A method for optimizing a network of pipelines for transporting woodchips
by Irving Campos Hoffman

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in Civil Engineering
Montana State University
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Abstract:
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transporting woodchips hydraulically by determining the concentration of woodchips and pipe diameter
for each line of the system which minimize the cost.

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characteristics provide a method for reducing to two the number of pipe diameters which could
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Optimization of three- and five-line networks utilizes the cost function of single lines and, in addition,
requires that the continuity of flow of the two-phase fluid be satisfied at the junctions.

The optimization technique is applied to an existing area.

Costs of pipeline transportation of woodchips from chipping areas to a processing plant are compared
with costs of moving the chips by rail and truck. The comparison shows rail costs are lowest in all
cases.
A METHOD FOR OPTIMIZING A NETWORK OF PIPELINES FOR TRANSPORTING WOODCHIPS

by

IRVING CAMPOS HOFFMAN

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MASTER OF SCIENCE

in

Civil Engineering

Approved:

[Signatures]

Head, Major Department

[Signature]

Chairman, Examining Committee

[Signature]

Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

December, 1967.
The optimizing techniques presented in this study are developed as part of the project investigating the transportation of woodchips in pipelines sponsored by the Forest Engineering Research Branch of the Intermountain Forest and Range Experiment Station, U.S. Forest Service, Department of Agriculture, as a cooperative aid project in the Department of Civil Engineering and Engineering Mechanics of Montana State University.


The author wishes to extend personal thanks to Dr. William A. Hunt for his encouragement and guidance, to the faculty of the Department of Civil Engineering and Engineering Mechanics, and to the research engineers of the U.S. Forest Service associated with the project.

Gratitude is expressed to Elizabeth A. Hoffman, the author's wife, for her help in completing this thesis.
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CHAPTER I
INTRODUCTION

Transportation of solids in pipelines is not a recent innovation. Successful applications have been made for over one hundred years in fields ranging from placer mining to grain handling. High costs of labor and maintenance in other transportation systems have intensified interest in pipelines in recent years. Presently, many successful pipeline installations exist, most of which occur in the mineral and mining industry (1), (2)*. A 72-mile pipeline is transporting 800 tons per day of gilsonite from a mine in northeastern Utah to a refinery in western Colorado. Copper concentrate is pumped 14 miles in Chile. The mines in South Africa have several pipelines, some up to 16 miles long, successfully transporting uranium-bearing gold tailings.

Since 1957, the Pulp and Paper Research Institute of Canada (3) has been investigating the possibility of using pipelines to transport woodchips to processing plants. In 1961, the U.S. Forest Service began a program to examine pipelines as a means of conveying woodchips. The Forest Service is seeking more economical methods of transporting wood to stimulate greater utilization of woodlands in this country. Reduction in transportation costs will allow low-value wood (cull and dead trees, slash, and residue from sawmills) now being discarded to be moved to the processing plant. Private processors are continuously searching

*Numbers in parentheses refer to numbered references in the Literature Cited.
for methods to lower handling and transportation costs of chips to increase production and profit. Pipelines may offer a means of reducing these costs.

The economic advantages of pipeline transportation are quite attractive:

Automation. Gas and oil pipelines have been automated for approximately twenty years. Once the fluid has been injected into the pipeline it is left unattended until the next input or discharge point. Pumping stations are controlled automatically from a central master station.

Dependability. Dependability of pipelines has been proven. The gilsonite pipeline in Utah has been operating for seven years (1). The Consolidated Coal Company operated a 108-mile pipeline in Ohio without a shutdown for three years (4).

Operating Costs. Operating costs are low; other costs are mostly fixed and remain nearly constant over the life of the installation. Other transportation systems have higher operating costs and are more easily affected by the rising costs of labor and personnel.

Maintenance Costs. Maintenance costs are low since pumping stations have few moving parts and the pipeline is buried and subject to little wear.

These advantages have been confirmed in the transport of single-phase fluids such as gas and oil. The advantages may potentially be applied to the transportation of solids by defining the hydraulics of two-phase flow.
The hydraulics of coal and gilsonite presently being transported long distances in pipelines are defined well enough to permit the design of pipeline systems. Although the mechanisms of flow for solids are not fully understood it is known that the small, uniform size of crushed coal and gilsonite produces a homogenous two-phase fluid at high velocities giving well defined friction loss relationships and allowing power requirements to be calculated and operating costs to be predicted. The hydraulic properties of woodchip mixtures are less well defined. Woodchips are relatively large and nonuniform in shape and size. The specific gravity of woodchips is lower than that of coal or gilsonite. Accurate relationships among head loss, which is shown to be the greatest economic factor, and other flow parameters, such as pipe size, velocity, and woodchip concentration, are required to predict the power requirements and, accordingly, the economics of woodchip pipeline transportation. Research is being conducted to investigate the mechanisms of motion of woodchips and to describe more accurately the head loss relationships involved.

A large research program sponsored by a group of ten interested companies was conducted in Marathon, Ontario (5). Friction loss tests for woodchip-water mixtures were conducted on 2,000 feet of 6-, 8-, and 10-inch steel pipe. Woodchip pipeline research projects at Montana State University (MSU) have been conducted to investigate the moisture absorptive properties of woodchips under pressure (6), the energy losses of woodchip-water mixtures passing through expansions and valves (7), the effect of woodchips on the performances of centrifugal pumps (8), and the economic feasibility of woodchip pipelines (9). Tests are
currently being conducted at MSU to define the energy losses due to friction by various woodchip mixtures. Queen's University in Kingston, Ontario (10), the Pulp and Paper Research Institute of Canada (3), and the Shell Pipeline Corporation have conducted head loss studies on transporting woodchips in pipelines.

Several equations exist for head loss in two-phase flow. Durand (11) proposed an equation giving head loss for sand and gravel. Elliott and de Montmorency (3) modified Durand's equation to express head loss for woodchip-water mixtures in pipelines. Faddick (12) developed an equation for woodchips quite similar to Durand's from tests conducted at Queen's University on 4-inch pipeline.

A mathematical model developed by Hunt (9) to investigate the feasibility and economics of woodchip pipelining uses the head loss equation proposed by Faddick (12) for determining energy requirements, number and size of the pumping units, and costs of the variable salaries and wages. The analysis gives the best operating concentration and pipe size along with costs for a given throughput and length of pipeline. Investigation of this economic model showed that pump efficiency, frictional loss coefficient, capital recovery factor, and the chip concentration are the variables having the greatest effect on pipeline economics. The results of the model for a single pipeline with the chip source at one end and the processing plant at the other show pipeline transportation costs to be competitive with rail and truck. The analysis was applied to an area in Alaska which had no existing transportation facilities;
a savings of 58 percent was anticipated over road construction and haul.

The disadvantage of Hunt's model is that it cannot be applied to a network of pipelines which many of its applications will require. A network model is more complicated than a single-line model since the total costs are influenced by the operating characteristics of each line. A model analysis which gives the pipe size and concentration of woodchips for each line of a pipeline network producing the lowest cost for the system is needed.

This thesis develops a technique for determining these optimum conditions and predicting the lowest cost for pipeline networks. The method of analysis uses the response surface (13) generated by the mathematical expression developed by Hunt (9) describing the economics of the pipeline system. This expression, called the objective function, contains all the information required for a rational decision and when minimized gives the optimum operating conditions for a pipeline network.
A technique of analyzing a response surface to determine the optimum operating conditions for a single-line pipeline system is described in this chapter. The response surface is produced from the objective function for the mathematical model developed by Hunt to describe the economics of a woodchip pipeline. This objective function contains three decision variables: (1) investment costs, (2) operating expenses, and (3) overhead costs. The three decision variables, in turn, are expressed by seven cost groups: (1) energy cost, $x_1$, (2) installed cost of the pipeline, $x_2$, (3) installed cost of the pump stations, $x_3$, (4) installed cost of injection and separation equipment, $x_4$, (5) cost of fixed salaries and wages, $x_5$, (6) cost of variable salaries and wages, $x_6$, and (7) cost of water treatment, $x_7$. The units for each of these groups are dollars per ton-mile which is the total expense of moving one ton of woodchips (oven-dry basis) one mile in a given transportation system. This unit was selected because it provides a basis for easily comparing rates of other transportation systems, such as rail and truck. The objective function for the single-line system is the summation of the seven cost groups and is expressed as

$$X_t = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

The equations developed by Hunt (9) and used in this thesis for these seven cost groups are summarized in this chapter on page 8 and are functions of the variables listed below:
C = Concentration of chips in the mixture by volume

crf = Capital recovery factor

D = Pipe diameter, feet

e = Combined efficiency of motor-pump drivers

f = Friction factor for Weisbach equation

H_t = Head due to friction and difference in elevation, feet/mile

L = Length of pipeline, miles

R_1 = Cost of electrical energy, $/kwh

R_2 = Installed cost of pipeline, including right-of-way, $/(in-mile)

R_3 = Cost of pump station and controls, $/(installed horsepower)

R_4 = Cost of chip injection system $/(ton per day of oven-dry chips)

R_5 = Cost of separation system $/(ton per day of oven-dry chips)

R_6 = Annual cost of fixed wages, salaries, operation maintenance; exclusive of pipeline maintenance and pump station operations, $/year

R_7 = Annual wages, salaries, etc. for pump station, $/(pump station)

R_8 = Annual maintenance cost of pipeline, $/mile

R_9 = Cost of water and treatment, $/million gallons

S_m = Specific gravity of water-chip mixture

S_odo = Specific gravity of oven-dry chips

W = Throughput, tons per day of oven-dry chips (TPD)

The variables S_m and H_t, which are developed in Appendix B, are functions
of the following additional variables listed in Appendix A:

\[ g \quad = \quad \text{Gravitational constant, ft/sec}^2 \]

\[ M \quad = \quad \text{Moisture content of chips, decimal fraction of oven-dry chips} \]

\[ Z_t \quad = \quad \text{Difference in elevation between the ends of pipe, feet} \]

The seven cost groups, expressed in units of dollars per ton-mile, are developed in Appendix B as functions of the above-listed variables and summarized by the following equations:

1. Energy cost,

\[ x_1 = 0.000753 \left( \frac{R}{e} \right) \left( \frac{S_{Ht}}{C} \right) \quad (1) \]

2. Installed cost of pipeline,

\[ x_2 = \left( \frac{R}{365 W} \right) \text{ crf} \quad (2) \]

3. Installed cost of pump stations,

\[ x_3 = 0.000000115 \left( \frac{R}{e} \right) \left( \frac{S_{Ht}}{C} \right) \text{ crf} \quad (3) \]

4. Installed cost of injection and separation systems,

\[ x_4 = \left( \frac{R}{365 L} \right) \text{ crf} \quad (4) \]

5. Cost of fixed salaries and wages,

\[ x_5 = \frac{R}{365 W L} \quad (5) \]
6. Cost of variable salaries and wages,

\[ x_6 = \frac{1}{365} \left( \frac{R \cdot H_t}{H_{sa}} + R_8 \right) \]  

(6)

7. Cost of water and water treatment,

\[ x_7 = 0.00024 \left( \frac{1 - C}{C} \right) \left( \frac{R_9}{S_{odc}} \right) \]  

(7)

The analytical expressions are based on the system operating 24 hours per day for 365 days per year. The optimization technique which is presented in the following pages determines the values of C and D which give the minimum cost; all other variables must be specified. The variables \( R_3 \) and \( R_7 \) for this analysis have been modified from those used by Hunt. \( R_3 \) and \( R_7 \) were defined as functions of additional variables by Hunt; in this analysis they are assigned a constant value determined from economic data recently acquired from Continental Pipe Line Company (4).

The objective function, \( X_t \), for the single-line system gives the total cost per ton-mile and can be expressed as a function of C and D in polynomial form by combining the seven cost groups algebraically. The polynomial expression, developed in Appendix C, is given by

\[ X_t = (K_1 C^{1.84} + K_2 C^{1.84}) D^{2.10} + (K_3 C^{-2} + K_4 C^{-3}) D^{5} + K_5 D + \frac{K_6}{G} + K_7 \]  

(8)

where the coefficients \( K_i \), \( i = 1 \) to \( 7 \), are combinations of the variables other than C and D.
The absolute minimum cost for the single-line system is obtained by solving the simultaneous equations

\[
\frac{\partial X_t}{\partial C} = 0
\]

and

\[
\frac{\partial X_t}{\partial D} = 0
\]

Hunt investigated the solution of these equations by plotting \(\frac{\partial X_t}{\partial C}\) and \(\frac{\partial X_t}{\partial D}\) for different pipe sizes versus concentration as shown in Figure 1. He observed that the \(\frac{\partial X_t}{\partial C}\) and \(\frac{\partial X_t}{\partial D}\) curves intersected close to but never exactly on the zero ordinate and interpreted the intersection of the \(\frac{\partial X_t}{\partial C}\) curves with the zero ordinate as sufficiently close to zero to describe a possible minimum condition although the equations for an absolute minimum were not satisfied. The reason these curves are not zero at the same concentration will be discussed later. Hunt was correct in selecting the points where \(\frac{\partial X_t}{\partial C} = 0\) as the points of possible minimum costs. He determined which combination of concentration and diameter produced the minimum cost by using a digital computer for:

1. Solving the value of concentration at which \(\frac{\partial X_t}{\partial C} = 0\) for a given pipe size
2. Computing the cost for this diameter at its optimum concentration using the seven cost groups
3. Repeating steps 1 and 2 for a given array of pipe sizes and comparing costs at the optimum conditions for each diameter.
Figure 1. PARAMETRIC CURVES OF $\frac{\partial X_t}{\partial D}$ and $\frac{\partial X_t}{\partial C}$
Analysis of the response surface (13) generated by plotting $X_t$, cost per ton-mile, as a function of the concentration, $C$, and diameter, $D$, in a three-coordinate system as shown in Figure 2 offers an improvement of Hunt's method of solution. All surfaces for the single-line model were found to have similar shapes which descend to lower costs with smaller diameters and higher concentrations. The shape of the surface shows no node (a point at which $\frac{\partial X_t}{\partial D}$ and $\frac{\partial X_t}{\partial C}$ are both equal to zero) in the region of physical meaning; therefore, the conditions for an absolute minimum do not occur which indicates why the intersection of the curves $\frac{\partial X_t}{\partial D}$ and $\frac{\partial X_t}{\partial C}$ plotted by Hunt and shown in Figure 1 do not occur at the zero ordinate.

The response surface must be limited to a feasible region describing physical applications with all their limitations and constraints. Such a region is necessary since the concentration of woodchips in a pipeline has a limiting maximum above which it may not be increased without compressing the chips and packing the pipe so that transport is stopped. Paddick found this limit to be 43 percent for four-inch pipe; however, Equation B-5*, which he suggested and on which the economic pipeline model is based, does not contain constraints. This physical limitation on the concentration requires that the feasible region of the response surface be bounded by the planes $C = 0$, $D = 0$, and $C = C_{\text{max}}$ where $C_{\text{max}}$ is the maximum allowable operating concentration. The value

---

*Equation numbers which contain letters refer to equations in the Appendix corresponding to the letter.
Response surface generated by plotting the objective function, $X_t$, as a function of the independent variables $D$ and $C$.

Figure 2. Single-Line Response Surface with Intersecting Planes
of \( C_{\text{max}} \) must be chosen sufficiently conservative to prevent any local concentrations occurring during operation from approaching the limiting concentration physically possible and stopping the flow of the mixture.

The lowest point on the response surface in the feasible region occurs at \( \frac{\partial X}{\partial D} \big|_{C = C_{\text{max}}} = 0 \) as shown in Figure 2. The pipe size which corresponds to the lowest cost is indicated as \( D_t \) in Figure 2. This theoretical diameter will seldom occur at a commercially available size. The minimum cost will then occur with a pipe whose diameter is either the next commercial size larger or the next one smaller than \( D_t \). These particular diameters are determined by considering the slope, \( \frac{\partial X}{\partial D} \big|_{C = C_{\text{max}}} \), of the curve formed by the intersection of the response surface and the plane \( C = C_{\text{max}} \) in the vicinity of \( D_t \). The slope, \( \frac{\partial X}{\partial D} \big|_{C = C_{\text{max}}} \), is negative for all diameters smaller than \( D_t \) and positive for all diameters larger than \( D_t \) as shown in Figure 2. The diameters giving the lowest negative and positive values of \( \frac{\partial X}{\partial D} \big|_{C = C_{\text{max}}} \) are the only two sizes which could possibly give the lowest cost and are shown as \( D_2 \) and \( D_3 \) in Figure 2. The planes defined by these two diameters intersect the response surface and describe curves shown in both Figure 2 and Figure 3.

A computer program, listed in Appendix E, was developed to select the commercially available pipe diameter with the lowest positive value and the one with the lowest negative value of the first partial derivative of the objective function with respect to diameter. This derivative, developed by the author in Appendix C, is given by
Only two possible curves which could give the lowest cost.

Optimum concentration for $D_2$

$\frac{\partial X}{\partial C} = 0$

Optimum concentration for $D_3$

Figure 3. Relationship Between Cost and Concentration for Particular Pipe Sizes
\[
\frac{3X_t}{3D} = 2.10 \left(K_1 C^{0.84} + K_2 C^{0.84} \right) D^{1.10} - 5 \left(K_3 C^{-2} + K_4 C^{-3} \right) D^{-6} + K_5
\]

The response surface analysis shortens the procedure used by Hunt by eliminating all but two possible pipe diameters.

Figures 2 and 3 show that the optimum concentration for the smaller size, \(D_2\), will always occur at \(C_{\text{max}}\) and for the larger size, \(D_3\), at a concentration, \(C\), less than \(C_{\text{max}}\) and where \(\frac{3X_t}{3C} \bigg|_{D_3} = 0\). The computer program was developed to find the optimum concentration for the latter case using the Newton-Raphson method of extracting real roots, as outlined by Scarborough (14), of the first derivative of \(X_t\) with respect to concentration for the larger diameter set equal to zero:

\[
\frac{3X_t}{3C} = (1.84 K_1 C^{0.84} + 0.84 K_2 C^{-0.16}) D^{2.10} - (2K_3 C^{-3} + 3K_4 C^{-4}) D^{-5} - K_5 C^{-2} = 0
\]

Sample results for the optimization of a pipeline with a specified length in miles, \(L\), throughput of woodchips in tons per day, \(W\), and difference in elevation of the pipe ends in feet, \(Z_t\), are shown in Table I. The other specified variables used in this sample are listed in Table VIII. Table I is divided into three sections.

The first section is a list of the variables appearing in the table and their definitions.

The second section lists in the second, third, and fourth columns the given quantities \(L\), \(W\), and \(Z_t\) for a designated line. The computed optimum diameter and concentration are listed in the fifth and sixth
columns. Using the optimum D and C, values are computed and listed in the remaining columns for (1) the velocity, V, given by Equations B-1 and B-2, (2) the make-up water, \( Q_w \), using Equation B-25 with units converted to a gallons per minute basis, (3) the energy loss due to friction, \( H_f \), from Equation B-9, and (4) the horsepower required per mile, HPPM, from Equation B-12.

The third section of the table gives a summary of the first costs for (1) pipeline installation computed by Equation B-15, (2) pumps, controls, and pump station construction given by Equation B-17, (3) the injection-separation equipment given by Equation B-21, and (4) the total for these three items. The sixth column in the third section lists the annual operating expense, ANOP, given by

\[
ANOP = 365 \left( W \right) (L) (x_1 + x_5 + x_6 + x_7)
\]

where \( x_1, x_5, x_6, x_7 \) are the operating expenses expressed by Equation 1, 5, 6, and 7. The final column presents the total costs on a ton-mile basis computed from the objective function for the optimum concentration and diameter given by Equation 8.

The logic and techniques used to investigate the single pipeline can be extended to analyze the economic feasibility of a three-line network which is considered in the next chapter.
Table I

Sample Results, Single-Line

L = Length of Line, Mile  
W = Throughput, Tons of Oven-Dry Chips per Day  
ZT = Difference in Elevation Between Inlet and Outlet End, Feet  
D = Diameter of Pipe, Inches  
C = Concentration of Woodchips in Mixture, by Volume, Percent  
V = Average Velocity of Flow, Feet per Second  
QW = Volumetric Flow Rate of Make-Up Water, Gallons per Minute  
HF = Head Loss Due to Frictional Resistance, Feet per Mile  
HPPM = Input Power Required, Horsepower per Mile of Pipeline.

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<th>Line</th>
<th>L</th>
<th>W</th>
<th>ZT</th>
<th>D</th>
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<td>FPS</td>
<td>GPM</td>
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First Costs and Annual Operating Expense for Optimum Conditions

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<td>$</td>
<td>$</td>
<td>$</td>
<td>$/Ton-Mile</td>
</tr>
<tr>
<td>1</td>
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<td>480229.</td>
<td>22500.</td>
<td>1795229.</td>
<td>567374.</td>
<td>0.05356</td>
</tr>
</tbody>
</table>
CHAPTER III
THREE-LINE NETWORK OPTIMIZATION

The objective function describing the transport of woodchips in a three-pipe network containing a junction with three branches is a linear combination of the objective function for each line of the system and will be optimized in this chapter by applying the method used for the analysis of a single pipe and satisfying the continuity relationships for the flow at the junction. The latter imposes an additional restriction which makes the analysis of pipeline networks more complex than for single lines.

A typical three-line network has two injection locations supplying chips to one plant as shown in Figure 4. The variables are subscripted to designate the line they describe.

The continuity conditions at the junction must be satisfied by the mass flow rates of both woodchips and the mixture of woodchips and water. The continuity relation for mass flow rate of the woodchips, developed in Appendix D, can be expressed as

\[ W_3 = W_1 + W_2 \]

that for the mixture as

\[ Q_{m3} = Q_{m1} + Q_{m2} \]

The equation of continuity for the mixture can be combined with Equation B-2 to give

\[ \frac{W_3}{C_3} = \frac{W_1}{C_1} + \frac{W_2}{C_2} \]
Figure 4. Schematic of the Three-Line Network
which is rearranged to give \( C_3 \) as a function of \( C_1 \) and \( C_2 \):

\[
C_3 = \frac{C_1 C_2 W_3}{C_1 W_2 + C_2 W_1} \quad (9)
\]

The objective function for the three-line network, which contains all the information required to make a rational decision about the optimum economic conditions, is now a function of the throughput and length of each line and is written as a "weighted" expression of the single-line costs to represent an average cost per ton-mile of all woodchips delivered in the system. The three-line objective function is expressed by

\[
X_t = \frac{X_1 W_1 L_1 + X_2 W_2 L_2 + X_3 W_3 L_3}{W_1 L_1 + W_2 L_2 + W_3 L_3} \quad (10)
\]

or more compactly

\[
X_t = \sum_{n=1}^{3} \frac{X_n W_n L_n}{W_n L_n} \quad (11)
\]

where \( X_n \) is the objective function for the \( n^{th} \) line of the system and is the same as defined for a single pipe in the previous chapter. The \( X_n \) are functions of the concentration and diameter in line \( n \):

\[
X_1 = \phi_1(C_1, D_1)
\]
\[
X_2 = \phi_2(C_2, D_2)
\]
\[
X_3 = \phi_3(C_3, D_3)
\]
but from Equation 9, \( C_3 \) is a function of \( C_1 \) and \( C_2 \); therefore,

\[
X_3 = \phi_3(C_1, C_2, D_3)
\]

which allows Equation 10 to be expressed as

\[
X_t = \psi_l(C_1, C_2, D_1, D_2, D_3)
\]  \hspace{1cm} (12)

The three-pipe network is optimized by determining the combination of \( C_1, C_2, D_1, D_2, \) and \( D_3 \) (\( C_3 \) is obtained from \( C_1 \) and \( C_2 \) by Equation 9) which reduces the objective function to a minimum.

Conditions for the absolute minimum cost can be found by solving the five simultaneous equations obtained by setting the first partial derivatives of the objective function with respect to the independent variables, \( C_1, C_2, D_1, D_2, \) and \( D_3 \) equal to zero:

\[
\frac{\Delta X_t}{\Delta C_1} = \left( \frac{\Delta X_1}{\Delta C_1} W_1 L_1 + \frac{\Delta X_2}{\Delta C_1} W_2 L_2 \right) \left( \frac{\Delta X_3}{\Delta C_3} W_3 L_3 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0\hspace{1cm} (13)
\]

\[
\frac{\Delta X_t}{\Delta C_2} = \left( \frac{\Delta X_2}{\Delta C_2} W_2 L_2 + \frac{\Delta X_3}{\Delta C_2} W_3 L_3 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0\hspace{1cm} (14)
\]

\[
\frac{\Delta X_t}{\Delta D_1} = \left( \frac{\Delta X_1}{\Delta D_1} W_1 L_1 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0\hspace{1cm} (15)
\]

\[
\frac{\Delta X_t}{\Delta D_2} = \left( \frac{\Delta X_2}{\Delta D_2} W_2 L_2 \right) \frac{1}{\sum_{n=1}^{3} W_n L_n} = 0\hspace{1cm} (16)
\]
The above simultaneous equations were solved by an iteration procedure on a digital computer and found to give concentrations far beyond the maximum physically possible and diameters which did not correspond to commercially available sizes. Because the solution for the theoretical minimum occurred at nonfeasible values of $C$ and $D$, the constraints $C_n = C_{\text{max}}$ and the limitation that diameters must be sizes commercially available were applied.

A method for reducing to two commercial sizes the number of diameters to be considered for each of the three lines is based on the following analysis. The objective function for the system, $X_t$ in Equation 11, will be in the minimum-value region when the objective function for each of the $n$ lines, $X_n$, is in its respective minimum-cost regions. As shown in Figure 2, these minimum values will occur in the vicinity of $C_n = C_{\text{max}}$. The two diameters for each line which could give the lowest cost are found by finding the commercial pipe sizes which give the lowest positive and negative values of

$$
\frac{\partial X_t}{\partial D_1} \bigg|_{C_1 = C_{\text{max}}} = 0 \quad (18)
$$

$$
\frac{\partial X_t}{\partial D_2} \bigg|_{C_2 = C_{\text{max}}} = 0 \quad (19)
$$
and
\[
\frac{\partial X_t}{\partial D^3} C_3 = C_{\text{max}} \quad (20)
\]
as was shown for the single-line system.

All combinations formed by the two pipe sizes for each of the three lines must be examined to find the one giving the lowest cost for the network. The number of combinations to be considered is determined by the mathematics of combinations and permutations described by Riordan (15) and is expressed by the equation
\[
(N_D)^{N_L} = N_C \quad (21)
\]
where \(N_D\) is the number of diameters tried for each line, \(N_L\) is the number of lines in the network, and \(N_C\) is the number of combinations of diameters which must be examined. Eight combinations are formed for the three-line network as computed by Equation 21:
\[
(2)^3 = 8
\]

The optimum concentration must be found for each line with each combination of diameters. The combination formed by the smallest size for each line will have optimum concentrations equal to \(C_{\text{max}}\). For the other combinations, the optimum concentrations are found by solving Equation 13 and 14 simultaneously with the constraints \(C_n < C_{\text{max}}\), giving the optimum concentrations at either \(\frac{\partial X_t}{\partial C_n} = 0\) or at \(C_{\text{max}}\). These two partial differential equations can be put in general polynomial form:
\[
\frac{3X_i}{3C_i} = \left[ (1.84 K_{2,i} C_i^{-0.84} + 0.84 K_{2,i} C_i^{-0.16}) D_i^{2.10} - (2K_{3,i} C_i^{-3} + 3K_{4,i} C_i^{-4}) D_i^{-5} - K_{6,i} C_i^{-2} \right] \frac{W_{i1}}{3} + \sum_{n \neq 1} \frac{W_{ln}}{n}
\]

\[
\left( \frac{c_1 c_2 w_3}{c_1 W_1 + c_2 W_1} \right)^{0.84} + 0.84 K_{2,3} \left( \frac{c_1 c_2 w_3}{c_1 W_2 + c_2 W_1} \right)^{-0.16} \right] D_i^{2.10}
\]

\[
\left[ 2K_{3,3} \left( \frac{c_1 c_2 w_3}{c_1 W_2 + c_2 W_1} \right)^{-3} - 3K_{4,3} \left( \frac{c_1 c_2 w_3}{c_1 W_2 + c_2 W_1} \right)^{-4} \right] D_i^{-5}
\]

\[
-K_{6,3} \left( \frac{c_1 c_2 w_3}{c_1 W_2 + c_2 W_1} \right)^{-2} \left\{ \left[ \frac{c_i^2 W_{i1} w_3}{(c_j W_{i1} + c_i W_{i1})^2} \right] \frac{W_{i1} L_{i1}}{3} \sum_{n \neq 1} \frac{W_{ln} L_{ln}}{n} \right\} = 0
\]

where for the first equation the subscript \( i \) is one and \( j \) is two; for the second equation the value of these subscripts are interchanged. The values for the diameters are given by the combination being examined and, therefore, are known.

These two equations can be solved simultaneously by any suitable method which imposes the constraints \( C_n \leq C_{\text{max}} \). The computer program developed for this analysis is listed in Appendix F and uses an iterative procedure combined with the Newton-Raphson method of extracting real roots. A general outline for the procedure used in the computer program is as follows:
1. Assume starting trial values of $C_1$ and $C_2$ for the first diameter combination at values less than $C_{\text{max}}$ to prevent the application of the concentration constraints and compute $C_3$ using Equation 9.

2. Solve for $C_1$ which satisfies $\frac{\partial X}{\partial C_1} = 0$ (Equation 22) by the Newton-Raphson method while holding $C_2$ constant.

3. Solve for $C_2$ which satisfies $\frac{\partial X}{\partial C_2} = 0$ holding $C_1$ constant at the value found in step 2.

4. With these "better" approximations for $C_1$ and $C_2$, begin the iteration again.

5. The constraints, $C_n \leq C_{\text{max}}$, are applied when either $C_1$ or $C_2$ becomes larger than $C_{\text{max}}$.

6. The process is continued until the change in the concentrations $C_1$ and $C_2$ for successive iterations are less than a predetermined value.

7. With $C_1$ and $C_2$ found, $C_3$ and the total costs are computed for each combination of diameters and compared to select the conditions which give the lowest cost.

Results from the optimization of a three-line network are shown in Table II which is divided into four sections. The first three sections are of the same format used in Table I and are explained in the previous chapter. The fourth section lists the total first costs and annual operating expense for the pipeline network. The final column in the fourth section gives the total cost of the system on a dollars per ton-
TABLE II
SAMPLE RESULTS, THREE-LINE NETWORK

L =LENGTH OF LINE, MILE
W =THROUGHPUT, TONS OF OVEN-DRY CHIPS PER DAY
ZT=DIFFERENCE IN ELEVATION BETWEEN INLET AND OUTLET END, FEET
D =DIAMETER OF PIPE, INCHES
C =CONCENTRATION OF WOODCHIPS IN MIXTURE BY VOLUME, PERCENT
V =AVERAGE VELOCITY OF FLOW, FEET PER SECOND
QW=VOLUMETRIC FLOW RATE OF MAKE-UP WATER, GALLONS PER MINUTE
HF=HEAD LOSS DUE TO FRICTIONAL RESISTANCE, FEET PER MILE
HPPM=INPUT POWER REQUIRED, HORSEPOWER PER MILE OF PIPELINE

- GIVEN QUANTITIES - | --- | COMPUTED OPTIMUM CONDITIONS ---
| LINE | L | W | ZT | D | C | V | QW | HF | HPPM |
| MILE | TPD | FT | IN. | PERCENT | FPS | GPM | FT/MI | HP/MI |
| 1 | 100.0 | 500. | 0. | 6.0 | .3000 | 7.87 | 485. | 328.3 | 59.5 |
| 2 | 50.0 | 1000. | 0. | 10.0 | .3000 | 5.66 | 971. | 294.2 | 106.7 |
| 3 | 50.0 | 1500. | 0. | 10.0 | .3000 | 8.50 | 1456. | 296.8 | 161.6 |

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS:

<table>
<thead>
<tr>
<th>LINE</th>
<th>PIPE INSTLN</th>
<th>$</th>
<th>PUMP STATIONS</th>
<th>$</th>
<th>INJ-SEP SYSTEM</th>
<th>$</th>
<th>TOTAL FST COST</th>
<th>$</th>
<th>ANNUAL OPER EXP</th>
<th>$</th>
<th>TOTAL COSTS $/TON-MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>536283.</td>
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</tr>
<tr>
<td>3</td>
<td>1292500.</td>
<td>727380.</td>
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<td>.04643</td>
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<td></td>
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</tbody>
</table>

TOTAL COST FOR THE PIPELINE TRANSPORTATION SYSTEM:

<table>
<thead>
<tr>
<th>PIPE INSTLN</th>
<th>$</th>
<th>PUMP STATIONS</th>
<th>$</th>
<th>INJ-SEP SYSTEM</th>
<th>$</th>
<th>TOTAL FST COST</th>
<th>$</th>
<th>ANNUAL OPER EXP</th>
<th>$</th>
<th>TOTAL COSTS $/TON-MI</th>
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</thead>
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<tr>
<td>4136000.</td>
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<td>5910211.</td>
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</tr>
</tbody>
</table>
mile basis as given by the objective function expressed in Equation 10. The line numbers and layout for the sample are shown in Figure 4. Variables describing the woodchips and the various costs were taken from Table VIII. The intermediate results from the computer program for the sample described in Table II are given in Appendix E for the eight combinations resulting from using two commercial diameters for each line and for the 27 combinations resulting from taking three sizes for each line, two of which are the next sizes larger than $D_L$ for the given line.

These techniques can be applied to higher order networks. Chapter IV outlines the optimization technique for the five-line network.
CHAPTER IV
FIVE-LINE NETWORK OPTIMIZATION

The procedure for optimizing the objective function describing the economics of transporting woodchips in a five-line network is outlined in this chapter using methods of analysis discussed in the preceding chapters for optimizing the single- and three-line objective functions.

The five-line objective function must satisfy two continuity relationships describing the mass flow rates at the two junctions which occur in the five-line network shown in Figure 5.

The continuity conditions at junction 1 are expressed by equations of the same form as given for the three-line network and are used to relate the mass flow rates of the woodchips and the woodchip-water mixture entering the junction to that leaving the junction. By changing the subscripts to conform to Figure 5, Equation 9 can be rewritten to express the concentration of woodchips in the mixture for line 4 which leaves junction 1 as:

\[
C_4 = \frac{C_1 W_4}{C_1 W_2 + C_2 W_1}
\]  

Equation 23

The continuity relationship at junction 2, which is fully developed in Appendix D, similarly gives the concentration of woodchips in line 5 leading from the junction as:

\[
C_5 = \frac{C_3 C_4 W_5}{C_3 W_4 + C_4 W_3}
\]  

Equation 24
$w_4 = w_1 + w_2$

$w_5 = w_3 + w_4$

Figure 5. Schematic of the Five-Line Network
and can be written as a function of $C_1$, $C_2$, $C_3$, $W_1$, $W_2$, $W_3$, and $W_5$ by substituting

$$W_5 = W_3 + W_4$$

and the expression for $C_4$, given by Equation 23, into Equation 24 giving:

$$C_5 = \frac{C_1 C_2 C_3 W_5}{C_1 C_3 W_3 + C_2 C_3 W_1 + C_1 C_3 W_2} \quad (25)$$

The objective function for the five-line network is a weighted expression of the single-line costs and is written in the same form as the three-line objective function given in Equation 10:

$$X_t = \frac{X_1 W_1 L_1 + X_2 W_2 L_2 + X_3 W_3 L_3 + X_4 W_4 L_4 + X_5 W_5 L_5}{W_1 L_1 + W_2 L_2 + W_3 L_3 + W_4 L_4 + W_5 L_5} \quad (26)$$

or

$$X_t = \sum_{n=1}^{5} \frac{X_n W_n L_n}{\sum_{n=1}^{5} W_n L_n} \quad (27)$$

where $X_t$ is given in dollars per ton-mile and $X_n$ is the single-line objective function. The $X_n$ are functions of the concentration and diameters in line $n$:

$$X_1 = \phi_1(C_1, D_1)$$

$$X_2 = \phi_2(C_2, D_2)$$
but from Equations 23 and 25, \( C_4 \) can be expressed as a function of \( C_1 \) and \( C_2 \) but \( C_5 \) is expressed as a function of \( C_1 \), \( C_2 \), and \( C_3 \); therefore, the objective function for the five-line network is expressed as

\[
X_t = \psi_2 (C_1, C_2, C_3, D_1, D_2, D_3, D_4, D_5)
\]  

(28)

The five-line network is optimized by determining the combination of \( C_1 \), \( C_2 \), \( C_3 \), \( D_1 \), \( D_2 \), \( D_3 \), \( D_4 \), and \( D_5 \) which reduce the objective function to a minimum. \( C_4 \) and \( C_5 \) are found by solving the continuity relationships given in Equation 23 and 25.

The conditions for the absolute minimum were examined by the same method used for the three-line objective function discussed in the previous chapter by solving the eight simultaneous equations formed by setting the first partial derivatives of the five-line network objective function with respect to the independent variables \( C_1 \), \( C_2 \), \( C_3 \), \( D_1 \), \( D_2 \), \( D_3 \), \( D_4 \), and \( D_5 \) equal to zero. The solution gives concentrations much greater than the maximum physically possible and diameters which were not sizes commercially available. Constraints were required to limit the concentration to \( C_{\text{max}} \) and the diameters to commercial sizes.

The number of commercially available pipe sizes for each line were reduced to two by the same method used for the three-line network by finding the commercial sizes which give the lowest positive and negative values of
\[ \frac{\partial X_t}{\partial C_i} = 0 \quad \text{for } n = C_{\text{max}} \] (29)

for the five individual lines.

The number of diameter combinations formed by these two pipe sizes for each line and which must be investigated to determine the lowest costs are found from Equation 21 to be 32.

The concentration which gives the lowest cost is \( C_{\text{max}} \) for the combination formed by the smaller pipe size for each line. The optimum concentrations for the other combinations are found by solving simultaneously the following three equations obtained by taking the partial derivatives with respect to \( C_1, C_2 \) and \( C_3 \) of the five-line network objective function:

\[
\frac{\partial X_t}{\partial C_i} = \frac{\partial X_i}{\partial C_i} \left( \frac{W_i}{n} \right) L_i + \frac{\partial X_i}{\partial C_i} \left( \frac{W_4}{n} \right) L_4 + \frac{\partial X_i}{\partial C_i} \left( \frac{W_5}{n} \right) L_5 = 0 \quad \text{for } i = 1 \text{ to } 3 \quad \text{and where} \]

\[
\frac{\partial C_4}{\partial C_3} = 0
\]

and

\[
C_i = C_{\text{max}}
\]

These equations, giving the optimum concentration, can be put in polynomial form similar to Equation 22 in terms of the unknowns \( C_1, C_2, \) and \( C_3 \); the diameters are given by the combination of sizes being investigated.
These three equations are solved simultaneously in the manner outlined in the previous chapter by using a numerical iterative procedure which converges to the optimum value of concentration.

Results from the optimization of a five-line network are shown in Table III using the same format at Table II for the three-line network. The line numbers and layout for the sample are shown in Figure 5.

The rational methods described in the last three chapters for determining the optimum concentrations and diameters and predicting the costs for pipeline networks can be extended to networks of more than five lines. The objective functions are constructed by using the pattern and form suggested by the similarity of the three- and five-line objective functions shown in Equations 11 and 27. The continuity relationships for each junction are developed to give the concentration in the line leaving in terms of the concentrations of the lines entering the junction in a manner similar to the method of developing the five-line continuity relationships. The two commercial pipe sizes for each line used in the combinations of diameters to be tried for lowest cost are found in the same manner used in the one-, three-, and five-line networks. The equations to be solved for optimum concentration show a pattern of development seen in Equations 13, 14 and 30 which can be used for networks of more than five lines.

The optimization techniques presented in the last three chapters to determine the best operating concentrations and pipe diameters to give the probable minimum costs for which woodchips can be transported are applied to a specific area in the next chapter.
-35-

TABLE III
SAMPLE RESULTS, FIVE-LINE NETWORK.

<table>
<thead>
<tr>
<th>LINE</th>
<th>L (MILE)</th>
<th>W (TPD)</th>
<th>ZT (FT)</th>
<th>D (IN)</th>
<th>C (PERCENT)</th>
<th>V (FPS)</th>
<th>QW (GPM)</th>
<th>HF (FT/MI)</th>
<th>HPPM (HP/MI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.0</td>
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<td>0</td>
<td>8</td>
<td>.3000</td>
<td>7.08</td>
<td>776</td>
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<td>84.3</td>
</tr>
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FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

<table>
<thead>
<tr>
<th>LINE</th>
<th>INSTLN COST ($100)</th>
<th>PUMP STATIONS</th>
<th>INJ-SEP SYSTEM</th>
<th>TOTAL FST COST ($100)</th>
<th>ANNUAL OPER- EXP ($100)</th>
<th>TOTAL COST ($100)</th>
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TOTAL COST FOR THE PIPELINE TRANSPORTATION SYSTEM

<table>
<thead>
<tr>
<th>PIPE INSTLN STATIONS</th>
<th>PUMP STATIONS</th>
<th>INJ-SEP SYSTEM</th>
<th>TOTAL FST COST ($100)</th>
<th>ANNUAL OPER- EXP ($100)</th>
<th>TOTAL COST ($100)</th>
<th>ANNUAL OPER- EXP ($100)</th>
</tr>
</thead>
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<td>4306943</td>
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</table>
CHAPTER V
CONCEPTUAL APPLICATION OF THE ECONOMIC MODEL

The study of the economics of pipelines as a method of transporting woodchips in a specific location in the northern Rocky Mountain area is presented in this chapter to demonstrate the technique developed in the preceding chapters for establishing the optimum conditions and predicting the cost of a pipeline system.

The location of the study area is in western Montana and northern Idaho as shown in Figure 6.

This area is under active consideration as a possible site for a woodpulp processing plant. The location is desirable because of the abundant volume of wood and the access to water for processing. The area is enhanced by the availability of rail transportation for shipment of the finished product. The Forest Service has available for this location data on timber volumes, timber growth, and local sawmills with their rated production capacity (16), (17), (18), (19).

The location is divided into four areas. For all federal, state, and private commercial forest lands, Table IV gives the timber volumes presently available and the estimated net annual growth (i.e., the net annual change in the volume of wood).

Existing sawmills are located throughout the study area as shown in Figure 6. Chip production from residue created by the mills operating at their rated capacities is calculated by applying the approximate conversion that 1,000 board feet mill production yields 1.68 tons of oven-dry chips from the mill residue. The data on residue production
Figure 6. Vicinity of Conceptual Application
<table>
<thead>
<tr>
<th>Area</th>
<th>Proposed Injection Location</th>
<th>Volume of Timber M cu ft</th>
<th>Estimated Net Annual Growth M cu ft</th>
<th>Estimated Possible Harvest M cu ft</th>
<th>Chips Available from Harvest Ton OCD/year</th>
<th>Possible Sustained Production of Chips Ton ODC/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Thompson Falls</td>
<td>1,218,908</td>
<td>158,458</td>
<td>75,268</td>
<td>234,836</td>
<td>643</td>
</tr>
<tr>
<td>II</td>
<td>Sandpoint</td>
<td>1,391,354</td>
<td>180,876</td>
<td>85,916</td>
<td>268,058</td>
<td>734</td>
</tr>
<tr>
<td>III</td>
<td>Bonners Ferry</td>
<td>1,358,855</td>
<td>176,651</td>
<td>83,909</td>
<td>261,796</td>
<td>717</td>
</tr>
<tr>
<td>IV</td>
<td>Libby</td>
<td>763,409</td>
<td>99,243</td>
<td>47,140</td>
<td>147,077</td>
<td>403</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td>4,732,526</td>
<td>615,228</td>
<td>292,233</td>
<td>764,690</td>
<td>2,497</td>
</tr>
</tbody>
</table>
at the existing sawmills are tabulated in Table V.

A hypothetical chip processing mill is assumed to be located at Thompson Falls with a rated capacity of 1,250 tons of product per day. Chips must be delivered at a rate of 2,500 tons per day for the mill to operate at capacity.

The study consists of analyzing the movement of woodchips in this area to the hypothetical processing plant at Thompson Falls. To allow the application of the single-, three-, and five-line optimization techniques, three delivery schedules will be examined and are given as follows:

1. Delivery of chips from Libby to Thompson Falls
2. Delivery of chips from Libby and Sandpoint to Thompson Falls
3. Delivery of chips from Libby, Sandpoint, and Bonners Ferry to Thompson Falls.

All chips are assumed to be produced from sawmill residue for this study. Only 47.5 percent of the annual growth must be harvested from the four areas to meet the supply demand of 2,500 tons per day. The amount of chips made from the residue from this percentage of harvest are computed by using the conversion that 1,000 cubic feet of wood delivered to the sawmill yields enough residue to produce 0.12 tons of chips. Table IV gives the chip yield for each area and the combined yield of 2,497 tons of oven-dry chips per day for all the areas.

Woodchips are assumed to be available at the injection points for each area at the design throughput rate shown in Table VI. Chips in
### TABLE V
PRESENT CHIP PRODUCTION CAPABILITY OF SAWMILLS IN THE APPLICATION AREA

<table>
<thead>
<tr>
<th>Area</th>
<th>Estimated Lumber Production MBF/year</th>
<th>Chip Production Capability Using Residue Tons ODC/year</th>
<th>Chip Production Capability Using Residue Tons ODC/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>63,875</td>
<td>107,310</td>
<td>294</td>
</tr>
<tr>
<td>II</td>
<td>61,300</td>
<td>102,984</td>
<td>282</td>
</tr>
<tr>
<td>III</td>
<td>58,000</td>
<td>89,040</td>
<td>244</td>
</tr>
<tr>
<td>IV</td>
<td>16,100</td>
<td>27,048</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td>894</td>
</tr>
</tbody>
</table>

### TABLE VI
INJECTION POINTS AND THROUGHPUT

<table>
<thead>
<tr>
<th>Injection Point</th>
<th>Location</th>
<th>Design Throughput, W tons/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Sandpoint</td>
<td>734</td>
</tr>
<tr>
<td>F</td>
<td>Bonners Ferry</td>
<td>717</td>
</tr>
<tr>
<td>D</td>
<td>Libby</td>
<td>403</td>
</tr>
<tr>
<td></td>
<td>Thompson Falls</td>
<td>643</td>
</tr>
<tr>
<td></td>
<td>Total chips</td>
<td>2,497</td>
</tr>
</tbody>
</table>
The Thompson Falls area are expected to be delivered directly from the sawmill to the processing plant. Rail, truck, and pipeline costs are not involved in the delivery of these chips.

The sawmills in the area could provide all the residue required to produce chips at the rate of 2,500 tons per day if their capacities were increased by a factor of three. This capability is shown by comparing the total estimated production available in Table IV with the present estimated mill output in Table V. A higher rate of chip supply could be obtained by increasing the percentage of harvest or by converting sawtimber into chips. Data verifying that enough wood is available for this increase in chip supply is shown in Table IV by the large volume of unharvested annual growth.

The data used in the computer program developed to optimize the various pipeline systems are given in Tables VII and VIII. Table VII gives the pipeline lengths and the difference in elevation of the ends of each line. These data were taken from Army Map Service topographic maps with a scale of 1:250,000. Table VIII gives the values of the variables used in each of the networks analyzed.

Tables IX, X, and XI give the results, in the same format used in Table II, of the analysis on each network whose locations are shown in Figure 6.

**Single-Line.** The single line, which begins at Libby and transports 403 tons of chips per day to Thompson Falls, has the optimum operating conditions of 30 percent concentration and a 6-inch pipe. The cost is
<table>
<thead>
<tr>
<th>Line</th>
<th>Length</th>
<th>Elevation Difference, $Z_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>miles</td>
<td>ft</td>
</tr>
<tr>
<td>DCBA</td>
<td>90.6</td>
<td>+365</td>
</tr>
<tr>
<td>BA</td>
<td>40.4</td>
<td>+260</td>
</tr>
<tr>
<td>DCB</td>
<td>50.2</td>
<td>+105</td>
</tr>
<tr>
<td>EB</td>
<td>44.4</td>
<td>+144</td>
</tr>
<tr>
<td>CB</td>
<td>34.5</td>
<td>+200</td>
</tr>
<tr>
<td>FC</td>
<td>35.5</td>
<td>+100</td>
</tr>
<tr>
<td>DC</td>
<td>15.7</td>
<td>-95</td>
</tr>
</tbody>
</table>
TABLE VIII
VALUES OF VARIABLES TO BE USED IN THE CONCEPTUAL APPLICATION

<table>
<thead>
<tr>
<th>Symbol of the Variable</th>
<th>Value Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>30. percent</td>
</tr>
<tr>
<td>crf</td>
<td>0.20</td>
</tr>
<tr>
<td>e</td>
<td>65. percent</td>
</tr>
<tr>
<td>f</td>
<td>0.018</td>
</tr>
<tr>
<td>g</td>
<td>32.2 ft/sec²</td>
</tr>
<tr>
<td>Hsa</td>
<td>1,200 ft</td>
</tr>
<tr>
<td>M</td>
<td>1.80</td>
</tr>
<tr>
<td>MTPU</td>
<td>1,500 TPD</td>
</tr>
<tr>
<td>R₁</td>
<td>$0.007/kwh</td>
</tr>
<tr>
<td>R₂</td>
<td>$2,585/in-mile</td>
</tr>
<tr>
<td>R₃</td>
<td>$90/installed HP</td>
</tr>
<tr>
<td>R₄</td>
<td>to be computed</td>
</tr>
<tr>
<td>R₅</td>
<td>to be computed</td>
</tr>
<tr>
<td>R₆</td>
<td>$45,000/year</td>
</tr>
<tr>
<td>R₇</td>
<td>$7,000/(pump station)</td>
</tr>
<tr>
<td>R₈</td>
<td>$100/mile-year</td>
</tr>
<tr>
<td>R₉</td>
<td>$80/million-gal</td>
</tr>
<tr>
<td>S₀dc</td>
<td>0.40</td>
</tr>
<tr>
<td>U₁</td>
<td>$7,500/MTPU</td>
</tr>
<tr>
<td>U₂</td>
<td>$15,000/MTPU</td>
</tr>
</tbody>
</table>
### TABLE IX

**SINGLE-LINE PIPELINE SYSTEM**

<table>
<thead>
<tr>
<th>LINE</th>
<th>L (MILE)</th>
<th>W (TPD)</th>
<th>ZT (FT)</th>
<th>D (IN.)</th>
<th>C (PERCENT)</th>
<th>V (FPS)</th>
<th>QW (GPM)</th>
<th>HF (FT/MI)</th>
<th>HPPM (HP/MI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCBA</td>
<td>90.6</td>
<td>403.1</td>
<td>365.7</td>
<td>6.0</td>
<td>0.3000</td>
<td>6.34</td>
<td>391.0</td>
<td>293.0</td>
<td>43.4</td>
</tr>
</tbody>
</table>

**FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS**

<table>
<thead>
<tr>
<th>LINE</th>
<th>PIPE INSTLN</th>
<th>PUMP STATIONS</th>
<th>INJ-SEP SYSTEM</th>
<th>FST COST</th>
<th>OPER.EXP</th>
<th>TOTAL COSTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCBA</td>
<td>1405206.0</td>
<td>354344.0</td>
<td>22499.0</td>
<td>1782050.0</td>
<td>539024.0</td>
<td>0.07004</td>
</tr>
</tbody>
</table>
### Table X

**Three-Line Pipeline Network**

<table>
<thead>
<tr>
<th>Line</th>
<th>L (Mile)</th>
<th>W (TPD)</th>
<th>ZT (FT)</th>
<th>D (In.)</th>
<th>C (Percent)</th>
<th>V (FPS)</th>
<th>QW (GPM)</th>
<th>HF (FT/Mi)</th>
<th>HPPM (HP/Mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB</td>
<td>50.2</td>
<td>403.0</td>
<td>105.0</td>
<td>6.0</td>
<td>3000</td>
<td>6.34</td>
<td>391.0</td>
<td>293.0</td>
<td>43.1</td>
</tr>
<tr>
<td>EB</td>
<td>44.4</td>
<td>734.0</td>
<td>144.0</td>
<td>8.0</td>
<td>3000</td>
<td>6.50</td>
<td>712.0</td>
<td>286.1</td>
<td>77.1</td>
</tr>
<tr>
<td>BA</td>
<td>40.4</td>
<td>1137.0</td>
<td>260.0</td>
<td>10.0</td>
<td>3000</td>
<td>6.44</td>
<td>1104.0</td>
<td>286.6</td>
<td>120.0</td>
</tr>
</tbody>
</table>

---

**First Costs and Annual Operating Expense for Optimum Conditions**

<table>
<thead>
<tr>
<th>Line</th>
<th>Instln Stations</th>
<th>System</th>
<th>Total Fst Cost</th>
<th>Oper. Exp Cost</th>
<th>Total</th>
<th>$/Ton-Mi</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB</td>
<td>778602.0</td>
<td>195056.0</td>
<td>7499.0</td>
<td>981158.0</td>
<td>307910.0</td>
<td>0.07110</td>
</tr>
<tr>
<td>EB</td>
<td>918192.0</td>
<td>308104.0</td>
<td>7499.0</td>
<td>1233796.0</td>
<td>381858.0</td>
<td>0.05562</td>
</tr>
<tr>
<td>BA</td>
<td>1044340.0</td>
<td>439744.0</td>
<td>14999.0</td>
<td>1499084.0</td>
<td>523812.0</td>
<td>0.05193</td>
</tr>
</tbody>
</table>

---

**Total Cost for the Pipeline Transportation System**

<table>
<thead>
<tr>
<th>Instln Stations</th>
<th>System</th>
<th>Total Fst Cost</th>
<th>Oper. Exp Cost</th>
<th>Total</th>
<th>$/Ton-Mi</th>
</tr>
</thead>
<tbody>
<tr>
<td>2741134.0</td>
<td>942905.0</td>
<td>29999.0</td>
<td>3714039.0</td>
<td>1213581.0</td>
<td>0.05707</td>
</tr>
</tbody>
</table>
TABLE XI

FIVE-LINE PIPELINE NETWORK

<table>
<thead>
<tr>
<th>LINE</th>
<th>L (MILE)</th>
<th>W (TPD)</th>
<th>ZT (FT)</th>
<th>D (IN.)</th>
<th>C (% Vol.)</th>
<th>V (FPS)</th>
<th>QW (GPM)</th>
<th>HF (FT/MI)</th>
<th>HPPM (HP/MILE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>15.7</td>
<td>403</td>
<td>-95</td>
<td>6.0</td>
<td>.3000</td>
<td>6.34</td>
<td>391</td>
<td>293.0</td>
<td>41.9</td>
</tr>
<tr>
<td>FC</td>
<td>35.5</td>
<td>717</td>
<td>100</td>
<td>8.0</td>
<td>.3000</td>
<td>6.35</td>
<td>696</td>
<td>285.7</td>
<td>75.0</td>
</tr>
<tr>
<td>EB</td>
<td>44.4</td>
<td>734</td>
<td>144</td>
<td>8.0</td>
<td>.3000</td>
<td>6.50</td>
<td>712</td>
<td>286.1</td>
<td>77.1</td>
</tr>
<tr>
<td>CB</td>
<td>34.5</td>
<td>1120</td>
<td>200</td>
<td>10.0</td>
<td>.3000</td>
<td>6.34</td>
<td>1087</td>
<td>287.1</td>
<td>119.0</td>
</tr>
<tr>
<td>BA</td>
<td>40.4</td>
<td>1854</td>
<td>260</td>
<td>12.0</td>
<td>.3000</td>
<td>7.29</td>
<td>1800</td>
<td>285.8</td>
<td>196.6</td>
</tr>
</tbody>
</table>

FIRST COSTS AND ANNUAL OPERATING EXPENSE FOR OPTIMUM CONDITIONS

<table>
<thead>
<tr>
<th>LINE</th>
<th>INSTLN</th>
<th>STATIONS</th>
<th>PUMP</th>
<th>INJ-SEP</th>
<th>TOTAL</th>
<th>ANNUAL</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>243507</td>
<td>59320</td>
<td>7499</td>
<td>310327</td>
<td>124999</td>
<td>.08375</td>
<td></td>
</tr>
<tr>
<td>FC</td>
<td>734140</td>
<td>239903</td>
<td>7499</td>
<td>981543</td>
<td>309070</td>
<td>.05716</td>
<td></td>
</tr>
<tr>
<td>EB</td>
<td>918192</td>
<td>308104</td>
<td>7499</td>
<td>1233796</td>
<td>381858</td>
<td>.05562</td>
<td></td>
</tr>
<tr>
<td>CB</td>
<td>891825</td>
<td>369770</td>
<td>0</td>
<td>1261595</td>
<td>409663</td>
<td>.04974</td>
<td></td>
</tr>
<tr>
<td>BA</td>
<td>1253208</td>
<td>715081</td>
<td>1854</td>
<td>1986829</td>
<td>768425</td>
<td>.04544</td>
<td></td>
</tr>
</tbody>
</table>

TOTAL COST FOR THE PIPELINE TRANSPORTATION SYSTEM

<table>
<thead>
<tr>
<th>INSTLN</th>
<th>STATIONS</th>
<th>PUMP</th>
<th>INJ-SEP</th>
<th>TOTAL</th>
<th>ANNUAL</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>4040872</td>
<td>1692181</td>
<td>41039</td>
<td>5774093</td>
<td>1994017</td>
<td>.05128</td>
<td></td>
</tr>
</tbody>
</table>
the highest of the three networks at $0.070 per ton-mile.

**Three-Line Network.** The three-line network has injection units at Libby and Sandpoint and delivers 1,137 tons of chips per day to Thompson Falls. The optimum concentration is 30 percent, and the best pipe sizes are 6-, 8-, and 10-inch. The cost is the median value of $0.057 per ton-mile.

**Five-Line Network.** The five-line network has three injection stations: Libby, Sandpoint, and Bonners Ferry. This network delivers 1,854 tons of chips per day to Thompson Falls at a concentration of 30 percent in pipe sizes of 6-, 8-, 10-, and 12-inch. The cost is the lowest of the three networks at $0.051 per ton-mile.

The lowest costs given in Tables IX, X, and XI for each of the three networks will be compared with costs for rail and truck in the next chapter.
CHAPTER VI
COMPARISON OF TRANSPORTATION COSTS

This chapter compares the costs of transporting woodchips in the single-, three-, and five-line network applications of the preceding chapter with transportation costs of rail and truck.

Freight rates for rail shipment of woodchips in western Montana can be obtained from either the railroad companies engaged in chip transport (20) or the Public Service Commission. The rates are posted in units of dollars per 200 cubic feet of chips but can be converted to dollars per ton-mile by using the conversion that 200 cubic feet of oven-dry chips weigh 2,400 pounds and dividing by the distance for the posted rates in miles. The rates for a variety of hauls ranged from $0.023 to $0.031. The latter rate will be used since it is the highest of a small sample and represents a conservative value.

Truck-haul costs are less easily determined. Because of the type of vehicle and the distances the freight is being moved, chip haulers are not required to register their rates with the Public Service Commission. Truck rates used in this comparison were taken from a 1961 Highway Research Board publication (21). These rates were adjusted to present rates, based on the rise in freight rates for the period 1961-1965 (22), (23), by increasing them 19.5 percent. With the adjustment the rate of $0.053 is used. The rate represents an operation in which the vehicle, a diesel engine tractor-trailer combination of 44,000 pounds gross weight, is loaded in one direction and returns empty.
Table XII gives the costs for each mode of transportation for each of the three networks. The line and lengths describe the routes along which each different mode of transportation must travel. The cost in dollars per ton-mile is given for each line and its throughput. The cost is converted to dollars per day for each line and then summarized for each method of transportation. The final column, listing total cost for each mode, gives a comparison of cost on a daily basis.

All modes of transportation in the study area have approximately the same route and distance to travel to deliver chips to Thompson Falls. Therefore, the mode which has the lowest cost in dollars per ton-mile will also give the lowest cost in dollars per day. In each of the three networks analyzed, rail transportation is the most economical. Although pipeline transportation costs are close to truck costs in all three networks, the pipeline shows an economic advantage only in the five-line system.

Transportation cost studies in areas where the methods of transportation do not parallel each other may produce results which show the pipeline to be most economical. Such an area could occur when the pipeline takes a route "cross country" and thereby reduces the distance chips must be moved. Pipelines with "cross country" routes may have a high rate on a dollars per ton-mile basis; but on a daily basis, reflecting the shortened route, the costs may be low and competitive with other modes of transportation. The distance the chips must be moved is an important consideration when comparing costs of several transportation systems.
### Table XII: Comparison of Costs

<table>
<thead>
<tr>
<th>Network</th>
<th>Mode of Transportation</th>
<th>Line</th>
<th>L (Miles)</th>
<th>W (Tons/day)</th>
<th>X ($/ton-mile)</th>
<th>(L)(W)(X) $/day</th>
<th>Total Cost $/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>Rail</td>
<td>DCFEBA</td>
<td>171.0</td>
<td>403</td>
<td>0.031</td>
<td>2,136</td>
<td>2,136</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>DCBA</td>
<td>90.6</td>
<td>403</td>
<td>0.051</td>
<td>1,935</td>
<td>1,935</td>
</tr>
<tr>
<td></td>
<td>Pipeline</td>
<td>DCBA</td>
<td>90.6</td>
<td>403</td>
<td>0.070</td>
<td>2,556</td>
<td>2,556</td>
</tr>
<tr>
<td>3-line</td>
<td>Rail</td>
<td>DCFEBA</td>
<td>86.2</td>
<td>403</td>
<td>0.031</td>
<td>1,077</td>
<td>2,989</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>DCBA</td>
<td>90.6</td>
<td>1,137</td>
<td>0.053</td>
<td>1,935</td>
<td>5,234</td>
</tr>
<tr>
<td></td>
<td>Pipeline</td>
<td>DCB</td>
<td>50.2</td>
<td>403</td>
<td>0.071</td>
<td>1,436</td>
<td>5,650</td>
</tr>
<tr>
<td></td>
<td></td>
<td>EB</td>
<td>44.4</td>
<td>734</td>
<td>0.056</td>
<td>1,825</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BA</td>
<td>40.4</td>
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Hunt showed that areas where no existing transportation facilities occurred the pipeline is very competitive with other systems. When the high installation and maintenance costs are considered for other transportation systems the economic advantages of pipeline transport of wood-chips are quite attractive.
Conclusions

The following statements can be concluded from the technique presented for optimizing the economics and operating conditions for the transportation of woodchips in pipelines:

1. The response surface generated by using the objective function offers a positive, visual analysis of the economic characteristics for the pipeline system.

2. The optimization technique presents a rational method using numerical procedures for determining the optimum pipe sizes and operating woodchip concentrations which can be extended to pipeline networks characterized by series of branching lines that do not loop back.

3. Any nonclosed-loop pipeline system can be described by properly defining the objective function for each network.

4. The seven cost groups and the objective functions are basic in describing the various costs and can be easily altered when new cost rates and head loss equations are developed.

The following conclusions are obtained from the economic analysis of the conceptual application of woodchip pipelines in northwestern Montana:

1. The proposed pipeline networks in northwestern Montana are not competitive with railroads.
2. The five-line network for transporting woodchips is competitive with trucking.

3. In general, woodchip pipelines cannot compete economically with rail when the routes are of the same distance.

4. Woodchip pipelines may be more economical than highway trucking for high throughputs and long distances.

**Recommendations**

The following recommendations are given for further refinement of the economic model:

1. Since the head loss equation is very significant in the development of the economic model, any improvements or developments of the head loss relationships should be incorporated in the model.

2. Tests should be conducted to define \( C_{\text{max}} \) more closely for various size pipes.

3. The variables used in the mathematical expressions for costs should constantly be updated to reflect current prices.
APPENDICES
APPENDIX A

LIST AND DEFINITION OF VARIABLES

ANOP  =  Annual operating expense, $/year.

C    =  Concentration of chips in mixture by volume.

C_{max}  =  Maximum concentration of chips permissible during operation.

c_{rf}  =  Charge on capital investment to cover interest, depreciation, etc., capital recovery factor.

D    =  Pipe diameter, inches.

D_t  =  Theoretical pipe size at which the lowest cost occurs in the feasible region of the response surface.

d    =  Pipe diameter, feet.

e  =  Combined efficiency of motor-pump drivers.

f    =  Friction factor for Weisbach equation.

g    =  Gravitational constant, ft/sec^2.

H_e    =  Head due to difference in elevation, ft/mile.

H_f    =  Head loss due to frictional resistance, ft/mile.

H_{sa}  =  Total head developed per pump station, ft.

H_t    =  Head due to friction and difference in elevation, ft/mile.

HFPM  =  Power requirements, horsepower per mile of pipeline.

h_m    =  Frictional head loss of the mixture, ft/ft of pipe.

h_w    =  Frictional head loss of clear water, ft/ft of pipe.

I_{pl}    =  Initial investment in the pipeline, $.

I_{tps}  =  Initial pump station investment, $.

I_{is}  =  Initial injection-separation equipment investment, $.

K    =  Constants used in the development of the objective functions.

L    =  Length of line, miles.
l = Length of line, feet.
M = Moisture content of chips, decimal fraction of oven-dry chips.
MTPU = Minimum throughput unit for injection and separation units.
Q_m = Volumetric discharge of mixture in the pipeline, cfs.
Q_w = Volumetric discharge of make-up water, cfs.
R_1 = Cost of electrical energy, $/kwh.
R_2 = Installed cost of pipeline, including right-of-way, $/(in-mile).
R_3 = Cost of pump station and controls, $/(installed hp).
R_4 = Cost of chip injection system, $/(ton per day oven-dry chips).
R_5 = Cost of separation system, $/(ton per day oven-dry chips).
R_6 = Annual cost of fixed wages, salaries, operation maintenance, exclusive of pipeline maintenance and pump station operation.
R_7 = Annual wages, salaries, etc. for pump station, $/(pump station).
R_8 = Annual maintenance cost of pipeline, $/mile.
R_9 = Cost of water and treatment, $/million gallons.
S_m = Specific gravity of water-chip mixture.
S_odc = Specific gravity of oven-dry chips.
U_1 = First cost of injection device $/unit of MTPU.
U_2 = First cost of separation system $/unit of MTPU.
V = Average velocity of flow, fps.
V_m = Average velocity of the mixture, fps.
W = Tons per day of oven-dry chips, TPD.
X = Total cost of a pipeline, $/ton-mile.
X_t = Objective function giving total cost of pipeline network, $/ton-mile.
\[ x \] = One of seven cost groups appearing as a term in \( X \), $/ton-mile.

\[ Z_t \] = Difference in elevation between inlet and discharge ends of pipe, ft.

\[ \lambda \] = Square root of pipe flow Froude Number.
APPENDIX B

DEVELOPMENT OF ECONOMIC COST GROUPS $x_1, x_2, \ldots, x_7$

Parts of this Appendix are taken from Hunt (9).

The detailed development of each cost group ($x_1, x_2, \ldots, x_7$) and other relationships among the variables describing the hydraulics and economics of a proposed woodchip pipeline are included in this section.

Volumetric discharge of mixture in the pipeline, $Q_m$, is the number of cubic feet per second of the woodchip-water mixture flowing through a pipeline and is given by the equation

$$Q_m = 0.00545 D^2 V_m$$  \hspace{1cm} (B-1)

where $D$ is the pipe diameter in inches and $V_m$ is the average velocity of the flowing mixture.

Throughput rate of woodchips on an oven-dry basis, $W$, is the number of ton per day of oven-dry chips carried by a pipeline and is given by the equation

$$W = \frac{62.4 \times 10^4}{2000} C Q_m S_{odc}$$  \hspace{1cm} (B-2)

where $C$ is the concentration of wood chips by volume in the mixture and $S_{odc}$ is the specific gravity of the oven-dry chips.

The specific gravity of the mixture, $S_m$, is given by the equation

$$S_m = C \left[ (M + 1) S_{odc} - 1 \right] + 1$$  \hspace{1cm} (B-3)

where $M$ is the moisture content of the chips on an oven-dry basis.

Frictional head loss per mile of line, $H_f$, in feet per mile is
the energy dissipated as heat by the fluid drag on the pipe walls and is expressed in foot-pounds per pound of fluid flowing. For water this is given by the Weisbach equation (given in any standard hydraulics or fluid mechanics text).

\[ h_w = f \frac{1}{d} \frac{V^2}{2g} \]  

(B-4)

where \( f \) is the friction factor, \( l \) and \( d \) are the pipeline length and diameter, respectively, in feet, and \( V \) is the average flow velocity.

The relation between the head loss, \( h_m \), for fluid transporting solids at a concentration \( C \) and that for water, \( h_w \), was found by Faddick (12) to be

\[ \frac{h_m}{h_w} = 18 \left( \frac{V^2}{C h_w g d} \right)^{-1.42} \]  

(B-5)

where \( g \) is the gravitational constant in feet per second squared. Solving equation B-1 and B-2 for \( V_m \) in terms of \( W, C, S_{odc} \) and using the diameter in inches (i.e., \( D = 12d \)), a parameter, \( \lambda \), is defined as

\[ \lambda = \frac{V^2}{gD} = \left( \frac{1}{216 g D^5} \right) \left( \frac{W}{C S_{odc}} \right)^2 \]  

(B-6)

Equation (B-5) may now be transposed and rewritten as

\[ h_m = (0.547 C' \lambda^{-1.42} + 1) h_w \]  

(B-7)
With the diameter in inches Equation B-4 becomes
\[ h_W = \frac{V^2}{2gD} = 6f\lambda \]  
 \hspace{1cm} (B-8)

With the length of pipeline in miles (i.e. \( l = 5280l \)), the head loss per mile for the mixture, \( H_f = h_m/L \) becomes
\[ H_f = 31,680 \left( 0.547 \lambda^{-1.42} + 1 \right) f\lambda \]  
 \hspace{1cm} (B-9)

Elevation head per mile of line, \( H_e \), in feet per mile, is the difference in elevation between the input and output installations of the pipeline system averaged over the length of the line and is given by the equation
\[ H_e = \frac{Z_f}{L} \]  
 \hspace{1cm} (B-10)

where \( Z_f \) is the total difference in elevation between the two ends of the pipeline. If the output end is higher than the input, this is a positive quantity; if lower, a negative quantity.

The total head per mile supplied by the pumps, \( H_t \), in feet per mile, is the sum of the frictional head loss and the elevation head and is given by the equation
\[ H_t = H_e + H_f = 31,680 \left( 0.547 \lambda^{-1.42} + 1 \right) f\lambda + \frac{Z_f}{L} \]  
 \hspace{1cm} (B-11)

The algebraic expressions for each of the seven cost groups \( (x_1, x_2, \ldots, x_7) \) are developed in the following paragraphs using the above-listed equation and relationships.

The energy cost per ton-mile for pumping, \( x_1 \), is based on the input
horsepower to the pumping stations required for each mile of pipeline. The latter is given by the equation

\[ HP = \frac{62.4 \cdot Q \cdot S \cdot H}{m \cdot m \cdot t} \quad (B-12) \]

where \( e \) is the combined motor-pump efficiency. This is reduced to cost per ton-mile by the following set of conversion factors:

\[ \$/\text{ton-mi} = \frac{HP}{m} \cdot \left( \frac{0.746 \text{ kw}}{\text{hp}} \right) \cdot \left( R \frac{\$}{\text{kwh}} \right) \cdot \left( \frac{24 \text{ hr}}{\text{day}} \right) \cdot \left( \frac{W \text{ tons}}{\text{day}} \right) \quad (B-13) \]

where \( R \) is the rate at which electrical energy may be obtained. Using this conversion and Equation B-2 and B-12, the cost per ton-mile of electrical energy becomes

\[ X_1 = 0.000753 \left( \frac{R}{e} \right) \cdot \frac{S \cdot H}{m \cdot t} \quad (B-14) \]

The cost per ton-mile for pipeline materials and installation, \( x_2 \), is based on the initial investment in the pipeline, including appropriate hardware. The latter is expressed by the equation

\[ I_{pl} = R_2 \cdot D \cdot L \quad (B-15) \]

where \( R_2 \) is cost of the installation in dollars per inch of diameter for each mile of pipeline. Representative values of \( R_2 \) were obtained from Continental Pipe Line Company, Billings, Montana (4).

If the annual return on the investment, amortization costs, taxes and insurance are combined together into one capital recovery factor, crf, the cost per ton-mile for capital investment in the pipeline itself
becomes

\[ x_2 = \left( \frac{R_D}{365 W} \right) (\text{crf}) \]  

(B-16)

The cost per ton-mile for pump stations, \( x_3 \), includes the initial investment in pumps, controls, and station construction costs. Cost data obtained from Continental Pipe Line Company indicate this investment to be a function of the installed horsepower at each station.

The pump station investment is

\[ I_{\text{ps}} = \frac{62.4 Q S H}{550 e} \]  

(B-17)

where \( R_3 \) is the cost of the pump station and controls in dollars per installed horsepower. Using Equation B-2 and the previously-defined capital recovery factor, the cost per ton-mile for pump stations becomes

\[ x_3 = 0.000000115 \left( \frac{R_3}{S H} \right) (\text{crf}) \]  

(B-18)

The cost per ton-mile for chip injection and separation systems, \( x_4 \), is based on the cost of fabricating a minimum capacity unit for each of these minimum capacity units, the cost of installation per ton per day for these is

\[ R_4 = \frac{U_1}{\text{MTPU}} \quad \text{for} \quad \frac{W}{\text{MTPU}} > 1 \]  

(B-19a)

\[ R_4 = \frac{U_1}{W} \quad \text{for} \quad \frac{W}{\text{MTPU}} < 1 \]  

(B-19b)
where \( R_4 \) and \( R_5 \) are the costs per ton per day of capacity of the injection and separation units respectively, \( U_1 \) and \( U_2 \) are the first costs of installing injection and separation systems, respectively, of a minimum throughput capacity, \( MTPU \).

The initial investment for the injection and separation equipment is given by

\[
I_{is} = (R_4 + R_5) W \quad (B-21)
\]

The cost per ton-mile for this phase of the pipeline system becomes

\[
x_4 = (R_4 + R_5) \frac{crf}{365 L} \quad (B-22)
\]

The cost per ton-mile for fixed wages, salaries, and operational expenses, \( x_5 \), includes those items of this nature which are necessary regardless of the length or size of the pipeline installed and is given by the equation

\[
x_5 = R_6 / (365 W L) \quad (B-23)
\]

where \( R_6 \) is the total annual cost of these fixed charges.

The cost per ton-mile for variable wages, salaries, and operational expenses, \( x_6 \), includes those items of this nature which vary with the length of the line and number of pumping stations required. If \( H_{sa} \) is
the total head developed per pump station in feet and the annual cost of maintaining the pipeline is \( R_8 \) dollars per mile, the total cost per ton-mile of the variable salaries and wages becomes

\[
x_6 = \frac{1}{365 \times 24} \left( R_7 \frac{H_t}{H_{sa}} + R_8 \right)
\]  

(B-24)

The cost per ton-mile for treatment of the water, \( x_7 \), includes purchase of water, if so required, in addition to any treatment which may be necessary to the influent and effluent to the pipeline. The cost of this depends upon the make-up water required for the mixture which is determined from the equation

\[
Q_w = (1 - C) Q_m
\]  

(B-25)

If the cost of the water and treatment is \( R_9 \) dollars per million gallons, the cost per day is

\[
C_{wt} = 24(3600) 7.48 \times 10^{-6} R_9 (1 - C) Q_m
\]  

(B-26)

The cost per ton-mile using Equation B-2 then is

\[
x_7 = 0.000240 \left( \frac{1 - C}{C} \right) \frac{R_9}{S_{odc} L}
\]  

(B-27)

\( R_9 \) is taken as zero except for the lines which terminate at a processing plant and have effluent.
APPENDIX C

DEVELOPMENT OF THE OBJECTIVE FUNCTION, $X_t$

The objective function for the single-line, $X_t$, is the sum of the seven cost groups given on page 8. These cost groups can be simplified for algebraic manipulation by redefining the constant coefficients:

- $x_1 = a_1 \frac{S \cdot H \cdot t}{c}$
- $x_2 = a_2 D$
- $x_3 = a_3 \frac{S \cdot H \cdot t}{c}$
- $x_4 = a_4$
- $x_5 = a_5$
- $x_6 = a_6 H \cdot t + a_6 A$
- $x_7 = a_7 \left( \frac{1}{c} - 1 \right)$

where

- $a_1 = \frac{0.000753 \cdot R_1}{e \cdot S_{odc}}$
- $a_2 = \frac{R_2 \cdot crf}{365 \cdot W}$
- $a_3 = \frac{0.000000115 \cdot R_2 \cdot crf}{e \cdot S_{odc}}$
- $a_4 = \frac{(R_4 + R_5) \cdot crf}{365 \cdot L}$
- $a_5 = \frac{R_6}{365 \cdot W \cdot L}$
- $a_6 = \frac{R_7}{365 \cdot W \cdot H_{sa}}$
- $a_7 = \frac{0.00024 \cdot R_9}{S_{odc} \cdot L}$
From Appendix B, Equation B-11

\[ H_t = 31,680 \, f \, (0.527 \, c \, \lambda^{-0.42} + \lambda) + Z_t / L \]

where

\[ \lambda = \frac{w^2}{216 \, g \, S_{odc} \, c^2 \, D^5} \]

let

\[ \lambda = \frac{b_1}{c^2 \, D^5} \]

where

\[ b_1 = \frac{w^2}{216 \, g \, S_{odc}} \]

giving

\[ H_t = 31,680 \, f \, (0.527) \, \frac{0.184}{b_1^{-0.42}} + \frac{31,680fb_1}{c^2 \, D^5} + \frac{Z_t}{L} \]

let

\[ b_2 = \frac{31,680f(0.527)}{b_1} \]

and

\[ b_3 = 31,680 \, f \, b_1 \]

\[ b_4 = \frac{Z_t}{L} \]

giving

\[ H_t = b_2 \, c^{1.84} \, D^{2.10} + b_3 \, c^{-2} \, D^{-5} + b_4 \]

From Appendix B, Equation B-3,

\[ S_m = C \left[ (M + 1) \, S_{odc} - 1 \right] + 1 \]
let
\[ b_5 = (M + 1) S_{\text{odc}} - 1 \]
giving
\[ S_m = C b_5 + 1 \]

The objective function, \( X_t \), can now be expressed in terms of these newly defined constants:
\[ X_t = a_1 \frac{S H_t}{C} + a_2 D + a_3 \frac{S H_t}{C} + a_4 + a_5 + a_6 H_t + a_6 A + a_7 \left( \frac{1}{C} - 1 \right) \]

rearranging and combining terms,
\[ X_t = \left( a_1 + a_2 \right) \frac{S H_t}{C} + a_2 D + a_6 H_t + a_7 + a_4 + a_5 + a_6 A - a_7 \]

Substituting for \( H_t \) and \( S_m \) from Equations B-11 and B-3 gives
\[ X_t = (a_1 + a_2) \left( b_5 + \frac{1}{C} \right)(b_2 C^{1.84} D^{2.10} + b_3 C^{-2} D^{-5} + b_4) + a_2 D + \]
\[ a_6 \left( b_2 C^{1.84} D^{2.10} + b_3 C^{-2} D^{-5} + b_4 \right) + a_7 + a_4 + a_5 + \]
\[ a_6 A - a_7 \]

Let
\[ d_1 = a_1 + a_2 \]

and multiply through:
\[ X_t = d_1 b_2 c^{1.84} D^{2.10} + d_1 b_3 c^{-2} D^{-5} + d_1 b_4 a_2 D + \]
\[ d_1 b_2 c^{0.84} D^{2.10} + d_1 b_3 c^{-3} D^{-5} + d_1 b_4 a_6 D^{1.84} D^{2.10} + \]
\[ a_6 b_3 c^{-2} D^{-5} + a_6 b_4 + \frac{a_7}{c} + a_4 + a_5 + a_6 a = a_7 \]

These terms can be combined in like powers of \(D\), giving:

\[ X_t = \left[ (d_1 b_2 c^{1.84} + d_1 b_3 c^{0.84}) \right] D^{2.10} + \left[ (d_1 b_2 + a_6 b_2) \right] c^{1.84} + d_1 b_3 c^{-2} D^{-5} + a_2 D + \frac{d_1 b_4 + a_7}{c} + d_1 b_4 a_6 + a_4 + a_5 + a_6 a = a_7 \]

The objective function can be put in the polynomial form \(X_t = \psi(K_i C^m D^n)\) by defining the following constants:

\[ K_1 = d_1 b_2 + a_6 b_2 \]
\[ K_2 = d_1 b_2 \]
\[ K_3 = d_1 b_3 + a_6 b_3 \]
\[ K_4 = d_1 b_3 \]
\[ K_5 = a_2 \]
\[ K_6 = d_1 b_4 + a_7 \]
\[ K_7 = d_1 b_5 b_4 + a_6 b_4 + a_4 + a_5 + a_6 a = a_7 \]
The objective function may now be written

\[ X_t = (K_1 C_{1.84} + K_2 C_{0.84}) D^{2.10} + (K_3 C^{-2} + K_4 C^{-3}) D^{-5} + K_5 D + \]
\[ \frac{K_6}{C} + K_7 \]  

(C-1)

The first and second derivative of \( X_t \) with respect to \( C \) are:

\[ \frac{\partial X_t}{\partial C} = (1.84 K_1 C^{0.84} + 0.84 K_2 C^{-0.16}) D^{2.10} - (2K_3 C^{-3} + 3K_4 C^{-4}) D^{-5} - K_6 C^{-2} \]  

(C-2)

\[ \frac{\partial^2 X_t}{\partial C^2} = (1.54 K_1 C^{-0.16} + 0.134 K_2 C^{-1.16}) D^{2.10} + (6K_3 C^{-4} + 12K_4 C^{-5}) D^{-5} + 2K_6 C^{-3} \]  

(C-3)

The first and second derivatives of \( X_t \) with respect to \( D \) are:

\[ \frac{\partial X_t}{\partial D} = 2.10 (K_1 C^{1.84} + K_2 C^{0.84}) D^{1.10} - 5(K_3 C^{-2} + K_4 C^{-3}) D^{-6} + K_5 \]  

(C-4)

\[ \frac{\partial^2 X_t}{\partial D^2} = 2.31(K_1 C^{1.84} + K_2 C^{0.84}) D^{0.10} + 30(K_3 C^{-2} + K_4 C^{-3}) D^{-7} \]  

(C-5)
APPENDIX D
DEVELOPMENT OF THE CONTINUITY RELATIONSHIPS

The continuity equations for the mass flow rates of the woodchips and the woodchip-water mixture must be satisfied at the junctions in pipeline networks shown in Figures 4 and 5. The continuity relationships developed to express these conditions are used to determine the concentration of woodchips in the mixture flowing out of the junction in terms of the concentrations of the lines flowing into the junction.

Three-Line Network

The volumetric flow of woodchip-water mixture entering a junction as shown in Figure 4 must equal that leaving. This statement can be expressed mathematically by

\[ Q_m^3 = Q_m^1 + Q_m^2 \]  \hspace{1cm} (D-1)

where \( Q_m^i \) is the volumetric flow rate in cubic feet per second for line \( i \). Similarly, the weight rate of flow of chips entering must equal that leaving:

\[ W_3 = W_1 + W_2 \]  \hspace{1cm} (D-2)

where \( W_i \) is the weight rate of flow of chips in tons per day. Equation B-2, giving the throughput in terms of concentration, flow rate of the mixture, and the specific weight of oven-dry chips, can be solved for \( Q_m^i \) and substituted in Equation D-1 giving

\[ \frac{W_1}{2695.7 C_1 S_{ocd}} = \frac{W_2}{2695.7 C_2 S_{ocd}} + \frac{W_3}{2695.7 C_3 S_{ocd}} \]
multiplying by 2695.7 $S_{oc}$:

$$\frac{W_3}{C_3} = \frac{W_1}{C_1} + \frac{W_2}{C_2}$$

which can be solved for $C_3$ giving

$$C_3 = \frac{C_1 C_2 W_3}{C_1 W_2 + C_2 W_1}$$

expressing the concentration of woodchips in line 3 as a function of the concentration in the other two lines and the throughputs.

The first and second partial derivatives of $C_2$ with respect to $C_1$ are found to be:

$$\frac{\partial C_3}{\partial C_1} = \frac{C_2^2 W_1 L}{(C_1 W_2 + C_2 W_1)^2}$$

$$\frac{\partial^2 C_3}{\partial C_1^2} = \frac{-2C_2^2 W_1 W_2 W_3}{(C_1 W_2 + C_2 W_1)^3}$$

The first and second partial derivatives of $C_3$ with respect to $C_2$ are similar to Equation D-4 and D-5 with the subscript 2 and 1 interchanged. The first derivatives are used in Equation 13 and 14 for the derivatives of the objective function for the three-line network. The second derivatives are used in the Newton-Raphson method for solving $\frac{\partial X_n}{\partial C_n} = 0$. 
Five-Line Network

Two junctions exist in the typical five-line network shown in Figure 5. Concentration in line 4 can be expressed as a function of $C_1$ and $C_2$; likewise, $C_5$ can be expressed as a function of $C_3$ and $C_4$ where $C_4$ is a function of $C_1$ and $C_2$. The three-line continuity relationships are used in developing the five-line expressions with the subscripts changed to conform to the notation in Figure 5 which was chosen to simplify the Fortran II programming.

The equation for $C_4$ takes the same form as Equation D-3:

$$
C_4 = \frac{C_1 C_2 W_4}{C_1 W_2 + C_2 W_1}
$$

(D-6)

similarly, for $C_5$

$$
C_5 = \frac{C_3 C_4 W_5}{C_3 W_4 + C_4 W_3}
$$

(D-7)

By substituting expressions for $C_4$ and $W_4$ and manipulating the equation, Equation D-7 can be rewritten:

$$
C_5 = \frac{C_1 C_2 C_3 W_5}{C_1 C_2 W_3 + C_2 C_3 W_1 + C_1 C_3 W_2}
$$

(D-8)

The partial derivatives $\frac{\partial C_4}{\partial C_1}$, $\frac{\partial C_4}{\partial C_2}$, $\frac{\partial^2 C_4}{\partial C_1^2}$, and $\frac{\partial^2 C_4}{\partial C_2^2}$ take the same form as Equations D-4 and D-5 with subscripts 3 and 4 interchanged.
The partial derivatives of \( \frac{C_5}{G_3} \) with respect to \( G_3 \) are expressed by:

\[
\frac{\partial C_5}{\partial G_3} = \frac{C_4^2 W_3 W_5}{(C_3 W_4 + C_4 W_3)^2}
\]  

(D-9)

and

\[
\frac{\partial^2 C_5}{\partial G_3^2} = \frac{2C_4^2 W_3 W_4 W_5}{(C_3 W_4 + C_4 W_3)^2}
\]  

(D-10)

which are of the same form as Equation D-4 and D-5 with the subscripts changed to apply to Figure 5.

The first and second derivative of \( C_5 \) with respect to \( C_1 \) are found to be

\[
\frac{\partial C_5}{\partial C_1} = \frac{C_2^2 C_3 W_1 W_5}{(C_1 W_3 + C_2 W_1 + C_3 W_2)^2}
\]  

(D-11)

and

\[
\frac{\partial^2 C_5}{\partial C_1^2} = \frac{-2C_2^2 C_3 W_1 W_5 (C_2 W_3 + C_3 W_2)}{(C_1 W_3 + C_2 W_1 + C_3 W_2)^3}
\]  

(D-12)

or simply

\[
\frac{\partial C_5}{\partial C_1} = \frac{3C_5}{3C_4} \frac{3C_4}{3C_1}
\]  

(D-13)
The partial derivative expressions on the right side of Equations D-13 and D-14 are of the forms given by Equations D-4 and D-5 with appropriate changes in the subscript notation.

The first and second partial derivatives of \( C_5 \) with respect to \( C_2 \) are of the same form as Equations D-11, D-12, D-13, and D-14 with the subscripts 1 and 2 interchanged.
APPENDIX E

INTERMEDIATE RESULTS FOR THE SAMPLE PROBLEM

The computer program listed in Appendix F produces intermediate data during the optimization procedure which gives the concentration and cost for each combination of pipe diameters being tried as given by Equation 21.

The following listing gives the intermediate data for the sample given in Table II for eight combinations given by two pipe sizes and for 27 combinations given by three pipe sizes for each line. The two diameters which give the eight combinations are the next smaller and larger pipe sizes than $D_t$. The three pipe sizes which give 27 combinations are the next size smaller and the next two sizes larger than $D_t$. The optimum conditions for each case are found to be identical and are listed in Table II.
### Description of the Network Being Analyzed

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<th>Pipeline Length (Miles)</th>
<th>Elevation Difference (Feet)</th>
<th>CMAX Percentage</th>
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### Costs for Each Combination of Diameters for the Network

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3-LINE NETWORK, 3 PIPE SIZES TRIED FOR EACH LINE
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COSTS FOR EACH COMBINATION OF DIAMETERS FOR THE NETWORK

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APPENDIX F

COMPUTER PROGRAM FOR OPTIMIZATION

The following is the listing of the program used to determine the optimum conditions for the hypothetical application of Chapter V. The program is divided into two basic groups: the first is the general group which reads in data, computes constants, selects the two pipe sizes for each line to be tried, computes costs, and prints the results. The second group is inserted into the center of the first group and after the two diameters for each line are selected, computes the optimum concentration for each combination of pipe sizes by using iteration and the Newton-Raphson method for extracting roots. The second group, which is inserted into the general group, can be modified to describe the system being analyzed. It is easy to make modifications to the second group to describe any system of pipelines (multiline networks, series, and networks with certain lines of given diameter). The inserts used for the analysis of the single-line, three-line network and five-line network are also listed.
C PROGRAM FOR OPTIMIZING PIPELINE NETWORKS
C FORTRAN II, IBM 1620
C BEGIN GENERAL GROUP

DIMENSION DC3DC(9), D2C3DC(9), DXIDC(10), D2XIDC(10)
1, D2C5DC(10), DC4DC(10), D2C4DC(10), P(10), DP(10)
2, Y(20), DOP(20), A2(9), X(10), R9(10), ZT(10), CTINV(10)
3, Q1(10), Q2(10), Q3(10), Q4(10), Q5(10), Q6(10), A6(10), A7(10)
DIMENSION DIAM(20), W(20), PL(20), C(9), CI(9)
1, COP(20), DXDC(20), D2XDC(20), DXDD(20), DIA(10, 10)
2, BET2(9), ALPHA1(9), B1(9), B2(9), B3(9), B4(9)
3, A4(9), A5(9), B6(10), BET7(10), D(10), XT(10), DC5DC(20)

DIMENSION HFT(10), HT(10), R4(10), R5(10), U1(9), U2(10)
1, V(10), HTOP(10), HPOP(10), HPPM(10), CPI(10), CPST(10), CISS(10)
2, ANOP(10), X1(10), X2(10), X3(10), X4(10), X5(10), X6(10)
3, X1OP(10), X2OP(10), X3OP(10), X4OP(10), CI(10), XTOP(10)
4, X5OP(10), X6OP(10), X7OP(10), QN(10), X7(10)

C COMPUT CONSTANTS FOR OBJECTIVE FUNCTION

AK1 = 7535E-03 * R1 / (SODC * E)
AK2 = R2 / CRF / 365.
A1 = 0.000753 * R1 / (E * SODC)
A3 = 0.15 * R3 * CRF / (E * SODC)
B5 = CW * (CM + 1) * SODC - 1.
D1 = A1 + A3
DO 25 I = 1, LINES
A6(I) = R7 / (365 * HSA * W(I))
A7(I) = 0.0024 * R9(I) / (PL(I) * SODC)
A2(I) = R2 / CRF / (365 * W(I))
B1(I) = W(I) * W(I) / (216.32 * 2 * SODC * SODC)
B3(I) = 31680. * F * B1(I)
B2(I) = 31680. * 527 / B1(I) * 42 * F
Q1(I) = (D1 * B5 + A6(I)) * B2(I)
Q2(I) = D1 * B2(I)
Q3(I) = (D1 * B5 + A6(I)) * B3(I)
Q4(I) = D1 * B3(I)
Q5(I) = A2(I)
Q6(I) = D1 * ZT(I) / PL(I) + A7(I)

C SELECT BEST TWO SIZES FOR EACH LINE
DO 20 J = 1, NDIA
DXDD(I)=2.1*(Q1(I)*CMA**1.84+Q2(I)*CMA**2.84)/DIA(J)**2
 1.1=5.0*(Q3(I)/CMA**2+Q4(I)/CMA**3)/DIA(J)**6+Q5(I)
IF(DXDD(I)<21,24,23)
21 DIA(I,1)=DIA(J)
   DIA(I,3)=DIA(J)
20 DIA(I,2)=DIA(J)
GO TO 25
23 IF(J-I)<22,24,22
22 DIA(I,1)=DIA(J)
   DIA(I,3)=DIA(J-1)
   DIA(I,2)=DIA(J+1)
GO TO 25
24 DIA(I,1)=DIA(J)
   DIA(I,3)=DIA(J)
   DIA(I,2)=DIA(J)
25 CONTINUE
XTOP =100
PRINT 8
8 FORMAT (41HINPUT VALUES FOR THIS 5 LINE NETWORK ARE/5X,
   1 4HLINE, 6X, 1HW, 9X, 1HL, 7X, 4HCDEV, 5X, 4HCMA)
PUNCH 111
111 FORMAT (12X,42HDESCRIPTION OF THE NETWORK BEING ANALYZED/,
   112X, 4HLINE, 2X, 10HTHROUGHPUT, 2X, 8HPipeline, 2X, 9HELEVATION,
   24X, 4HCMA/31X, 6HLENGTH, 3X, 10HDIFERENCE/19X, 7HTON/DAY,
   35X, 5HMILES, 7X, 4HFEET, 5X, 7HPERCENT/)
DO 4 I=1,LINES
   PUNCH 112, I+WI(I), PL(I), ZT(I)
4 PRINT 5, I, WI(I), PL(I)
PUNCH 116, CMA
116 FORMAT (1H*, 51X, F6.4/)
112 FORMAT (14X, I1, F10.0, F11, I1, F10.0)
5 FORMAT (7X, I1, F10.0, F10.1)
PRINT 6, CDEV, CMA
6 FORMAT (1H*, 28X, 2F9.3/)
PRINT 7
7 FORMAT (34H THE INVESTIGATED COMBINATIONS ARE/5X, 4HLINE,
   15X, 4HDIAM, 7X, 4HCNc, 9X, 4HXTOL)
PUNCH 113
113 FORMAT (12X, 4HCOSTS FOR EACH COMBINATION OF DIAMETERS/,
   115HFOR THE NETWORK/12X, 4HLINE, 3X, 7HOPTIMUM, 5X, 7HOPTIMUM,
   2 5X, 8HCOST FOR, 4X, 10HTOTAL COST/18X, 8HDiameter, 2X, 3HCnC,
   310HCENTRATION, 2X, 9HEACH LINE, 3X, 11HFOR NETWORK/20X, 4HINCH
   4, 7X, 7HPERCENT, 5X, 10HS/Ton-MILE, 3X, 10HS/Ton-MILE/)
C SINGLE LINE INSERT
DO 61 J=1,2
   D(I)=DIA(1,J)
I = 1
11 C(I) = CMAX + 0.05
84 CI(I) = C(I)
   IF(C(I) - CMAX) 85, 32, 32
32 C(I) = CMAX
   GO TO 18
85 M = 1
   DXIDC(M) = (1.84*Q1(M)/C(M)**1.64 + 0.84*Q2(M)/C(M)**1.16) * D(M)
   1**2 + (0.2*Q3(M)/C(M)**4 + 3.4*Q4(M)/C(M)**1.16) * 2
   D2XIDC(M) = (1.54*Q1(M)/C(M)**1.64 + 0.134*Q2(M)/C(M)**1.16) * 1D(M)**2 + 4.2*Q3(M)/C(M)**4 + 12.3*Q4(M)/C(M)**1.16) * D(M)
   2
   2**2
   P(I) = W(I)*PL(I) * DXIDC(I)
   DP(I) = W(I) * PL(I) * D2XIDC(I)
   DELC = -P(I)/DP(I)
   IF(C(I) - DELC) 80, 80, 81
80 C(I) = C(I) - C(I)/2
   GO TO 84
81 C(I) = C(I) + DELC
   IF(C(I) - CMAX) 37, 36, 36
36 C(I) = CMAX
   GO TO 18
37 IF(ABS(C(I) - CI(I)) - CDEV) 18, 18, 84
18 CONTINUE
C END SINGLE-LINE INSERT
C CONTINUE GENERAL GROUP; COMPUTE COSTS
DO 64 N = 1, LINES
   CONS2 = ZT(N)/PL(N)
   SM = C(N) * (C + 1.0) * SODC - 1.0 + 1.
   CONS3 = W(N)/W(N) / (216.0 * SODC * SODC * C(N) * (C + 1.0) * D(N)**5 * 32.2)
   HF(N) = 31680.0 * F**((527.0 * C(N) * CONS3**2) + CONS3)
   HT(N) = HF(N) + CONS2
   IF(W(N) - STPU) 70, 71, 71
70 R4(N) = U1(N)/W(N)
   R5(N) = U2(N)/W(N)
   GO TO 72
71 R4(N) = U1(N)/STPU
   R5(N) = U2(N)/STPU
72 AK4 = 0.2739E-02*(R4(N) + R5(N)) * CRF
   IF(HT(N)) 88, 88, 89
88 HT(N) = 0
89 X1(N) = AK1 * SM * HT(N)/C(N)
   X2(N) = AK2 * D(N)/W(N)
   X3(N) = 0.1150E-06 * R3 * SM * CRF/(E*SODC*C(N)) * HT(N)
   X4(N) = AK4/PL(N)
\[ X_5(N) = \frac{R_6}{365 \cdot W(N) \cdot PL(N)} \]
\[ X_6(N) = \frac{R_7 \cdot HT(N)}{HSA + R_8} \cdot \frac{1}{365 \cdot W(N)} \]
\[ X_7(N) = \frac{2.46 \cdot 10^{-3} \cdot (I - C(N))}{C(N) \cdot R_9(N)} \cdot \frac{1}{SODC \cdot PL(N)} \]
\[ X_T(N) = X_1(N) + X_2(N) + X_3(N) + X_4(N) + X_5(N) + X_6(N) + X_7(N) \]

\[ SN = 0 \]
\[ SD = 0 \]
\[ DO \ 63 \ II = 1, LINES \]
\[ SN = W(II) \cdot PL(II) \cdot XT(II) + SN \]
\[ SD = W(II) \cdot PL(II) + SD \]
\[ XTOL = SN / SD \]

C. SELECT LOWEST COST

\[ IF(XTOL < XTOP) \]
\[ DO 45 \]
\[ XTOL = XTOL \]
\[ DO 46 \ N = 1, LINES \]
\[ XTOP(N) = XT(N) \]
\[ HTOP(N) = HT(N) \]
\[ HFOP(N) = HF(N) \]
\[ X1OP(N) = X1(N) \]
\[ X2OP(N) = X2(N) \]
\[ X3OP(N) = X3(N) \]
\[ X4OP(N) = X4(N) \]
\[ X5OP(N) = X5(N) \]
\[ X6OP(N) = X6(N) \]
\[ X7OP(N) = X7(N) \]
\[ DOP(N) = D(N) \]
\[ DO 65 \ N = 1, LINES \]
\[ PUNCH 114, N, D(N), C(N), XT(N) \]
\[ PRINT 47, N, D(N), C(N) \]
\[ 114 FORMAT (14X, I1, F10.3, F12.5, F14.5) \]
\[ PUNCH 115, XTOL \]
\[ 115 FORMAT (1H*, 51X, F12.5/) \]
\[ PRINT 67, XTOL \]
\[ 47 FORMAT (7X, I1, F11.3, F11.5) \]
\[ 67 FORMAT (1H*, 33X, F11.5/) \]

C. SS = 3 TO LEAVE LOOP

\[ IF(SEXNE SWITCH 3) \]
\[ 69 CONTINUE \]

\[ DO 86 \ N = 1, LINES \]
\[ V(N) = W(N) / (14.7 \cdot C(N) \cdot DOP(N) \cdot DOP(N) \cdot SODC) \]
\[ QW(N) = (I - C(N)) \cdot 2.447 \cdot DOP(N) \cdot DOP(N) \cdot V(N) \]
\[ SM = C(N) \cdot (SODC - 1.0) + 1.0 \]
\[ HPPM(N) = 6.15 \cdot 10^{-5} \cdot DOP(N) \cdot DOP(N) \cdot V(N) \cdot SM \cdot HTOP(N) \]
\[ CPI(N) = R_2 \cdot DOP(N) \cdot PL(N) \]
CPST(N) = R3*HPPM(N)*PL(N)
CISS(N) = (R4(N)+R5(N))*W(N)
CTINV(N) = CPI(N)+CPST(N)+CISS(N)
ANOP(N) = 365*(X10P(N)+X50P(N)+X60P(N)+X70P(N))*W(N)*PL

SUM1 = SUM1 + CPI(N)
SUM2 = SUM2 + CPST(N)
SUM3 = SUM3 + CISS(N)
SUM4 = SUM4 + CTINV(N)

SUM5 = SUM5 + ANOP(N)

PRINT 101
PUNCH 101

101 FORMAT(12X,'HOPTIMUM CONDITIONS FOR THE SPECIFIED W,L,  
122HZT AND OTHER CONSTANTS//12X,4HLINE,4X,1HL,5X,1HW,5X,  
22HZT,5X,1HD,6X,1HC,5X,1HV,5X,2HQW,5X,2HFF,4X,4HPPM /18X  
3,4HPIPE,3X,3HTPD,4X,2HFT,4X,2HHP,3X,4HCONC,3X,3HPFS,3X,  
43HPMP,3X,5HTFT/MI,2X,5HHP/MI )
DO 180 N=1,LINES ,
PUNCH 105 ,N,PL(N),W(N),ZT(N),DOP(N),COP(N),V(N),QW(N),  
1HFOP(N),HPPM(N)
180 PRINT 105 ,N,PL(N),W(N),ZT(N),DOP(N),COP(N),V(N),QW(N),  
1HFOP(N),HPPM(N)

105 FORMAT (14X,1I,F7.1,F7.0,F6.0,F6.1,F7.4,F6.2,F7.0,F7.1,  
1F7.1)
PRINT 102
PUNCH 102

102 FORMAT(12X,'FIRST COSTS AND ANNUAL OPERATING EXPENS  
1,24HE. FOR OPTIMUM CONDITIONS//12X,4HLINE,4X,4HPIPE,6X,  
24HPUMP,4X,7HINJ-SEP,4X,5HTOTAL,4X,6HANNUAL,4X,5HTOTAL )
PRINT 103
PUNCH 103

103 FORMAT(19X,'INSTLN STATIONS SYSTEM FIRST COST OPER  
1,12H.EXP COSTS /22X,1HS,9X,1H$,8X,1H$,8X,1HS,9X,1H$,6X  
2,H$/TON-MI/ )
DO 181 N=1,LINES ,
PUNCH 106 ,N,CPI(N),CPST(N),CISS(N),CTINV(N),ANOP(N),  
1XTOPT(N)
181 PRINT 106 ,N,CPI(N),CPST(N),CISS(N),CTINV(N),ANOP(N),  
1XTOPT(N)

106 FORMAT (14X,1I,F11.0,F10.0,F9.0,F10.0,F10.0,F9.5)
PUNCH 104
PRINT 104

104 FORMAT(12X,'TOTAL COST FOR THE PIPELINE TRANSPORTAT  
1,10ION SYSTEM /20X,4HPIPE,6X,4HPUMP,4X,7HINJ-SEP,4X,  
25HTOTAL,4X,6HANNUAL,4X,5HTOTAL )
PRINT 103
PUNCH 103
PRINT 107, SUM1, SUM2, SUM3, SUM4, SUM5, XTOP
PUNCH 107, SUM1, SUM2, SUM3, SUM4, SUM5, XTOP
107 FORMAT (16X, F10.0, F10.0, F9.0, F10.0, F10.0, F9.5)
GO TO 401
400 CALL EXIT
END

C THIS BEGINS THE INSERT FOR THE 3-LINE NETWORK

C THE NEXT 3 CARDS ARE CHANGED TO TRY 3 D-S DEEP

DO 61 J=1,2
DO 61 K=1,2
DO 61 L=1,2
D(I)=D(J)
D(2)=DTA(2,K)
D(3)=DIA(3,L)
DO 11 I=1,2
11 C(I)=CMAK=0.05
91 DO 12 N=1,2
12 C(I,N)=C(I,N)
34 DO 18 I=1,2
84 C(I)=CMAX
91 DO 12 N=1,2
32 C(I,N)=C(I,N)
85 C(I)=C(I)*C(I)*W(3)/(C(I)*W(2)+C(2)*W(1))

C PARTIAL DERIVATIVES OF THE CONTINUITY EQUATIONS
DC3DC(I)=C(2)**2*W(I)**2/(C(1)*W(2)+C(2)*W(1))**2
DC3DC(2)=C(I)**2*W(2)**2/(C(2)*W(1)+C(2)*W(2))**2
D2C3DC(I)= -2*C(2)*C(2)*W(2)**2*W(1)*W(3)/(C(2)*W(1)+C(1)**2)
1*W(2)**3
D2C3DC(2)= -2*C(I)**2*W(1)**2*W(2)**2*W(3)/(C(2)*W(1)+C(1)**2)
1*W(2)**3
DO 3 M=1,2
DX(IDC(M)=(1.84*Q1(M)*C(M)**84+0.84*Q2(M)/C(M)**16)*D(M)**2
1**2,1-(2*Q3(M)/C(M)**2.3*Q4(M)/C(M)**4)/D(M)**5
2**Q6(M)/C(M)**3
3 D2X(IDC(3)=(1.54*Q1(M)/C(M)**16+0.134*Q2(M)/C(M)**16)**2
1D(M)**2,1+(6*Q3(M)/C(M)**4+12*Q4(M)/C(M)**4)/D(M)**5
2*Q6(M)/C(M)**3
33 P(I)=W(I)*PL(I)/DX(IDC(I)+W(3)*PL(3)*DX(IDC(3))*DC3DC(I)
DP(I)=W(I)*PL(I)*D2X(IDC(I)+W(3)*PL(3)*(DX(IDC(3))*D2C3DC(I)
+DC3DC(I)*DC3DC(I)*D2X(IDC(3)))
DELCC=-P(I)/DP(I)
IF(C(I)+DELCC)80,80,81
80 C(I)=C(I)-C(I)/2.
GO TO 84
81 C(I)=C(I)+DELCC
IF(C(I)-CMA) 37,36,36
36 C(I)=CMA
GO TO 18
37 IF(ABSF(C(I)-CI(I))-CDEV)18,18,84
18 CONTINUE
   DO 19 I=1,2
      IF(ABS(C(I)-CI(I))-CDEV)18,19,84
19 CONTINUE
35 C(3)=C(1)*C(2)*W(3)/(C(1)*W(2)+C(2)*W(1))
C END 3-LINE NETWORK INSERT

C THE FOLLOWING IS AN INSERT FOR THE 5-LINE NETWORK:
   DO 61 J=1,2
      DO 61 K=1,2
         DO 61 L=1,2
            D(1)=DIA(I,J)
            D(2)=DIA(2,K)
            D(3)=DIA(3,L)
            D(4)=DIA(4,LL)
            D(5)=DIA(5,LL)
   61 CONTINUE
11 C(I)=CMA-.05
91 DO 12 N=1,3
12 C(N)=C(N)
34 DO 18 I=1,3
84 CI(I)=C(I)
   JJ=21
      IF(C(I)-CMA)32,32
32 C(I)=CMA
GO TO 18
85 C(4)=C(1)*C(2)*W(4)/(C(1)*W(2)+C(2)*W(1))
   C(5)=C(1)*C(2)*C(3)*W(5)/(C(1)*C(2)*W(3)+C(2)*C(3)*W(1))
1+C(1)*C(3)*W(2))
C PARTIAL DERIVATIVES OF THE CONTINUITY EQUATIONS
   DC4DC(1)=C(2)**2*W(1)*W(4)/(C(1)*W(2)+C(2)*W(1))**2
   DC4DC(2)=C(1)**2*W(2)*W(4)/(C(2)*W(1)+C(1)*W(2))**2
   DC4DC(3)=0
   D2C4DC(1)=-2*C(2)*C(2)*W(2)*W(1)*W(4)/(C(2)*W(1)+C(1)
1*W(2))**3
   D2C4DC(2)=-2*C(1)*C(1)*W(1)*W(2)*W(4)/(C(2)*W(1)+C(1)
1*W(2))**3
   D2C4DC(3)=0
DC5DC(4) = C(3)*C(3)*C(3)*W(4)*W(5)/(C(3)*W(3)+C(4)*W(4))**2
DC5DC(1) = DC5DC(4)*DC4DC(1)
DC5DC(2) = DC5DC(4)*DC4DC(2)
DC5DC(3) = (C(4)*C(4)*W(3)*W(5)/(C(3)*W(4)+C(4)*W(3))**2
D2C5DC(3) = -2*C(4)*C(4)*W(3)*W(5)/(C(3)*W(4)+C(4)*W(3))
1*W(3)**3
D2C5DC(4) = -2*C(3)*C(3)*W(3)*W(4)*W(5)/(C(3)*W(3)+C(4)*W(4))
1*W(3)**3
D2C5DC(1) = D2C5DC(4)*DC4DC(1)+D2C4DC(1)*DC5DC(4)
D2C5DC(2) = D2C5DC(4)*DC4DC(2)+D2C4DC(2)*DC5DC(4)
D2C5DC(3) = D2C5DC(4)*DC4DC(3)+D2C4DC(3)*DC5DC(4)
D2C5DC(4) = 2*C(4)*C(4)*W(3)/(C(3)*W(4)+C(4)*W(3))

IF YOU'VE READ THIS FAR, HAVE A BEER

C IF YOU'VE READ THIS FAR, HAVE A BEER
33 P(I) = W(I)*PL(I)*DXIDC(I)+W(4)*PL(4)*DXIDC(4)*DC4DC(I)
1+W(5)*PL(5)*DXIDC(5)*DC5DC(I).
DP(I) = W(I)*PL(I)*DXIDC(I)+W(4)*PL(4)*DXIDC(4)*DC4DC(I)
1+D2C4DC(I)*D2XIDC(4)+W(5)*PL(5)*DXIDC(5)*D2C5DC(I)
D2C5DC(I) = D2XIDC(5)*DC5DC(I)*DC5DC(I)
DEL = P(I)/DP(I).
IF(C(I)+DEL) GO TO 80, 80, 81
80 C(I) = C(I)-C(I)/2.
GO TO 84
81 C(I) = C(I)+DEL.
IF(C(I)-CMAX) GO TO 37, 36, 36
36 C(I) = CMAX.
GO TO 18
37 IF(ABS(C(I)-C(I)-CDEV)) GO TO 18, 18, 84
18 CONTINUE.
DO 19 I = 1, 3.
IF(ABS(C(I-I)-C(I)-CDEV)) GO TO 19, 19, 91
19 CONTINUE.
35 C(4) = C(1)*C(2)*W(4)/(C(1)*W(2)+C(2)*W(1))
C(5) = C(1)*C(2)*C(3)*W(5)/(C(1)*C(2)*W(3)+C(I)+C(1)*C(3)*W(2))
LITERATURE CITED


A method for optimizing a network of pipelines for transporting woodchips.