The maximum deflection of a blast loaded cantilever beam by the modified Galerkin method by Daniel Michael Koszuta

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University
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Abstract:
This paper presents an analytical method for calculating the maximum deflection of a rectangular cantilever beam subjected to blast loading. The method is based on the modified Galerkin method and uses a nonlinear stress-strain relation together with the nonlinear geometry changes of large deflection motion.

The paper also presents a method whereby the approximate solution can be improved. By variation of the shape functions used the corresponding equation residuals are studied. A preferred or "better residual" is defined and then used to select the shape function which best satisfies the governing equation of motion.

Results show that a one term Galerkin solution gives satisfactory accuracy. Both the linearized and nonlinear beam equations are solved. Residual properties of both the linear and nonlinear problems are found to be identical. This suggests first solving the linearized version of a physical system by this method and using the shape functions which give good results in the linear case to solve the nonlinear version.

The investigation revealed that combining shape functions resulted in an averaging of their residuals which can be used to good advantage when attempting to reduce equation residuals.
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Date 11-21-69
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BY THE MODIFIED GALERKIN METHOD

by

DANIEL MICHAEL KOSZUTA

A thesis submitted to the Graduate Faculty in partial
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of
MASTER OF SCIENCE
in
Aerospace and Mechanical Engineering

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CHAPTER I: INTRODUCTION

The classical linearized theory of beam deflections is known well and has been used extensively with good results in some engineering problems encountered in the past. This theory is based on the restrictive assumptions of small deflections and a linear stress-strain relation. Both assumptions eliminate highly nonlinear terms from the governing equations of motion. Previous works \([1,2]\) have included some nonlinear aspects of beam motion but still only approach the general form of the equation of motion as developed by Eringen \([3]\). Another popular approach used in the past \([4-8]\) has been the Rigid-Plastic theory, but in most cases it is again restricted to small or moderately large deflections. The literature presently available to the author suggests a need to investigate more of the nonlinear terms involved in the general governing equations.

The purpose of this paper is to present an approximate method of solution for the large deflection motion of a blast loaded rectangular cantilever beam. Fewer restrictive assumptions have been made in the derivation of an equation of motion which is a better approximation to the general form. An inverse tangent function was used to approximate a true stress-strain relation. An approximate solution for the maximum deflection is obtained by the modified Galerkin method. This method involves choosing deflection shape functions which reduce the continuous system to one of a finite number of degrees of freedom. The difficulty

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1 Numbers in brackets refer to literature consulted.
of solving nonlinear equations led to a one term solution of the motion. A procedure to improve the approximate solution by mode shape variation is then described and evaluated. To aid this study the method is applied first to the classical beam equation using the corresponding exact eigenfunctions with variation. The results of mode shape variation in the linear case are studied, then the same technique is used on the nonlinear equation of motion. In this case the deflection shape of the nonlinear motion [9] is assumed to be approximately that of the linearized beam.
2.1 The Galerkin Method as a Weighted Residual Method

Many physical problems encountered in engineering, such as the large deflections of a beam, lead to nonlinear mathematical models. The present methods available do not provide exact solutions in such cases. The engineer, quite practically, turns to methods which yield approximate solutions. The method of weighted residuals, of which the Galerkin method is one type, is such a method. The method involves assuming a particular form of the solution which is a sum of products of space and time functions. The space or shape functions are known from assumptions made through experience and intuition in most cases. Nonlinear problems make the choice of shape functions much more difficult. The time functions are unknown. The assumed solution, being inexact, will not satisfy the system model but will yield an error termed the equation residual and/or boundary residuals. These residuals are weighted in different fashions resulting in differential equations to be solved for the time functions.

The accuracy of the resulting approximate solution depends on the accuracy of the chosen shape functions. Finlayson and Scriven point out the difficulties involved and the scant knowledge available on convergence, especially in a nonlinear problem. Crandl tacitly assumed that convergence occurs with an increasing number of terms in the assumed solution.
Consider the mathematical model in two independent variables for \( y(x,t) \):

\[
N[y(x,t)] - \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad \text{for } x \text{ in } V, \ t > 0 \quad \text{(Eq. 1)}
\]

where \( N[y] \) denotes a general differential operator involving spatial derivatives of \( y \) in the three-dimensional domain \( V \) with boundary \( S \), and \( t \) represents time.

The initial and boundary conditions are:

\[
y(x,0) = y_0(x) \quad \text{for } x \text{ in } V
\]

\[
y(x,t) = f_s(x,t) \quad \text{for } x \text{ on } S \quad \text{(Eq. 2)}
\]

A trial solution is assumed of the form:

\[
y^*(x,t) = y_s(x,t) + \sum_{i=1}^{n} a_i(t) \Theta_i(x) \quad \text{(Eq. 3)}
\]

where \( y_s(x,t) \) satisfies all the nonhomogeneous boundary conditions and \( \Theta_i(x) \) satisfies all homogeneous boundary conditions; i.e.,

\[
y_s = f_s \quad \text{for } x \text{ on } S
\]

\[
\Theta_i = 0 \quad \text{(Eq. 4)}
\]

Substitution of the assumed solution Equation (3) into the governing Equation (1) yields the equation residual:

\[
R[y^*(x,t)] = N(y^*) - \frac{\partial^2 y^*}{\partial t^2} \quad \text{(Eq. 5)}
\]
The residual is a measure of the exactness of the assumed solution. When the assumed solution is the exact solution, the residual will be identically zero throughout the domain V. It is a premise of this paper that the magnitude and distribution of the residual can be used to judge the merit of the assumed solution.

The next step in the procedure is an approximation to the ideal case of a zero residual. The weighted integrals of the residual are set equal to zero over the domain of interest:

\[ \int_V W_j R(y^*) \, dV = 0 \quad j = 1, 2, 3, \ldots, n \]  
(Eq. 6)

where the \( W_j \) are prescribed weighting functions and can be chosen in several different ways \([10]\). Each criterion for choosing the weighting functions corresponds to a particular weighted residual method. The Galerkin method uses the chosen space modes as weighting functions;

\[ \int_V \Theta_j R(y^*) \, dV = 0 \quad j = 1, 2, 3, \ldots, n \]  
(Eq. 7)

Equations (7) represent a system of \( n \) ordinary second order differential equations in time for the time modes \( q_i(t) \). They may be linear or nonlinear, coupled or uncoupled, depending on the spacial operator \( N(y) \) and the orthogonality of the space modes with respect to the residual. The original initial conditions are imposed on the \( q_i(t) \), term by term, thus completing the approximate solution, Equation 3.
2.2 The Modified Galerkin Method

The modified Galerkin method is nearly identical to the Galerkin method. As discussed by Anderson [12], it is used for problems which present great difficulty in choosing shape functions which satisfy all boundary conditions. The modified Galerkin method permits the use of functions which satisfy only the displacement boundary conditions. As an example, let \( y(x,t) \) represent the nonlinear large deflection motion of a cantilever beam fixed at \( x = 0 \) and free at \( x = L \). The displacement boundary conditions at the wall are:

\[
\begin{align*}
y(0,t) &= 0 \quad \text{(zero displacement)} \\
\frac{\partial y(0,t)}{\partial x} &= 0 \quad \text{(zero slope)} \quad \text{(Eq. 8)}
\end{align*}
\]

which can be easily satisfied. The force boundary conditions of zero moment and shear at the free end are very difficult to satisfy since these expressions are highly nonlinear. Using space modes \( \Phi_j(x) \) which satisfy only the displacement boundary conditions will result in errors in force at the free end termed boundary residuals. The modified Galerkin method includes the boundary errors in the weighted residual condition, Equation (7):

\[
\int_V \psi_j \, R(y^*) \, dV + E_M \, \Phi_j + E_V \, \phi_j = 0 \quad j = 1, 2, 3, \ldots, n \quad \text{(Eq. 9)}
\]

where \( E_M \) and \( E_V \) are the moment and shear force errors at the free end.
$\Phi_j$ and $\theta_j$ are weighting terms usually chosen as a rotation and displacement of the free end in the $j$th space mode. Equations (9) similarly result in $n$ differential equations in the time modes $q_i$. 
CHAPTER III: FORMULATION OF SYSTEM MODELS

3.1 The Blast Loaded Linear Beam

The derivation of the linearized equation of motion of a cantilever beam is well known and is presented here only for easy comparison with that of the nonlinear large deflection equation of motion.

Figure 1. Linear Beam Deflection and Elemental Forces

Referring to Figure 1, it is assumed that there exists a neutral plane at the beam center, initially on the x-axis, which does not deflect longitudinally. Neglecting shear deformation and assuming that cross-sectional planes remain plane, let $y(x,t)$ represent the motion of the neutral plane. The beam is fixed at $x = 0$ and free at $x = L$. The external loading is represented by $w$(lbs/in). Referring to the free body diagram (Figure 1-b) of a beam element of length $dx$, we can write the equation of motion from a Newtonian force balance in the $y$ direction:
\[ \sum \text{External Forces} - \text{Inertia Force} = 0 \]

\[
w \, dx + V - (V + \frac{dV}{dx} \, dx) - \mu \, dx \frac{d^2y}{dt^2} = 0
\]

or:

\[
w - \frac{dV}{dx} - \mu \frac{d^2y}{dt^2} = 0 \quad \text{(Eq. 10)}
\]

where \( V = \) Shear Force (psi)

\( \mu = \) Linear Beam Density (lbsm/in)

\( t = \) Time (sec)

Summing moments and neglecting rotational inertia yields:

\[
\frac{dM}{dx} = V \quad \text{(Eq. 11)}
\]

where \( M = \) Internal Moment (in-lbs).

Substituting Equation (11) into Equation (10):

\[
\sum w - \frac{dM}{dx} - \mu \frac{d^2y}{dt^2} = 0 \quad \text{(Eq. 12)}
\]

Note that it was assumed that geometry changes do not effect the direction of forces. Assuming a linear stress-strain relation and small deflection, from elementary beam theory we have:

\[
\frac{dy}{dx} = \tan \phi = \phi
\]

and

\[
M = EI \frac{d^2y}{dx^2} \quad \text{(Eq. 13)}
\]
where \( \phi \) = Deflection Angle at \( x \) (Radians)

\( I \) = Cross-Sectional Area Moment of Inertia

\( E \) = Constant Modulus of Elasticity (psi)

Substituting Equation (13) into Equation (12) results in the final equation of motion

\[
+ w(x,t) - EI \frac{\partial^4 w}{\partial x^4} - \mu \frac{\partial^2 w}{\partial t^2} = 0
\]  

(Eq. 14)

An exponential function is used to describe the pressure loading caused by the blast shock wave as presented in [14].

\[ P_{\text{max}} = \text{Maximum Blast Pressure (psi)} \]

\[ \beta = \text{Blast Decay Constant (sec}^{-1}) \]

Figure 2. Blast Load Approximation
Note that we have assumed that the blast pressure is independent of \( x \).

The external loading function is now:

\[
    w(t) = D \text{ Pmax} \ e^{-\beta t} \quad \text{(lbsf/in)}
\]

where \( D = \text{Beam Width (in.)} \). The governing equation of motion becomes:

\[
    \alpha e^{-\beta t} - EI \frac{\partial^4 y}{\partial x^4} - \mu \frac{\partial^2 y}{\partial t^2} = 0 \quad \text{(Eq. 15)}
\]

where \( \alpha = D \text{ Pmax} \).

3.1.2 Exact Solution of the Linear Beam Motion

The linear mathematical model describing the beam motion due to blast loading is a nonhomogeneous, initial-boundary value problem:

Boundary Conditions:

\[
    y(0,t) = 0 \quad \text{(zero displacement at wall) B.C. (1)}
\]

\[
    \frac{\partial y(0,t)}{\partial x} = 0 \quad \text{(zero slope at wall) B.C. (2)}
\]

\[
    \frac{\partial^2 y(L,t)}{\partial x^2} = 0 \quad \text{(zero moment at free end) B.C. (3)}
\]

\[
    \frac{\partial^3 y(L,t)}{\partial x^3} = 0 \quad \text{(zero shear at free end) B.C. (4)}
\]
The solution is assumed to be a sum of a homogeneous part and a particular part:

\[ y(x,t) = y_H(x,t) + y_p(x,t) \]  

(Eq. 16)

The particular part has the form

\[ y_p(x,t) = \Phi(x) e^{-\beta t} \]  

(Eq. 17)

where \( \Phi(x) \) is an unknown function of \( x \).

We next impose the conditions that \( y_H \) satisfies the homogeneous equation of motion and \( y_p \) satisfies the nonhomogeneous equation of motion. Substituting the particular solution Equation (17) into Equation (15), applying the appropriate transformed boundary conditions and De Moivre's theorem \([15]\) for the extraction of roots, results in:

\[ y_p(x,t) = \left\{ e^{\xi x} \left( D_1 \sin \xi x + D_2 \cos \xi x \right) + e^{-\xi x} \left( D_3 \sin \xi x + D_4 \cos \xi x \right) \right\} \cdot e^{-\beta t} \]

where

\[ \xi = \frac{\sqrt{2}}{2} \left( \frac{\mu \beta^2}{EI} \right)^{1/4} \]

and the constants \( D_1, D_2, D_3, \) and \( D_4 \) are found from the boundary conditions on \( \Phi(x) \):

\[ \Phi(0) = \Phi'(0) = \Phi''(L) + \Phi'''(L) = 0 \]
The technique of separation of variables is used to obtain the homogeneous solution presented by Wylie [16],

\[ y_R(x,t) = \sum_{n=1}^{\infty} X_n(x) T_n(t) \]

and

\[ X_n(x) = (\cos z_n + \cosh z_n)(\sin \gamma_n x - \sinh \gamma_n x) \]

\[ - (\sin z_n + \sinh z_n)(\cos \gamma_n x - \cosh \gamma_n x) \quad \text{(Eq. 17a)} \]

\[ T_n(t) = E_n \sin \omega_n t + F_n \cos \omega_n t \]

where the \( z_n \) are the roots to the frequency equation:

\[ \cos z \cosh z = -1 \]

and

\[ \omega_n = \sqrt{\frac{EI}{\mu}} \left( \frac{z_n}{L} \right)^2 \quad \text{(natural frequencies)} \]

\[ \gamma_n = L \sqrt{\frac{\mu \omega_n^2}{EI}} \]

Application of initial conditions gives the coefficients \( E_n \)

and \( F_n \):

\[ F_n = -\frac{\int_0^L \tilde{f}(x) X_n(x) \, dx}{\int_0^L x_n^2(x) \, dx} \quad \text{(Eq. 17b)} \]
thus resulting in the complete solution

\[
y(x,t) = \sum_{n=1}^{\infty} X_n(x) T_n(x) + \phi(x) e^{-\beta t}
\]

3.2 Equation of Motion for Large Deflections of a Blast Loaded Cantilever Beam

Referring to Figure 3, the effect of geometry changes due to large deflections will now be included. Unlike the general form of the equations of motion developed by Eringen [3], normal elemental forces are neglected and a nonlinear stress-strain relation is used. Cross-sectional planes are assumed to remain plane. Again let \( y(x,t) \) represent the motion of the neutral plane of a beam fixed at \( x = 0 \) and free at \( x = L \), neglecting shear deformation and longitudinal deflection. Since the duration of the blast load is very short it is assumed that geometry changes do not affect the direction of the pressure force. Results justified this. End deflections of only \( 0.01L \) occurred after a 99% decay of the pressure function.

Consider the forces on the beam element of length \( \Delta x \) in the deformed position. Summing forces in the positive \( y \) direction we get:

\[
V \cos \phi + \frac{\partial V}{\partial x} \Delta x - (V + \frac{\partial V}{\partial x} \Delta x) \cos (\phi + \frac{\partial \phi}{\partial x} \Delta x) - \mu \Delta x \frac{\partial^2 y}{\partial t^2} = 0
\]

(Eq. 18)
where: 
\( V \) = Total Shear Force (psi)
\( \phi \) = Deflection Angle at \( x \) (radians)
\( w \) = Loading Function, \( w = \alpha e^{-\beta t} \) (lbs/in)
\( \mu \) = Linear Beam Density (lbs/in)
\( t \) = Time (sec)
\( M \) = Internal Moment (in-lbs)

Summing moments about the right face, using counterclockwise as positive, and neglecting the effect of rotational inertia, we get:

\[ M + V \Delta x \cos \phi + V \Delta y \sin \phi + (w \Delta x) \frac{\Delta x}{2} - (M + \frac{\partial M}{\partial x} \Delta x) = 0 \]

rearranging,

\[ V = \frac{\partial M}{\partial x} \cos \phi \]  \hspace{1cm} (Eq. 19)

therefore

\[ \frac{\partial V}{\partial x} = - \frac{\partial M}{\partial x} \frac{\partial \phi}{\partial x} \sin \phi + \frac{\partial^2 M}{\partial x^2} \cos \phi \]  \hspace{1cm} (Eq. 20)

Substitution of Equations (19) and (20) into (18) gives the governing equation of motion in terms of the displacement \( y \) and the internal moment \( M \):

\[ w(t) - \left\{ \frac{\partial^2 M}{\partial x^2} \cos^2 \phi - \frac{\partial M}{\partial x} \frac{\partial \phi}{\partial x} \sin 2 \phi \right\} - \mu \frac{\partial^2 y}{\partial t^2} = 0 \]

\hspace{1cm} (Eq. 21)

Note that neglecting the rotational inertia force enabled elimination of the shear force from Equation (18).
It is now desirable to eliminate the internal moment $M$ from Equation (21). To accomplish this, a relation between the moment and displacement is needed. Referring to Figure 4, we begin by defining the strain ($\epsilon$) of a longitudinal fiber at a distance $z$ from the neutral plane. From the definition of the neutral plane and assuming symmetrical deformation about it, it can be seen that

$$\epsilon (x, z) = \frac{z \phi'}{\Delta x} \, \text{ (in/in)}$$  \hspace{1cm} (Eq. 22)
which conforms to the popular sign convention of positive for tension and negative for compression.

\[
\sigma = \frac{1}{b} \tan^{-1} (a) \quad \text{(psi)} \\
\text{(Eq. 23)}
\]

where \( \sigma = \text{stress (psi)} \),

Figure 4. Fiber Deformation Defining Strain.

The true stress-strain relation of a material (shown in Figure 8) can be approximated by the relation
and a and b are constants which are selected for best fit. The internal moment can now be defined

\[ M = D \int_{-c}^{c} \sigma z dz = \frac{D}{b} \int_{-c}^{c} \tan^{-1} (a'c) z dz \quad (\text{Eq. 24}) \]

Integrating Equation (24) results in

\[ M = \frac{D}{b} \left\{ c^2 \tan^{-1} a'c + (\tan^{-1} a'c - a''c)/a'' \right\} \quad (\text{Eq. 25}) \]

where \( \phi = \text{Deflection Angle} \)
\[ \phi' = \frac{d\phi}{dx} \]

Differentiation of Equation (25) gives the necessary relations to eliminate the internal moment from Equation (21):

\[ \frac{dM}{dx} = \frac{2D\phi''}{a'^2} \left\{ \frac{a'c - \tan^{-1} a'c}{\phi' \phi''} \right\} \quad (\text{Eq. 26}) \]

and

\[ \frac{d^2M}{dx^2} = \frac{2D}{a'^2} \left\{ \phi''^2 \left[ 3(1 + a'^2c^2) \tan^{-1} (a'c) - 3a'c 
- 2(a'c)^3 \right]/\phi' \right. 
\left. + \phi'' \left[ a'c - \tan^{-1} a'c \right]/\phi' \right\} \quad (\text{Eq. 27}) \]
The final elimination to obtain an equation of motion only in terms of the displacement is to define the deflection angle $\phi$ as:

$$\phi = \tan^{-1} \frac{\partial y}{\partial x}$$  \hspace{1cm} (Eq. 28)

Substitution of Equations (26), (27), and (28) into Equation (21) gives the final governing equation of motion in terms of the displacement $y$ only, which for brevity is not written here. Needless to say, the complete equation of motion is highly nonlinear, thus an approximate solution is the most reasonable approach.

To complete the nonlinear mathematical model, the initial and boundary conditions are defined as:

**Initial Conditions:**

- $y(x,0) = 0$ (zero initial displacement) I.C. (1)
- $\frac{\partial y(x,0)}{\partial t} = 0$ (zero initial velocity) I.C. (2)

**Boundary Conditions:**

- $y(0,t) = 0$ (zero displacement at wall) B.C. (1)
- $\frac{\partial y(0,t)}{\partial x} = 0$ (zero slope at wall) B.C. (2)
- $M(L,t) = 0 = \frac{D}{b} \left[ c^2 \tan^{-1} \left( a\phi ' c \right) + \left( \tan^{-1} (a\phi ' c) - a\phi ' c \right) \right]_{x=L}$ (zero moment at free end) B.C. (3)
Note the complexity of the moment and shear relations compared with that of the linear case. Shape functions which satisfy these conditions are difficult to find, hence the modified Galerkin method was used. It was found that the static deflection shape of a linearized cantilever beam with a constant loading function did satisfy all boundary conditions. Further investigation showed that the condition for satisfying B.C. (3) and B.C. (4) was:

$$V(L,t) = 0 = \left[ \frac{\partial M}{\partial x} \cos \phi \right]_{x=L}$$

(zero shear at free end) \hspace{1cm} \text{B.C. (4)}

$$\left( \phi' \right)_{x=L} = \left( \frac{\partial \phi}{\partial x} \right)_{x=L} = 0$$
CHAPTER IV: APPLICATION OF THE MODIFIED GALERKIN METHOD AND COMPUTATIONAL PROCEDURE.

4.1 The Modified Galerkin Method Applied to the Linearized Problem.

A one term Galerkin solution of the linearized beam problem was studied so that comparison of the results to the exact solution could be made. The availability of an exact solution gave a foundation for evaluation of the method itself and the mode shape variation technique to be presented. We begin by restating the initial-boundary value problem which models the motion of the beam.

Equation of motion:

\[ \alpha e^{-\beta t} - EI \frac{\partial^4 y}{\partial x^4} - \mu \frac{\partial^2 y}{\partial t^2} = 0 \]  
(Eq. 15)

Initial conditions:

\[ y(x,0) = 0 \quad \text{I.C. (1)} \]

\[ \frac{\partial y(x,0)}{\partial t} = 0 \quad \text{I.C. (2)} \]

Boundary conditions:

\[ y(0,t) = 0 \quad \text{B.C. (1)} \]

\[ \frac{\partial y(0,t)}{\partial x} = 0 \quad \text{B.C. (2)} \]
First, a form of the solution is assumed corresponding to Equation (3) of Chapter II. Since the boundary conditions are all homogeneous, $y_g$ is zero. As was shown in Chapter II, the final step in the process, Equation (9), is the solution of a system of differential equations in $q_1(t)$. A two term solution in the nonlinear case would result in two coupled nonlinear equations in $q_1$ and $q_2$. The difficulty of solving such a system of equations led to using a one term approximation which was used in both the linear and nonlinear cases.

$$y^*(x,t) = q(t) \psi(x) \quad (Eq. 29)$$

As can be seen from the linear boundary conditions, selection of shape functions which satisfy all of them can be made with relative ease. Again, to aid the linear-nonlinear comparison, mode shapes satisfying only the displacement boundary conditions were used.

Substitution of Equation (29) into Equation (15) results in the equation residual:

$$R(x,t) = \alpha e^{-\beta t} - EI \frac{\partial^4 y^*}{\partial x^4} - \mu \frac{\partial^2 y^*}{\partial t^2}$$
or

\[ R(x,t) = \alpha e^{-\beta t} - EI q \theta'' - \mu \dot{\theta} \]  
(Eq. 30)

where \( \dot{q} = \frac{d^2 q}{dt^2} \) and \( \theta'' = \frac{d^2 \theta}{dx^2} \)

![Figure 5. Boundary Residuals](image)

Referring to Figure 5, consider a beam element at \( x = L \). Since the shape function \( \theta(x) \) does not satisfy the force boundary conditions of zero moment and shear at \( x = L \), there will be boundary residuals. Summing forces on the end element, using the same sign convention as used in deriving the equation of motion, we have

\[
\left( \sum \right) E_M = M(y^*)_{x=L} - 0 = EI \frac{\partial^2 y}{\partial x^2} \bigg|_{x=L} = EI q \theta''_{x=L}  \quad \text{(Eq. 31)}
\]
Note that each term in the equation of motion and the boundary conditions are in units of force. The residuals can then be thought of as errors in force. The first term in the equation residual, Equation (30), is an inertial force error,

\[ E_{IN} = \dot{\ddot{\mu}} q \theta \]  

(Eq. 33)

The second term is an error force related to internal beam forces,

\[ E_{IF} = -EI q \theta''' \]  

(Eq. 34)

The weighted residual condition, Equation (9), can be interpreted as the total virtual work \((\Delta W_T)\) of the error forces resulting from a virtual displacement of the coordinate \(q\); i.e.,

\[ \delta y = \delta q \cdot \theta \]

and

\[ \Delta W_T = \int_0^L R(x,t) \delta y \, dx + E_M(\delta \phi) + E_V \delta y = 0 \]  

(Eq. 35)

corresponding to the principle of virtual work. Another interpretation of Equation (35) is that it is a condition which distributes the error over the domain of interest \((0 \leq x \leq L)\). Note the weighting terms for the boundary residuals. The moment error is weighted by the
resulting virtual rotation at \( x = L \). Considering positive virtual work:

\[
\delta \Phi = - \delta \left( \frac{\partial y^*}{\partial x} \right)_{x=L} = - \delta q \, \theta_L^* 
\]  

(Eq. 35a)

and the shear error is weighted by the virtual displacement at \( x = L \); both terms having units of work.

The weighted residual condition, Equation (35), then leads to a second order differential equation in the unknown \( q(t) \),

\[
\left\{ \mu \int_0^L \theta^2 \, dx \right\} \ddot{q} + \left\{ \int_0^L \theta'' \, dx + \int_0^L \theta' \, dx - \int_0^L \theta'' \, dx \right\} q = \left\{ \int_0^L \theta \, dx \right\} \alpha e^{-\beta t} 
\]  

(Eq. 36)

where

\[
\bar{M} = \mu \int_0^L \theta^2 \, dx 
\]  

(Eq. 37)

is the generalized mass term in the \( q \) coordinate system, and

\[
\bar{K} = EI \left\{ \int_0^L \theta''' \, dx + \theta'' \, \theta' \, dx - \theta' \, \theta'' \, dx \right\} 
\]  

(Eq. 38)

is the generalized stiffness.
The differential equation in \( q \) has the form:

\[
\ddot{q} + K q = A e^{-\beta t} \tag{Eq. 39}
\]

where

\[
A = \alpha \int_0^L \theta \, dx
\]

Solution of Equation (39) with the initial conditions

\[
q(0) = 0
\]

\[
\frac{dq(0)}{dt} = 0
\]

completes the approximate solution of the motion

\[
y^*(x,t) = q(t) \theta(x)
\]

### 4.2 Linear Beam Computer Program

Fortran IV-H level computer programs for the linear and non-linear problems were written to aid the analysis. An incremental plotter was used to plot equation residuals for evaluation of mode shape variations. Listings of the plotting routines are not included. The plotting routines were based on an XDS Sigma VII machine language character generator which undoubtedly would not be compatible with other machines. Plot data is array stored before plotting, allowing easy insertion of compatible plot statements.
The necessary input is read in by statements 200 and 206 (see Appendix A). The program was designed for ease of mode shape changes. Statement functions were used to define the mode shape and its first four derivatives. Changing the mode shape requires changing only statements 51 through 59. Shape functions normalized to 1 at x = L were used, thus q(t) and q(t) represents the displacement and velocity of the free end. Integrations were performed by Simpson's 1/3 rule \cite{17}. Refer to Appendix A for more detail on the linear program.

Referring to Equation (36), the program first performs the necessary integrations for Φ̇, Φ, and Φ̈. Note that the first term in (Φ̇q) represents the virtual work of internal error forces (WI):

\[ WI(q) = \int_0^L (EI \dfrac{d''}{dx^2} q \varphi) \, dx \quad \text{(Eq. 40)} \]

The integrand of WI is a weighted internal force residual

\[ F_{YY}(q,x) = (EI \dfrac{d''}{dx^2} \varphi)q = K_1q \quad \text{(Eq. 41)} \]

The remaining two terms are the virtual work of boundary force errors:

\[ WM(q) = (EI \left[ \varphi'' \right]_L \varphi'_L)q = K_2q \quad \text{(virtual work of boundary moment error)} \quad \text{(Eq. 42)} \]

\[ WV(q) = -(EI \left[ \varphi'' \right]_L \varphi'_L)q = K_3q \quad \text{(virtual work of boundary shear error)} \quad \text{(Eq. 43)} \]
The total virtual work of internal forces (DW) is then linear in q:

\[ DW(q) = \bar{K}q = (K_1 + K_2 - K_3)q \]  

(Eq. 44)

The solution of \( q(t) \) and \( \dot{q}(t) \) from Equation (39) is then generated:

\[ q(t) = \frac{A\alpha}{M\beta^2 + \bar{K}} \left\{ \beta \sqrt{\frac{M}{\bar{K}}} \sin \sqrt{\frac{\bar{K}}{M}} t + e^{-\beta t} \cos \sqrt{\frac{\bar{K}}{M}} t \right\} \]  

(Eq. 45)

\[ \dot{q}(t) = \frac{A\alpha}{M\beta^2 + \bar{K}} \left\{ \beta \left[ \cos \sqrt{\frac{\bar{K}}{M}} t - e^{-\beta t} \right] + \sqrt{\frac{\bar{K}}{M}} \sin \sqrt{\frac{\bar{K}}{M}} t \right\} \]  

(Eq. 46)

The maximum deflection of the free end is found by noting the end displacement (q) corresponding to the velocity (\( \dot{q} \)) being zero. The approximate solution being complete, the equation residual \( R(x,t) \), Equation (30), can be generated:

\[ R(x,t) = R_1 + R_2 - R_3 \]

where

\[ R_1 = \mu \ddot{q} \theta \]  

(inertial residual)

\[ R_2 = EI \dddot{q} \theta'' \]  

(internal force residual)

\[ R_3 = \alpha e^{-\beta t} \]  

(forcing term)
4.3 The Modified Galerkin Method Applied to the Nonlinear Problem.

Solution of the large deflection motion of a cantilever beam by the modified Galerkin method proceeds in like manner to that of the linearized problem. The integrations performed and the computer programming is somewhat more complex.

We begin with the equation of motion (21); after substitution of Equations (26), (27), and (28), it has the form:

\[- \mu \frac{d^2 y}{dt^2} - F_x(y) + \alpha e^{-\beta t} = 0 \]  

(Eq. 47)

where \( F_x(y) \) is a nonlinear differential operator operating on \( y(x,t) \).

As mentioned, a one term solution was assumed:

\[ y^*(x,t) = q(t) \theta(x) \]  

(Eq. 48)

Substitution of Equation (48) into (47) results in the equation residual:

\[ R(x,t) = -\mu \ddot{q} \theta - f(q,x) + \alpha e^{-\beta t} \]  

(Eq. 49)

where \( f(q,x) \) is a result of \( F_x(y^*) \).

The boundary force residuals are similar to Equations (31) and (32) and are obtained from Equations (19) and (25).

\[ E_M = M(y^*) \]  

(Eq. 50)

\[ E_V = V(y^*) \]  

(Eq. 51)
Similarly, the inertia force error is

\[ E_{IN} = -\mu \ddot{q} \theta \]

(Eq. 52)

and the internal force error is

\[ E_{LP} = -f(q, x) \]

(Eq. 53)

The weighted residual condition is:

\[
\int_0^L R(x,t) \delta y \, dx + E_M (\delta \phi) + E_V \delta y = 0
\]

where

\[ \delta \phi = \delta (\tan^{-1} \frac{\delta y}{L}) \]

(Eq. 53b)

\[ \delta y = \delta q \theta \]

which results in a nonlinear differential equation in \( q \)

\[
\left\{ \int_0^L \mu \theta^2 \, dx \right\} \ddot{q} + \left\{ \int_0^L f(q, x) \theta \, dx + E_M (1 + q^2 \theta^2) \right\}_L
\]

\[- E_V \theta_L \right\} = \left\{ \int_0^L \theta \, dx \right\} \alpha e^{-\beta t}
\]

(Eq. 54)

where similarly

\[ \overline{M} = \int_0^L \mu \theta^2 \, dx \]

\[ A = \alpha \int_0^L \theta \, dx \]
But note that the stiffness term is now a nonlinear function of \( q \),

Equation (54) has the form

\[
\ddot{M} q + \delta W(q) = A e^{-\beta t} \quad \text{(Eq. 55)}
\]

where

\[
\delta W(q) = \int_{0}^{L} f(q,x) \vartheta \, dx + E_M/(1 + q^2 \vartheta^2)_{L} - E_V \vartheta_L \quad \text{(Eq. 56)}
\]

with the initial conditions

\[
q(0) = \dot{q}(0) = 0 \quad \text{(Eq. 57)}
\]

Solution of Equation (55) completes the approximate solution,

\[
y^*(x,t) = q(t) \vartheta(x)
\]

The corresponding weighted internal force function is

\[
F_{YY}(q,x) = f(q,x) \vartheta \quad \text{(Eq. 58)}
\]

The virtual work of the internal force error is:

\[
W_I = \int_{0}^{L} f(q,x) \vartheta \, dx \quad \text{(Eq. 58a)}
\]

The virtual work of the boundary moment error is:

\[
W_M = M(y^*)/(1 + q^2 \vartheta^2)_{x=L} \quad \text{(Eq. 58b)}
\]

and the virtual work of the boundary shear error is:

\[
W_V = V(y^*) \vartheta \bigg|_{x=L} \quad \text{(Eq. 58c)}
\]
4.4 Computer Program for the Nonlinear Problem

4.4.1 Nonlinear Program Input and Output

Input and output of the nonlinear program follows that of the linear program except for an exact solution. Mode shape changes are made in an identical manner. Input of stress constants is required for the inverse tangent fit of the true stress-strain relation.

4.4.2 Computational Procedure of Nonlinear Beam Program

The main difference of the nonlinear program is in the computational procedure for \( q(t) \). The solution of Equation (55) required a numerical approach as a result of the nonlinearity of the virtual work term \( \delta W(q) \). Referring to Figure 6, a small interval \( \Delta q \) was chosen and \( \delta W \) was approximated by a straight line within the interval.

\[ \delta W(q) \]

\[ q_i, q_{i+1} \]

\[ \delta W_i, \delta W_{i+1} \]

\[ \gamma_i \]

\[ \Delta q \]

\[ K_1 \]

Figure 6. Approximation of Virtual Work Function
\[ \delta W(q) = K_i q + \gamma_i \quad \text{for} \quad q_i \leq q(t) \leq q_{i+1} \quad (\text{Eq. 57}) \]

where

\[ K_i = \frac{\delta W_{i+1} - \delta W_i}{\Delta q} \]

Within the interval \( q(t) \) approximately satisfies,

\[ M \ddot{q} + K_i q = A e^{-\beta t} - \gamma_i \quad \text{for} \quad q_i \leq q(t) \leq q_{i+1} \quad (\text{Eq. 58}) \]

with the initial conditions at \( q_i \):

\[ q(t_i) = q_i \]
\[ \dot{q}(t_i) = \dot{q}_i \]

The program steps through the solution, Equation (59), of \( q(t) \) in this manner stopping the calculation at the maximum deflection.

\[ q(t) = E_1 \sin \sqrt{\frac{K_i}{M}} t + E_2 \cos \sqrt{\frac{K_i}{M}} t + c_1 e^{-\beta t} + c_2 \quad (\text{Eq. 59}) \]

where

\[ c_1 = \frac{A}{M\beta^2 + K_i} \quad , \quad c_2 = -\frac{\gamma_i}{K_i} \quad \text{for} \quad q_i \leq q(t) \leq q_{i+1} \]

and \( E_1 \) and \( E_2 \) are determined from initial conditions at \( q_i \). Refer to Appendix B for more detail on the nonlinear program.
CHAPTER V: RESULTS AND CONCLUSIONS

5.1 The Mode Variation Technique

As mentioned in Chapter II, if the assumed solution chosen for the modified Galerkin method is the true solution then the residual, Equation (5), will be identically zero. It seems to follow, therefore, that the smallest residual magnitude, distributed uniformly over the interval $0 \leq x \leq L$, would denote the best approximation. The "best" refers to that which best satisfies the governing equation of motion. Four mode shape functions were chosen, three of which could be varied. All shape functions were normalized to 1 at $x = L$. The functions used were:

1) $\theta_1(x) = \frac{(x^4 - 4Lx^3 + 6L^2x^2)}{3L^4}$

which is the static deflection shape of a linearized beam;

2) $\theta_2(x) = \frac{X_1(x) + a_2 X_2(x)}{X_1(L) + a_2 X_2(L)}$

where $X_1(x)$ and $X_2(x)$ are the first two eigenfunctions, Equation (17a), of the linearized beam motion, and $a_2$ is the variation parameter;

3) $\theta_3(x) = \frac{X_1(x) + a_3 X_1^2(x)}{X_1(L) + a_3 X_1^2(L)}$

and the final shape function used was:

4) $\theta_4(x) = \frac{z_1(x) + a_4 z_2(x)}{z_1(L) + a_4 z_2(L)}$
where
\[ z_1(x) = \theta_1(x) \] (the static deflection shape)

and
\[ z_2(x) = 1 - \cos \frac{n\pi x}{2L} \]

Shape functions \( \theta_2 \), \( \theta_3 \), and \( \theta_4 \) could be varied by variation of the constants \( a_2 \), \( a_3 \), and \( a_4 \). Evaluation of the particular shape function was made by the size of the residual and the stiffness of the reduced system reflected by the magnitude of the maximum deflection.

5.2 Problem Data

The same problem data was used for all runs for uniformity of results. Experimental results were not available; therefore an arbitrary problem was chosen. Figure 7 shows the dimensions of the beam used.

![Cross Section](image)

Figure 7. Beam Dimensions
Figure 8 shows the stress-strain relation used. The modulus of elasticity of aluminum was used for the linear problem \( E = 10^7 \) psi. The nonlinear relation used had the same initial slope and an ultimate stress of 65,500 psi.

The impulse \( I \) of the blast pressure was chosen to insure deflections into the nonlinear range:

\[
I = \frac{P_{\text{max}}}{\beta} = \frac{6000}{3000} = 2 \text{ psi-sec}
\]

5.3 Results of Linear Beam Problem

Table I is a summary of maximum end deflection results for both the linear and nonlinear problems. The mode shape variation investigation proceeded as indicated by the order in the table.

First, the accuracy of the method was tested. The normalized exact first eigenfunction, \( X_1 \), of the linear beam problem was used as a shape function in the Galerkin approximation. As expected, this yielded the true first mode motion:

\[
y(x,t) = X_1(x) T_1(t) + y_p(x,t)
\]

Figure 9 compares the Galerkin results of end deflection with the true motion, illustrating the results that can be expected from a correct choice of the shape function in a one term approximation.
$\sigma = E \varepsilon$

$\sigma = \frac{1}{b} \tan^{-1} (a \varepsilon)$

$a = 257$

$b = 2.4 \times 10^{-7}$ (psi$^{-1}$)

Max. Stress $= 65,500$ psi

$E = 10^7$ psi

Figure 8. Stress-Strain Approximation
Table I. Linear-Nonlinear Comparison of Deflection Results

<table>
<thead>
<tr>
<th>Shape Function $\theta(x)$</th>
<th>Max. End Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
</tr>
<tr>
<td>$\theta(x) = \frac{X_1(x)}{X_1(L)}$</td>
<td>24.24</td>
</tr>
<tr>
<td>$\theta(x) = \frac{X_1(x) + 0.3 X_2(x)}{X_1(L) + 0.3 X_2(L)}$</td>
<td>15.36</td>
</tr>
<tr>
<td>$\theta(x) = \frac{X_1(x) + X_2(x)}{X_1(L) + X_2(L)}$</td>
<td>3.62</td>
</tr>
<tr>
<td>$\theta(x) = \frac{X_1(x) + 0.1 X_1^2(x)}{X_1(L) + 0.1 X_1^2(L)}$</td>
<td>24.36*</td>
</tr>
<tr>
<td>$\theta(x) = \frac{X_1(x) + 0.25 X_1^2(x)}{X_1(L) + 0.25 X_1^2(L)}$</td>
<td>24.12</td>
</tr>
<tr>
<td>$\theta(x) = \frac{X_1(x) + 0.5 X_1^2(x)}{X_1(L) + 0.5 X_1^2(L)}$</td>
<td>23.15</td>
</tr>
<tr>
<td>$\theta(x) = \frac{X_1(x) - 0.2 X_1^2(x)}{X_1(L) - 0.2 X_1^2(L)}$</td>
<td>22.91</td>
</tr>
<tr>
<td>$\theta(x) = \frac{x^4 - 4Lx^3 + 6L^2x^2}{3L^4}$</td>
<td>24.02</td>
</tr>
<tr>
<td>$\theta(x) = 1 - \cos \left( \frac{\pi x}{2L} \right)$</td>
<td>23.81</td>
</tr>
<tr>
<td>$\theta(x) = \frac{z_1(x) + 5z_2(x)}{z_1(L) + 5z_2(L)}$</td>
<td>24.09</td>
</tr>
<tr>
<td>$\theta(x) = \frac{z_1(x) + z_2(x)}{z_1(L) + z_2(L)}$</td>
<td>24.36*</td>
</tr>
</tbody>
</table>

* Denotes "best" shape function.
Figure 9. Approximation of True Linear Beam Motion by the Modified Galerkin Method.
The beam, though, is not being forced in a manner which results in only the first mode motion. It was then theorized that a shape function, other than $X_1(x)$, could be chosen which would give "better" results. As suggested by Anderson [12], a "better" shape function will result in a reduced system stiffness, thus resulting in larger deflections. The criterion used here to select the best shape function was to choose that which resulted in a smaller and "better" shaped equation residual. The "better residual" will be defined in this Section.

The first attempt at an improvement over $X_1$ was the selection of a linear combination of the first and second eigen functions $X_1(x)$ and $X_2(x)$, respectively.

$$\theta(x) = \frac{X_1(x) + a_2 X_2(x)}{X_1(L) + a_2 X_2(L)}$$

As the shape variation parameter $a_2$ was increased, including more of the second mode, the equation residual increased tremendously and oscillated wildly over the interval of interest: $0 \leq x \leq L$. Maximum magnitudes were of the order of $10^5$ lbs/in. The system became stiffer, resulting in smaller deflections. From the magnitude of the residual and the decreased deflections it was deduced that this combination was a poor choice and that shape functions similar to $X_1(x)$ would be more desirable.
Final deformation shapes shown by Bodner and Symonds [5] suggested that a shape having a larger curvature at the wall (x = 0) would be a better choice. A linear combination of $X_1(x)$ and $X_1^2(x)$ was chosen, namely:

$$
\theta(x) = \frac{X_1(x) + a_3 X_1^2(x)}{X_1(L) + a_3 X_1^2(L)}
$$

Figure 10 illustrates the residual changes resulting from variation of $a_3$. Note that the maximum magnitudes of the residuals have been reduced greatly over that of the previous combination. Referring back to Table I, it can be seen that variation of the parameter $a_3$ resulted, in one case ($a_3 = 0.1$), in a larger deflection than that obtained when using only $X_1(x)$, ($a_3 = 0$). All other variations of $a_3$ resulted in smaller deflections.

With the aid of Figure 10 the "better residual" can now be defined. As an approximation to the ideal case of an identically zero residual, the preferred residual is of smaller magnitude over the interval $0 \leq x \leq L$. Thus the approximate solution exhibits less error over the interval and better satisfies the governing equation of motion. When judging the relative merits of two residuals, the best is that which conforms to the definition over a greater portion of the interval.
Referring to Figure 10, consider the residuals resulting from the variations $a_3 = 0.1$ and $a_3 = 0.5$. The residual resulting from $a_3 = 0.1$ has a smaller magnitude than that of $a_3 = 0.5$ over the entire length of the beam. In keeping with our definition, the shape function with $a_3 = 0.1$ is then judged as a better approximation. Deflection results showed $a_3 = 0.1$ giving the largest deflection, which was also slightly greater than that obtained from using $X_1(x)$ or $a_3 = 0$. It can be seen from Figure 10 that the $a_3 = 0.1$ residual is of smaller magnitude over a greater portion of the interval than that resulting from $a_3 = 0$, thus again $a_3 = 0.1$ is judged best. Similar arguments can be made for each variation on $a_3$ resulting in $a_3 = 0.1$ as best.

Two shapes, which through physical considerations were chosen as being reasonable approximations, were then used giving favorable results. They were the static deflection shape

$$z_1(x) = \frac{x^4 - 4Lx^3 + 6L^2x^2}{3L^4}$$

and

$$z_2(x) = 1 - \cos \frac{\pi x}{2L}$$

Figure 11 shows their corresponding residuals.
Figure 10. Comparison of Equation Residuals of the Shape Variation

\[ \theta = x_1 + a_3 x_1^2 \] (Linear Beam).

\[ t = 0.01 \text{ sec} \]
Figure 11. Residual Averaging -- Linear Beam

\[ z_1 = \text{Static} \]

\[ \theta = z_1 + z_2 \]

\[ \theta = z_1 + 5z_2 \]

\[ z_2 = 1 - \cos \frac{\pi x}{2L} \]
A final linear combination was then used, namely:

$$\theta(x) = \frac{z_1(x) + a_4 z_2(x)}{z_1(L) + a_4 z_2(L)}$$

Referring to Figure 11, by our definition the above combination with $a_4 = 5$ is a better approximation than either $z_1$ or $z_2$ alone. Deflection results show that this combination gives a larger deflection.

Note that the residual resulting from $a_4 = 5$ is approximately 0.6 of that resulting from $z_2(x)$ alone. This suggested decreasing the amount of the cosine function in the combination $\theta(x)$. The parameter $a_4$ was decreased to 1.0, resulting in a greatly decreased residual.

Note that the combination of the shape functions resulted in an averaging of residuals in some sense. This property, if repeatable, could be used to good advantage when attempting to minimize equation residuals. Further variation of $a_4$ could result in further decrease of the residual. Again large deflection results coincided with the selection of $a_4 = 1$ being best.

It is interesting to note that a maximum end deflection of 24.36 inches resulted from two different mode shapes having differently shaped residuals. Figure 12 illustrates this point. From our definition the $z_1 + z_2$ residual is better, yet $X_1 + .1 X_1^2$ gives the same maximum deflection results. It is speculated that the latter may give poorer results for deflections at other points on the beam. Higher
order deflections such as slope or curvature may not be approximated as well, either.

5.4 Results of Nonlinear Beam Problem

In general, results of the nonlinear beam problem followed the same patterns as the linear problem, using exactly the same shape function variations. Maximum deflections occurred for different shapes as a result of the expected different deflection shape. The overall maximum deflection achieved by mode variation was that of the static deflection shape whose residual was determined as best according to the definition.

Figure 13 illustrates residuals resulting from the variation on the combination

\[ \theta(x) = \frac{X_1(x) + a_3 X_1^2(x)}{X_1(L) + a_3 X_1^2(L)} \]

for the nonlinear problem. Similarly, the variation \(a_3 = 0.5\) giving the smallest deflection resulted in the "worst" residual, while the variation \(a_1 = 0\) giving the largest deflection of this combination resulted in the "best" residual. All nonlinear deflections were larger than the linear case. This is a result of the system stiffness decreasing as deflection increased approaching the fully plastic state for the nonlinear problem. This is reflected in the decreasing slope of the nonlinear stress-strain relation used.
Figure 12. The "Better" Residual -- Linear Beam
Figure 13. Residuals of Variation on $X_1^2 + a_1 X_1^2$ -- Nonlinear Beam
The nonlinear problem exhibited the same averaging of residuals from the combination of $z_1(x)$ and $z_2(x)$ as the linear problem. This is illustrated in Figure 14. Note the small magnitude of the static shape residual which gave the largest deflection.

Introduction of the second eigenfunction $X_2(x)$ similarly gave small deflections and residuals of large magnitude.

5.5 Conclusions and Further Studies

From comparison of the approximate solution of the linearized beam deflection to the true motion (Figure 9), it appears that the one term modified Galerkin solution has merit. Selection of a shape function similar to the first mode shape is preferred. Inclusion of a shape similar to the second mode shape is not. This corresponds to forcing a deflection in the second mode at a frequency near the first natural frequency requiring a large amount of energy, thus increasing the stiffness of the reduced system.

In both the linear and nonlinear cases, separately, differences in deflection results were small between shape functions judged best. Refer to Table I. This suggests that the method is a stable one where reasonable shapes will give reasonable answers.

Comparison of the deflection results presented here should be made to experimental results. It would be a simple task to modify the computer programs to the experimental beam dimensions and cross-
Figure 14. Residual Averaging -- Nonlinear Beam
sectional geometry. The "better residual" criterion presented may give more dependable analytic results.

The method presented for selection of a best shape function by the preferred residual definition also appears to have merit. The averaging property of the system residual found in this investigation could prove to be valuable if this is a general property. This suggests a technique to reduce equation residuals. Two shape functions which resulted in reasonably flat residuals, one of positive value and the other negative, must first be found. These shapes could then be combined in some manner to reduce the residual, as illustrated in Figure 11. The large deflections resulting from shape functions judged best corresponds to the previously implied shape function criterion. That is, the best shape function gives the largest deflection. This correspondence strengthens the merit of both approaches.

The similarity of residual properties and deflection results between the linear and nonlinear cases suggests an approach to solving nonlinear system models. First investigate a linearized version which in general will have an exact solution available. Solve the linear problem by the modified Galerkin method noting the shape functions which give the best results. Use these same functions for a modified Galerkin solution of the nonlinear system. If both solutions exhibit similar properties, as did the beam problems, it does not seem unreas-
onable to assume that the solution of the nonlinear system will possess the same merit.

The shape function selection technique presented should be investigated further to determine its merits in different problems. Other preferred residual definitions could prove to be more desirable. Possibly the minimization of area under the absolute value of the residual is more significant. Another possibility is minimizing the area under the square of the residual. This corresponds to the method of Least Squares (see [10]) which is difficult to apply in a nonlinear case.
APPENDIX A

A-1. Flow Chart of Linear Beam Program.

1. Define Mode Shape and Derivatives (51-59)
2. Read Input Data (200-206)
3. Perform Integrations for M and A (222-253)
4. Generate Internal Force Residual FYY (255-316)
5. Perform Integration for $\bar{K}$ (355-410)
6. Generate $DW(Q) = \bar{K}Q$ (449-490)
7. Generate $q$, $\dot{q}$, $\ddot{q}$ (505-543)
8. Generate Residual $R(x,t)$ (570-675)
9. Plot Mode Shape $\Theta(x)$ and Blast Pressure (705-1026)
10. Calculate Exact Solution of End Deflection (1100-1430)
### Linear Program Nomenclature

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>Augmented Coefficient Matrix, Exact Solution</td>
</tr>
<tr>
<td>ALPHA</td>
<td>$\alpha = P_{\text{max}} D$</td>
</tr>
<tr>
<td>BETA</td>
<td>$\beta = \text{Blast Decay Constant}$</td>
</tr>
<tr>
<td>BL</td>
<td>$L = \text{Beam Length (inches)}$</td>
</tr>
<tr>
<td>BM</td>
<td>$\bar{M} = \text{Generalized Mass}$</td>
</tr>
<tr>
<td>C</td>
<td>$2C = \text{Height of Beam Cross-section (inches)}$</td>
</tr>
<tr>
<td>CAPX</td>
<td>Exact Eigenfunctions $X_n(x)$</td>
</tr>
<tr>
<td>D</td>
<td>$D = \text{Thickness of Beam (inches)}$</td>
</tr>
<tr>
<td>DD</td>
<td>Coefficients of Particular Solution</td>
</tr>
<tr>
<td>DT</td>
<td>Time Increment (sec)</td>
</tr>
<tr>
<td>DQ</td>
<td>Deflection Increment (inches)</td>
</tr>
<tr>
<td>DEF</td>
<td>Exact End Deflection</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of Elasticity</td>
</tr>
<tr>
<td>EP</td>
<td>$\varepsilon = \sqrt{\frac{2}{\pi}} \sqrt{\frac{\mu \beta^2}{EI}}$</td>
</tr>
<tr>
<td>EPL</td>
<td>$\varepsilon(L) = L$</td>
</tr>
<tr>
<td>P</td>
<td>$P = A = \alpha \int_0^L \theta , dx$</td>
</tr>
<tr>
<td>PT</td>
<td>Blast Pressure Function</td>
</tr>
<tr>
<td>PMAX</td>
<td>Maximum Blast Pressure</td>
</tr>
<tr>
<td>QT</td>
<td>End Displacement (inches)</td>
</tr>
<tr>
<td>QDT</td>
<td>End Velocity (in/sec)</td>
</tr>
<tr>
<td>QDDT</td>
<td>End Acceleration (in/sec$^2$)</td>
</tr>
</tbody>
</table>
A-2. Linear Program Nomenclature (continued)

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHO</td>
<td>Material Density (lbs/in$^3$)</td>
</tr>
<tr>
<td>SK</td>
<td>$\bar{K}$ = Generalized Stiffness</td>
</tr>
<tr>
<td>THETA, Y</td>
<td>$\theta(x), y(x) = q(t) \theta(x)$</td>
</tr>
<tr>
<td>THETA1, YX1</td>
<td>$\theta', y'$</td>
</tr>
<tr>
<td>THETA2, YX2</td>
<td>$\theta'', y''$</td>
</tr>
<tr>
<td>THETA3, YX3</td>
<td>$\theta''', y'''$</td>
</tr>
<tr>
<td>THETA4, YX4</td>
<td>$\theta''', y'''$</td>
</tr>
<tr>
<td>U</td>
<td>$\mu = \text{Linear Beam Density (lbs/in)}$</td>
</tr>
<tr>
<td>X</td>
<td>Beam Length Coordinate (inches)</td>
</tr>
<tr>
<td>YP</td>
<td>Exact Particular Solution</td>
</tr>
</tbody>
</table>
A-3. Linear Program Listing

C MAXIMUM DEFLECTION OF A CANTILEVER BEAM UNDER BLAST LOAD
C MODIFIED GALERKIN METHOD USED WITH LINEAR STRESS-STRAIN RELATION
C AND SMALL DEFLECTION GEOMETRY.

DIMENSION XA(50),YA(50),YAA(50),ZYAA(50),YA(500),YB(500),AA(4,5),DD(4)

REAL BLX,YA(YA),YAA(YA),ZYAA(YA),YB(YB),AA(AA),DD(DD)

CZ1(BLX)*=(X**4+4*BLX*X**3+6*BLX**2*X+2*X**2)/3/BLX**4
CZ11(BLX)*=(X**3+3*BLX*X**2+3*BLX**2*X+1)/BLX**4
CZ12(BLX)=4*(X-BLX)**2/BLX**4
CZ13(BLX)=8*(X-BLX)/BLX**4
CZ14(BLX)=(8/9L**4)*(X+1-X)
CZ2(BLX)=1-8*1415923*X/2/BLX)
CZ21(BLX)=(3*1415923/2/BLX)*SIN(3*1415923*X/2/BLX)
CZ22(BLX)=(3*1415923/2/BLX)*COS(3*1415923*X/2/BLX)
CZ23(BLX)=COS(3*1415923*X/2/BLX)
CZ24(BLX)=COS(3*1415923*X/2/BLX)

THETA(BLX)=CZ1(BLX)*0.50*CZ2(BLX))

THETA2(BLX)=(CZ12(BLX))*0.50*CZ22(BLX)

THETA3(BLX)=(CZ13(BLX)*0.50*CZ23(BLX)

THETA4(BLX)=(CZ14(BLX)*0.50*CZ24(BLX)

60 PT=PMAK*(BETA+T)*PMAX*EXP(-BETA+T)
70 CPRXGZX)=COS(SQRT(Z)*)&COSH(SQRT(Z))&X=SIN(SQRT(X-X))&=

100 Y(Q,6L,X)=Q*THETA(BLX)
105 YX1(Q,6L,X)=Q*THETA2(BLX)
110 YX2(Q,6L,X)=Q*THETA3(BLX)
115 YX3(Q,6L,X)=Q*THETA4(BLX)
120 YX4(Q,6L,X)=Q*THETA5(BLX)

125 F(E,C,D,BLX)=2*E*D*X**3*THETA4(BLX)+THETA(BLX)/3
130 C1(P,ALPHA,BM,BETA,SK)=ALPHA/(BM+BETA**2+SK)
190 QT(P,ALPHA,BM,BETA,SK,T)=C1(P,ALPHA,BM,BETA,SK)*T

192 QDT(P,ALPHA,BM,BETA,SK,T)=C1(P,ALPHA,BM,BETA,SK)*T

195 QDDT(P,ALPHA,BM,BETA,SK,T)=C1(P,ALPHA,BM,BETA,SK)*T

198 PHI(EP,D1,D2,D3,D4,C0,X)=EXP(EP*X),(D1*E*E*(EP*X)+D2+COS(EP*X))

199 YP1(EP,D1,D2,D3,D4,C0,BETA,X,T)=PHI(EP,D1,D2,D3,D4,C0,X)*EXP(-BETA+T)

200 READ(105,201) E,C,D,BL,RHO,PMAX,BETA
201 FORMAT(F7.4)

C N=EVEN NUMBER OF INTEGRATION STEPS -- J=NO DATA POINTS IN Q

205 IF(E) 2000,2000,266
206 READ(105,207) N,J
207 FORMAT(B1S)
208 PT=-3*1415927
210 CALL 'P/init

C PLAT INTERNAL FORCE FUNCTION FYY(Q,X)

211 CALL AXIS(1,10,150.,200.,300.,6.5,4.5)
212 WRITE(108,213)
213 FORMAT(1H1,1H2X/28X,1HMAXIMUM DEFLECTION OF A CANTILEVER 1BEAM/",2X/28X,1H1BYI DANIEL M. KOZLUTZKA/34X,AEROSPACE & MECHANICAL E
2NGINEERING DEPT./1.34X,MONTANA STATE UNIVERSITY/)
214 WRITE(108,216) BL,CD,E,RHO,PMAX,BETA
215 FORMAT(1H1,1H2X/23X,1HBEAM LENGTH ********* BL="F10
1.4/4.0X,1HHALF BEAM HEIGHT ***** C="F10.4/4.0X,1HBEAM R1DTR "********
2X,"D="F10.4/4.0X,1HMODULUS OF ELASTICITY, E="F10.4/4.0X,1HMATERIA

C
3L DENSITY 
RHE=1, F7.4,40X, MAX BLAST PRESSURE = 1, F8.2/40 
4X, BLAST DECAY CONST = !, F8.2 //////////

222 D=0.0001
227 NP1=K+1
230 H=BL/X
235 X1=D*
240 X=BL/C
241 QMAX=BL
242 BM=H*(THETA(BL,X1)**2+THETA(BL,XN)**2)/3
243 P=H*(THETA(BL,X1)+THETA(BL,XN))/3
244 DB 249 K=1/2
245 N=K+1
246 D6=249 L=K,NK+2
247 X=H
248 P=H*(THETA(BL,X)**2)/3
249 BM=BM+H*THETA(BL,X)**2/3/K
250 DQ=GMAX/J
252 J1=1+1
253 BM=C*D+H*BM/16*085/12*
254 ALPHA=PMAX=Q
255 DB 316 L=13
256 IF(L=1) 200,270,265
257 IF(L=2) 200,280,290
270 Q=DQ
275 G=70 295
280 Q=(J/6)*DG
285 G=70 295
290 Q=(J/4)*DG
295 D6 310 I=1, NP1
300 XA(I)=(I-1)*H
305 YA(I)=Q*(E,C,D,Bl,XA(I))
310 WRITE(108,315) XA(I),YA(I),Q
312 CALL PLOTS(1,1,RP10+0,150,E=200,800,6,5,4,5,XA,YA)
313 IF(L=1) 200,316,314
314 LKK=K+1
315 CALL PLOTS(LKK,1,1,RP10+0,150,E=200,800,6,5,4,5,XA,YA)
316 CONTINUE
318 FORMAT(10X,1X,E12.5,10X,1FYY=1E12.5,10X,1Q=1E12.5)>
350 WRITE(108,351)
351 FORMAT(10H1)
355 SK1=H*(F(E,C,D,Bl,X1)+F(E,C,D,Bl,XN))/3
356 DB 380 K=1/2
365 NK=K+K
370 D6 380 L=K,NK+2
375 X=H
380 SK1=SK1+4*H*(E,C,D,Bl,X)/3/K
400 SK2=2*E*D*C**3*THETA2(BL,Bl)*THETA1(Bl,Bl)/3
405 SK3=2*E*D*C**3*THETA3(BL,Bl)*THETA1(Bl,Bl)*(-3)
410 SK=SK1+SK2+SK3
415 WRITE(108,416) SK,SK2,SK3,SK
416 FORMAT(10X,K1=1,E12.5,10X,K2=1,E12.5,10X,K3=1,E12.5,10X,K=1,E12.5)
418 WRITE(108,419) SK,SK2,SK3,SK
419 FORMAT(5X,14=1,E12.5,14=1,E12.5,14=1,E12.5,14=1,E12.5)
449 JJ2=J/2+1
450 D9 480 I=1, JJ2
455 XA(I)=(I-1)*D2
CALL AXIS(1,0.0,75.0,0.0,75000.0,6.5,4.5)
CALL PLOTS(1,1.0,75.0,0.0,75000.0,6.5,4.5,XA,YA)
WRITE(10,351)

DO 675 L=1,IAPl
   IF(L-1) 200,590,580
      T=50-DT
   580   YMIN=4000
   590   YMAX=16000
   595   G3=TE 615
   600   T=40-CDT
   605   YMIN=4000
   610   YMAX=16000
   615   G3=TE 615
   620   T=100-CDT
   625   YMIN=4000
   630   YMAX=16000
   635   G3=TE 615
   640   T=500-CDT
   645   YMIN=4000
   650   YMAX=16000
   655   GD=645 I=1,IAPl
   660   XA(I)=I-1+H
   665   YA(I)=RHO*COS(THECTOR(BL),XA(I))=QCDT(P,ALPHA,BM,BETA,SK,T)
   670   YYA(I)=RHO*SINC(THECTOR(BL),XA(I))=QCDT(P,ALPHA,BM,BETA,SK,T)
   675   YYYA(I)=RHO*SINC(THECTOR(BL),XA(I))=QCDT(P,ALPHA,BM,BETA,SK,T)
   680   WRITE(108,351)
   685   DO 725 I=1,IAPl
      XA(I)=(I-1)-H
   700   CONTINUE
   705   WRITE(108,351)
   710   XA(I)=I-1+H
   720   WRITE(108,351)
   725   XA(I)=I-1+H
720 YA(I)=THETA(BL,XA(I))
725 WRITE(108,726) XA(I),YA(I)
726 FORMAT(10X, 'XA(I),YA(I)'
730 CALL AXII(I,0.0,150.,0.0,1.25,6.5,4.5)
731 CALL PLOTS(I,I,110.0,150.0,0.0,1.25,6.5,4.5,XA,YA)
1000 DXA=0.002/50.
1005 DB=1020 I=I+1
1010 XA(I)=(I-1)*DXA
1015 YA(I)=P(T)(PHAX,BETA,XA(I))
1020 WRITE(108,1021) I,XA(I),YA(I)
1021 FORMAT(10X, 'X',I2,'E12.5,10X,'THETA',I2,'E12.5)
C '  ■ '  PLBT'PRESS
C '  ■ '  D=
PLBT'PRESS
U=CD*RHO/193.02
AA(1,1)=ALPHA/U/BETA**2
EP=0.5*D.5*(3*U*BETA**2/2/E/D/C**3)**0.25
EPL=EP*BL
EX=EXP(EPL)
EXM=EXP(-EPL)
CS=CS(EPL)
S=SN(EPL)
AA(3,1)=EX*CS
AA(3,2)=EX*SN
AA(3,3)=EX*CS
AA(3,4)=EX*SN
AA(3,5)=0.
AA(4,1)=EX*(CS*SN)
AA(4,2)=EX*(SN*CS)
AA(4,3)=EX*(SN*CS)
AA(4,4)=EX*(CS*SN)
AA(4,5)=0.
CO=AA(2,5)
1130 CALL SOLTN(AA,CO,DD,4,5)
C '  ■ '  FIRST FIND COEFFICIENTS OF PARTICULAR SOLUTION
C 'PRINT COEFFICIENTS OF PARTICULAR SOLUTION & THE PARTICULAR
C ' SOLUTION EVALUATED AT THE BEAM-END'
1135 WRITE(108,1136)
1136 FORMAT(1H1,///20X,'PARTICULAR SOLUTION!///)
1140 DO 1141 I=1,4
1141 WRITE(108,1142) I,DD(I)
1142 FORMAT(10X, 'I',I2, 'I',I2, 'E12.5)
1150 WRITE(108,1136)
1160 DO 1175 I=1,175
1175 WRITE(108,1176) T,YPL
1176 FORMAT(10X, 'T',I2,'E12.5,10X,YP(BL,T),E12.5)
C CALCULATE NATURAL FREQUENCIES OF HOMOGENEOUS SOLUTION

1220 IF(NV-1) 1500,1210,1215
1210 ZO=1.9
1211 GE T6 123C
1215 IF(NV=2) 1500,1220,1225
1220 ZO=4.67
1221 GE T6 1230
1225 ZO=Z(NN-1)+PI
1230 IC=0
1235 ZN=ZO+(COS(ZO)+1/COSH(ZO))/(SIN(ZO)+SINH(ZO)/COSH(ZO))*E)
1240 IC=IC+1
1245 DZ=ABS(ZN-ZO)
1250 IF(DZ-105) 1265,1266,1265
1255 IF(ZN>12) 1260,1265
1260 ZO=ZN
1265 GE T6 1235
1266 ZNN=ZN
1270 DZN=Z(NN)=Z(NN-1)
1275 W(NN)=SORT(2D+D**3/E/3/U)*(Z(NN)/BL)**2
1280 G(NN)=(W(NN)**2/2/E/D/E**3)**0.25
1285 WRITE(108,1286) NN,W(NN),NN,HN,HN,ZN,DZ,DZN
1286 FORMAT(2X,1R1('i',12,1),E12.5,5X,'f1('i',12,1)-',E12.5).....
1290 WRITE(108,1301)
1300 FORMAT(1H1///20X,'HOMOGENEOUS SOLUTION COEFFICIENTS (ic)'///
1305 DO 1355 NN-1,50 ....................... 
1310 FF1=PFI(EP,DD(1),DD(2),DD(3),DD(4),CO,0,0)*CARX(Z(NN),G(NN),0,0)
1315 FF2=PFI(EP,DD(1),DD(2),DD(3),DD(4),CO,BL)*CAPX(Z(NN),G(NN),BL)**E/3 
1320 DO 1345 X-I2 .............
1325 NK=NN
1330 DO 1345 L=K,NK,2
1335 X=LN/X
1340 FF1=FF1*E-PFI(EP,DD(1),DD(2),DD(3),DD(4),CO*X)*CAPX(Z(NN),G(NN),X)**E/3 
1345 FF2=FF2*E*CAPX(Z(NN),G(NN),X)**2/E/3 
1350 FF(NN)=FF1/FF2
1355 WRITE(108,1356) NN,FF(NN)
1356 FORMAT(2X,1R1('i',12,1)-',E12.5/3)
1360 WRITE(108,1361)
1361 NT=I
1365 NT=I
1370 WRITE(108,1391) NT
1391 FORMAT(1H1///10X,'DEFLECTION OF BEAM END USING 10X,20X,30X,40X,50 TERMS'/
1395 DO 1430 I=1,5
1380 D=1.430
1385 NT=I
1390 NT=I
1391 FORMAT(1H1///10X,'DEFLECTION OF BEAM END USING 10X,20X,30X,40X,50 TERMS'/
1395 D=1.425 L=1,250
1400 T=(L-1)*DT
1401 XA(L)=T
1405 DO 1400 DEF=0.0
1410 D=1.425 J=1,NT
1415 DEF=DEF+FF(J)*CAPX(Z(J),G(J),BL)*COS(W(J)*T)+BETA*SIN(W(J)*T)/H(J)
1420 YA(J)=DEF
1425 WRITE(108,1426) T,DEF
1426 FORMAT(2X,1R1('i',12,1)-',E12.5,10X,1DEF=1'E12.5)
SOLUTION OF SIMULTANEOUS EQUATIONS BY GAUSSIAN ELIMINATION

N = NUMBER OF SIMULTANEOUS EQUATIONS
M = NUMBER OF COLUMNS IN THE AUGMENTED MATRIX
L = NUMBER OF EQUATIONS
A(I,J) = ELEMENTS OF THE AUGMENTED MATRICES
I = MATRIX ROW NUMBER
J = MATRIX COLUMN NUMBER
JJ = TAKES ON VALUES OF THE ROW NUMBERS WHICH ARE POSSIBLE PIVOT ROWS, EVENTUALLY TAKING ON THE VALUE IDENTIFYING THE ROW HAVING THE LARGEST PIVOT ELEMENT
BIG = TAKES ON VALUES OF THE ELEMENTS IN THE COLUMN CONTAINING POSSIBLE PIVOT ELEMENTS, EVENTUALLY TAKING ON THE VALUE OF THE ELEMENT USED
TEMP = TEMPORARY NAME USED FOR THE ELEMENTS OF THE ROW SELECTED TO BECOME THE PIVOT ROW, BEFORE THE INTERCHANGE IS MADE
K = INDEX OF A DO LOOP TAKING ON VALUES FROM 1 TO N-1. IT IDENTIFIES THE COLUMN CONTAINING POSSIBLE PIVOT ELEMENTS
KPI = K + 1
AB = ABSOLUTE VALUE OF A(I,K)
QUOT = QUOTIENT A(I,K)/A(K,K)
X(J) = UNKNOWNS OF THE SET OF EQUATIONS BEING SOLVED
SUM = SUMMATION OF A(I,J)*X(J) FROM J=1 TO N
IP1 = INDEX OF A DO LOOP TAKING ON VALUES FROM 1 TO N-1

SUBROUTINE SOLTN(A,X,N,M)
DIMENSION A(N,M),X(N)

DO 12 K=1,L
    JJ=K
    BIG=ABS(A(K,K))
    KPI=K+1
    DO 7 J=KPI,N
        AB=ABS(A(I,J))
        IF (BIG>AB) 6,7,7
    6 BIG=AB
    JJ=I
    CONTINUE
    IF (JJ=K) GOTO 10
    DO 9 J=K+1,M
        TEMP = A(J,J)
        A(J,J)=A(K,J)
    9 A(K,J)= TEMP
    DO 11 I=KPI,N
        QUOT = A(I,K)/A(K,K)
    11 A(I,J)=A(I,J)*QUOT*A(K,J)
    DO 12 I=KPI,N
        A(I,K)=0
    12 X(N)=A(N,M)/A(N,N)
    DO 14 NN=1,L
        SUM=0
        I=NN
        IP1=I+1
        DO 13 J=IP1,N
            SUM=SUM+A(I,J)*X(J)
    13 X(I)=(A(I,M)*SUM)/A(I,I)
    RETURN
END
1429 CALL AXIS(1.0:0.0:0.0:0.0:50.0:5.6:5.4:5)
1430 CALL PLOTS(1.0:250.0:0.0:0.0:50.0:5.6:5.4:5:XA,YA)
1500 GO TO 200
2000 END

SUBPROGRAMS

<table>
<thead>
<tr>
<th>BFIPIN</th>
<th>BFISL</th>
<th>BFIFN</th>
<th>BFIFF</th>
<th>COS</th>
<th>SIN</th>
<th>EXP</th>
<th>PINIT</th>
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PROGRAM ALLOCATION

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<tr>
<th>EOC+0</th>
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<th>E0D+0</th>
<th>E0E+0</th>
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<th>E10+0</th>
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<tr>
<td>E11+0</td>
<td>PMAX</td>
<td>E12+0</td>
<td>E13+0</td>
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<td>E14+0</td>
<td>J</td>
<td>E15+0</td>
<td>X1</td>
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<td>E16+0</td>
<td>DT</td>
<td>E17+0</td>
<td>NP1</td>
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<td>H</td>
<td>E19+0</td>
<td>X1</td>
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<td>E1C+0</td>
<td>BM</td>
<td>E1D+0</td>
<td>P</td>
<td>E1E+0</td>
<td>K</td>
<td>E1F+0</td>
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<td>E20+0</td>
<td>L</td>
<td>E21+0</td>
<td>X</td>
<td>E22+0</td>
<td>DQ</td>
<td>E23+0</td>
<td>J1</td>
<td>E24+0</td>
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<td>E25+0</td>
<td>J</td>
<td>E26+0</td>
<td>I</td>
<td>E27+0</td>
<td>LKK</td>
<td>E28+0</td>
<td>Sk1</td>
<td>E29+0</td>
</tr>
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<td>SK3</td>
<td>E2B+0</td>
<td>SK</td>
<td>E2C+0</td>
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<td>E31+0</td>
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<td>E32+0</td>
<td>YMAX</td>
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<td>IC</td>
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<td>ZN</td>
<td>E41+0</td>
<td>DZ</td>
<td>E42+0</td>
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<td>F1</td>
<td>E44+0</td>
<td>F2</td>
<td>E45+0</td>
<td>NT</td>
<td>E46+0</td>
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<td>E47+0</td>
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<td>1038+0</td>
<td>YA</td>
<td>122F+0</td>
<td>YYA</td>
<td>1423+0</td>
<td>YYYA</td>
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<td>AA</td>
<td>181F+0</td>
<td>DD</td>
<td>1823+0</td>
<td>W</td>
<td>1855+0</td>
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PROGRAM SIZE 13EB

PROGRAM END
### Appendix B

#### B-1. Nonlinear Program Nomenclature

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<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Stress Constant $a$, Eq. (23)</td>
</tr>
<tr>
<td>APC</td>
<td>Argument of $\tan^{-1} a$ (argument of Eq. (23))</td>
</tr>
<tr>
<td>ALPHA</td>
<td>$\alpha = P_{\max} D$, Eq. (15)</td>
</tr>
<tr>
<td>B</td>
<td>Stress Constant (psi$^{-1}$), Eq. (23)</td>
</tr>
<tr>
<td>BMX1</td>
<td>$\frac{\partial M}{\partial x}$, Eq. (26)</td>
</tr>
<tr>
<td>BMXL</td>
<td>$\frac{\partial M}{\partial x}$ with series for $\tan^{-1}$</td>
</tr>
<tr>
<td>BMX2</td>
<td>$\frac{\partial^2 M}{\partial x^2}$, Eq. (27)</td>
</tr>
<tr>
<td>BMX2S</td>
<td>$\frac{\partial^2 M}{\partial x^2}$, with series for $\tan^{-1}$</td>
</tr>
<tr>
<td>BMML</td>
<td>Moment at $x = L$</td>
</tr>
<tr>
<td>BMMS</td>
<td>Moment with series for $\tan^{-1}$</td>
</tr>
<tr>
<td>BETA</td>
<td>Blast Decay Constant (sec$^{-1}$)</td>
</tr>
<tr>
<td>BL</td>
<td>Beam Length (in.)</td>
</tr>
<tr>
<td>C</td>
<td>$2C =$ Beam Height (in.)</td>
</tr>
<tr>
<td>Cl, C2</td>
<td>Constants of Eq. (59)</td>
</tr>
<tr>
<td>D</td>
<td>Beam Thickness (in.)</td>
</tr>
<tr>
<td>DT</td>
<td>Time Increment</td>
</tr>
<tr>
<td>DW</td>
<td>Virtual Work, Eq. (56)</td>
</tr>
<tr>
<td>DX</td>
<td>Beam Length Coordinate Increment, (in.)</td>
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<tr>
<td>DQ</td>
<td>Displacement Increment, (in.)</td>
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<tr>
<td>DWN &amp; DWN1</td>
<td>$\delta W_i$ &amp; $\delta W_{i+1}$, Fig. 6</td>
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<tr>
<td>E1, E2</td>
<td>Constants of Eq. (59)</td>
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<td>Variable Name</td>
<td>Description</td>
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<tr>
<td>---------------</td>
<td>-------------</td>
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<tr>
<td>FYY</td>
<td>Weighted Internal Force, Eq. (58)</td>
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<tr>
<td>FIN</td>
<td>Internal Force Error, Eq. (53)</td>
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<tr>
<td>GN</td>
<td>Straight Line Intercept, Fig. 6</td>
</tr>
<tr>
<td>$P$</td>
<td>$P = \int_0^L \theta , dx$</td>
</tr>
<tr>
<td>PHI</td>
<td>Deflection Angle $\theta$, Fig. 3</td>
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<tr>
<td>PHI1</td>
<td>$\phi / x$</td>
</tr>
<tr>
<td>PHI2</td>
<td>$\delta^2 \phi / \delta x^2$</td>
</tr>
<tr>
<td>PHI3</td>
<td>$\delta^3 \phi / \delta x^3$</td>
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<tr>
<td>PHIL</td>
<td>$\delta \phi$, Eq. (53b)</td>
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<tr>
<td>PT</td>
<td>Blast Pressure Function (psi), Fig. 2</td>
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<tr>
<td>QNT</td>
<td>Free End Deflection (in.)</td>
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<tr>
<td>QDNT</td>
<td>Free End Velocity (in/sec)</td>
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<tr>
<td>QDDT</td>
<td>Free End Acceleration (in/sec^2)</td>
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<tr>
<td>QDOTO</td>
<td>Initial Velocity</td>
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<tr>
<td>QO</td>
<td>Initial Displacement</td>
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<tr>
<td>RHO</td>
<td>Material Density (lbm/in^3)</td>
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<tr>
<td>RES</td>
<td>Equation Residual, Eq. (49)</td>
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<tr>
<td>T</td>
<td>Time (sec)</td>
</tr>
<tr>
<td>THETA, Y</td>
<td>$\theta(x), y^*(x,t)$</td>
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</table>
### Nonlinear Program Nomenclature (continued)

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
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<tbody>
<tr>
<td>THETA1, YX1</td>
<td>$\theta^{'}, y^{*}$</td>
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<tr>
<td>THETA2, YX2</td>
<td>$\theta^{''}, y^{*''}$</td>
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<tr>
<td>THETA3, YX3</td>
<td>$\theta^{''''}, y^{*''''}$</td>
</tr>
<tr>
<td>THETA4, YX4</td>
<td>$\theta^{'''''}, y^{*'''''}$</td>
</tr>
<tr>
<td>U</td>
<td>Linear Density (lbsm/in)</td>
</tr>
<tr>
<td>WI</td>
<td>Internal Work, Eq. (58a)</td>
</tr>
<tr>
<td>WM</td>
<td>Boundary Moment Work, Eq. (58b)</td>
</tr>
<tr>
<td>WV</td>
<td>Boundary Shear Work, Eq. (58c)</td>
</tr>
</tbody>
</table>
B-2. Nonlinear Program Listing

MAXIMUM DEFLECTION OF A CANTILEVER BEAM ASSUMING LARGE DEFLECTIONS AND A NONLINEAR STRESS-STRAIN RELATIONSHIP.

DIMENSION XA(500),YA(500),XXA(500),YXA(500),XB(500),YB(500),

C MAXIMUM DEFLECT IGN OF A CANTILEVER BEAM ASSUMING LARGE DEFLECTIONS' AND A NONLINEAR STRESS-STRAIN RECATJON.

DIMENSION XA(500),YA(500),XXA(500),YXA(500),XB(900),YB(500),

CZ1(BL,X) = (X**2*4*BL**2*X**3 + 6*BL**2*X**2)/3/BL**4
CZ11(BL,X) = (X**3*3*BL**2*X**2 + 2*BL**2*X)/4/3/BL**4
CZ12(BL,X) = (X**2*3*BL**2*X**2)/2/BL**4
CZ13(BL,X) = 8*(X**8*L)/BL**4
CZ14(BL,X) = (8/BL**4)*(X + 1.5*L)

THETA(BL,X) = (CZ1(SC,X) + 0.50*CZ2(BL,X))/(V 0.56)
THETA1(BL,X) = (CZ11(SC,X) + 0.50*CZ21(BL,X))/(V 0.56)
THETA2(BL,X) = (CZ12(SC,X) + 0.50*CZ22(BL,X))/(V 0.56)
THETA3(BL,X) = (CZ13(BL,X) + 0.50*CZ23(BL,X))/(V 0.56)
THETA4(BL,X) = (CZ14(BL,X) + 0.50*CZ24(BL,X))/(V 0.56)

PT(PFA*X,BETA,T) = PMAX*EXP((-BETA*T)

100 Y(P1,BL,X) = Y(Q,BL,X) + 0*THETA(BL,X)
105 YX1(Q,BL,X) = Q*THETA2(BL,X)
110 YX2(Q,BL,X) = Q*THETA3(BL,X)
115 YX3(Q,BL,X) = Q*THETA4(BL,X)
120 YX4(Q,BL,X) = Q*THETA(BL,X)
125 PRI(Q,BL,X) = ATAN(YX1(Q,BL,X))
130 PH1(Q,BL,X) = YX1(Q,BL,X)**2
135 PH11(Q,BL,X) = YX1(Q,BL,X)**2
140 PH12(Q,BL,X) = YX1(Q,BL,X)**2
145 PH13(Q,BL,X) = YX1(Q,BL,X)**2

150 BMX1(Q,BL,X,A,B,C,D) = 2*D*A*C**2*PHI2(Q,BL,X)**2/R(A,PHI)/B

170 THETA1(BL,BL) = THETA(BL,BL)
175 PHII(0,BL) = THETA1(BL,BL)/(1 + Q*Q*THETA1(BL,BL))
180 WM(Q,BL,X,A,B,C,D) = VM(Q,BL,X,A,B,C,D) = THETA(BL,BL)
185 WM(Q,BL,X,A,B,C,D) = BMX(Q,BL,X,A,B,C,D) = PHI(Q,BL)
190 SKN(DWN1,DWN,DQ) = (DWN1 - DWN)*(SKN(DWN1,DWN,DQ)
187 GN(DWN1,DWN,DQ) = GN(DWN1,DWN,DQ) = SKN(DWN1,DWN,DQ)
188 CI(P,ALPHA,BH,BETA,DWN1,DWN,DQ) = P*ALPHA/(BM + BETA**2)*SKN(DWN1,DWN,DQ)
189 CE(DWN1,DWN,DQ) = GN(DWN1,DWN,DQ) / SKN(DWN1,DWN,DQ)
190 E1(DWN1,DWN,DQ) = BM + TN + DT + P*ALPHA + BETA = SIN(SORT) + SKN(DWN1,DWN,DQ)
-68-

1DO1/BM1*TY) = (2Q1 = C1(P, ALPHA, BM, BETA, DWNI, DWN2, DOQ) * EXP(-BETA * N) * C2(D
2Q1 = DWN2, DWN3, DO1, QO, BM, BM, BM, DWNI, DWN3, DOQ) * 1 = (2Q1 = C1(P, ALPHA, BETA, DWNI, DWN3, DOQ) * EXP(-BETA * N) +
3) / BM1 * TY) = (QD0 TO + BETA * C1(P, ALPHA, BM, BETA, DWNI, DWN3, DOQ) * EXP(-BETA * N)
4))

192 FIN(Q, BL, X, A, B, C, D) =
(B'MX2(Q, BL, X, A, B, C, D) * COS(PHI(Q, BL, X))
1) * 2 + BMX1(Q, BL, X, A, B, C, D) * PHI1(Q, BL, X) * SIN(2 * PHI(Q, BL, X))

193 BMS(3, C, D, APC) = 2 * D * C * 2 / B * (APC / 3 - APC * 3 / 15 + APC * 1 / 35 + APC / 7 / 63
1 + APC / 9 / 99)

194 BMS1(Q, BL, X, A, B, C, D, APC) = 2 * A * C * 3 * D * PHI2(Q, BL, X) / B / (16 / 3 - APC * 2 / 5
1 + APC * 8 / 7 / 9 + APC * 8 / 9)

195 FIN(0, BL, X, A, B, C, D, APC) = BMX2(Q, BL, X, A, B, C, D, ARC) / COS(PHI(Q, BL, X))
1) * 2 + BMX1(Q, BL, X, A, B, C, D, APC) * PSI1(Q, BL, X) * SIN(2 * PHI(Q, BL, X))

196 WMS(Q, BL, X, A, B, C, D, APC) = BMS1(Q, BL, X, A, B, C, D, APC) * COS(PHI(Q, BL, X))
1 * PHI2(Q, BL, X)

197 WMS(Q, BL, X, A, B, C, D, APC) = BMS(C, B, C, D, APC) * PHI1(Q, BL, X)

200 READ(105, 201) A, B, C, D, BL, RHBD, MAXX, BETA

201 FORMAT(8F10.0)

NAME NUMBER OF INTEGRATION STEPS --- J = # DATA POINTS FOR STIFFNESS K

205 IF(A) 2000, 2000

206 READ(105, 207) N, J

207 FORMAT(215)

210 CALL PINIT

212 WRITE(108, 213)

253 BM=C*D#RH0=BM/16*085/12*
254 ALPHA#PHAX#D
255 DB 900 M=1,1
260 APC=A*C*PHI1(Q, BL, X1)
261 IF(APC=0.1) 262,262,263
262 FYY=FYS(Q, BL, X1, A, B, C, D, APC)
264 G0 TH 264
268 FYY=FYS(Q, BL, X1, A, B, C, D)
269 WI=H#FYY/3
270 XA(1)=X1
271 YA(1)=FYY
272 IF(Q#4&D2) 275,275,276
273 WRITE(108, 996) Q, X1, FYY, APC
274 DB 299 K=1,2
275 NB=K=N
276 XI=L#H
277 APC=A*C*PHI1(Q, BL, X)
278 IF(APC=0.1) 279,279,275
279 FYY=FYS(Q, BL, X, A, B, C, D, APC)
280 G0 TO 276
281 FYY=FYS(Q, BL, X, A, B, C, D)
282 WI=I#H#FYY/3
283 XA[NP1]=XN
284 YA[NP1]=FYY
285 IF(Q#4&D0) 289,289,287
286 WRITE(108, 996) Q, XN, FYY, APC
287 DB 299 K=1,2
288 N=N+K
289 XI=L#H
290 APC=A*C#PHI1(Q, BL, X)
291 IF(APC=0.1) 292,292,291
292 FYY=FYS(Q, BL, X, A, B, C, D)
293 WI=I#H#FYY/3
294 XA[NP1]=XN
295 YA[NP1]=FYY
296 FXX=IF(280, 896) Q, X, FYY, APC
297 DB 299 K=1,2
298 N=N+K
299 CONTINUE
300 IF(M=2) 315,310,315
310 CALL AXIS(1, 01, 0, 150, 200, 100, 6, 5, 4, 5)
311 MS=1
312 G0 TO 324
313 IF(M=J/6=1) 320,316,320
314 MS=3
315 G0 TO 324
316 MS=1
317 G0 TO 324
318 IF(M=J/4=1) 350,321,350
319 MS=4
320 WRITE(108, 998)
321 D0 330 JL#W1,NP1
322 WRITE(108, 311) Q, JL, XA(JL), JL, YA(JL)
323 FORMAT(20X,1G9.1,E12.5,10X,1X1,12.1)=1,E12.5,10X,1FXY(1,12.1)=1,E1
324 12.5)
325 G0 TO 326
326 IF(MS=1) 327,327,331
327 CALL PLOT1MS(1, NP1, 0, 0, 150, 200, 800, 6, 5, 4, 5, XA, YA)
328 IF(MS=1) 327,327,331
329 CALL PLOT(1, WP1, 0, 0, 150, 200, 800, 6, 5, 4, 5, XA, YA)
330 APCL=A#PHI1(Q, BL, BL)=C
331 CALL INTERNAL FORCE FUNCTION
332 IF(MS=1) 333,336,333
333 CALL PLOTS(1, NP1, 0, 0, 150, 200, 800, 6, 5, 4, 5, XA, YA)
334 IF(MS=1) 333,336,333
335 CALL PLOT(1, WP1, 0, 0, 150, 200, 800, 6, 5, 4, 5, XA, YA)
336 APCL=A#PHI1(Q, BL, BL)=C
337 CALL PLOT1MS(1, NP1, 0, 0, 150, 200, 800, 6, 5, 4, 5, XA, YA)
IF (APCL < 0) 351, 351, 352
351 WVV=WVS(Q, BL+ , A, C, D, APCL)
WPR=WPR(Q, BL+ , A, C, D, APCL)
GO 'T' 353
352 WVV=WV(Q, BL+ , A, B, C, D)
WPR=WPR(Q, BL+ , A, B, C, D)
353 DWH=VWV+WMR
354 STIF=DW/DW
355 WRITE (108, 356) Q, W, WVV, WMM, DW, STIF
356 FORMAT (4X, 5(E12:5, 4X), E12:5)
360 XXX(M)=Q
361 YYY(M)=DW
480 DW=OWN
490 IF (Q) 510, 510, 420
420 SKB=SKN(DWN1, DWN2, DQ)
430 IF (SKB) 560, 445, 445
445 CI=CI (P, ALPHA, BM, BETA, DWN1, DWN2, DQ)
450 CI=CI (DWN1, DWN2, DQ, QO)
455 EI=EI (DWN1, DWN2, DQ, GO, BM, TN, QD8TO, P, ALPHA, BETA)
460 EL=EL (DWN1, DWN2, DQ, GO, BM, TN, QD8TO, P, ALPHA, BETA)
465 WRITE (108, 466) BM, P, ALPHA, BETA, DWN1, DWN2, SKB, QD8TO, QO, TN, CI, CL, EI,
466 FBRMAT (/10X, PM, 5, 10X, ePub, E12:5, 10X,
10X, TBM), E12:5, 10X, 1DWN=1, E12:5, 10X, TBM, BM, TN, QD8TO, P, ALPHA, BETA)
470 QNT=EI*1+SIN(SORT (SKB/BM)), T)+EL*COS (SORT (SKB/BM)), T)+CI*EXP (BETA,T)
471 QDDT=CI*BETA+2*EXP (BETA=T)=SKB/BM*(EI+1*SIN (SORT (SKB/BM)), T)+EL*COS (SORT (SKB/BM)), T))
472 WRITE (108, 473) QNT, QDNT, T
473 FORMAT (10X, IQNT=1, E12:5, 10X, !QDNT=1, E12:5, 10X) !T=1, E12:5
474 IF (T) 475, 475
475 QNT=00
476 QDNT=00
477 IB=1
478 IA=1
479 IF (IB=1) 480, 480, 487
480 XB (IA) = T
481 YB (IA) = QNT
482 YB (IA*250) = QDNT
483 XB (IA*250) = QDNT
484 IF (IA=250) 483, 483, 487
485 T=T+DT
486 G9 'T' 470
487 IB=1
488 G9 'T' 485
489 IF (QNT=Q) 490, 490, 495
490 IF (QDNT) 550, 550, 479
495 QO=QNT
500 QDQT=QDNT
505 TN=T
510 DWN=OWN1
515 G9 'T' 900
545 WRITE (108, 546)
550 WRITE(108,551) QNT,QDNT,T
551 FORMAT(1H1,10X,1QNT=1,E12.5,10X,T=1,E12.5)
555 GO TO 904
560 WRITE(108,561) SKB
561 FORMAT(1H1,10X,1NEGATIVE STIFFNESS SKB=1,E12.5)
565 GO TO 950
570 Q=M*CO
574 WRITE(108,998)
580 DB 906 ML#1,M
590 WRITE(108,907) ML,XXA(ML),ML,YYA(ML)
597 FORMAT(20X,'Q(I,I2)'),E12.5,10X,'DW(I,I2),E12.5)

C PLOT VIRTUAL 4BRK FUNCTION DW,
CALL AXIS(100,0.0,100,7500.0,6.5,4.5)
CALL PLBTS(111,1,M,C,0.0,75000.0,6.5,4.5,XXA,YYA)
909 WRITE(108,998)
910 IP=1*A=1
911 DO 915 M#1,M
912 IS=MM+250
913 WRITE(108,914) MM,XXB(MM),MM,YYB(MM),IS,YYB(IS),IS,XXB(IS)
914 FORMAT(5X,'T(I,L2)'),E12.5,5X,'DW(I,L2),E12.5)

C PLOT DEFLECTION 4F BEAM END Q(T),
CALL AXIS(100,0.0,0.0,0.05,6.5,4.5)
CALL PLBTS(111,1,IP,0.0,0.0,2500.0,6.5,4.5,XXB,YYB)
916 DB 915 IP=1*IP
915 YB(I)=YB(I+250)

C PLOT VELOCITY 4F BEAM ENQ,
CALL AXIS(100,0.0,0.0,0.05,6.5,4.5)
CALL PLBTS(111,1,IP,0.0,0.0,2500.0,6.5,4.5,XXB,YYB)
U=RHS*C*D/193.02
916 DX=D0
920 DO 949 K=1,3
921 IF(K=1) 200,922,925
922 T=2+DT
923 YMIN=400.0
924 YMAX=1600.0
925 IA=2
926 GO TO 933
925 IF(K=2) 200,926,930
926 IM=IP/2
927 T=2+IM+DT
928 IA=1+1
929 GO TO 933
930 T=2*(IP-1)*DT
931 IA=IP
932 YMIN=400.0
933 YMAX=1600.0
934 QDT=XB(IA+250)
935 CALL PRT(PMAX,BETA,T)*D
935 DO 943 I=1,J1
936 X=I*1.0*DX
937 APC=A*C*PHI1(Q,BL,X)
938 IF(APC>0) 939,939,940
939 FIN=FIN(Q,BL,X,A,B,C,D,APC)
940 GO TO 941
940 FIN=FIN(Q,BL,X,A,B,C,D)
941 XXA(I)=U=THETA(BL,X)*QDDT
XXA(I+250) = FIN
YYA(I) = PRS
RES = XXA(I) + XXA(I+250)

942 XXA(I) = X
943 YYA(I) = RES

C PLOT RESIDUAL AND COMPONENTS
CALL AXIS(I,0.0,150.0,YMIN,YMAX,6.5,4.5)
WRITE(108,998)
DO 944 I=1,J1
944 WRITE(108,999) T,I,XXA(I),I,XXA(I),ISS,XXA(ISS),I,YYA(I)
WRITE(108,998)
CALL PLTS(1,1,J1,0.0,150.0,YMIN,YMAX,6.5,4.5,XXA,XXA)
CALL PLTS(3,1,J1,0.0,150.0,YMIN,YMAX,6.5,4.5,XXA,XXA)
DO 945 I=1,J1
945 XXA(I) = XXA(I+250)
CALL PLTS(1,1,J1,0.0,150.0,YMIN,YMAX,6.5,4.5,XXA,XXA)
CALL PLTS(3,1,J1,0.0,150.0,YMIN,YMAX,6.5,4.5,XXA,XXA)
CALL PLTS(5,1,J1,0.0,150.0,YMIN,YMAX,6.5,4.5,XXA,XXA)
DO 946 I=1,J1
946 WRITE(108,947) X,I,XXA(I),I,YYA(I),T,I,XXA,XXA
15X,YB(I13)=E12.5)
948 CALL PLTS(1,1,J1,0.0,150.0,YMIN,YMAX,6.5,4.5,XXA,YYA)
949 CALL PLTS(9,1,J1,0.0,150.0,YMIN,YMAX,6.5,4.5,XXA,YYA)
950 DXA=0.5/10
954 WRITE(108,950)
955 DO 970 I=1,J1
960 XI(I) = (I=1) * DXA
965 YYA(I) = THETA(BETA,XA(I))
970 WRITE(108,971) T,XA(I),I,YYA(I)
971 FORMAT(35X,T12,15X,E12.5,10X,THETA(I12,1),15X,E12.5)
C PLOT MODE SHAPE
975 CALL AXIS(1,0.0,150.0,0.0,1.25,6.5,4.5)
976 CALL PLTS(1,1,J1,0.0,150.0.0.1.25,6.5,4.5,XXA,YYA)
998 FORMAT(I1H1)
999 FORMAT(1X,15X,T12,15X,E12.5,4X,1X(13),15X,E12.5,4X,1R1(13),15X,E12.5,
14X,R2(13),15X,E12.5,4X,1R3(13),15X,E12.5)
1000 DO XA(I)=0.002/50.
1004 WRITE(108,998)
1005 DO 1010 I=1,J1
1010 XI(I) = (I=1) * DXA
1015 YYA(I) = PT(BETA,XA(I))
1020 WRITE(108,1021) T,XA(I),I,YYA(I)
1021 FORMAT(35X,T12,15X,E12.5,10X,PT(BETA,I12,1),15X,E12.5)
C PLOT PRESSURE FUNCTION OF BLAST
1025 CALL AXIS(1,0.0,150.0,0.0,1000.0,6.5,4.5)
1026 CALL PLTS(1,1,J1,0.0,150.0,0.1.000.0,6.5,4.5,XXA,YYA)
1500 END
2000 END


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cop. 2  The maximum deflection of a blast loaded cantilever beam...