differential equations reduces these equations to a set of ordinary differential equations which can be solved by numerical methods.

V. L. Streeter (6) applied the method of characteristics to Eqs. (3) and (4). He has obtained excellent correlation between experimental results and theoretical results for simple pipelines. Because of the availability of a computer and because of the simplicity of application of the characteristic equations to physical systems, the method of characteristics was used to study the hydraulic transients developed by the valving operation in the wood chip injection system previously described.

CHAPTER III

DEVELOPMENT OF EQUATIONS

The basic equations and methods of analysis used in determining a method of operation for the valves in the injection system are developed in this chapter. The development proceeds from the basic differential equations, through the characteristic solution to these equations, to the equations and methods of analysis used in two basic applications of the characteristic equations.

A. Basic Water Hammer Equations

The basic differential equations which describe transient flow in a closed conduit in a two-dimensional plane are given by the equation of motion and the equation of continuity, Eqs. (3) and (4). The forms of these equations adopted for this analysis are

$$gH_x + V_t + \frac{fV|V|}{2D} = 0 \qquad (5)$$

and

$$H_t + \frac{a^2}{g} V_x = 0 \qquad (6)$$

Rather than discuss the development of the basic equations and their simplified forms in this paper, the reader is referred to the development by Streeter (6).

B. Solution by Method of Characteristics

The method of characteristics provides a powerful method of solving sets of linear partial differential equations. This method, outlined briefly below, converts the partial differential equations to ordinary differential equations which describe the action of the dependent variables along given characteristic directions in the independent variable plane. The reader is referred to the discussion of Mary Lister (4) and S. H. Crandall (2) for the details of the mathematical analysis of the method of characteristics. The method of characteristics is applied to Eqs. (5) and (6) in the following paragraphs.

Eqs. (5) and (6) can be combined linearly be using an unknown multiplier, Z. If Eq. (6) is multiplied by Z and added to Eq. (5), the result, after rearrangement of terms, is:

$$\left(\frac{g}{Z} H_x + H_t\right) + \left(\frac{Z a^2}{g} V_x + V_t\right) + \frac{fV|V|}{2D} = 0 \tag{7}$$

The method of characteristics involves the selection of two values of Z that result in converting Eqs. (5) and (6) into a pair of total differential equations. Any two real, distinct values of Z will yield two equations in terms of H and V that are equivalent in every respect to Eqs. (5) and (6). If V and H are functions of x and t only, their total derivatives are:

$$\frac{dH}{dt} = H_x \frac{dx}{dt} + H_t \tag{8}$$

and

$$\frac{dV}{dt} = V_x \frac{dx}{dt} + V_t \tag{9}$$

Examination of Eq. (7) shows that if

$$\frac{dx}{dt} = \frac{g}{Z} \tag{10a}$$

and if

$$\frac{dx}{dt} = \frac{Za^2}{g} \tag{10b}$$

Eq. (7) becomes an ordinary differential equation:

$$Z \frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 \qquad (11)$$

Solving Eqs. (10a) and (10b) yields two particular values of Z which reduce the original equations to the ordinary differential equation given by Eq. (11). The solutions are:

$$Z = \frac{\pm g}{a} \qquad (12)$$

Substituting these values into Eqs. (10a) and (10b) yields:

$$\frac{dx}{dt} = \pm a \qquad (13)$$

Eq. (11) will satisfy Eqs. (5) and (6) only when Eq. (13) is satisfied. Thus there are two characteristic directions in the x-t plane along which Eq. (11) is applicable.

For the positive characteristic direction

$$C^{+} = \frac{dx}{dt} = a \qquad (14)$$

Eq. (11) becomes

$$g \frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 \qquad (15)$$

For the negative characteristic direction

$$C^{-} = \frac{dx}{dt} = -a \qquad (16)$$

Eq. (11) becomes

$$-\frac{g}{a} \frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 \qquad (17)$$

Eqs. (15) and (17) are applicable only along the characteristic directions given by Eqs. (14) and (16). These characteristic directions are illustrated on the x-t diagram shown in Figure 2.

The ordinary differential Eqs. (15) and (17) are not amenable to an

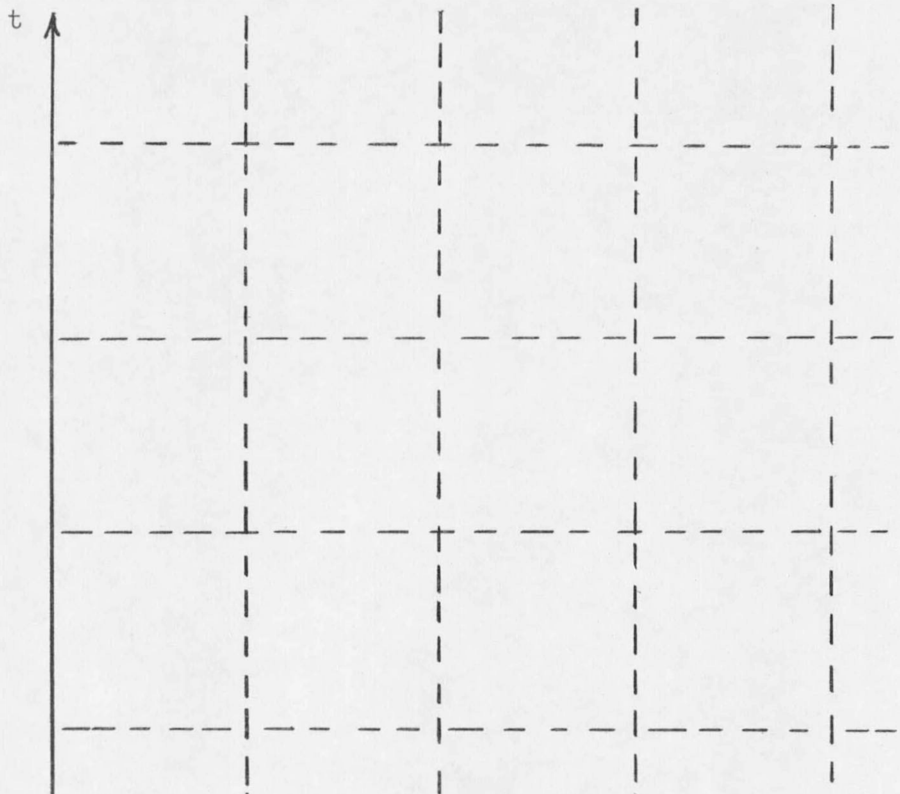

Figure 2. Characteristics in the x-t plane

exact mathematical solution, but may be solved by finite difference techniques. The equation written in finite difference form for the C^+ direction is:

$$V_P - V_A + \frac{g}{a}(H_P - H_A) + \frac{fV_A|V_A|}{2D}(t_P - t_A) = 0 \tag{18}$$

and for the C^- direction is:

$$V_P - V_B - \frac{g}{a}(H_P - H_B) + \frac{fV_B|V_B|}{2D}(t_P - t_A) = 0 \tag{19}$$

The subscripting notation refers to Figure 2. With the use of the digital computer, relatively small time and distance increments can be taken on the x-t plane, and a very close approximation to an exact solution can be made.

C. Application of the Method of Characteristics

The transient analysis for a pipeline of constant diameter and constant thickness is effected using Eqs. (18) and (19). The method of applying these equations to a physical system is shown in the following example. The system consisting of a single pipe with a constant head reservoir at one end and a valve at the other is shown in Figure 3. The x-t diagram for this system is shown directly beneath the pipe. The grid is organized by setting $dx = L/N$ where L is the pipe length and N is an arbitrary integer. The time increment dt equals dx/a. The object is to determine the heads and velocities at all points in the grid as the valve is closed.

Initially the flow is in a steady state condition, and the heads