Analysis of thermal response of an experimental air preheater
by Geoffrey Charles Herrick

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
A model was developed to simulate the thermal response of an experimental high-temperature fixed-bed cored-brick regenerative air preheater. The experimental air preheater has been designed and built to study the problems associated with the use of air preheaters in magnetohydrodynamic power generation. The model simulates the experimental preheater core, where the gas to ceramic heat transfer occurs, by the use of finite-difference equations. The insulation layers surrounding the core are thermally coupled with the core using a second set of finite-difference equations. The core finite-difference equations describe the heat transfer and the temperature distribution in nineteen elements which comprise a thirty-degree segment of the core cross section. The finite-difference equations modeling the insulation include the vertical expansion gaps between insulation layers which provide additional resistance to the heat transfer.

At cyclic equilibrium conditions the transient thermal response of the core and insulation and the instantaneous heat transfer between the core and insulation were calculated over a complete equilibrium cycle. Also calculated was the total heat loss from the core to the insulation over one cycle.

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by

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A thesis submitted in partial fulfillment
of the requirements for the degree

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NOMENCLATURE

Symbol	Description
A	Area [m^2]
$\bar{A}$	Average area
A_B	Area of boundary surface between two elements
A_C	Area of plane convective surface
A_i	Area of centroid plane, element i
$A(x)$	Convective heat transfer area
C_p	Specific heat [Joules/kg-K]
C_{p_c}	Core specific heat
C_{p_g}	Gas specific heat
D	Flow channel diameter [m]
dT	Differential temperature [K]
dt	Differential time [sec]
dx	Differential distance [m]
dZ	Differential axial distance
F_{ij}	Radiation view factor
f	Darcy-Weisbach friction factor
f_o	Darcy-Weisbach friction factor for smooth pipe
f_r	Darcy-Weisbach friction factor for rough pipe
Gr_δ	Grashof number based on gap width
g	Acceleration of gravity [m/sec^2]
h	Heat transfer coefficient [W/m^2-K]

Symbol	Description
k	Thermal conductivity [W/m-K]
k_g	Gas thermal conductivity
k_i	Thermal conductivity, element i
k_j	Thermal conductivity, element j
k_s	Sand grain roughness height [m]
L	Length of vertical gap [m]
$\dot{m}$	Mass flow rate [kg/sec]
$\dot{m}_k$	Mass flow rate, flow channel k
Nu	Nusselt number
Nu_g	Nusselt number based on gap
$\overline{Nu}_g$	Average Nusselt number based on gap
Pr	Prandtl number
QI13	Radiative-convective heat transfer from element 13 [W]
QI18	Radiative-convective heat transfer from element 18
QI19	Radiative-convective heat transfer from element 19
$\dot{q}$	Heat transfer rate [W]
$\dot{q}_{ck}$	Heat transfer rate from convective surface to flow channel K
$\dot{q}_{cv}$	Convective heat transfer rate
$\dot{q}_{iB}$	Heat transfer rate from element i centroid plane to boundary plane
$\dot{q}_{ic}$	Heat transfer rate from element i centroid plane to convective surface

Symbol	Description
$\dot{q}_{ij}$	Heat transfer rate from element i to element j
$\dot{q}_{ik}$	Heat transfer rate from element i to flow channel k
$\dot{q}_{jB}$	Heat transfer rate from element j centroid plane to to boundary plane
$\dot{q}_{s_i}$	Rate of energy stored in element i
Re	Reynolds number
R_i	Radiosity of surface i [W/m^2]
S	Conduction shape factor [m]
Sc_{ij}	Modified conduction shape factor [W/K] from element i to element j
$\bar{Sc}_{ii}$	Modified conduction shape factor from position J-1 to J
$\overset{+}{Sc}_{ii}$	Modified conduction shape factor from position J to J+1
Scv_{ik}	Modified convection shape factor from element i to flow channel k [W/K]
T_B	Temperature of boundary plane [K]
T_c	Temperature of convective surface
T_i	Temperature of centroid element i
T_j	Temperature of centroid element j
T_i'	Updated temperature of centroid element i
T_1	Temperature of surface 1
T_2	Temperature of surface 2
Tg_k	Temperature of gas stream k
T(I,J,K)	Element centroid temperature; element I, axial position J, time K

Symbol	Description
Tg(I,J)	Gas stream temperature; flow channel I, axial position J
V_i	Volume element i [m^3]
X	Parameter defined in Equations (26) [W/m^2]
Z	Axial distance from flow channel entrance [m]
α_i	Parameter defined in Equations (31)
β	Volume coefficient of expansion
β_i	Parameter defined in Equation (35)
ΔT	Finite-difference temperature
Δt	Finite-difference time
ΔZ	Finite-difference axial distance
δ	Width of vertical gap [m]
ε_i	Emissivity of surface i
μ	Fluid dynamic viscosity
ρ_c	Core density [kg/m^3]
ρ_g	Gas density
ρ_i	Reflectivity of surface i

ABSTRACT

A model was developed to simulate the thermal response of an experimental high-temperature fixed-bed cored-brick regenerative air preheater. The experimental air preheater has been designed and built to study the problems associated with the use of air preheaters in magnetohydrodynamic power generation. The model simulates the experimental preheater core, where the gas to ceramic heat transfer occurs, by the use of finite-difference equations. The insulation layers surrounding the core are thermally coupled with the core using a second set of finite-difference equations. The core finite-difference equations describe the heat transfer and the temperature distribution in nineteen elements which comprise a thirty-degree segment of the core cross section. The finite-difference equations modeling the insulation include the vertical expansion gaps between insulation layers which provide additional resistance to the heat transfer.

At cyclic equilibrium conditions the transient thermal response of the core and insulation and the instantaneous heat transfer between the core and insulation were calculated over a complete equilibrium cycle. Also calculated was the total heat loss from the core to the insulation over one cycle.

CHAPTER I

INTRODUCTION

The proposed open-cycle magnetohydrodynamics (MHD) of the ECAS study [1] require a gas temperature of approximately 2800 K, well above the 1900 K available from the combustion of air and a fossil fuel. There are several methods that can be used to achieve the gas temperatures required for open-cycle MHD. One is the use of oxygen enrichment, but this method is objectionable economically because of the cost and amount of the oxygen required. A second method is the preheating of the combustion gasses prior to the combustion process.

Pre-combustion heating of the gasses by passing them through a preheater, which has been heated by the MHD channel exhaust gasses, is one method of achieving the desired temperatures. The types of preheaters suggested include: falling bed preheaters, pebble bed regenerators, and fixed-bed cored-brick regenerative preheaters. The third type, the fixed-bed cored-brick regenerative preheater, is the subject of this thesis.

There are two basic problems associated with the fixed-bed cored-brick regenerative preheater. First, the MHD channel exhaust gasses are laden with highly corrosive particulates, coal slag and alkali compounds, due to combustion and the injection of seed material. Second, as the exhaust gasses cool, the particulates solidify and deposit in the passages of the preheater. To study these effects, an experimental fixed-bed cored-brick air preheater has been constructed.

The experimental air preheater consists of a column of hexagonal ceramic bricks. The bricks are symmetrically cored with nineteen holes, 19.1 mm in diameter, through which hot and cold gasses alternately flow. Surrounding the cored brick are a ceramic containment tube, five layers of insulation, a steel shell and, finally, a layer of fiberglass insulation. Figures 1 and 2 show the details of the preheater design.

Because of the size difference, the extension of results from the experimental preheater to a full-size preheater must be done with caution. In a full-size preheater the ratio of core mass to insulation mass and the corresponding ratios of thermal energies is much greater than one. In the experimental preheater these ratios are less than one. Consequently, the heat transfer between the core and insulation in the experimental preheater is of considerable interest.

The primary purpose of this thesis is the determination of temperature histories in the ceramic core and insulation, and the subsequent mapping of heat flows. Of prime importance is the central flow channel and the immediately surrounding ceramic material, as it is this flow channel that would best approximate a flow channel in an operating facility.

CHAPTER II

LITERATURE REVIEW

The earliest investigations of regenerative air preheaters were all restricted to a bed with no radial temperature gradients, but even with this simplifying assumption the solution to the differential equations was unwieldy. Nusselt [2] obtained the first closed form solutions for some simplified cases, to which Heiligenstaedt [3] made some simplifying mathematical approximations. Rummel [4] used emperical values for the heat transfer coefficients. The most successful treatment of a purely mathematical nature is accredited to Hausen [5].

With the advent of the digital computer there have been a number of studies done using finite difference techniques to solve the differential equations. Appropriately, one of the first, by Butterfield, Schofield and Young [6], confirmed a method proposed by Hausen [5] some twenty years earlier. Numerous types of preheaters have been analyzed numerically. A study of a checkerwork regenerator has been done by Manrique and Cardenas [7], a pebble bed regenerator by El Rifai et al. [8] and a cored hexagonal brick regenerator by Zakkay et al. [9].

Cook [10] studied the insulation of a fixed-bed air preheater, but he did so only to optimize the insulation cost and was unconcerned with the preheater core. The method used equated the heat transfer per unit heat exchanger length with the sum of the radiative and convective heat transfer to the surroundings. The equations used described the insulation thickness as a function of the thermal conductivities of the insulation, the diameter of the heat exchanger matrix and the emissivities of the metal shell.

All of the preceding studies have dealt with either the heat exchanger matrix or the insulation layer. None of them have coupled the core and the insulation in a simulation of the entire regenerator. The exception to this is an analysis done by Ameel [11]. Ameel modeled the same facility as this thesis using a lumped-mass model derived by Reihman et al. [12] to simulate the core matrix, and a finite difference analysis of the insulation blanket. This analysis established the significance of heat fluxes from the core to the insulation but gives no insight into the radial temperature gradients in the core. The present investigation models radial heat transfer across the core, considering the contribution of each core flow channel. With this addition, cross-sectional temperature histories are possible.

CHAPTER III

THEORY

Operational simulation of the experimental air preheater requires models describing the energy transfer in both the core and insulation. In this investigation the model used to describe the energy transfer in the insulation was developed by Ameel [11]. The model describing energy transfer in the core is developed herein.

The model developed by Ameel describes the energy transfer in the insulation as one-dimensional heat transfer and storage in the radial direction at an arbitrary number of discrete axial positions. Radial temperature distributions at the remaining axial positions were found by linearly interpolating between adjacent calculated profiles. The insulation model, which was developed in finite-difference form, includes the vertical expansion gaps that exist between adjacent radial layers of insulation.

In the core, the energy transfer from the hot fluid to the cold fluid occurs through intermediate energy storage in the ceramic core material. During the reheat phase of operation, the hot exhaust gasses from the MHD channel pass through the preheater, transferring energy to the ceramic core. During the blowdown phase of operation, energy stored in the ceramic core is transferred to the cooler counterflow gas stream. In each phase of operation the known conditions are the inlet gas temperature, pressure and mass flow rate, the respective cycle times and the ambient air temperature.

Energy transfer in the core was modeled using two sets of finite-difference equations developed from energy balances performed on elemental lengths of the heat exchanger core and gas flow channels. One set of equations describes the energy transfer in the gas flow channels and the other describes energy transfer in the ceramic matrix.

Because the gas properties are known as initial conditions, the gas nodal structure positions the first node at the very top of the bed. The core equations, on the other hand, were developed with the first node located half an axial increment from the top of the bed to simplify the energy balances. As both equations use the same axial step this results in the gas and ceramic nodes being staggered as depicted in Figure 3.

Assuming homogeneity of the ceramic core material and neglecting the anti-rotation notches, the cross section of the core shown in Figure 4 was divided into thirty-degree segments separated by the adiabatic (dashed) lines. The thirty-degree core segment was then broken up into the nineteen elements shown in Figure 5. The geometries of the elements were fixed by restricting the thermal volume coefficient of expansion to zero. Each element was characterized by the temperature of the element centroid.

The three modes of heat transfer considered in the ceramic core were: conduction, convection and radiation. Conduction heat transfer was considered between adjoining elements and axially along the bed.

The heat transfer from the gas flow channels to the elements was approximated by forced convection relations while the heat transfer from the external surfaces of elements 13, 18 and 19 was approximated by free convection relations. Radiation heat transfer was considered only over the external surfaces of elements 13, 18 and 19 and was neglected across the flow channels.

The ceramic finite-difference equations were developed from one-dimensional heat transfer relations describing the heat transfer across the plane and flow channel boundaries of each element and a term describing the transient energy storage in each element. The equations describing the element to element heat transfer, the element to gas heat transfer and the transient energy storage were derived in a general form for all elements. The equations describing the radiative and convective heat transfer across the external surfaces of elements 13, 18, and 19 were derived as coupled non-linear equations and were solved by an iterative method.

The one-dimensional element to element heat transfer equations were developed by assuming the heat transfer occurs between two constant temperature planes, passing through adjacent element centroids, and parallel to the common heat transfer boundary. The centroid planes were assumed to be at the temperature of the element centroids and the boundary plane was assumed to be at some intermediate temperature, T_B. The typical geometry used to develop the element to element heat transfer equations is shown in Figures 6a and 6b.

Conduction heat transfer from the element centroid plane to the boundary plane can be described by the one-dimensional Fourier heat transfer equation.

$$\dot{q} = -kA(x)\frac{dT}{dx} \tag{1}$$

where $\dot{q}$ = heat transfer rate (W)

k = thermal conductivity (W/m-K)

$A(x)$ = heat transfer area (m^2)

dT = differential temperature (K)

dx = differential distance (m).

By applying Fourier's equation to the geometry of Figure 6a and integrating Equation (1) becomes

$$\dot{q}_{iB} = -k_i(T_i - T_B)\frac{1}{\int_0^{x_1}\frac{dx}{A(x)}} \tag{2}$$

where $\dot{q}_{iB}$ = heat transfer rate from centroid surface, i, to the boundary surface, B, (W)

k_i = thermal conductivity of element i (W/m-K)

T_i = temperature of surface i (K)

T_B = temperature of boundary surface (K)

The conduction shape factor is introduced from the following one-dimensional conduction heat transfer equation given by Holman [14].

$$\dot{q} = -kS(\Delta T) \tag{3}$$

where S = conduction shape factor (m)

ΔT = finite difference temperature (K).

Solving Equations (2) and (3) for the shape factor yields

$$S = \frac{1}{\int_0^{x_1} \frac{dx}{A(x)}} \tag{4}$$

Evaluating Equation (4) for the geometry of Figure 6b gives

$$S = \frac{A_i - A_B}{x_1 \ln(\frac{A_i}{A_B})} \tag{5}$$

where A_i = area of centroid plane (m^2)

A_B = area of boundary plane (m^2)

Equation (5) was evaluated for all possible centroid to boundary plane geometries for the four basic element types shown in Figures 7a through 7d.

Writing Equation (3) for the general element to element interation of Figure 6b yields the equations

$$\dot{q}_{iB} = -k_i S_{iB}(T_i - T_B) \tag{6a}$$

$$\dot{q}_{jB} = -k_j S_{jB}(T_j - T_B) \tag{6b}$$

Because the heat transfer rate across the common boundary must be continuous $\dot{q}_{iB}$ and $\dot{q}_{jB}$ are of equal magnitude and opposite sign. Therefore, the simultaneous solution of Equations (6) gives the equation for element to element conduction heat transfer as

$$\dot{q}_{ij} = \left[\frac{1}{\dfrac{1}{k_i S_{iB}} + \dfrac{1}{k_j S_{jB}}}\right](T_i - T_j) \tag{7}$$

The parameter enclosed in brackets was defined as the modified conduction shape factor, Sc_{ij}, which has the units (W/K). The modified conduction shape factors are tabulated in Table 1 for all element to element interactions.

Using the modified conduction shape factor the general form of the element to element conduction heat transfer equation is written as

$$\dot{q}_{ij} = Sc_{ij}(T_i - T_j) \tag{8}$$

The general form of the element to gas flow channel heat transfer equation was, like the element to element heat transfer equation, developed from two one-dimensional equations. The first equation describes the conduction heat transfer from the element centroid to the flow channel surface and the second describes the convective heat transfer from the gas stream to the flow channel surface.

Element centroid to flow channel surface conduction heat transfer equations were developed by assuming the heat transfer occurs between a centroid plane and a plane approximating the convective surface as shown in Figure 8. This equation, written in the form of Equation (6), is

$$\dot{q}_{ic} = S_{ic}(T_i - T_c) \tag{9}$$

where $\dot{q}_{ic}$ = heat transfer rate centroid plane from element i to convective plane (W)

S_{ic} = modified conduction shape factor from centroid plane to convective surface plane (W/K)

T_c = temperature of convective surface (K)

The convective heat transfer from the gas flow channel to the convective surface was calculated using the equation

$$\dot{q}_{ck} = hA_c(T_c - Tg_k) \tag{10}$$

where $\dot{q}_{ck}$ = heat transfer rate from gas stream to convective surface of element k (W)

h = heat transfer coefficient (W/m^2-K)

A_c = convective area (m^2)

Tg_k = temperature of gas stream (K)

The heat transfer coefficient is essentially an aerodynamic property of the gas flow channel system and the correlations used to calculate the heat transfer coefficients are presented in conjunction with the gas flow channel equations.

The heat transfer rates across the common boundary are equated and the simultaneous solution of Equations (9) and (10) yields

$$\dot{q}_{ik} = \left[\frac{1}{\dfrac{1}{k_i S_{ic}} + \dfrac{1}{hA_{c_i}}}\right](T_i - Tg_k) \qquad (11)$$

where $\dot{q}_{ik}$ = heat transfer rate from element i to flow channel k (W).

The bracketed term was defined as the modified convection shape factor, Scv_{ik}, which has units watts per degree Kelvin. A tabulation of the modified convection shape factors for all element to flow channel interactions is given in Table 2.

Consequently, the general form of the element to gas flow channel heat transfer equation was written as

$$\dot{q}_{ik} = Scv_{ik}(T_i - Tg_k) \qquad (12)$$

Elements 13, 18, and 19 are subject to two modes of heat transfer, convection and radiation, across their external surfaces. As in the previous developments, the heat transfer from the element centroid to the radiative-convective boundary was represented by a one-dimentional conduction heat transfer equation. Heat transfer from the radiative-convective surfaces was represented by two one-dimensional heat transfer equations. One equation describing convective heat transfer in the enclosed space and the other representing radiative heat transfer between the surfaces.

The conduction heat transfer from the element centroids to the radiative-convective boundaries was again represented by a form of Equation (6) as

$$\dot{q}_{ir} = S_{ir}(T_i - T_r) \tag{13}$$

where the subscript r represents the radiative-convective surface, and the subscript i refers to elements 13, 18, or 19.

The free convection heat transfer from the external surfaces of elements 13, 18, and 19 was approximated by an empirical equation for natural convection in the air space between two isothermal walls as presented by Jakob [13]. It was assumed, primarily, that the hexagonal core could be approximated by a cylinder of the same surface area. Then the approximation of a cylindrical air space by an air space between two flat plates was used. Jakob [13] suggests the use of this approximation when, as is the case for the air preheater geometry,

$$L/\delta >> 1 \tag{14}$$

where L = length of the vertical gap (m) and
δ = width of the gap (m).

The Nusselt number, Nu_δ, for natural convection in a vertical gap was defined a

$$Nu_\delta = h\delta/k_g \tag{15}$$

where k_g = thermal conductivity of the gas in the gap (W/m-k).

The Nusselt number is a function of the Grashof number, Gr_δ, for heat transfer in vertical spacings. The Grashof number is given by

$$Gr_\delta = \rho_g^2 g\beta(T_1 - T_2)\delta^3/\mu^2 \quad (16)$$

where ρ_g = gas density (kg/m^3)

g = acceleration due to gravity (9.8 m/sec)

β = volume coefficient of expansion for an ideal gas

T_1 = temperature of wall 1 (K)

T_2 = temperature of wall 2 (K)

μ = dynamic viscosity (Pa-sec).

Correlations for the free convection heat transfer coefficients are given by Jakob [13] as

$$\overline{Nu}_\delta = \begin{cases} 0.18\ Gr_\delta^{1/4}(L/\delta)^{-1/9} & \text{for } 2\text{x}10^3 < Gr_\delta < 2\text{x}10^4 \quad (17) \\ 0.065\ Gr_\delta^{1/3}(L/\delta)^{-1/9} & \text{for } 2\text{x}10^4 < Gr_\delta < 11\text{x}10^6 \quad (18) \end{cases}$$

where $\overline{Nu}_\delta$ denotes an average Nusselt number. For a Grashof number less than 2000, the process is simple conduction and the following equation applies.

$$\overline{Nu}_\delta = h\delta/k_g = 1.0. \quad (19)$$

Equations (17), (18) and (19) include the effects of conduction and natural convection. Therefore, the heat transferred across a vertical gap by conduction and natural convection is given by

$$\dot{q}_{cv} = h(T_1 - T_2)\overline{A} \qquad (20)$$

where $\overline{A}$ = the average area of the two surfaces; and

$\dot{q}_{cv}$ = convective heat transfer rate (W).

The radiative heat flux from the external surfaces of elements 13, 18, and 19 was determined by performing a simplified zonal analysis on the enclosure shown in Figure 9. (Surfaces 1, 2, and 3 represent the external surfaces of elements 13, 18, and 19 respectively and surface 4 represents the containment tube.) The enclosure is composed of one flat plate broken into surfaces 1, 2, and 3 which are at temperatures T_1, T_2, and T_3, have hemispherical emissivities ε_1, ε_2, and ε_3 and diffuse hemispherical reflectivities ρ_1, ρ_2, and ρ_3 respectively. The fourth surface is a sixty degree segment of a cylinder which is at temperature T_4, has hemispherical emissivity ε_4, and diffuse hemispherical reflectivity ρ_4. The medium between the surfaces was assumed to be non-participating.

The equations for the radiosities were obtained from Ozisik [19] as

$$R_i = \varepsilon_i \sigma T_i^4 + \rho_i \sum_{j=1}^{4} R_j F_{ij} \qquad (21)$$

where R_i = radiosity of surface i

σ = Stephan-Boltzman constant

F_{ij} = view factor between surfaces i and j

By inspection the view factor matrix reduces to

$$F_{ij} = \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ F_{41} & F_{42} & F_{43} & F_{44} \end{matrix} \qquad (22)$$

Using the algebraic view factor equations:

$$A_i F_{ij} = A_j F_{ji} \qquad (23)$$

$$\sum_{j=1}^{4} F_{ij} = 1 \qquad (24)$$

and the known geometry of the elements the remaining view factors were found. The complete view factor matrix is

$$F_{ij} = \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.3438 & 0.3438 & 0.2591 & 0.0534 \end{matrix} \qquad (25)$$

The values for the view factor were substituted into Equation (21), which were then solved simultaneously to give

$$R_1 = \varepsilon_1 \sigma T_1^4 + \rho_1 X \qquad (26a)$$

$$R_2 = \varepsilon_2 \sigma T_2^4 + \rho_2 X \qquad (26b)$$

$$R_3 = \varepsilon_3 \sigma T_3^4 + \rho_3 X \qquad (26c)$$

$$R_4 = X \qquad (26d)$$

where

$$X = \frac{\varepsilon_4 \sigma T_4{}^4 + \rho_4[0.3438(\varepsilon_1 \sigma T_1{}^4 + \varepsilon_2 \sigma T_2{}^4) + 0.2591 T_3{}^4]}{1 - \rho_4[1 - 0.3438(\varepsilon_1 + \varepsilon_2) - 0.2591\ \varepsilon_3]}$$

The heat flux from each zone is given by Ozisik [19] as

$$\dot{q}_i'' = R_i - \sum_{j=1}^{4} R_i\ F_{ij} \tag{27}$$

and the heat transfer rate is simply the product of the heat flux and the corresponding heat transfer area. Therefore the heat transfer rates from areas 1, 2 and 3 are

$$\dot{q}_1 = (\varepsilon_1 \sigma T_1{}^4 - \varepsilon_1 X)(A_1) \tag{28a}$$

$$\dot{q}_2 = (\varepsilon_2 \sigma T_2{}^4 - \varepsilon_2 X)(A_2) \tag{28b}$$

$$\dot{q}_3 = (\varepsilon_3 \sigma T_3{}^4 - \varepsilon_3 X)(A_3). \tag{28c}$$

Because the equations describing the radiative heat transfer from the external surfaces of elements 13, 18, and 19 are coupled non-linear equations an iterative procedure was used to solve for radiative-convective heat transfer from these elements. The heat transfer rates described by Equations (13) must, in this case, be equal to the sum of the convective and radiative heat transfer rates described by Equations (20) and (28) for each element. Using the average of the centroid and containment tube temperatures as initial values for the radiative-convective surface temperatures, Equations (20) and (28) were solved and subsequently summed to give the radiative-convective heat transfer rates, which were then used in Equations (13) to solve for the radiative-convective surface temperatures. Convergence was assumed to occur when the new radiative-convective surface temperatures varied by less than 0.001 degree from the previous iterative value.

The heat transfer rates found by this procedure were, after convergence, designated as QI13, QI18, and QI19, and represent the heat transfer rate in watts from the external surfaces of element 13, 18, and 19 respectively.

The last term needed for the energy balance on the ceramic core elements describes the transient energy storage in each element. The rate of heat storage in an element i can be written in differential form as

$$\dot{q}_{s_i} = \rho_c C_{p_c} V_i \frac{dT}{dt} \qquad (29)$$

where $\dot{q}_{s_i}$ = rate of heat stored (W)

ρ_c = density of ceramic (kg/m^3)

C_{p_c} = heat capacity of ceramic (J/kg-K)

V_i = volume of element (m^3).

In finite difference form Equation (29) becomes

$$\dot{q}_{s_i} = \frac{\rho_c C_{p_c} V_i}{\Delta t} (T_i' - T_i) \qquad (30)$$

where T_i' = element centroid temperature at $t + \Delta t$(K).

Performing energy balances on the ceramic elements results in the following finite-difference equations

$$T(1,J,2) = T(1,J,1) + \alpha_1[Scv_{1-1}\ Tg_1 + S\bar{c}_{11}$$
$$T(1,J-1,1) + S\overset{+}{c}_{1-1}\ T(1,J+1,1) + Sc_{1-2}\ T(2,J,1) -$$
$$T(1,J,1)(Scv_{1-1} + S\bar{c}_{1-1} + S\overset{+}{c}_{1-1} + Sc_{1-2})] \quad (31a)$$

$$T(2,J,2) = T(2,J,1) + \alpha_2[Scv_{2-2}\ Tg_2 + S\bar{c}_{2-2}\ T(2,J-1,1)$$
$$+ S\overset{+}{c}_{2-2}\ T(2,J+1,1) + Sc_{1-2}\ T(1,J,1) + Sc_{2-3}\ T(3,J,1)$$
$$- T(2,J,1)(Scv_{2-2} + S\bar{c}_{2-2} + S\overset{+}{c}_{2-2} + Sc_{1-2} + Sc_{2-3})] \quad (31b)$$

$$T(3,J,2) = T(3,J,1) + \alpha_3\ [Scv_{3-2}\ Tg_2 + S\bar{c}_{3-3}\ T(3,J-1,1)$$
$$+ S\overset{+}{c}_{3-3}\ T(3,J+1,1) + Sc_{2-3}\ T(2,J,1) + Sc_{3-4}\ T(4,J,1)$$
$$- T(3,J,1)(Scv_{3-2} + S\bar{c}_{3-3} + S\overset{+}{c}_{3-3} + Sc_{2-3} + Sc_{3-4})] \quad (31c)$$

$$T(4,J,2) = T(4,J,1) + \alpha_4\ [Scv_{4-2}\ Tg_2 + S\bar{c}_{4-4}\ T(4,J-1,1)$$
$$+ S\overset{+}{c}_{4-4}\ T(4,J+1,1) + Sc_{3-4}\ T(3,J,1) + Sc_{4-5}\ T(5,J,1)$$
$$- T(4,J,1)(Scv_{4-2} + S\bar{c}_{4-4} + S\overset{+}{c}_{4-4} + Sc_{3-4} + Sc_{4-5})] \quad (31d)$$

$$T(5,J,2) = T(5,J,1) + \alpha_5\ [Scv_{5-2}\ Tg_2 + S\bar{c}_{5-5}\ T(5,J-1,1)$$
$$+ S\overset{+}{c}_{5-5}\ T(5,J+1,1) + Sc_{4-5}\ T(4,J,1) + Sc_{5-8}\ T(8,J,1)$$
$$+ Sc_{5-6}\ T(6,J,1) - T(5,J,1)(Scv_{5-2} + S\bar{c}_{5-5} + S\overset{+}{c}_{5-5}$$
$$+ Sc_{4-5} + Sc_{5-8} + Sc_{5-6})] \quad (31e)$$