Variation of the resistance of copper with potential energy of deformation

by Walter T Kerttula

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Engineering Physics
Montana State University

Abstract:
The variation of the resistance of copper wire with potential energy of deformation is studied. Equations for resistivity and resistance in terms of potential energy of deforma-tion are derived oa the assumption that resistance varies linearly with tension, as found by other observers. The change In resistance is measured, and the results plotted against the potential energy of deformation. The experimental data are found to conform to the equations derived, which state that resistance varies directly with the square root of the potential energy of deformation within the elastic limit.

A determination of the unit change in resistivity for a stress of one kilogram per sgqare centimeter is made from the data, using Bridgman's equation, which remalts incidentally from the derivation of the resistance-energy equation. The value is in close agreement with the results of Bridgman and Rolnick* Data on the behavior beyond the elastic limit are includ-ed, but no definite conclusions are drawn from them.

An equation for computing the strain sensitivity factor (the ratio of unit change of resistance to unit strain) used in calibrating commercial strain gages, is included in an appendix, and the strain sensitivity factor for copper is computed from the experimental data*

VARIATION OF THE RESISTANCE OF COPPER WITH POTENTIAL ENERGY OF DEFORMATION

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ABSTRACT

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HISTORICAL BACKGROUND

The effect of tension on the resistance of metals parallel to the direction of the tensile force was studied as long ago as 1883 by Tomlinson[1], and a considerable amount of work has been done since that time, consisting principally of checking the original results and developing new techniques. Possibly the most extensive work was done by Bridgman[2], who used methods somewhat similar to those adopted for the present experiments; that is, the sample was subjected to a known tensile stress, and the resistance measured by means of a potentiometer, using a null method. The change in resistance due to geometrical changes was subtracted from the total resistance change, and the remainder was assumed to be the change due to stress.

Later determinations were made by Rolnick[3], using the method of vibrations, i.e., the sample was made to vibrate longitudinally, and the change in potential between the two points measured by means of a calibrated amplifier. The amplitude of the vibrations was measured by means of a light beam reflected from a mirror vibrating with the apparatus. Young's Modulus and Poisson's Ratio were used to compute the stress and lateral contraction. Values of these constants were taken from handbooks, or computed from values of the shear modulus.

For most metals, resistivity has been found to vary linearly with tension within the elastic limit. For most metals, the resistivity increases with strain, but in some, notably strontium and bismuth, the tension coefficient of resistivity is negative. Still others, particularly nickel, behave even more abnormally. In this metal, the resistivity decreases to a minimum, and then begins to increase again with increasing tension[4].

Copper has a normal positive tension coefficient of resistivity, which has been measured as 1.6×10^{-6} (4), 1.33×10^{-6} (2), and 1.32×10^{-6} (3) ohm-centimeters per ohm-centimeter per kilogram per square centimeter by different investigators.

Recently, this phenomenon has been put to practical use in the analysis of strain by means of resistance wire strain gages[5]. The strain sensitivity factor of these gages, defined as the ratio of unit change of resistance to unit strain, $\left(\frac{\Delta R/R_o}{\Delta L/L_o}\right)$* must, at present, be determined experimentally. Regarding this strain sensitivity factor, D. M. Nielson[5] states:

"For most metals, the strain sensitivity ... would be expected to be between 1.5 and 1.8 from a consideration of only the dimensional changes of the wire. Instead, most metals and alloys have sensitivity factors considerably higher. This has never been completely explained. The reason probably lies in an actual change of volume resistivity of the metal."

The fact that there is a change in volume resistivity (referred to herein as "resistivity") with strain has been established in the investigations mentioned above, but a method of determining this change theoretically, and thereby eliminating some of the tedious control procedures involved in the manufacture of strain gages, has not been developed. The basic

*See appendix

nature of the phenomenon is still not understood sufficiently to enable one to propound any workable theory concerning it. It is hoped that the present work, considering the resistance change from the viewpoint of potential energy of deformation may help to throw light upon the problem.

THEORY

Since resistivity varies linearly with stress, we may write for the resistivity, 'ρ', of a specimen under a tensile stress 'F',

1) $\rho = \rho_o(1+\beta F)$

where 'β' is the tension coefficient of resistivity, and 'ρ_o' the resistivity of the unstressed specimen.

The potential energy of deformation, 'W', is the work done by a tension 'T' acting through a displacement 'e', and may be written $W = \int_o^e T de$. Assuming Hooke's Law, this may be written:

2) $W = \int_o^e ke \cdot de = \frac{ke^2}{2} = \frac{Te}{2}$.

Young's Modulus is defined by the equation $E = \frac{T/A_o}{e/L_o} = \frac{F}{e/L_o}$, whence $e = \frac{L_oF}{E}$. Substituting these values in 2) we obtain

$W = \frac{A_oL_oF^2}{2E}$, or:

3) $F = \sqrt{\frac{2E}{A_oL_o}} \sqrt{W}$.

Substituting the value of 'F' from equation 3) in equation 1), we obtain for the resistivity in terms of the potential energy of deformation:

4) $\rho = \rho_o(1+\beta \sqrt{\frac{2E}{A_oL_o}} \sqrt{W})$

A curve of resistivity versus energy of deformation should, therefore, fit equation 4), while resistivity versus the square root of the energy of deformation would be a straight line, the slope being equal to $\rho_o\sqrt{\frac{2E}{A_oL_o}}\beta$. Since the plotting of a "theoretical" curve against which to compare an experimental curve would be of little value, because the constants 'β', 'E', 'L_o', and 'A_o' would have to be taken from the data, we have here a method of determining whether the experimental data fit a curve of the form of equation 4). If the curve of resistivity versus the square root of the energy of deformation is a straight line, we will know that the data satisfy an equation of the form of 4).

An approximate relationship between the total resistance and the energy of deformation may be derived from consideration of 'β', 'E', and Poisson's Ratio, 'μ', as follows.

The resistance of a piece of metal is given by the well-known relation

5) $R = \rho\frac{L}{A}$

where, for a sample under a tension of 'T' dynes, the length, 'L' is equal to L_o+e, and the area, 'A' equals $A_o - \Delta A$, while the resistivity 'ρ' is given by substituting $\frac{T}{A_o} = F$ in equation 1).

Poisson's Ratio is defined by $\mu = \frac{\Delta r/r_o}{e/L_o}$ whence $\Delta r = \frac{\mu r_o e}{L_o} = \frac{\mu T}{\pi r_o E}$, where '$r_o$' is the original radius, (see p. 7).

Now, $A_o-\Delta A = \pi(r_o-\Delta r)^2 = \pi(r_o^2 - 2r_o\Delta r + \Delta r^2)$

Since 'μ' is less than 1/2[6], and 'T' in the present experiment never exceeded 2×10^7, the order of magnitude of μT is not more than 10^7. 'E' is of the order of magnitude 10^{12}, while 'r_o' was never below the order of 10^{-1}. Hence, the second term of the expression for $(A_o - \Delta A)$ is of the order of magnitude 10^{-5}, while the third term is at most 10^{-8}. We therefore neglect the term Δr^2 for the purposes of this experiment and write:

$$A_o - \Delta A = \frac{\pi E r_o^2 - 2\mu T}{E}$$

With these values for 'ρ', 'L', and 'A', the equation 5) may be written:

$$R = \rho_o (1 + \beta \frac{T}{A_o}) \frac{L_o + \frac{L_o T}{\pi r_o^2 E}}{\frac{\pi r_o^2 E - 2\mu T}{E}}$$

Since $R_o = \rho_o \frac{L_o}{A_o}$, and $A_o = \pi r_o^2$, the expression for R becomes:

$$R = R_o \frac{A_o E + (1+\beta E)T + \frac{\beta T^2}{A_o}}{A_o E - 2\mu T}$$

Performing the indicated division:

$$6)\ R = R_o \left[1 + \frac{1+\beta E+2\mu}{A_o E} T + \frac{2\mu\beta E+6\mu+\beta E}{A_o^2 E^2} T^2 + \frac{2\mu(2\mu\beta E+6\mu+\beta E)}{A_o^3 E^3} T^3 + \ldots \right]$$

For the values of 'T' mentioned above, the series converges very rapidly, in fact, the second term is more than 10^5 times the third term, so that it is permissible to neglect all terms above the second. In this form, the equation for resistance in terms of tension becomes:

$$7)\ R = R_o \left[1 + \frac{1+\beta E+2\mu}{A_o E} T\right]$$

Substituting for 'T' its value in terms of 'W' from eq. 3):

$$8)\ R = R_o \left[1 + \frac{\sqrt{2}\,(1+\beta E+2\mu)}{\sqrt{A_o L_o E_o}} \sqrt{W}\right]$$

By comparison with equation 4), it may be seen that a curve of 'R' versus 'W' would resemble the curve of 'ρ' versus 'W', differing in this approximate form only by the terms brought in because of geometrical changes. Again, the plot of 'R' versus the square root of 'W' would be a straight line, the slope of which yields an expression for 'β'. Within limits, equation 8) is as accurate as 4), and is simpler for the purposes of this experiment, since it eliminates the necessity of computing the resistivity, 'ρ', (see pp. 15 - 16).

An interesting incidental result may be obtained from equation 7), by noting that the term $R_o \frac{1+\beta E+2\mu}{A_o E}$ is the slope of the curve of 'R' versus 'T', or

$$\frac{\Delta R}{\Delta T} = R_o \frac{1+\beta E+2\mu}{A_o E}$$

Solving for 'β', and remembering that $\beta = \frac{1}{\rho_o}\frac{\Delta\rho}{\Delta F}$, and $\Delta F = \frac{\Delta T}{A_o}$, we obtain

$$9)\ \frac{\Delta\rho}{\rho_o \Delta F} = \frac{\Delta R}{R_o \Delta F} - \frac{1+2\mu}{E}$$

This is Bridgman's equation as it is referred to in an abstract of the work of E. Lopuhkin in Techn. Phys. U. S. S. R., 4, 1, pp 25-35, 1937[7]. (The derivation of this equation could not be found in any of the available literature.) Bridgman[2] and Rolnick[3] both use the quantity $\frac{1+2\mu}{E}$ as a correction factor for the effect of dimensional changes in the computation of 'Δp', but no derivation is given by either in the literature consulted. As may be seen in the above derivation, this is but a first order approximation, holding only for low values of 'T' and fairly high values of 'r', the radius of the wire.

EXPERIMENTAL PROCEDURE

In order to obtain data relating the potential energy of deformation to resistivity, one of two variations of method might be used. One might simply measure the resistance and tension, and compute the elongation from Young's Modulus and the measured length of the specimen. Otherwise, the elongation could be measured directly. The latter method was chosen here, since data on the material used varies considerably as given in handbooks. Apparatus used and methods for preparing the data for presentation are outlined below.

The apparatus consisted of a sheet metal box, roughly 150 centimeters in length, and of square cross section, 5x5 centimeters, reenforced with one-half inch angle iron, and insulated on the sides, bottom and ends with about two inches of asbestos. A longitudinal section and a top view, showing the specimen in place, are given in figures 1A and 1B.

The clamp 'E' was a piece of iron rod, drilled longitudinally, and fitted with set screws for clamping the specimen. This was brazed securely to the angle iron reenforcements at one end of the box. The clamp 'G', constructed similarly, was used to fasten the specimen to a steel wire leading through the pulleys 'F' and 'H' to the weight hanger 'I'. The pulley 'F' was mounted on a horizontal shaft, running the width of the box. The mounting of this pulley was insulated from the box with strips of sheet mica, thus one end of the specimen could be grounded through the box to eliminate static charges without otherwise affecting the resistance measurements.

The resistance between the points 'A' and 'B' of the specimen was measured by means of a self-contained Kelvin Bridge, the current clamps of which were at 'C' and 'D', and the potential terminals were knife-edge clamps at 'A' and 'B'.

Two traveling microscopes, '1' and '2', were used to measure the elongation. These were mounted separately from the rest of the equipment on a 2" pipe running the length of the box, held in vertical supports fastened to the table with wood screws. To hold the microscopes, holes were drilled through the pipe to fit the support rods of the microscopes, the rods inserted, and clamped in place with set screws.

Transparent transformer oil was then poured into the box to submerge the specimen and clamps. During the experiment, the oil was agitated manually, it being necessary to allow the oil to come to rest before taking readings on the microscopes. Temperature was measured with ordinary mercury thermometers during the preliminary experiments, and with a Beckmann vacuum thermometer calibrated in hundredths of a degree C. on the final experiments. However, there was little difference in the data obtained in the two cases, since the temperature fluctuation was not very great as a rule.

It was necessary to start with the wire taut so that it would be and remain in focus under the microscopes. The starting tension was taken as the zero point. Microscope No. 2 measured the motion of the point 'B' of the specimen, farthest from the fixed end at 'E', i.e., the elongation of the entire wire between the point 'B' and the clamp, as well as any slipping in the clamp, and other rigid-body motions. Microscope No. 1 measured the motion of the point 'A', which consisted of the elongation of the wire between 'A' and 'E', and the other possible motions mentioned above. As each weight was added to the hanger, readings were taken on each microscope. The difference between these and the readings for the previous tension gave the motion caused by the added tension. The difference between the motion measured by No. 2 and the motion measured by No. 1 gave the elongation due to the added tension of the portion of the specimen under test (AB, figure 1), since the rigid-body motions and the elongation of EA (measured by both microscopes) was thus cancelled. To obtain the total elongation for any tension, it was necessary to sum up the elongations for each increase in tension from the zero point to the required tension.

The resistance measured as above, was recorded for each tension and elongation, together with the temperature of the oil bath at the time. The direct experimental data, then, consisted of the readings of the microscopes, the tension in kilograms, Kelvin Bridge readings, temperature, and the original length and radius of the specimen.

The method of determining the elongation 'e' has already been described. The tension was simply converted to dynes, (the weight of the connecting wire being neglected), and this tension was plotted against the elongation on ordinary graph paper, (figures 2 and 8), and the area under the curves measured with a planimeter for each point corresponding to a given elongation. The energy of deformation, 'W', was obtained from this area by multiplying by the value of 'Te' represented by each square inch.

In computing the resistivity 'ρ', the following method, which eliminates the approximations used in deriving equations 8) and 9) was adopted. The equation $\Delta r = \frac{\mu T}{\pi E r_o}$, (see p. 7) was used to compute the lateral contraction, a value of 0.33 being used for Poisson's Ratio[8]. The radius 'r_n' corresponding to the particular tension 'T_n', where the subscript refers to the number of the reading (see Tables I, II and III, and p. 20), was computed by subtracting the Δr_n, from r_o, i.e., $r_n = r_o - \Delta r_n$. In the case where the elastic limit was exceeded sufficiently to cause a measurably permanent change in radius (figures 8, 9 and 10), the average radius, 'r_a', measured after the tension had been removed, was used in place of 'r_o' for decreasing tension. The length, 'L_n' and the area 'A_n', for each tension were then computed by the equations $L_n = L_o + e_n$