Basic heat transfer using EAI TR-10 electronic analog computer
by Michael Epifanio Hilario

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
The electronic analog computer is a very useful problem solving tool to the engineering student. The Montana State University Engineering Department has two analog computers for the use of the faculty and students. These are the EAI (Electronic Associates, Incorporated) TR - 48 and the smaller EAI TR - 10. The Mechanical Engineering Department has 4 TR -10 patchboards and the necessary wires and resistors to set up computer problems. Heat transfer is one field of engineering, among others, in which this department will use the analog computer for problem analysis at the undergraduate and graduate level.

An introductory course in analog computer programming should enable the student to use the computer for various problem solutions even without a strong background in electronics or servomechanisms. This thesis is presented as a guide to the student wishing to utilize the analog computer for heat transfer work. It demonstrates general problem analysis and some necessary procedures for obtaining solutions for some of the basic heat transfer equations.

The literature search in preparation of this thesis left the impression that there has not been much work in heat transfer using the analog computer at the undergraduate level. Rather, it appears that analog-simulation systems; such as the resistance analog, conductive-solid analog, and conductive-liquid analog, have been used more often for analysis.

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Abstract

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CHAPTER I

INTRODUCTION

There are two types of electronic computers in general use in industry for purposes of research, data processing, control systems, to name a few. These are the analog computer and the digital computer. The digital computer is able to store problem data and instructions. Through various related units, the digital finally supplies the results of computations. Generally, the digital computer performs only addition and there is no relationship between the computing time and the problem variables. On the other hand, analog computers do not count in terms of numbers, but use continuously variable data. Problem solution by the analog computer is accomplished by analogy; the computer is programmed so that its circuit equations have the same mathematical form as the equations of the problem. Thus voltages represent various physical quantities such as distance, temperature, force, displacement, and so on. The voltage magnitudes are related to the physical quantity magnitudes by a scale factor. The analog computer performs its operations in a continuous process with respect to time; therefore, there is a relationship between the actual computer run time and the problem time or some other independent variable.

Analog computers make it possible to simulate entire system problems, components, or subsystems. It may be that parameters of a system would be difficult, expensive, or time consuming to actually change in real equipment. However, if simulated on an analog computer, paramenters could be varied by simply using a dial in a well designed analog circuit. Process effects that take hours or even days can be simulated on an analog computer so that data can be obtained in a matter of seconds.

Although a knowledge of electronics is very valuable to a person utilizing the analog computer, it is not necessary. An introductory course in circuit analysis will suffice. The engineering student will find that using an analog computer is fairly easy and very applicable to engineering problems. For example, mechanical vibration differential equations have time as the independent variable and the analog computer produces solutions to differential equations with the independent variable being time. This then involves the direct solution of the equations. Many heat transfer problems also have time as the independent variable and the analysis and solution of these problems is fairly straightforward.

Any discussion of heat transfer solutions would not be complete without including the solution of partial differential equations. The mathematical description of any continuous media usually involves partial differential equations, i.e., differential equations having derivatives with respect to more than one independent variable. Since the electronic analog computer can integrate with respect to only one variable, namely time, it can only be used to solve ordinary differential equations. Thus it is necessary to convert a partial differential equation to one or more ordinary differential equations in order to solve it on the analog computer. There are two popular techniques for doing this: (1) the method of separation of variables, leading to an eigenvalue problem; (2) the finite difference method wherein derivatives with respect to all variables but one are replaced by finite-difference approximations. The first method will only be discussed and the second method will be demonstrated by analysis and analog computer solution.

The basis of analog computer operation is presented in the Appendix. No attempt is made to prove or explain any of the computer operations on the basis of electronic theory. The reader is instead referred to one of the texts listed in the literature consulted. The operation of the electronic analog computer itself is presented in Chapter 2. The EAI TR-10 analog computer was used exclusively. This is a relatively small computer and thus is somewhat limited in the size of problems which can be solved. The limiting factor is the number of operational amplifiers available for the problem solution.

The answers from the computer are obtained in the form of observations or recordings of continuous time variables. If the problem is static, that is, if the answers have one and only one value when the solution has been reached, all the variables will have ceased to change and their values can be read with a simple voltmeter. In most cases, however, the problem solution involves variables that change continuously with time and are recorded continuously in time. The problem solutions presented herein were recorded on a two-dimensional X -Y plot on paper measuring 10 inches x 15 inches then re-plotted on standard 8 1/2 inch x 11 inch paper. Regarding the solution of an equation or a set of equations, it should be understood that very seldom is just one numerical answer sought. Rather a better understanding of the situation or system being studied is more important.

A logical question arises as to when to use a digital computer and when to use an analog computer. Basically, the following statements concerning when to use a digital computer could be stated.

1. Problems involving large amounts of tabulated data in discrete-number form.

2. Problems with precise input data requiring precise solutions.

Similarly, the following statements apply concerning when to use an electronic analog computer.

1. Solution of simultaneous differential equations.

2. When an analogy of a mechanical, or other type of system is required. That is, a mathematical model will represent the physical problem, thus solution of the equations will be analogous to the physical system.

CHAPTER 2

EAI TR - 10 ELECTRONIC ANALOG COMPUTER

The TR - 10 patchboard contains all of the computer component connections which are explained in the Appendix. The patchboard is removable from the computer so that full use can be made of the computer. Problems can be "patched" or wired before putting the patchboard on the computer; then potentiometer settings can be made when the operator is ready for the problem solution.

The control panel of the TR-10 contains the following control switches.

Mode Control Switch; positions are RESET, HOLD, and OPERATE.

Reset — The programmed problem is at the conditons corresponding to time zero.

Hold — This position will stop and hold the problem solution instantly for observation.

Operate — This will start the problem solution. Operation can start from the reset or hold position.

Meter Mode Selector Switch; positions are POT BUS, NULL, V.M., AMP, BAL.

Pot Bus — This position is used to set values on pots. This is a precision potentiometer which is bridged so that when the pot which is being set reaches the same value as the precision potentiometer, the reading on the meter will show a null indication.

Null — A position used for setting potentiometers other than the ones which are found on the Montana State University analog computer.

V.M. This connects the meter for use as a voltimeter. The V.M. switch was not used for any problems.

Amp This connects the meter as a voltmeter to the amplifier selector switch so that the output voltage of each amplifier can be read on the meter. This is used as a check to determine what amplifier is overloaded (> 10 volts) when there is an overload indication.

Bal Used to balance, to zero, the stabilizer outputs of all amplifiers.

The important switch positions for problem solution are the pot bus and amp settings. The pot bus is necessary in order to obtain accurate pot settings. The amp setting enables the operator to read the voltage output of each amplifier. The computer will hum distinctly audibly when there is an amplifier which is overloaded.

This is a brief presentation of the more important control components of the TR-10 analog computer. A more extensive explanation of controls can be found in the operator's handbook which is always located in the computer room. The operator's handbook will also illustrate how to make the patch-board connections required from the computer diagram. The connections are very easy to make; the handbook being easy to follow and also, the operator soon learns the basic connections. The most important aspect of using the analog computer is to obtain the correct computer diagram.

CHAPTER 3

STEADY-STATE HEAT CONDUCTION: UNIFORM CROSS-SECTION FIN

Consider a fin of uniform circular cross-section, Figure 1, whose base is attached to a plane section which is maintained at the constant temperature T_b. The fin exchanges heat along its surface with a fluid at a bulk temperature T_a. A heat balance for a differential element of the fin must consider heat flow into the element by conduction, heat generated by the element, heat flow out of the element by conduction, and the heat capacity of the element. In steady-state heat transfer the heat capacity of the element is unimportant and if the fin is highly conducting the temperature along the fin is a function of the axial distance alone. Now a heat balance for a differential element in such a fin will be considered in order to derive an equation for the temperature distribution along its length. Heat flows by conduction into one face of the element while heat flows out of the opposite face by conduction and from the surface by convection. In the absence of energy generation within the element and under steady-state conditions:

Rate of heat flow by conduction into element at x	=	Rate of heat flow by conduction out of element at (x + dx)	+	Rate of heat flow by convection from surface

$$-kA\frac{dT_x}{dx} = \left[-kA\frac{dT_x}{dx} + \frac{d}{dx}\left(-kA\frac{dT_x}{dx}\right)dx + \ldots\right] + hA_s\ (T_x - T_a)$$

$$-kA\frac{dT_x}{dx} = -kA\frac{dT_x}{dx} - kA\frac{d^2T_x}{dx^2}dx + h(Pdx)\ (T_x - T_a)$$

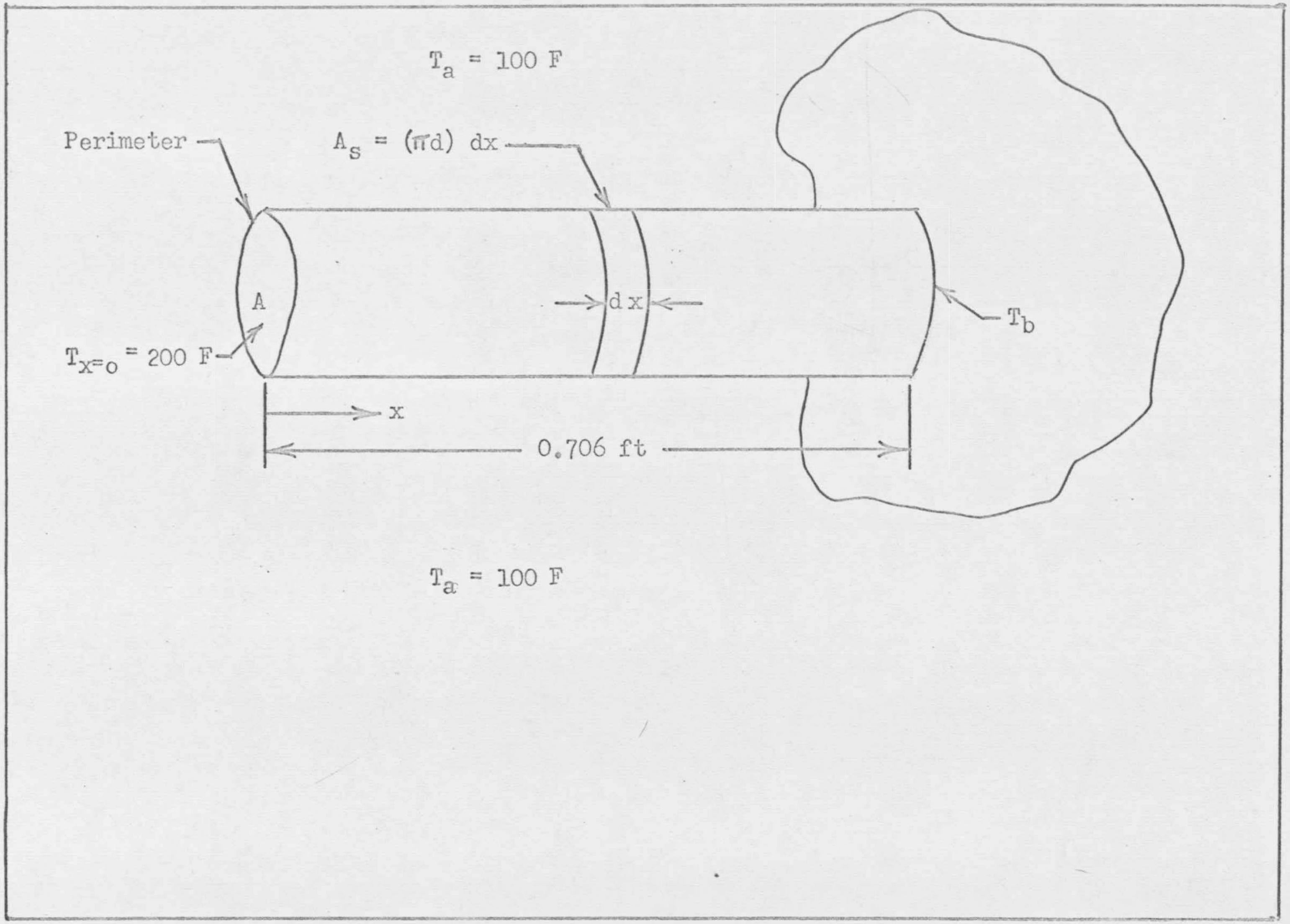

Figure 1. Uniform Cross-Section Rod Fin

$$kA \frac{d^2T_x}{dx^2} = hP(T_x - T_a)$$

(1) $$\frac{d^2T_x}{dx^2} = \frac{hP}{kA} (T_x - T_a)$$

Where q = Rate of heat flow

k = Thermal conductivity

h = Heat transfer coefficient

A_s = Surface area

A = Area of fin cross-section

P = Perimeter

Let $T = (T_x - T_a)$

T_a = Constant

Therefore

$$\frac{d^2T_x}{dx^2} = \frac{d^2T}{dx^2}$$

and eq. 1 can be written

(2) $$\frac{d^2T}{dx^2} = \frac{hP}{kA} T$$

Independent Variable Other Than Time

The independent variable of eq. 2 is distance rather than time. This does not mean that is is impossible to solve it on the analog computer, it does mean a transformation between time and distance must be defined. The degree of the correspondence between the distance x and the time t will depend on the units in which the equation is written. If distance x is in inches, a possible correspondence may be 1 inch = 1 second or if x is in feet, the correspondence may be 1 foot = 1 second, and so forth. Considering eq. 2, the units of p, h, k, and A must be consistent with the units of x. The relation of time and distance for the solution of the rod fin heat transfer equation, eq. 2, can be written:

$$x\ (\mathrm{ft}) = a \cdot t\ (\mathrm{sec})$$

$$\frac{dx}{dt} = a$$

also $$x = f(t)$$

$$T = g(x)$$

By using the chain rule twice and substituting

$$\frac{dT}{dt} = \frac{dx}{dt} \cdot \frac{dT}{dx}$$

$$\frac{dT}{dt} = a\,\frac{dT}{dx}$$

$$\frac{d^2T}{dt^2} = a\left[\frac{d}{dx}\left(\frac{dT}{dx}\right)\right] \quad \frac{dx}{dt}$$

$$\frac{d^2T}{dt^2} = a\,\frac{d^2T}{dx^2} \cdot \frac{dx}{dt}$$

$$\frac{d^2T}{dt^2} = a^2\,\frac{d^2T}{dx^2}$$

and (3) $$\frac{d^2T}{dx^2} = \frac{1}{a^2}\,\frac{d^2T}{dt^2}$$

Analog Computer Solution of a Steady-State Heat Conduction Problem

Analysis of the uniform cross-section rod fin will continue by considering an example problem.

Problem: An engine is air-cooled by circular-rod fins. The surrounding ambient air temperature is 100 F. The temperature of the end of the fin that is entirely in the air is 200F. Determine the temperature distribution along the length of the fin. The problem is illustrated in Figure 1. Let

$h = 5 \frac{Btu}{hr\ ft^2\ F}$ $\qquad$ d = 1 inch diameter

L = 0.706 ft $\qquad$ $k = 25 \frac{Btu}{hr\ ft\ F}$

Equation 2 has been found to be the equation describing the heat flow for the rod fin. Let

$$m = \frac{Ph}{kA} = \frac{(\pi d)\ h}{K(\pi/4\ d^2)} = \frac{(\pi/12\ ft)\left(5\ \frac{BTU}{hr \cdot FT^2 \cdot F}\right)}{\left(25\ \frac{BTU}{hR \cdot FT \cdot F}\right)\left(\frac{\pi}{4} \cdot \frac{1}{144}\ fT^2\right)} = 9.6\ fT.^{2^{-1}}$$

Then

$$\frac{d^2T}{dx^2} = mT = 9.6T\left(\frac{F}{fT^2}\right) \tag{4}$$

A substitution must be made since the equation involves an independent variable other than time. The method is explained previously. Equation 3 will be substituted into Eq. 4.

$$\frac{d^2T}{dx^2} = mT \tag{4}$$

$$\frac{1}{a^2}\frac{d^2T}{dt^2} = mT$$

or

$$\frac{d^2T}{dt^2} = a^2mT \tag{5}$$

In order to obtain a numerical value for a, t is arbitrarily selected as 1 second of problem time.

$$a = \frac{x}{t} = \frac{0.706\ ft}{1\ second}$$

$$a^2 = (0.706)^2 = 0.50$$

Using dot notation for the derivatives so that $\frac{d^2T}{dt^2}$ and $\frac{dT}{dt}$ will be $\ddot{T}$ and and $\dot{T}$ respectively results in:

(6) $$\ddot{T} = a^2 mT = 0.50\ (9.6)\ T = 4.80\ T$$

This then is the equation which must be solved on the analog computer. The only boundary condition is:

$$T_{x=0} = (200 - 100)F = 100F$$

Three steps will be followed before wiring the computer patchboard. These are: (1) magnitude scaling, (2) time scaling, (3) draw a complete computer diagram. From knowledge of the problem, it is possible to estimate maximum temperatures and problem duration. It should be kept in mind that should an estimate be wrong, it is an easy thing to change the scale factors by changing the pot settings. For this problem, it will be assumed that T_{max} = 1000F. Therefore, on a "per volt" basis, the scale factor will be:

$$10v = 1000F$$

and the scaled variable is:

$$\left[0.01\ \frac{v}{F}\right] \qquad 1\ \text{volt} = 100F$$

The computer time constant is:

$$B = \frac{t_c}{t}$$

where t_c = computer time

t = actual problem time

For the rod fin problem, a satisfactory value of computer run time is 10 seconds, so that B = 10.

In this problem, T is a function of x so that there is no problem time involved in this steady-state condition, but a value of B must be selected to give a satisfactory computer time.

The computer diagram is shown in Figure 2. The initial condition, or known boundary value, is at the exposed end of the rod fin where $x = 0$. The computer diagram is a solution to Equation 6. The output Y is shown on the computer diagram and represents T. A voltage must be generated to represent distance x or time t on the X axis. The voltage is generated by integrating a voltage as is shown in Figure 2. A voltage of 10 volts will be generated in 10 seconds. An arm scale setting on the X - Y plotter of 1 volt/inch will generate 10 inches on the plotting paper in 10 seconds.

In order to show the computer solution (volts and time) and the conversion to problem solution (temperature and distance), both graphs are shown as Figure 3 and Figure 4 respectively. The conversion factors used:

1 volt = 100 F (per volt basis)

1 second of problem time = 0.706 feet

10 seconds of computer time = 1 second of problem time

The computer time was 10 seconds; therefore, the problem time was 1 second. This 1 second represented 0.706 feet of rod fin length.