Heat transfer from a horizontal bundle of continuous, helical finned tubes in an air fluidized bed
by Michael Todd Kratovil

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE
in Chemical Engineering
Montana State University
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Abstract:
Heat transfer coefficients were measured from a horizontal bundle of electrically heated finned tubes to
a rectangular air fluidized glass bead bed. Continuous, helical copper finned tubes were studied.
Experimental parameters included fin height, fin spacing, bed particle diameter and air fluidizing velocity.

• Results indicated that the coefficient generally increased with increasing fluidizing velocity. A
  maximum coefficient was observed in some cases. The coefficient increased with decreasing particle
  size. Increases of up to 50 percent were observed between the large and small particles. The coefficient
  increased with decreasing fin height and increasing fin spacing. The coefficient was very sensitive to
  fin spacings as large as 30 particle diameters. At less than 10 particle diameters the coefficient became
  less sensitive to fin spacing. The best performing tube as rated on heat delivery was a tube of
  intermediate fin spacing and maximum fin height. A particle mode heat transfer mechanism was
  successfully used to explain all trends observed.

A correlation independent of fin material was developed relating experimental variables to the particle
Nusselt number. Deviation of the data from the correlation was within the estimated experimental error.
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HEAT TRANSFER FROM A HORIZONTAL BUNDLE OF CONTINUOUS, HELICAL FINNED TUBES IN AN AIR FLUIDIZED BED

by

MICHAEL TODD KRATOVIL

A thesis submitted in partial fulfillment of the requirements for the degree of

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Heat transfer coefficients were measured from a horizontal bundle of electrically heated finned tubes to a rectangular air fluidized glass bead bed. Continuous, helical copper finned tubes were studied. Experimental parameters included fin height, fin spacing, bed particle diameter and air fluidizing velocity.

Results indicated that the coefficient generally increased with increasing fluidizing velocity. A maximum coefficient was observed in some cases. The coefficient increased with decreasing particle size. Increases of up to 50 percent were observed between the large and small particles. The coefficient increased with decreasing fin height and increasing fin spacing. The coefficient was very sensitive to fin spacings as large as 30 particle diameters. At less than 10 particle diameters the coefficient became less sensitive to fin spacing. The best performing tube as rated on heat delivery was a tube of intermediate fin spacing and maximum fin height. A particle mode heat transfer mechanism was successfully used to explain all trends observed.

A correlation independent of fin material was developed relating experimental variables to the particle Nusselt number. Deviation of the data from the correlation was within the estimated experimental error.
INTRODUCTION

Use of fluidized beds in carrying out commercial unit operations is a growing dynamic field today. Applications in drying, calcining, mixing, cooling towers, and chemical catalytic reactors have been demonstrated and a myriad of additional applications are under development.

A fluidized bed consists of a column, a porous distributor plate supporting the particulate bed material and a fluidizing medium, either gaseous or liquid. The term fluidized comes about by the visual appearance and physical characteristics of the fluidized bed.

At low fluidizing mass velocities the particulate bed acts just like a packed bed. The fluidizing medium flows around the particles through the interstices between the particles and out of the bed. Particles tend to remain stationary under these conditions.

As the fluidizing mass velocity increases the pressure drop across the bed eventually increases so that its value is equal to the weight of the bed of particles. This condition is called the point of minimum fluidization (M.F.) and the value of the fluidization medium velocity is known as the minimum fluidization velocity (M.F.V.). At M.F. bed particles exhibit limited motion and the bed tends to expand. Each particle tends to "float" in the fluidizing medium and the entire bed takes on the appearance of a granular liquid. The actual physical properties of the bed are those of a highly viscous Bingham-plastic fluid (1).
As the fluidizing medium velocity is increased beyond the point of M.F., bubbles of fluidizing medium are seen to form at the porous support plate and rise upward through the bed. The bubbles expand and coalesce as they rise in the bed and burst as they reach the upper bed surface. The bubble action tends to agitate the bed increasing the random motion of the particles. This condition of a freely bubbling or percolating bed is known as aggregative fluidization.

As the fluidizing medium velocity is increased still further the size of the bubbles increases until the bubble diameter approaches the size of the column. Layers of fluidizing medium and particles are seen rising in the column in piston-like action. This condition of fluidization is known as slugging. Maximum particle motion is achieved with slugging but physical equipment damage, excessive particle degradation and entrainment may result. Most commercial fluidized beds operate in the aggregative regime. Both aggregative and slugging regimes were observed in this study.

The fluid-like particle motion of the bed produces desirable characteristics that can be commercially utilized. The fluid-like qualities of the bed allow conversion of less desirable batch processes to continuous operations. The random particle movement promotes good particle mixing providing nearly isothermal, and uniform composition conditions throughout the bed allowing easy control of physical bed parameters. The rate of heat transfer between the bed particles and an
immersed bed exchanger is high compared to a fixed bed arrangement thus reducing the surface requirements and cost of the heat exchanger. The scrubbing action of the particles on exposed surfaces virtually eliminates surface fouling. Heat and mass transfer rates between fluidizing gas and bed particles are high compared to other contacting means because of the high surface area provided by the particles. The small diameter particles in a fluidized bed are generally of a smaller order of magnitude than in a fixed bed thus providing a smaller diffusion resistance to mass transfer. This is important in catalytic reactor applications.

It should be realized however that there are also some undesirable physical characteristics of fluidized beds that prevent commercial applications. The fluidized bed is by no means the universal answer to industry's problems. Channeling and by-passing of solids by bubbles presents an inefficient contacting scheme. The fluidized bed is not applicable in caking conditions or processes involving "sticky" reactants or products. Particle degradation and entrainment may prove too costly when fragile or expensive catalyst particles are used. The scrubbing action of the particles promote erosion of vessel walls and piping. The fluidized bed acts like a back-mixed reactor reducing the driving force for mass and heat transfer and chemical reaction. Fluidized beds can however be arranged and baffled to exhibit limited counter current behavior.
There have been many important situations that prove favorable to fluidized bed processing. Some applications are listed below.

(1) Fritz Winkler's gasification of powdered coal (2). This was the first significant use of fluidized beds in a commercial operation. It has proved inefficient and uneconomical in today's petroleum world (1922).

(2) U. S. Petroleum industry's continuous fluidized bed catalytic cracking unit (1). It replaced the fixed bed Houdry process and made possible economical conversion of heavier crude oil into lighter more desirable petroleum fractions (1939). (still in use)

(3) Standard Oil Development Company's continuous fluidized-bed catalytic reformer (2). Isomerizes light hydrocarbon feed in contact with a suitable catalyst to produce a higher octane number gasoline (1953). (still in use)

(4) Fluidized-bed drying of iron-ore concentrate (3) reduces moisture content of wet concentrate iron-ore from 4.5% to 1.5% to allow handling of the ore in subzero conditions (1974). (still in use)

(5) Calcination of nuclear wastes (4). Solution of radioactive fission products and solvent are sprayed into hot fluidized bed. The solvent is vaporized and radioactive components solidify on bed particles thus greatly reducing the volume
and changing the phase of nuclear wastes. (still in use)

(6) Fluidized-bed wet-dry towers in waste heat disposal (5) are an alternative to cooling ponds and cooling water towers using shallow air fluidized bed wet-dry cooling units. (under development)

The above list is not intended to be complete in any way. It is intended to show some of the significance and diversity of application that fluidized beds have.

As can be seen from the above list, many applications of fluidized beds require heat transfer to or from the bed. This heat transfer is accomplished either through the column walls or more commonly through immersed surfaces within the bed. The random motion of the particles and the complex fluidizing medium flow path make analytical analysis for determining heat transfer coefficients to or from the bed virtually impossible. Consequently, experimental correlations are being developed relating the heat transfer surface geometry to the heat transfer coefficient. With the development of these correlations it is hoped that our understanding of what is really going on in a fluidized bed will be enhanced, enabling successful design and scale up of columns meeting required specifications.

The purpose of this investigation was to develop a correlation relating surface geometry parameters of horizontal, continuous, helical, copper finned tubes in an air fluidized-bed to the corresponding
heat-transfer coefficient of the tube. The bed material consisted of glass beads of controlled diameters. Experimental variables included fin height, fin spacing, particle diameter, and fluidizing gas mass velocity.
THEORY AND PREVIOUS RELATED RESEARCH

The theory and previous related research from extended horizontal surfaces in dense phase fluidized beds is presented in three parts. The first section describes the internal bubble, fluidizing medium, and particle behavior in the bed; the second section presents proposed mechanisms for heat transfer from immersed surfaces; the third section describes previous related research.

Bed Dynamics

As mentioned previously, it is noted that the rising bubbles through the bed induce the particle motion within the bed. Various researchers (2,6,7,8) have investigated bubble behavior and the associated particle motion in fluidized beds and the following is a summation of their findings.

As a first approximation all of the gas in excess of that needed to just fluidize the bed passes through the bed as bubbles while the emulsion phase (dense phase bed excluding bubbles) remains at minimum fluidizing conditions. Small bubbles form at the distributor plate, coalesce, grow, and speed up as they rise through the emulsion phase. The rising bubbles are not only composed of gas but also contain from 0.2 percent to 1.0 percent solids.

Bubbles are in general spherical with the base of the bubble concave. The reason for the concave base is that the pressure in the lower part of the bubble is less than in the adjacent emulsion phase. Gas
therefore flows into the base of the bubble and out the top resulting in an instability and partial bubble collapse at the base with turbulent mixing behind the bubble. This turbulent mixing results in solids being drawn up behind the bubble forming a particle wake. As the bubble rises the particle wake is drawn upward at the bubble velocity and is continually exchanged with fresh emulsion solid as it rises. The particle exchange in the bubble wake is the primary mechanism for particle mixing.

The bubble roof stability is maintained by the upward flowing gas in the bubble. Two to three times as much gas flows through the bubble cross-section as through an equivalent section of emulsion phase in the same time interval. The flow pattern of this gas is a circulating one. Gas enters the bubble base flows out the bubble roof and sweeps around the outside periphery of the rising bubble back to the base. The circulating gas forms a "cloud" around the bubble. The thickness of the cloud and the amount of gas recirculated is a function of the bubble velocity. The rest of the percolating gas in the bed does not mix with the circulating gas but is pushed aside with the solid emulsion as the bubble passes by. It is noted that bubble size rather than number increases with increased gas velocity.

The induced particle motion from the rising bubbles and gas is as follows. Individual particles wander everywhere in the bed. There is a definite up and down movement, the upward being rapid, the downward
being relatively slow. Thus solids spend most of their time moving downward slowly but are occasionally swept upward in the bed.

Two studies from the literature dealing with bubble and particle motion around immersed horizontal tubes are pertinent.

Keirns (9) observed fluidization behavior in a rectangular fluidized bed containing horizontal bare tubes. Visual observations indicated that uniform fluidization and temperature distributions around horizontal tubes are inhibited by stagnant "caps" of particles on the top of the tubes and by defluidized regions between the tubes and the walls and in the corners of a rectangular bed. In a subsequent paper he reported heat transfer coefficients on the top of a 2 inch diameter para-dichlorobenzene cylinder to be 7-12 times smaller than coefficients at the bottom.

Hager and Thomson (10) did x-ray and flow visualization studies of bubble behavior around immersed tubes including a horizontal helical finned tube. They observed an air boundary layer below the central core of the tube with no defluidized cap on top of the tube. The entire region within the fins appeared to be defluidized. Most bubbles were either diverted to one side or disappeared within the fins and reemerged at the top of the fin. Bubbles did not appear to penetrate completely to the fin base.
Mechanisms for Heat Transfer

Bed to surface heat transfer coefficients are many times larger than corresponding surface to gas or surface to packed bed coefficients. Several models based on various controlling heat transfer resistances have been presented to explain this phenomenon. Refer to Figure 1 for a schematic representing each model discussed.

Levenspiel and Walton (11) presented a 'film' model. In the film model a thin laminar film of fluidizing gas is adjacent to the heat transfer surface. The major resistance to heat flow is considered to be through this laminar film. The scouring action of the fluidized particles against the film decreases its thickness, hence decreasing the resistance to heat flow. Both the particle velocity adjacent to the surface and the particle concentration at the surface affect the film thickness. These two factors have opposite effects on heat transfer (with increasing particle velocity, bed voidage increases and surface particle concentration decreases), therefore a maximum heat transfer coefficient is obtained when it is plotted against gas mass velocity. This agrees with experimental findings.

Mickley and Fairbanks (12) viewed the mode of heat transfer as unsteady heating of 'packets' of emulsion phase ('packet model'). In their model a packet of particles from the core of the emulsion at the bulk bed temperature \( t_b \) moves into contact with a flat surface of temperature \( t_w \). Unsteady state conduction begins on contact. Heat
FIGURE 1. MODELS FOR HEAT TRANSFER
transfer rate is largest at initial contacting and decreases exponentially with residence time. After a certain residence time the packet leaves the surface and breaks up dissipating its excess thermal energy to the core of the bed. The packet is replaced by a fresh packet from the bed core and the process is repeated.

Modifications to the 'packet model' i.e. a 'particle model' was presented by Ziegler, Koppel, and Brazelton (13) and later extended by Genetti and Knudsen (14). In the extended 'particle model' a particle from the bed core at bulk medium temperature \( t_b \) comes into contact with the surface at temperature \( t_w \). The particle receives energy by unsteady state convection from the thin laminar film of gas (temperature \( t_b = (t_b + t_{wall})/2 \) adjacent to the surface. Heat transfer by conduction at the point of contact is considered negligible. After some residence time the particle returns to the bulk of the bed where it dissipates its excess energy.

This model is the one used to correlate data from this study. The resulting expression for the Nusselt number from the model and the modified expression relating my experimental variables as well as further elaboration on the model is presented in the correlation section of this paper.

One important experiment performed by Ziegler and Brazelton (15) supports the 'particle model' heat transfer mechanism for their experimental conditions. Simultaneous heat and mass transfer rates were
measured from a 1-1/2 inch diameter celite sphere saturated with water. Transfer rates were measured in both an air stream and in a fluidized bed that contained particles with negligible absorptivity for water.

Consequently the only mechanism of importance for transfer of mass is diffusion through the film. Without fluidized particles mass and heat transfer modes are analogous i.e. film diffusion. If particles contribute significantly in heat transfer then the rate of heat transfer should increase much more than the mass transfer rate in the fluidized bed study. Increases in heat transfer coefficients from 10 to 20 times were observed, whereas, increases in mass transfer rates were only 1-1/2 to 2 times the previous air study. They concluded that 80 to 95 percent of the heat is transferred by the particle mode.

Wicke and Pettin (16) proposed a model which accounts for both thin-film and emulsion resistances. Heat is first conducted from the surface through a laminar gas layer of thickness, \( l_g \). This heat is absorbed by solids flowing parallel to the surface in a zone of thickness \( l_e \). Some of this heat, \( q_l \), goes into sensible heat of the solids, while the rest, \( q_r \), is transferred to the bed core by interchange of solids.

Kunii and Levenspiel (2) examined the previous mentioned models for heat transfer and decided the models were really not in conflict but rather that each one described a different controlling resistance to heat transfer. They therefore postulated a general theory that
allows all four mechanisms to be considered. In their general model a thin film of gas of thickness $l_g$ coats the surface, some solids are in direct contact with the surface, and the emulsion of equivalent thickness $l_e$ flows past the surface and is replaced occasionally by fresh emulsion. The four mechanisms represented are the following:

1. Heat transfer through a thin gas film of the order of $d_p$ or less.

2. Heat transfer by convection from the laminar film by particles in contact with the surface with frequent particle replacement.

3. Unsteady-state absorption of heat by fresh emulsion which is swept away from the surface. This represents a surface renewal model for the emulsion.

4. Steady-state conduction through the emulsion layer that is seldom swept away.

They developed criteria to suggest which mechanism controls and which type of model should be used to represent a particular situation. This will not be discussed here. The reader is referred to their book (2) for further details.
Previous Related Research

Heat transfer from the walls of the column to the bed and from the bed particles to the fluidizing medium have received much attention in the past. Zabrodsky (17) in chapters 8 and 10 of his book on fluidization surveys and summarizes this work. Since these subjects are not directly pertinent to this investigation, they will not be discussed further.

Many more recent studies have dealt with heat transfer from immersed surfaces. These studies will be summarized here.

Vreedenberg (18) performed some of the initial experiments with immersed tubes. He measured heat transfer coefficients of horizontal water-cooled bare tubes. Variables in his study were bed temperature,
mass velocity of the fluidizing air, particle diameter, particle shape, particle density, and tube diameter. He correlated the Nusselt number in terms of the Reynolds number, void fraction, and fluid and solid properties. Deviations of experimental values from his correlation were about 43 percent. Vreedenberg, like most future investigators, observed a maximum in heat transfer coefficient with increasing gas mass velocity.

Genetti et. al (19) studied the effect of bare and serrated fin tube orientation in an air fluidized bed. Variables studied included particle size, fluidizing mass velocity and orientation angle. A serrated carbon steel finned tube with fin height of 0.75 in., fin thickness of 0.025 in. and tube O.D. of 0.625 in. was used. A minimum heat transfer coefficient was observed at orientation angles of 45 and 60 degrees (measured from the horizontal) for the bare and finned tubes, respectively. Maximum values of the coefficient for the finned tube occurred at 30 degrees; the horizontal orientation produced a coefficient near the maximum value, whereas, the vertical orientation produced a coefficient of about 18 percent lower than the horizontal position.

Petrie, Freeby, and Buckham (20) studied horizontal bundles of 19 tubes arranged on 2.25 in. center-to-center triangular and square pitch arrays. Three tube bundles were used, plain tubes, 5 fin per inch and 11 fin per inch tubes. Fins were transverse 0.4 inch long helical
fins. All tubes were aluminum. They found that tubes acted independently from one another for tube spacings greater than 43 particle diameters. The critical distance for tube independence was not determined. Bare tube data was correlated into a single equation relating the coefficient to the experimental variables. Experimental results from their investigation are compared with my correlation values in the results and discussion section of this thesis.

A study by Ziegler, Koppel, and Brazelton (13) looked at the effect of particle heat capacity and thermal conductivity on the heat transfer coefficient. Copper, solder, and nickel particles were used. Results indicated that the solid thermal conductivity had a negligible effect on the surface-to-bed heat transfer coefficient. The coefficient did increase with increasing solid heat capacity but in a less than linear fashion. Using their 'particle model' theory for relating the particle Nusselt number to the variables studied, particle residence times at the heat transfer surface were back-calculated from heat transfer coefficients. These contact times were in agreement with previous observations and with results of simple theoretical models.

Ozkaynak and Chen (21) measured directly the residence time of emulsion phase on the surface of an immersed tube in a fluidized bed. A specially designed fast response capacitance probe was used. The validity of the 'packet model' for heat transfer was investigated. Satisfactory agreement for both the small and large particles used
was obtained when the 'packet model' was modified to account for
the change of void fraction near the surface of packets. Gamma and
log-normal packet surface residence time distributions were found to
give good representation of the data.

Generally the heat transfer coefficient increases with decreasing
particle sizes. Baerns (22) observed that as the particle size was
reduced the heat transfer coefficient increased, passed through a max-
imum and then decreased. The decreasing heat transfer rates corresponded
with the region where the interparticle adhesive forces (van der Waals
forces) affected the quality of fluidization. These forces came into
play when the particle size is smaller than about 50 \( \mu \) (.002 in.) and
cause agglomeration and channeling. Particles used in this study
were much larger than this.

Bartel and Genetti (23) measured the rate of heat transfer from a
horizontal bundle of carbon steel bare tubes and finned tubes to a bed
of glass spheres fluidized with air. The experimental variables were
fin height, distance between tubes, particle diameter, and fluidizing
air velocity. The same fluidizing column was used as in this investi-
gation. A correlation based on the 'particle model' was developed
relating the particle Nusselt number to the experimental variables.
The final correlation was independent of fin material. Experimental
data agreed with the correlation to within + or - 15 percent. They
found that the rate of heat transfer increased with fin height, but the
rate leveled off near a fin height of about one inch. For a bundle of short finned tubes (1/8 inch fins) the rate of heat transfer is sensitive to tube spacings until there is about 2 inches between tube centers. The rate of heat transfer for the 7/8 inch fin tubes was independent of tube spacing for all center-to-center distances.  

Priebe and Genetti (24) studied heat transfer coefficients of horizontal discontinuous finned and spined tubes in an air fluidized bed. Heat flux, fin spacing, particle diameter, and gas mass velocity were the variables for the discontinuous finned tubes. Spine height, spines per turn, spine material, and gas mass velocity were the variables for the spined tubes. The same fluidizing column was used as in this study.

Results indicated that the coefficient began to fall rapidly for fin spacings less than 10 particle diameters. Coefficients obtained for copper spines were greater than for stainless steel (lower thermal conductivity than copper). There was little difference in coefficients with larger number of spines per turn, but the increased area yielded higher heat transfer rates.

Each tube type led to a correlation relating the particle Nusselt number to the variables studied. Deviation from the correlations was less than ±12.5 percent. By modeling the heat conduction within the spines the effect of spine thermal conductivity on the coefficient was eliminated.
EXPERIMENTAL EQUIPMENT

The equipment used in this research was already available having been twice previously used for similar heat transfer investigations of other geometry finned tubes. Only a brief description of the equipment will be given here. The reader is referred to Bartel's Ph.D. thesis (4) for a more detailed description.

The discussion of the equipment will be divided into three sections: 1) the column, 2) the fluidizing system and 3) the electrical system. An overall schematic of the equipment is shown in Figure 2.

Column

Figure 3 shows a detailed view of the fluidizing column. The fluidizing column is rectangular in shape 94 inches high, 15-1/2 inches wide, 8 inches deep (outside dimensions). It is fabricated of clear 3/4 inch plexiglas. A rectangular column was chosen to permit easy installation and removal of uniform lengthed finned tubes. Clear plexiglas was used to allow observation of the degree of fluidization within the bed.

The distributor plate consisted of two layers of 140-mesh brass wire cloth sandwiched between two 1/32 in. thick steel perforated plates (perforations were 1/32 in. diameter and 1/5 in. center-to-center). A particle drain pipe, 1 in. O.D., was silver soldered flush near one corner of the distributor plate. The drain pipe extended
FIGURE 2. OVERALL VIEW OF EQUIPMENT

1 PLEXIGLASS COLUMN, 2 MICARTA PLATE, 3 AIR BLOWER, 4 MAIN AIR LINE VALVE, 5 BYPASS VALVE, 6 ORIFICE, 7 ORIFICE MANOMETER, 8 PRESSURE DROP ACROSS TUBES, 9 TUBE AND BED THERMOCOUPLES, 10 CHART RECORDER, 11 HIGH TEMPERATURE LIMIT THERMOCOUPLE, 12 HEATER LEAD WIRES, 13 HIGH TEMPERATURE LIMIT PROTECTOR, 14 WATTMETER, 15 RHEOSTAT, 16 VARIAC, 17 POWER SOURCE
EXIT AIR PORTS

BED THERMOCOUPLES
FINNED TUBES
MICARTA PLATE
PRESSURE TAPS
HEATERS
DISTRIBUTOR PLATE
FLOW STRAIGHTENERS
PARTICLE DRAIN
MAIN AIR LINE

FIGURE 3. DETAILS OF FLUIDIZING COLUMN
through the bottom of the column. A quick opening valve was installed in the pipe just below the column. The distributor plate provided adequate support as well as uniform fluidization of the glass sphere bed. The drain pipe permitted easy removal of the bed material from the column.

The column extended 18 in. beneath the distributor plate. Nine 3/4 in. O.D. tubes were installed in two rows in this section of the column to provide a smoothing section for the fluidizing air.

The column extended 74 in. above the distributor plate. This allowed ample room for the bed to expand and fluidize freely. This part of the column did impose limitations on the air flow rates permissible. At higher air flow rates the bed expanded so that pluggage of the exit screens occurred.

Two micarta plates (laminated canvas) 3/4 in. thick were employed to support the heater assemblies within the bed. In one plate seven 1 in. holes in a hexagonal array (5 in. center-to-center spacing) were drilled completely through and fitted with swagelock fittings to hold one end of the heaters securely in place. In the other plate holes 2/5 in. deep were drilled to support the other end of the heaters. The micarta plates were held in place with "trunk-lid" type clamps.

The column top was a removable 3/4 in. plexiglas lid with a large rectangular hole in its center. The rectangular hole was covered with 200 mesh stainless steel cloth. A 3 in. diameter hole was cut near
the top on each side of the column and covered with wire mesh. The lid was removable to allow filling the column with the glass beads and to allow direct observation of the alignments of the tube bundle. The mesh cloth prevented the bed material from leaving the column with the fluidizing air.

To aid in removing the bed material between runs two plug fitted 3 in. diameter vacuum ports were cut on opposite sides of the column (short sides) just above the distributor plate.

The column was supported by four angle irons bolted to the floor with wooden blocks wedged between the column and the supports.

Fluidizing System

Air was used as the fluidizing medium. Air was supplied by a positive displacement Sutorbilt blower powered by a 7-1/2 H.P. Brown-Brockmeyer electrical motor. 2-1/2" O.D. schedule 40 steel pipe was used both for an air supply line to the column and for a column bypass line.

Two gate valves were used in the supply system, one in the supply line and one in the by-pass line. Air flow rate to the bed was controlled by throttling the by-pass valve; the supply valve was left open for all runs. Air flow rate to the bed was measured with an orifice in the supply line. The orifice had a 1-1/2 in. diameter opening and vena contracta pressure taps. Water manometers were used.
to measure the pressure drop across the orifice.

Back pressure from the column was measured with a Duragauge pressure gauge located downstream of the orifice.

Three sizes of Blast-O-Lite glass beads were used as the bed material. The average sphere diameters were 0.0068 in., 0.0103 in., and 0.0217 in. The specific gravity of the beads was 2.5 (156 #/ft^3). Viewing the beads under a microscope showed that they had a sphere count of about 80+ percent. Bead diameters were determined by measuring bead diameters of a random sample of beads and determining the average values. A microscope with a scaled ocular was used for these measurements. A stagnant bed height of 23 in. was used in each run. Table I represents a summary of the bead size analysis.

| TABLE I. BLAST-O-LITE BEAD SIZE ANALYSIS |
|-----------------|----------------|---------|---------|--------|--------|
| (in) Diameter   | (in) Standard  | Sample  | Sphere  | Nominal |        |
|                 | Deviation      | Size    | Count (%)| Name   |        |
| 0.0217          | 0.0032         | 300     | 80      | Large  |        |
| 0.0103          | 0.00104        | 300     | 84      | Medium |        |
| 0.0068          | 0.00105        | 350     | 88      | Small  |        |

To determine the particle fraction in the bed pressure taps for a water manometer were placed on both sides of the tube bundle.

The slugging action of the fluidized bed made reading the water
manometers difficult. At high gas mass velocities, fluctuations of up to ±30 percent in readings were noted for the orifice manometer and ±50% in readings were noted for the bed manometer. Average values of a number of high and low readings were used as the representation of the 'true' pressure drops. Reproducibility of the air flow rates was surprisingly easy to attain.

Electrical System

The electrical system consists of a power supply, cartridge heaters for the finned tubes, and a thermocouple system.

Power was supplied to the heaters from a 240 V outlet, through a powerstat, a rheostat, a bed temperature limit protector, a wattmeter, through fuses to the seven cartridge heaters wired in parallel.

The powerstat was provided to compensate for normal building power fluctuations and to maximize the temperature difference between the finned tube and the bed. The bed temperature limit protector was a safety circuit to prevent the bed from overheating. The sensing device for this circuit was a bare thermocouple inserted in the bed. The wattmeter was used to determine the power input to each cartridge heater.

Watlow fire rod cartridge heaters of appropriate diameters were used as the heat source for the finned tubes. As shown in Figure 4 each cartridge heater was 9-2/5 in. long comprised of a 6-1/2 inch
FIGURE 4. DETAILS OF A CARTRIDGE HEATER
heated section with cold, insulated ends. The 2/5 in. cold end fit into the partially drilled micarta plate the longer cold end with electrical leads extended out of the bed and was held by the previously mentioned swagelock fittings. A 1/8 in. diameter, 3 in. longitudinal hole was drilled from the lead end into the cartridge to allow passage of the tube thermocouple out of the bed.

This design minimized end heat losses from the heater. A 6-1/2 in. finned tube was centered over the heated section of each cartridge. Therefore most of the energy supplied to the heater leaves by way of the finned tube. In fact, in this study all of the heat supplied to the heater is assumed to leave by way of the finned tube. This is not a bad assumption since at steady state conditions the temperature of a finned tube surface ranged as high as 280°F whereas the protruding end of the heater was only warm to the touch. To promote good contacting between the heater and the tube, the annular space was filled in with layers of copper anti-sieze compound and aluminum foil. The annular space was in general less than 1/16 in. wide.

Six of the tubes studied were provided by the Rome-Turney Radiator Company, Rome, New York. All tubes studied had continuous, helical copper fins. Rome Turney fins were alloy bonded to the tube wall. Three other tubes studied were supplied by the Wolverine Trufin Company. These tubes had integral fins with thicker tube walls and fins. All fins were slightly tapered. Fin spacings, fins per inch
and fin heights were the geometric parameters investigated. Table II lists the physical dimensions of the nine tubes used.

### TABLE II. TUBE DIMENSIONS

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.453</td>
<td>0.625</td>
<td>0.414</td>
<td>0.016</td>
<td>9</td>
<td>0.0951</td>
<td>1.1449</td>
</tr>
<tr>
<td>2</td>
<td>1.375</td>
<td>0.625</td>
<td>0.375</td>
<td>0.024</td>
<td>9</td>
<td>0.0871</td>
<td>1.0608</td>
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<tr>
<td>3</td>
<td>1.328</td>
<td>0.625</td>
<td>0.352</td>
<td>0.016</td>
<td>9</td>
<td>0.0951</td>
<td>0.9314</td>
</tr>
<tr>
<td>4</td>
<td>1.094</td>
<td>0.625</td>
<td>0.234</td>
<td>0.016</td>
<td>5</td>
<td>0.1840</td>
<td>0.3540</td>
</tr>
<tr>
<td>5</td>
<td>1.094</td>
<td>0.625</td>
<td>0.234</td>
<td>0.016</td>
<td>9</td>
<td>0.0951</td>
<td>0.6004</td>
</tr>
<tr>
<td>6</td>
<td>1.094</td>
<td>0.625</td>
<td>0.234</td>
<td>0.016</td>
<td>14</td>
<td>0.0554</td>
<td>0.8893</td>
</tr>
<tr>
<td>7</td>
<td>1.094</td>
<td>0.625</td>
<td>0.234</td>
<td>0.016</td>
<td>18</td>
<td>0.0396</td>
<td>1.0463</td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>0.453</td>
<td>0.274</td>
<td>0.025</td>
<td>5</td>
<td>0.1750</td>
<td>0.3541</td>
</tr>
<tr>
<td>9</td>
<td>1.500</td>
<td>0.750</td>
<td>0.375</td>
<td>0.020</td>
<td>7</td>
<td>0.1229</td>
<td>0.9372</td>
</tr>
</tbody>
</table>

Tubes 2, 8 and 9 were provided by Wolverine Tube Div. All other tubes were provided by Rome-Turney Radiator Company.

As shown in Figure 5 each finned section used was 6-1/2 in long. Each section had a 1/8 in. diameter hole drilled at the base of the fins from one end to the tube center. A single iron constantan thermocouple was wired and soldered into a filed groove in the tube wall at
FIGURE 5. TUBE DETAILS AND NOMENCLATURE
the tube center and threaded through the base of the fins out of the bed through the drilled cartridge heater. The other end of the finned tube had a 3/8 in. section of fins removed with a set screw attached to hold the tube in place on the heater.

Ten iron constantan thermocouples were used. Seven thermocouples, one for each tube of the tube bundle, were used to measure the tube wall temperatures. Three thermocouples placed above, below and within the tube bundle were used to measure the bed temperature.

The in bed thermocouples were threaded into 1/8 in diameter copper tubing with wire mesh over the tube end to protect the thermocouple from the bed action. Consequently the bed fluid temperature was measured rather than the particle temperature. An unprotected thermocouple indicated that the particle-fluid temperature was less than 3°F higher than the protected thermocouples.

All thermocouples were connected through a switching box to a Honeywell-Brown chart recorder. A chart recorder was used so that an average tube wall temperature could be obtained. All tube temperatures showed some tendency for cycling. This is expected as particles and fluid arrive and leave the tube surface. Some fluctuations were as high as ±3°F. Satisfactory average values were obtainable in most cases to within ±1.5°F.

The physical soldering of the thermocouples to the tube walls was critical to this study. Small errors in temperature measurement
result in large errors in the experimentally determined heat transfer coefficient. In an effort to minimize this error seven tubes of the same kind are used in each tube bundle. The average of all seven tube results is used as representative of the tube type. In this way it was hoped that errors in reading the charts and errors caused by variations in thermocouple to wall contacting would tend to average out. Indeed deviations of ±20 percent in individual tube heat transfer coefficients were observed.
EXPERIMENTAL PROCEDURE

Bead Size

Initially 0.05 inch average diameter particles were used as one kind of bed material. These 0.05 inch particles were approaching the size of the fin spacings on the 9 or more fin per inch (F.P.I.) tubes. Problems resulted in particles lodging between the fins, preventing free particle movement in and out of the fin spaces, thus shielding much of the tube surface from particle mode heat transfer. Erratic and unpredictable heat transfer coefficients resulted. The tubes actually proved to be a good particle sieve since in a bed of very dilute concentration of the larger 0.05 inch beads the tubes would selectively trap these larger beads.

Bead size is an important parameter to be considered for any application of finned tubes. Fin spacing should be of the order of 2 or more times the largest particle diameter of the bed to prevent loss of heat transfer surface from particles lodging between the fins preventing penetration of the particles to the depths of the fin spaces.

Minimum Fluidization Velocities

Minimum fluidization velocities (M.F.V.'s) of the 3 particle sizes were determined. The first tube bundle was assembled in the column. The column was filled to a fluffed 23 in. height of sized beads. The air and heaters were turned on and the column was allowed to heat up for several hours. This was to drive off any moisture from the bed.
that would cause the beads to stick together.

The M.F.V. was determined by throttling the air flow rate until initial fluidization or defluidization of the bed was visually verified. A micro water-manometer was used to increase precision in measuring these M.F.V.'s. This procedure was repeated with different tube bundles. Average values of all readings were used as the M.F.V. value. Deviations of less than 10 percent were observed for all cases. M.F.V.'s for the three bead sizes are shown in Figure 6.

**Tube Thermocouple Location**

Bartel (4) investigated tube surface temperature variation with serrated-fin carbon steel tubes. At a tube temperature of 200°F, he found the greatest temperature difference between any 2 points on a tube was only about 2°F. He concluded that there was essentially no temperature gradient on the tube surface in either the longitudinal or angular direction.

In this investigation angular temperature gradients were looked at. This was done by allowing the column to come to steady state and taking tube surface temperature readings as the tubes were rotated 360° in 15° increments. The largest observed temperature difference with a tube temperature of 270°F was ±3°F. No definite pattern of temperature variation with angular position could be determined. Since no angular dependency was observed, the longitudinal dependency was not examined.
FIGURE 6. PARTICLE MINIMUM FLUIDIZATION VELOCITIES
As mentioned previously (9, 10) stagnant caps (areas of little particle movement) may be present at the top and bottom of horizontal tubes in a fluidized bed. No corresponding temperature gradient was observed with these tubes. If the caps were present the high thermal conductivity of the copper apparently 'conducted away' any observable temperature gradients on or within the tubes.

It was decided for this work to place a single thermocouple centered on the tube wall and facing upward. This is believed to give heat transfer coefficients that may be conservative. The possible stagnant cap on the top of the tube would mean less particle movement, a hotter local surface temperature, a larger tube-bed $\Delta T$, and a smaller coefficient.

Heater Power Input

Priebe (23) looked at the effect of heat flux variation on the heat transfer coefficient in serrated fin, carbon steel tubes. He observed that the coefficient increased less than 10 percent with a 2-fold increase in heat flux. To account for this increase he postulated that with increased heat flux the thermal boundary layer adjacent to the tube wall increased in thickness. Hence, more particles adjacent to the wall were immersed in this hot film thus increasing the rate of heat transfer.

As was mentioned previously small errors in temperature measurement
result in large errors in experimental coefficients. With increased particle size (greater M.F.V.) larger air mass velocities were used, significantly decreasing the tube wall temperature. In an effort to maximize the tube wall-bed ΔT the tubes were run at higher voltages when the larger beads were used.

<table>
<thead>
<tr>
<th>Bead Diameter</th>
<th>Heater Voltage</th>
<th>Heater Wattage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0068 in.</td>
<td>115v</td>
<td>230</td>
</tr>
<tr>
<td>0.0103 in.</td>
<td>120v</td>
<td>255</td>
</tr>
<tr>
<td>0.0217 in.</td>
<td>125v</td>
<td>270</td>
</tr>
</tbody>
</table>

With a 17 percent increase in heat flux between the 0.0068-in. and 0.0217 in. particles the effect on the heat transfer coefficient due to this increased heat flux was less than 5 percent (see Figure 7), whereas tube wall-bed ΔT increased by 16 percent. Thus some increased accuracy of experimental results was obtained by this method.

Procedure for a Typical Run

The same procedure was used for all runs. A set of tubes 6-1/2 in. long was selected; thermocouples and set screws were attached to the tubes; tubes were mounted on cartridge heaters. Layers of copper-anti-sieze compound and aluminum foil were used to promote contacting between the heaters and the tubes.

The tube bundle was assembled in the column with all tube thermo-
Heat Transfer Coefficient (BTU/hr·ft·sq°F)

Volts | Heat Flux (Btu/hr·sq.ft)
--- | ---
125    | 980  
115    | 840  

Fin Height = 0.375 in
Fin Spacing = 0.1229 in
F.P.I. = 7 (Large Particles)

FIGURE 7. HEAT TRANSFER COEFFICIENT VERSUS AIR MASS VELOCITY VERSUS HEAT FLUX
couples facing upward. The column micarta plates positioned and held the tubes in place. The interface between the micarta plates and the plexiglass column was sealed with silicone latex sealant to prevent air and particle leakage.

Particles of the desired size were poured in from the top of the column to a fluffed static bed height of 23 inches. The column lid was secured in place and the blower and heaters were turned on and set.

The column was allowed to reach steady state. Steady state was determined by observing the center bed thermocouple history on the recorder.

Data included tube wall and bed temperatures; power input to the heaters; and supply air and bed manometer readings. Steady state readings were repeated for different supply air flow rates. Steady state for the initial heat up of the bed required about 4 hours and for successive readings about 2 hours each.

To change particles, the particle drain pipe was used along with the vacuum ports. To change the tube bundle, particles were drained and the outer micarta plate with the tube bundle was removed. A new tube bundle was assembled as before and the procedure was repeated.
CORRELATION

A single equation relating the experimental parameters to the heat transfer coefficient was developed. Recall that the experimental parameters were fin height, fin spacing, particle diameter and gas mass velocity. The form of the equation used to correlate the experimental results was based on the 'particle' mode heat transfer model presented by Ziegler, Koppel, and Brazelton (13) and extended by Genetti and Knudsen (14). This section is presented in three parts. First, the 'particle' mode heat transfer theory and resulting mathematical model is discussed. Second, the fin thermal conductivity parameter is eliminated from the data. Finally, the procedure used in correlating the data is presented.

Particle Model for Heat Transfer

Figure 8 is a schematic showing the details of the proposed particle heat transfer mechanism. A particle from the bulk of the bed at temperature $t_b$ moves adjacent to the heat transfer surface. While at the surface the particle receives thermal energy by unsteady state convection with the laminar fluid layer adjacent to the wall. Temperature of this fluid $t_f$ is the arithmetic average of $t_w$ and $t_b$. After some residence time the particle returns to the bulk of the bed where its excess thermal energy is dissipated.

In developing a mathematical expression describing this model the following assumptions were made:
Particle at bulk temperature

Particle resides at wall receiving heat by unsteady state convection from surrounding fluid.

Hot particle returns to bulk bed

FIGURE 8. PARTICLE MODEL FOR HEAT TRANSFER
1. The fluidized particles are spheres of uniform diameter.

2. The physical and thermal properties of the solid and gas are constant.

3. The fluid temperature adjacent to the surface is at the arithmetic mean of the wall and bulk bed temperature.

4. The major portion of heat transfer occurs by the mechanism outlined above.

5. Radiant heat transfer from the surface to the particle is neglected. Baddour and Yoon (25) have shown this effect to be negligible in packed beds at temperatures below 600°C.

6. Conduction at the point of contact of the particles and the surface is negligible.

The boundary value problem describing the temperature profile in the particle while it is near the surface is presented by Genetti and Knudsen (14). Assuming a gamma distribution for particle-surface residence times and a hexagonal packing of spheres next to the surface the following equation was developed describing the rate of heat transfer from a surface in a fluidized bed:

\[ \text{Nu}_p = \frac{hD_p}{k_g} = \frac{7.2}{\left[1 + \left(\frac{6k_g \bar{v}}{\rho_s c_s D_p^2}\right)^2\right]} \]  \tag{a}

Note: Genetti and Knudsen (14) suggest replacing the 7.2 in (a) with
the quantity $10(1-\varepsilon)^{0.5}$

where,

- $\text{Nu}_p =$ particle Nusselt number, dimensionless, $hD_p / \kappa$
- $h =$ heat transfer coefficient, BTU/hr-ft$^2\text{°F}$
- $D_p =$ average particle diameter, inches
- $\rho_s =$ density of bed particles, lbs/ft$^3$
- $k_g =$ thermal conductivity of fluidizing medium, BTU/hr-ft$^2\text{°F}$
- $C_s =$ heat capacity of bed particles, BTU/lbs°F
- $\bar{\theta} =$ particle-surface contact time, hr
- $1-\varepsilon =$ particle fraction, dimensionless

In this study air was the only fluidizing medium and the various sized glass beads were of the same density and heat capacity. Consequently, the denominator of (1) can be written as:

$$1 + C_1 \frac{\bar{\theta}}{D_p^2}$$

where $C_1$ is a constant $= \frac{6k_g}{\rho_s C_s}$ (b)

Particle residence time at the surface, $\bar{\theta}$, was not measured directly, however, it is a function of fin geometry, particle diameter, and particle Reynolds number, $Re_p$. Using dimensional analysis to represent $\bar{\theta}$ with parameters that affect it, equation (1) can be written in the following form:
\[ \text{Nu}_p = \frac{7.2}{1 + C_2 (Re_p)^a \left( \frac{D_p}{S} \right)^b \left( \frac{L}{S} \right)^c} \]  

where,

- \( L \) = fin height, inches
- \( S \) = fin spacing, inches
- \( D_p \) = particle diameter, inches

Quantities \( C_2, a, b, \) and \( c \) are evaluated from the data from this study.

Raw data from this study included the tube wall temperature, \( t_w \), the bulk bed temperature, \( t_b \), and the energy input to each finned tube, \( q \). From this information an experimental heat transfer coefficient for convection from a surface, \( h_{\text{exp}} \), was determined with the following expressions:

\[ h_{\text{exp}} = \frac{q}{A_T} (t_w - t_b) \]  

where

- \( A_T \) = total finned tube area, \( \text{ft}^2 \)

If at this point values of \( h_{\text{exp}} \) and other pertinent information were substituted into (c) to obtain 'best fit' values for \( C_2, a, b, \) and \( c \) the correlation would be good only for copper tubes since only copper tubes were used. The \( h_{\text{exp}} \) defined in (d) is a conservative value and is a function of the fin material. The actual driving force for the
transfer of heat from the finned area of the tube is less than \( (t_w - t_b) \) because of the temperature drop along the length of the fin. Consequently, the 'true' value of the coefficient over the entire tube surface is somewhat higher than \( h_{\text{exp}} \) and would be different for different fin materials. The higher the thermal conductivity of the fin material the smaller the temperature gradient along the fin and the larger the value for \( (t_w - t_b) \).

If the temperature gradient in the fin is accounted for when determining the heat transfer coefficient then that coefficient is independent of the fin thermal conductivity. The coefficient determined in this manner is defined as \( \bar{h} \). If \( \bar{h} \) values and other pertinent information are substituted into (c) to obtain 'best fit' values for \( C_2, a, b, \) and \( c \) the resulting correlation would describe the effects of the experimental parameters on \( \bar{h} \) and would be valid for any tube material.

The next section discusses how the temperature gradient in the fin is accounted for when determining \( \bar{h} \). The above discussion and following discussion assumes \( \bar{h} \) to be constant over the entire surface of the finned tube. This is probably not strictly true, especially near the depths of the fin spaces, however the analysis is too complicated without this assumption.
To obtain $h$ from $h_{exp}$ the following procedure is followed:

1) develop an expression for the temperature distribution in a fin.

2) determine the mean fin temperature.

3) use an area weighted temperature difference for $(t_w - t_b)$ in the convection equation to obtain $h$.

The helical fins were slightly tapered; this taper was taken into account in determining the fin temperature distribution. It was later found that the taper was so slight as to not significantly affect the final values for $h$.

To obtain the temperature distribution in the fin a steady state energy balance is taken around a differential fin element. Assuming angular symmetry, the problem becomes 1-dimensional in the $r$-direction (see sketch). Heat flows by conduction into the left face of the element, while heat flows out of the element by conduction through the right face and by convection from the surface.
To account for the change in surface area of the fin due to the fin taper the Pythagorean theorem was used to express $l$ (surface length) in terms of $r$ (distance from fin base) and a (1/2 total change in fin thickness).

\[
\Delta l = \sqrt{\Delta r^2 + \Delta a^2}
\]

where $\Delta a = \Delta r \frac{(w_o - w_e)}{2L}$, $w_o$ = base fin thickness

$w_e$ = end fin thickness

substituting for $da$:

\[
\Delta l = \Delta r \sqrt{\frac{4L^2 + (w_o - w_e)^2}{4L^2}} = A_1 \Delta r
\]

where

\[
A_1 = 1 + \frac{(w_o - w_e)^2}{4L^2}
\]

Under steady state conditions the energy balance becomes:

\[-k 2\pi r w \frac{dt}{dr} \bigg|_r + k 2\pi r w \frac{dt}{dr} \bigg|_{r+\Delta r} = -2 h2\pi rA_1\Delta r(t-t_b) = 0\]

Rate of heat flow by conduction into - out of element = rate of heat flow by convection from surfaces between r and (r+â€œr)

accumulation of energy within the element
where:

\( w = \) local fin width (function of \( r \)), \( \text{ft} \)

\( t = \) local fin temperature, \( ^\circ \text{F} \)

\( k = \) thermal conductivity of the fin, \( \text{BTU/hrft}^\circ \text{F} \)

Dividing through by \( \Delta r \) and taking the limit as \( \Delta r \) goes to zero and simplifying, the following differential equation for the temperature distribution in the fin is obtained:

\[
\frac{d}{dr} \left( rw \frac{dt}{dr} \right) - \frac{2\pi A_1 r (t-t_b)}{k} = 0
\]  

To get the above expression into dimensionless form the following dimensionless variables are defined:

\[
T = \frac{t-t_b}{t_w-t_b} \quad R = \frac{r}{r_o} \quad W = \frac{w}{w_o}
\]

substituting the dimensionless variables into the differential equation, the following expression is obtained:

\[
\frac{1}{r_0} \frac{d}{dR} \left( r_o R w_o W \left( t_w-t_b \right) \frac{dT}{dR} \right) - \frac{2\pi A_1 r_0 R (t-t_b)}{k} = 0
\]

Simplifying,

\[
\frac{d}{dR} \left( RW \frac{dT}{dR} \right) - \frac{2\pi A_1 r_0^2 R}{w_o k} T = 0
\]
Let \( A_2 = \frac{2\pi A_1 r_0^2}{w_0 k} \) \hspace{1cm} (1a)

substituting in \( A_2 \)

\[
\frac{d}{dR} (RW \frac{dT}{dR}) - A_2 RT = 0
\]

Expanding out the derivative term,

\[
W \frac{dT}{dR} + R \frac{dT}{dR} \frac{dW}{dR} + RW \frac{d^2 T}{dR^2} - A_2 RT = 0
\]

Rearranging,

\[
\frac{d^2 T}{dR^2} = -[\frac{1}{R} + \frac{1}{W} \frac{dW}{dR}] \frac{dT}{dR} + \frac{A_2 T}{W}
\] \hspace{1cm} (2)

Now determine a substitution for \( 1/W \) in terms of \( R \). Looking at the sketch,

\[
\frac{dW}{dR} = -\frac{r_0}{L} \frac{w_0 - w_e}{w_0} \frac{dR}{dR} = -\frac{r_0}{L} \frac{w_0 - w_e}{w_0}
\]

Define \( \frac{dW}{dR} = -A_3 = -\frac{r_0}{L} \frac{w_0 - w_e}{w_0} \)

\[
W = \frac{w}{w_0} = 1 - \left(\frac{w_0 - w_e}{w_0}\right) \frac{r_0}{L} (R-1) = 1 - A_3 (R-1)
\]

so:

\[
\frac{1}{W} \frac{dW}{dR} = -\frac{r_0}{L} \frac{w_0 - w_e}{w_0} \frac{1}{1 - \left(\frac{w_0 - w_e}{w_0}\right) \frac{r_0}{L} (R-1)}
\]
Substituting $A_3$ for $-dW/dR$,

$$\frac{1}{W} \frac{dW}{dR} = \frac{A_3}{A_3 (R-1) - 1} = \frac{1}{R - 1 - 1/A_3}$$

Now substituting for $\frac{1}{W} \frac{dW}{dR}$ into (2),

$$\frac{d^2T}{dR^2} = - \left[ \frac{1}{R} + \frac{1}{R-1 - 1/A_3} \right] \frac{dT}{dR} + \frac{A_2T}{1-A_3(R-1)}$$

Finally, upon rearranging, the following expression describing the temperature distribution in the fin in terms of the dimensionless variables is obtained:

$$\frac{d^2T}{dR^2} = - \left[ \frac{2R - 1 - 1/A_3}{R(R-1-1/A_3)} \right] \frac{dT}{dR} + \frac{A_2T}{1-A_3(R-1)} \quad (3)$$

Two boundary conditions are needed to solve this second order differential equation.

The first one is that $T = \frac{t - t_b}{t_w - t_b} = 1$ when $R = \frac{r}{r_0} = 1$

The second one is obtained by making an overall energy balance on the finned tube. In words this balance states that the heat supplied to the finned tube is equal to the heat leaving the bare tube surface plus the heat entering the fins. In symbols, the following expression is obtained:
\[ q = \tilde{h} A_b (t_w - t_b) - N_T k 2\pi r_o w_o \frac{dt}{dr} \bigg|_{r=r_0} \]

where,

\[ N_T = \text{number of fins on the tube}. \]

In dimensionless form this equation becomes,

\[ q = \tilde{h} A_b (t_w - t_b) - N_T k 2\pi r_o w_o \frac{t_w - t_b}{r_o} \frac{dT}{dR} \bigg|_{R=1} \]

simplifying,

\[ \frac{q}{(t_w - t_b)} = \tilde{h} A_b - N_T k 2\pi w_o \frac{dT}{dR} \bigg|_{R=1} \quad (4) \]

Now \( \tilde{h} = h_{\text{exp}} = \frac{q}{(t_w - t_b) A_t} \)

Using \( h_{\text{exp}} \) as a first approximation for \( \tilde{h} \) equation (4) is written in the following form:

\[ \tilde{h} A_t = h A_b - N_T k 2\pi w_o \frac{dT}{dR} \bigg|_{R=1} \]

Solving for \( \frac{dT}{dR} \bigg|_{R=1} \),

\[ \frac{dT}{dR} \bigg|_{R=1} = \frac{\tilde{h} A_t - \tilde{h} A_b}{-N_T k 2\pi w_o} \]

Substituting \( A_f = A_t - A_b \), the final form for the second boundary condition becomes,
The 'true' $\bar{h}$ value is defined as follows:

$$\bar{h} = \frac{q}{A_t (t-t_b)_m}$$  \hspace{1cm} (6)

$(t-t_b)_m$ is an area weighted $\Delta T$ distribution equal to the fraction of tube surface area that is bare times the wall-bed temperature difference plus the fraction of the tube surface that is finned times the average fin-bed temperature difference.

$$ (t-t_b)_m = \frac{A_b}{A_t} (t_w-t_b) + \frac{A_f}{A_t} (t-t_b)_{mf} $$

where $(t-t_b)_{mf}$ is the mean fin-bed temperature difference defined as follows:

$$ (t-t_b)_{mf} = \frac{\int_{r_o}^{r_e} 2\pi r(t-t_b)dr}{2\pi (r_e^2 - r_o^2)} $$

To get into dimensionless form both sides are divided by $(t_w-t_b)$ and a new dimensionless variable is defined,

$$ T_{mf} = \frac{(t-t_b)_{mf}}{(t_w-t_b)} = \frac{\int_{r_o}^{r_e} 2r\pi r T dr}{(r_e^2 - r_o^2)} $$
Now substituting \( r \rightarrow R \) for \( r \) and changing the limits on the integral, the final form for \( T_{\text{mf}} \) in terms of dimensionless variables is obtained,

\[
T_{\text{mf}} = \frac{1}{\frac{r_e^2}{r_o^2} - 1} \int \frac{2RT}{r_o^2} dR
\]  

(7)

Equation (6) can now be written as follows:

\[
h = \frac{q}{A_t} \left( \frac{1}{A_b(t_w-t_b) + A_f(t_w-t_b)T_{\text{mf}}} \right)
\]

or

\[
h = \frac{q}{(t_w-t_b)(A_b+A_fT_{\text{mf}})}
\]  

(8)

A computer program involving finite differenced expressions and numerical integration subroutines was used to determine the temperature \( \bar{h} \) value by trial and error. (27). The procedure followed in the program is listed below.

1) Using \( h_{\text{exp}} \) as the first guess for \( \bar{h} \) an \( A_2 \) and \( \frac{dT}{dR} \) \( R=1 \) value using equations (1a and 5) respectively were determined and the temperature distribution in the fin was solved for using equation (3).

2) From the fin temperature distribution a \( T_{\text{mf}} \) value is obtained using equation (7).
3) Next, a new \( h \) value is obtained using equation (8).

4) If this new \( h \) is not within \( \pm 0.01 \) units of the old \( h \) value, steps 1 through 3 are repeated with the newly determined \( h \).

5) Steps 1 through 4 are repeated until \( h \) converges. Convergence criteria is \( h_{\text{new}} = h_{\text{old}} \pm 0.01 \) units.

All \( h_{\text{exp}} \) values were corrected to \( h \) values by this method. These \( h \) values are the ones used to correlate the data from this investigation. The correlation procedure is outlined in the next section.

Correlation Development

As an initial attempt in correlating the data, the following form of an equation was used. This, as mentioned previously, is based on the 'particle' mode mechanism for heat transfer.

\[
\text{Nu}_p = \frac{7.2}{[1 + C_2(Re_p)^a \left( \frac{D_p}{S} \right)^b \left( \frac{L}{S} \right)^c]^2}
\]

1) To obtain a value for \( a \), rearrange and take the log of both sides:

\[
\log \left[ \sqrt{\frac{7.2}{\text{Nu}_p}} - 1 \right] = a \log \text{Re}_p + \log \left[ C_2 \left( \frac{D_p}{S} \right)^b \left( \frac{L}{S} \right)^c \right]
\]

From the experimental data 26 straight lines of \( \log \left[ \sqrt{\frac{7.2}{\text{Nu}_p}} - 1 \right] \)
versus log Re<sub>p</sub> were constructed using method of least squares.

The slopes of these lines are the 'a' values. The intercepts of these lines are the log [C<sub>2</sub> (D<sub>p</sub>/S)<sup>b</sup>(L/S)<sup>c</sup>] values.

The dependence of 'a' with Re<sub>p</sub> and Nu<sub>p</sub> was unclear; 'a' was assumed to be some function of (D<sub>p</sub>/S) and (L/S) i.e.

\[ a = f \left( \frac{D_p}{S}, \frac{L}{S} \right) \text{ or } -a = -C_3 \left( \frac{D_p}{S} \right)^d \left( \frac{L}{S} \right)^e \]

The negative signs are introduced above because 'a' values were negative, which present difficulties when taking the log of both sides.

Taking the log of both sides:

\[ \log (-a) = d \log \left( \frac{D_p}{S} \right) + \log (-C_3 \left( \frac{L}{S} \right)^e) \]

Three particle diameters were used with each tube. Consequently 9 least square lines (3 points per line) are obtained when log (-a) is plotted against log (D<sub>p</sub>/S) for constant values of (L/S).

The slopes of these lines are the 'd' values. 'd' was found to be a straight line function of (L/S) i.e. \( d = -.685 \left( \frac{L}{S} \right) + 1.96 \).

The intercepts of these lines are the log (-C<sub>3</sub>(L/S)<sup>e</sup>) values.

Expanding this term out gives:

\[ \text{intercepts} = e \log \left( \frac{L}{S} \right) + \log (-C_3) \]

Plotting the log (L/S) versus the intercepts gives a straight line fit
to the 9 points. The 'e' value is -3.4 and the antilog of the intercept is the $C_3$ value which is -8.57.

So the 'a' value turns out to be the following:

$$a = -8.57 \left( \frac{L}{S} \right)^{-3.4}$$

2. To obtain values for $C_2$, $b$, and $c$, the intercepts from the 26 curves of $\log \left[ \sqrt{\frac{\text{Nu}_D}{7.2}} - 1 \right]$ versus $\log \text{Re}_p$ equals $\log \left[ C_2 \left( \frac{D}{S} \right)^b \left( \frac{L}{S} \right)^c \right]$. Expanding this term out gives:

$$\text{intercepts} = b \log \left( \frac{D}{S} \right) + \log (C_2 \left( \frac{L}{S} \right)^c)$$

9 least square lines are obtained by plotting intercept versus $\log (D_p/S)$ for constant values of $(L/S)$. The slopes of these lines are values of 'b'. The values for 'b' turns out to be relatively constant; the average value is -.77. The intercepts of these lines are values of $\log (C_2(L/S)^c)$.

Expanding this term out gives,

$$\text{intercepts} = c \log (L/S) + \log C_2$$

A straight line is obtained when the intercepts are plotted against $\log (L/S)$. The slope of this line is the 'c' value which is .75. The antilog of the intercept of this line is the $C_2$ value which is .074.
The correlation is now determined. It is:

\[ \frac{Nu_p}{7.2} = \frac{1 + 0.074 \, \text{Re}_p}{[1 - 8.57 \left( \frac{L}{S} \right) - 3.4 \, \frac{D_p}{S} - 0.685 (L/S) + 1.96 \left( \frac{L}{S} \right)^2]} \]

The data from this investigation was plotted on a log-log plot of \( \frac{Nu_p}{7.2} \) versus the denominator on the right hand side of the equation \([[]]^2\). The plot took on a fan shape with a definite \((L/S)\) dependency. A further refinement in the correlation is necessary. Leaving the results within the brackets in the denominator on the right hand side alone \([[]]\), the following form of the correlation is assumed:

\[ Nu_p = \frac{A}{B} \]

where \(A\) and \(B\) are some functions of \((L/S)\). Taking the log of both sides:

\[ \log Nu_p = -B \log [ ] + \log A \]

Plotting \(\log Nu_p\) versus \(\log [ ]\) for constant \((L/S)\) values, 9 least square lines were obtained. The slopes of these lines are 'B' values. The intercepts of these lines are values for \(\log A\). Least square straight lines were then obtained from plots of \((L/S)\) versus 'B' values and \((L/S)\) versus 'A' values. The results:
The final form of the equation is then,

$$\text{Nu}_p = \frac{-0.26 \frac{L}{S} + 7.1}{1 + 0.074 \text{Re}_p \left[ -8.57 \left( \frac{L}{S} \right) -3.4 \frac{D_p}{S} -0.685 \frac{L}{S} + 1.96 \right]}$$

$$\left( \frac{D_p}{S} \right)^{-0.77} \left( \frac{L}{S} \right)^{0.75} \frac{0.24(L/S)+1}{\left( \frac{L}{S} \right)}$$

All of the experimental data is plotted in Figure 9 with the correlation line. Most of the data lies within ±20% of the correlation. This ±20% is the error determined in the error analysis on page 84.

The range of correlation applicability is given below.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle diameter</td>
<td>0.0068, 0.0103, 0.0217 inches</td>
</tr>
<tr>
<td>Fin height</td>
<td>0.234, 0.274, 0.352, 0.375, 0.414 in.</td>
</tr>
<tr>
<td>Fin spacing</td>
<td>0.0396 to 0.1840 inches</td>
</tr>
<tr>
<td>Tube diameter</td>
<td>-0.453, 0.625, 0.750 inches</td>
</tr>
<tr>
<td>Fin thickness</td>
<td>0.016 to 0.025 inches</td>
</tr>
<tr>
<td>Fins per inch</td>
<td>5 to 18</td>
</tr>
<tr>
<td>Fluidizing velocity</td>
<td>100 - 700 lb/hrft²</td>
</tr>
<tr>
<td>$G/G_{mf}$</td>
<td>1 to 7</td>
</tr>
<tr>
<td>Bed temperature</td>
<td>160 to 260°F</td>
</tr>
<tr>
<td>Tube wall temperature</td>
<td>185 to 275°F</td>
</tr>
</tbody>
</table>
FIGURE 9. CORRELATION (ALL TUBES INCLUDED)
VARIABLE

Heat flux

RANGE

700 to 2600 BTU/hrft^2

Bed material
glass (sp.g. = 2.5)

A Design Problem

The correlation developed accounts for the effect of fin geometry on \( \bar{h} \), independent of fin thermal conductivity. A typical design problem would be to determine the number of continuous, helical finned tubes of a certain material required to deliver a given heat load at a specified \((t_w-t_b)\). Additional specifications would include fin height, fin spacing, tube area, particle diameter, and gas mass velocity. From the correlation a value for \( \text{Nu}_p \) is obtained and subsequently a value for \( \bar{h} \). The standard equation is used to obtain the heat delivery capability per tube.

\[
q = \bar{h} A_t \eta_t (t_w-t_b)
\]

where

\( \eta_t \) is the total efficiency of the surface, accounting for the thermal conductivity of the fin material

\[
A_t \eta_t = A_b + A_f \eta_f
\]

\( \eta_f = \frac{\text{actual heat transferred by the fin}}{\text{heat transferred if the entire fin were at } t_w} \)
Values of $n_F$ versus a parameter accounting for fin thermal conductivity and fin geometry are presented graphically in Figure 10 (26). Once the heat delivery of a single tube is determined the number of tubes required for the job is easily determined.
Figure 10. Efficiency of Continuous Helical Fins

\[ \frac{(r_0 + \frac{t}{2} - r_i)}{r_i} = \frac{3.00}{1.25} \]

\[ (r_0 + \frac{t}{2} - r_i)^{3/2} \]

\[ \frac{2h}{kt} (r_0 - r_i) \]
RESULTS AND DISCUSSION

Note: All average heat transfer coefficients, $\bar{h}$, reported in this section allow for the temperature gradient from the base to the top of the fin.

In some early pioneering work Petrie, Freeby, and Buckham (20) reported heat transfer coefficients obtained using continuous, helical, aluminum finned tubes in an air fluidized bed. Bare, 5-F.P.I., and 11-F.P.I. tubes were used. Fin height for both finned tubes was 0.406 inches. The bed material was sand with an average diameter of 0.02 inches.

A maximum deviation of 64 percent from Vreedenberg's correlation (18) was observed with the bare tubes. Vreedenberg's maximum reported deviation from his own correlation was 29 percent. Petrie, Freeby and Buckham observed a 43 percent deviation from the bare tube correlation developed from their data.

Their reported coefficients for the finned tubes were compared with values obtained using their fin dimensions and the correlation determined from my experimental results (see page 58). The $\bar{h}$ values from my correlation were adjusted for the aluminum fin material by using fin efficiency values from Figure 10, page 62.

My calculated $\bar{h}$ values differed from 15 to 69 percent from their reported experimental coefficients. In all cases my correlation reported a higher coefficient. Bartel (4) reported lower coefficients of the
order of 10 percent when irregular sand particles were used instead of spherical glass beads in his studies with serrated finned tubes. Therefore, it seems reasonable that my results should be higher than their reported values in all cases.

Using their reported deviations from Vreedenberg's correlation and their own correlation as a yard stick it is suggested that the \( \bar{h} \) values obtained in my investigation are 'reasonable' giving some confidence to my later reported results.

**\( \bar{h} \) versus \( G \)**

It has generally been observed that \( \bar{h} \) increases, reaches a maximum value and then decreases with increasing fluidizing gas mass velocities, \( G \). This maximum is a result of two opposing factors. First, particle movement increases as \( G \) increases resulting in shorter particle-surface residence times and a resulting higher coefficient. Second, the void fraction of the bed increases as \( G \) increases resulting in a lower particle concentration adjacent to the surface and consequently lower heat transfer coefficients.

In this study the small particles (0.0068 inch diameter) exhibited steeper positive slopes followed by the medium particles (0.0103-inch diameter), followed by the large particles (0.0217 inch diameter) on the \( \bar{h} \) versus \( G \) plots. Occasionally a maximum value of \( \bar{h} \) was obtained with the small and medium beads. A negative slope was observed with two
different finned tubes with the large beads indicating that the values of $h$ obtained were on the downward side of the maximum.

The coefficient, $h$ increased with decreasing particle sizes. The increase was larger between the large, and medium beads than between the medium and small beads. Increases in $h$ of up to 50 percent were noted in the coefficient between the large and small beads. The sensitivity of the coefficient on particle size diminished as the fin spacing decreased and as the fin height increased.

A representative plot of $h$ versus $G$ with the 3 different bead sizes is shown in Figure 11. This shows the performance of tubes with fin height (L) of 0.375 inches, fin spacing (S) of 0.0871 inches, fins per inch (F.P.I.) of 9.

$h$ versus Fin Height

Four tubes with F.P.I. values of 9 were used to determine the dependency of $h$ with fin height. Fin heights varied from 0.234 inches to 0.414 inches.

Figure 12 is a representative plot showing the effect on $h$ with increasing fin height as the fluidizing gas mass velocity is increased. Large beads were used in this case.

Note that the coefficient increased with decreasing fin height. Little increase in $h$ occurred as the fin height was reduced from 0.414 inches to 0.352 inches. A larger increase in $h$ is noted as the
F.P.I. = 9, S = 0.0871 in.,
L = 0.375 in.

FIGURE 11. $h$ VERSUS G VERSUS PARTICLE DIAMETER
FIGURE 12. $h$ VERSUS AIR MASS VELOCITY VERSUS FIN HEIGHT ($L$)
fin height is reduced from 0.352 inches to 0.234 inches.

This increasing coefficient with decreasing fin height is what is predicted from 'particle' mode heat transfer theory. Particle motion into and out of the depths of the fin space is more hindered with longer fins. Hindering particle motion increases particle to surface contact times reducing the rate of heat transfer per unit of surface area.

\( \bar{h} \) versus Fin Spacing

Four tubes with fin heights of 0.234 inches were used to determine the dependence of \( \bar{h} \) with fin spacing. Fin spacings varied from 0.0396 inches to 0.1840 inches. Nominal F.P.I. values were 5, 9, 14 and 18.

Figure 13, is a representative plot showing the effects of fin spacing on \( \bar{h} \) as \( G \) is varied. Medium beads were used for this set of data. It is noted that as the fin spacing increases the coefficient increases accordingly. It is also apparent that the magnitude of the increase of the coefficient is proportional to the magnitude of the fin spacing change.

These trends are consistent with 'particle' mode heat transfer theory. As the distance between fins increases particle motion into and out of the fin space depths becomes easier thus reducing particle-surface contact residence time, increasing the rate of heat transfer per unit surface area.
FIGURE 13. $\bar{h}$ VERSUS AIR MASS VELOCITY VERSUS FIN SPACING (S)
Figure 14 shows the effect of the dimensionless ratio $D_p/S$ (particle diameter to fin spacing ratio) on $h$. This data was obtained with tubes with 0.234-inch fin height at a reduced gas mass velocity ratio ($G/G_{\text{min}}$) equal to 4.0. A constant $G/G_{\text{min}}$ value was used to obtain dynamic similarity of the 3 particle sizes, thus eliminating the particle diameter parameter from the figure.

The curve is steep for fin spacings greater than 10 particle diameters and begins to flatten out for fin spacings less than 10 particle diameters. This indicates that $h$ is quite sensitive to fin spacings greater than 10 particle diameters but becomes less sensitive as the fin spacing is reduced further. As the fin spacing is reduced particle movement into and out of the depths of the fin gaps becomes more and more hindered creating substantial defluidization in the inner regions of the fin gaps. This region of the finned tube begins to assimilate a packed bed with its corresponding lower coefficients. Finally as the fin spacing is reduced further particles become lodged in the fin spaces reducing particle packing adjacent to the surface and particle movement within the fin space thus reducing the coefficient further.

Figure 15 is also a plot of $D_p/S$ versus $h$ at $G/G_{\text{min}}$ equal to 4.0 only now all 9 tubes are included in the plot. The various tube fin heights have added some scatter to the curve but the same general trends as previously discussed are still apparent. In the range of fin
Fin Height = 0.234-in.

\( G/G_{\text{min}} = 4.0 \)

Note: \( h \) for a bare tube \( (D_p/S=0) \approx 120 \text{ Btu/hrftsq°F} \)

**FIGURE 14.** \( h \) VERSUS \( D_p/S \) (CONSTANT FIN HEIGHT)
Note: $h$ for a bare tube ($D_p/S=0$) = 120 Btu/hr ft$^2$°F (4)

$G/G_{\text{min}} = 4.0$

FIGURE 15. $\bar{h}$ VERSUS $D_p/S$ (ALL TUBES)
heights and fin spacings looked at in this thesis, fin spacing seems to have a more dominant influence on $\bar{h}$ than does fin height.

Priebe (24) looked at the effect of fin spacing on the coefficient with serrated finned tubes. He observed little effect when the fin spacing was greater than 10 particle diameters. At less than 10 particle diameters the coefficient fell rapidly up to a point where the spacing is less than 2 particle diameters. From that point on the curve fell very slowly.

The observations from my study are not in conflict with Priebe's results. A continuous fin presents much more interference to particle motion much earlier than does a serrated fin. Particle motion is mostly restricted to an in and out motion in the fin space with a continuous fin whereas, particles can move into and around the serrated fins. Consequently for a fin spacing as high as 30 particle diameters marked particle motion interference is noted with a continuous finned tube. The fin spacing of a continuous finned tube is essentially defluidized at a fin spacing of 10 or less particle diameters hence fin spacings in this range have little effect on $\bar{h}$.

$q/\Delta T$ for the Tubes

A convenient way of rating the performance of all the tubes irrespective of fin geometry is by determining $q/\Delta T$ values. $q/\Delta T$ values are indicative of how much heat a given tube can transmit to the bed
per unit degree of temperature driving force. An equivalent expression for \( \frac{\text{q}}{\Delta T} \) is the product of \( h_{\text{exp}} \) and the tube area. Two opposing factors determine the \( \frac{\text{q}}{\Delta T} \) value. Increasing the tube area either by decreasing the fin spacing or increasing the fin height results in a lower heat transfer coefficient. Designing a tube to obtain a maximum \( \frac{\text{q}}{\Delta T} \) value involves juggling the fin height and fin spacing parameters.

Figure 16 is a plot of \( \frac{\text{q}}{\Delta T} \) versus \( G \) with four tubes with equal fin spacing but different fin heights. Large beads were used for this data. As can be seen \( \frac{\text{q}}{\Delta T} \) increased with increasing fin height. The magnitude of the \( \frac{\text{q}}{\Delta T} \) change decreased with increasing fin height. It is clear that a maximum \( \frac{\text{q}}{\Delta T} \) value would be obtained at a higher fin height.

Figure 17 is a plot of \( \frac{\text{q}}{\Delta T} \) versus \( G \) with four tubes of equal fin height but different fin spacings. Medium particles were used for this data. \( \frac{\text{q}}{\Delta T} \) increased with decreasing fin spacing. Again it is clear that a maximum \( \frac{\text{q}}{\Delta T} \) value would be reached at some smaller fin spacing.

Figure 18 is a representative plot of \( \frac{\text{q}}{\Delta T} \) versus \( G \) for all tubes studied. Large particles were used for this data. The results of the 3 tubes shown, span the range of data for all 9 tubes. Only 3 of the nine tubes are shown to give clarity to the figure.

In Table III all nine tubes are listed in order of decreasing average \( \frac{\text{q}}{\Delta T} \) values for all 3 particle sizes. It was found that the order of ranking is independent of particle diameter. The best
FIGURE 16.  $q/\Delta T$ VERSUS G VERSUS FIN HEIGHT
FIGURE 17. \( \frac{q}{\Delta T} \) VERSUS G VERSUS FIN SPACING
FIGURE 18. $\frac{q}{\Delta T}$ VERSUS G
<table>
<thead>
<tr>
<th>Rank</th>
<th>q/ΔT</th>
<th>F.P.I.</th>
<th>Fin Spacing (in)</th>
<th>Fin Height (in)</th>
<th>q/ΔT BTU/hr F (small beads)</th>
<th>q/ΔT BTU/hr F (Med. Beads)</th>
<th>q/ΔT BTU/hr F (Large Beads)</th>
<th>Ranking Based on h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>0.0951</td>
<td>0.414</td>
<td></td>
<td>56.16</td>
<td>47.55</td>
<td>38.77</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>0.0871</td>
<td>0.375</td>
<td></td>
<td>54.85</td>
<td>49.00</td>
<td>38.33</td>
<td>5</td>
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<tr>
<td>3</td>
<td>7</td>
<td>0.1229</td>
<td>0.375</td>
<td></td>
<td>51.02</td>
<td>42.35</td>
<td>33.28</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>0.0396</td>
<td>0.234</td>
<td></td>
<td>45.83</td>
<td>42.24</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>0.0951</td>
<td>0.352</td>
<td></td>
<td>44.77</td>
<td>40.96</td>
<td>32.23</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>0.0554</td>
<td>0.234</td>
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<td>43.67</td>
<td>39.57</td>
<td>30.77</td>
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</tr>
<tr>
<td>7</td>
<td>9</td>
<td>0.0951</td>
<td>0.234</td>
<td></td>
<td>39.27</td>
<td>33.54</td>
<td>26.17</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>0.1840</td>
<td>0.234</td>
<td></td>
<td>31.69</td>
<td>26.95</td>
<td>20.78</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>0.1750</td>
<td>0.274</td>
<td></td>
<td>24.98</td>
<td>20.42</td>
<td>15.65</td>
<td>2</td>
</tr>
</tbody>
</table>
performer was a tube of intermediate fin spacing with maximum fin height.

**Tube Diameter**

Two tubes studied had identical fin heights with different fin spacings and tube diameters. The specific dimensions are:

<table>
<thead>
<tr>
<th>Tube</th>
<th>F. P. I.</th>
<th>L(in)</th>
<th>S(in)</th>
<th>Tube Diameter (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
<td>0.375</td>
<td>0.0871</td>
<td>0.625</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>0.375</td>
<td>0.1229</td>
<td>0.750</td>
</tr>
</tbody>
</table>

The larger diameter tube (tube 9) had the same fin height but larger fin spacing. From previous noted trends, $h$ would be expected to be larger for tube 9 than tube 2 because of the greater degree of freedom of particle motion within the larger fin spacing. This was not observed; within experimental error both tubes had the same heat transfer coefficients for all three particle diameters.

One qualitative conclusion can be drawn from these results. The heat transfer coefficient tends to decrease with increasing tube diameter. This phenomenon can be explained with the particle theory for heat transfer. Fin surface area increases with increasing tube diameter for a constant fin height. The more fin surface that the random moving particles have to interact with the more hindered they become. Again, the more hindered particle motion becomes the smaller
the coefficient.

It should be noted that the tube diameter effect on the coefficient was not significant enough that tube diameter had to be included as a parameter in the correlation.
CALCULATIONS

Air Mass Velocity

As mentioned previously a vena contracta orifice with a water manometer were used to determine the air mass velocity to the column. A standard equation for an orifice is used.

\[
G = \frac{3600 \, C_0 \, Y \, S_c}{A_c} \sqrt{\frac{2g_c \, (P_1-P_2) \, \rho_1}{1-\beta^4}}
\]

where,

- \( G \) = air mass velocity, lbm/hr-ft²
- \( C_0 \) = orifice coefficient
- \( Y \) = expansion factor, dimensionless
- \( S_c \) = cross sectional area of the orifice, ft²
- \( A_c \) = cross sectional area of the column, ft²
- \( g_c \) = gravitational constant, ft-lbm/hr²-lbf
- \( P_1-P_2 \) = pressure drop across the orifice, lbf/ft²
- \( \rho_1 \) = density of air at the upstream pressure, lbm/ft³
- \( \beta \) = ratio of orifice diameter to inside pipe diameter, dimensionless

For a square edged orifice, the expansion factor is given as:

\[
Y = 1 - \frac{P_1 - P_2}{P_1 k_r} (0.41 - 0.35 \beta^4)
\]

where

\[
k_r = \frac{C_p}{C_v}
\]

The orifice coefficient is a function of the Reynolds number, and
was found by trial and error. It was found to be very nearly constant and equal to 0.6 \( (4) \). This value was used in all calculations.

**Bed Temperature**

Bed temperature was determined by averaging the 3 in bed thermocouple readings. All 3 in bed thermocouples read within 1 or 2 degrees of each other.

**Tube Temperature**

Each tube temperature was read directly off the chart recorder.

**Heat Input to Each Tube**

Electrical power input to each tube was measured with a Simpson Wattmeter. A conversion factor of 3.413 BTU/watt-hr was used to convert the measured watts to BTU/hr.

**Area of Each Tube**

The surface area of each tube was determined by calculating the bare tube area and adding on the finned area. Finned area was determined by multiplying the area of a fin by the number of fins on the tube.

**\( h_{\text{exp}} \) for Each Tube**

The experimental heat transfer coefficient for each tube was
calculated from the standard equation for convection from a surface.

\[ h_{\text{exp}} = \frac{q}{A_T (t_w - t_b)} \quad \text{BTU/hr-ft}^2{^\circ\text{F}} \]

For the Bundle of Tubes
\[ h_{\text{exp}} \]

For the bundle of tubes was determined by averaging the 7 individual \( h_{\text{exp}} \) values.

Air Viscosity and Thermal Conductivity

Air viscosity was determined by using an equation fit to experimental data.

\[ \mu_f = [2.45(t_b-32) + 1538.1] \times 2.688 \times 10^{-5}, \text{lb/ft-hr} \]
\( t_b \) is in \(^\circ\text{F} \).

Air thermal conductivity was determined by linear interpolation between selected listed values in Kreith (26). Evaluation temperature was \((t_w + t_b)/2\).

Particle Reynolds Number

\[ \text{Re}_p = \frac{D_p G}{\mu_f} \quad \text{dimensionless} \]

Particle Nusslet Number

\[ \text{Nu}_p = \frac{\bar{h} D_p}{k_g} \quad \text{dimensionless} \]
**ERROR ANALYSIS**

Assuming $h_{\text{exp}}$ is only affected by the experimental determinations of $q$ and $(t_w - t_b)$, the error analysis is performed on the following equation:

$$h_{\text{exp}} = \frac{q}{A_t(t_w - t_b)}$$

The wattmeter is assumed to be accurate to within ±5 percent. No error is assumed in determining the area of the fin. The bed temperature is assumed to be measured to within ±0.5°F. The tube wall temperature could be measured to within ±1.5°F. A minimum $(t_w - t_b)$ value of 15°F was experimentally observed.

Using the above assumed experimental accuracies maximum and minimum errors for $h_{\text{exp}}$ are determined. This analysis is based on an $h_{\text{exp}}$ 'true' value of 1.0.

**Maximum $h_{\text{exp}}$**

$$h_{\text{exp}} = \frac{1.05}{1 - 2/15} = 1.21$$

Error $= (1.21 - 1.0) \times 100 = +21$ percent

**Minimum $h_{\text{exp}}$**

$$h_{\text{exp}} = \frac{0.95}{1 + 2/15} = 0.84$$

Error $= (0.84 - 1.0) \times 100 = -16$ percent

Therefore the experimental error range bracketing all results is +21 percent, -16 percent. Therefore, maximum deviations from 'true' values
should be about ±20 percent.
SUMMARY OF RESULTS AND CONCLUSIONS

The following are based on results and conclusions from this study.

1) It was generally observed that the heat transfer coefficient increased with increasing air mass velocity. A maximum was observed in some cases with small and medium beads.

2) The heat transfer coefficient increased with decreasing particle size. The increase was larger between the large and medium beads than between the medium and small beads. Increases in the coefficient of up to 50 percent were observed when going from large to small beads.

3) Some increased experimental accuracy resulted by using higher heat fluxes with the tubes when larger beads were used.

4) No radial tube wall temperature gradient was observed.

5) The heat transfer coefficient increased with decreasing fin height. Little increase is observed in the coefficient as the fin height was reduced from 0.414 to 0.352 inches. A larger increase is noted as the fin height is reduced from 0.352 to 0.234 inches.

6) The heat transfer coefficient increased proportionally with increased fin spacing.

7) The heat transfer coefficient was very sensitive to fin spacings as large as 30 particle diameters. The coefficient was less sensitive to fin spacings of less than 10 particle diameters. It is believed that defluidization in the fin spaces was significant for fin spacings less than 10 particle diameters.
8) The heat transfer coefficient decreased with increasing tube diameter.

9) The best performer as far as heat delivery capabilities was a tube of intermediate fin spacing and maximum fin height.

10) A correlation relating experimental parameters to the heat transfer coefficient was developed. Most of the data fell within ±20 percent of the correlation. This was the maximum error determined from an error analysis.
RECOMMENDATIONS

Three studies (23,24) including this one have now been completed. Serrated, spined, and continuous finned tubes have been examined. Parameters have included tube spacings, fin geometries, fin materials, bed material, sizes and shapes. The 'particle' heat transfer theory has satisfactorily accounted for all trends observed. This is significant in that what is actually going on in fluidized beds is being realized.

The correlations developed are based on narrow ranges of operating conditions. Furthermore extension of the correlations to other conditions would probably give unsatisfactory results. Research of this kind is interesting and somewhat pioneering but not directly useful. Studies of this nature are performed in the hopes that eventually fluidized bed dynamics will be fully realized. Then, perhaps, more general and more useful correlations with wide design applicability will be developed. We have a long way to go.

One specific recommendation for further study is to extend the ranges of applicability of the existing correlations. This would bring us closer to a 'general' correlation. Areas for further investigation include using different fin dimensions, distributor plates, column dimensions, tube locations, bed materials, shapes, and sizes. Determining means to successfully scale up results from previous studies would be a real break through for the designer.
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>1/2 total change in fin thickness from base to tip</td>
<td>ft</td>
</tr>
<tr>
<td>( a, b, c, d, e, C_2 )</td>
<td>Functions of experimental parameters in correlation equation</td>
<td>dimensionless</td>
</tr>
<tr>
<td>( A, B )</td>
<td>Functions of ((L/S)) in correlation equation</td>
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</tr>
<tr>
<td>( A_b )</td>
<td>Bare tube area</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( A_c )</td>
<td>Inside cross-sectional area of column</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( A_f )</td>
<td>Finned area of tube</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( A_t )</td>
<td>Total tube area (fin + bare areas)</td>
<td>ft(^2)</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>Orifice coefficient</td>
<td>dimensionless</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Heat capacity at constant pressure</td>
<td>BTU/hr-ft(^2)-°F</td>
</tr>
<tr>
<td>( C_v )</td>
<td>Heat capacity at constant volume</td>
<td>BTU/hr-ft(^2)-°F</td>
</tr>
<tr>
<td>( C_s )</td>
<td>Solid particle heat capacity</td>
<td>BTU/hr-ft(^2)-°F</td>
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<tr>
<td>( D_p )</td>
<td>Particle diameter</td>
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</tr>
<tr>
<td>( G )</td>
<td>Air mass velocity</td>
<td>lbs/hr-ft(^2)</td>
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<tr>
<td>( g_c )</td>
<td>Constant</td>
<td>lbs-ft/lb force-hr(^2)</td>
</tr>
<tr>
<td>( G_{\text{min}} )</td>
<td>Air mass velocity at minimum fluidizing conditions</td>
<td>lbs/hr-ft(^2)</td>
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<tr>
<td>( h )</td>
<td>Heat transfer coefficient, average</td>
<td>BTU/hr-ft(^2)-°F</td>
</tr>
<tr>
<td>( h_{\text{exp}} )</td>
<td>Experimental heat transfer coefficient, average</td>
<td>BTU/hr-ft(^2)-°F</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Dimension</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>$h$</td>
<td>Heat transfer coefficient independent of fin material, average</td>
<td>BTU/hr-ft²-°F</td>
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<tr>
<td>$k$</td>
<td>Thermal conductivity of fin</td>
<td>BTU/hr-ft-°F</td>
</tr>
<tr>
<td>$k_g$</td>
<td>Thermal conductivity of fluiding medium</td>
<td>BTU/hr-ft-°F</td>
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<td>Ratio of $C_p/C_v$</td>
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<tr>
<td>$l$</td>
<td>Fin surface length</td>
<td>ft</td>
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<tr>
<td>$L$</td>
<td>Fin height</td>
<td>inches</td>
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<tr>
<td>$l_e$</td>
<td>Equivalent thickness of emulsion layer</td>
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<tr>
<td>$l_g$</td>
<td>Thickness of gas film</td>
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<td>$N_T$</td>
<td>Number of fins on tube</td>
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<tr>
<td>$Nu_p$</td>
<td>Particle Nusselt number=$hD_p/k_g$</td>
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<tr>
<td>$(p_1-p_2)$</td>
<td>Pressure drop across the orifice</td>
<td>lb/ft²</td>
</tr>
<tr>
<td>$q$</td>
<td>Rate of heat transfer</td>
<td>BTU/hr</td>
</tr>
<tr>
<td>$q_l$</td>
<td>Rate of sensible heating of moving solids</td>
<td>BTU/hr</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Rate of heat transfer to bulk bed by particle exchange</td>
<td>BTU/hr</td>
</tr>
<tr>
<td>$r$</td>
<td>Distance from base of fin</td>
<td>ft</td>
</tr>
<tr>
<td>$R$</td>
<td>$r/r_o$</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$r_e$</td>
<td>Radius of tube at fin tip</td>
<td>ft</td>
</tr>
<tr>
<td>$Re_p$</td>
<td>Particle Reynolds number, $GD_p/\mu_f$</td>
<td>dimensionless</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Dimension</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>-----------</td>
</tr>
<tr>
<td>( r_o )</td>
<td>Radius of tube at fin base</td>
<td>ft</td>
</tr>
<tr>
<td>( S )</td>
<td>Fin spacing</td>
<td>inches</td>
</tr>
<tr>
<td>( S_c )</td>
<td>Cross-sectional area of the orifice</td>
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</tr>
<tr>
<td>( t )</td>
<td>Local tube temperature</td>
<td>°F</td>
</tr>
<tr>
<td>( T )</td>
<td>( \frac{(t-t_b)}{(t_w-t_b)} )</td>
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</tr>
<tr>
<td>( t_b )</td>
<td>Bulk bed temperature</td>
<td>°F</td>
</tr>
<tr>
<td>( t_f )</td>
<td>Fluid temperature adjacent to heat transfer surface. Arithmetic average of ( t_b ) and ( t_w )</td>
<td>°F</td>
</tr>
<tr>
<td>( T_{mf} )</td>
<td>( \frac{(t-t_b)_{mf}}{(t_w-t_b)} )</td>
<td>dimensionless</td>
</tr>
<tr>
<td>( t_p )</td>
<td>Particle temperature</td>
<td>°F</td>
</tr>
<tr>
<td>( t_w )</td>
<td>Heat transfer surface temperature</td>
<td>°F</td>
</tr>
<tr>
<td>( t(\theta) )</td>
<td>Unsteady state particle temperature</td>
<td>°F</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>Temperature difference</td>
<td>°F</td>
</tr>
<tr>
<td>( (t-t_b)_m )</td>
<td>Mean finned tube-bed ( \Delta T )</td>
<td>°F</td>
</tr>
<tr>
<td>( (t-t_b)_{mf} )</td>
<td>Mean fin-bed ( \Delta T )</td>
<td>°F</td>
</tr>
<tr>
<td>( u_s )</td>
<td>Velocity of solids</td>
<td>ft/sec</td>
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<tr>
<td>( w )</td>
<td>Fin thickness</td>
<td>ft</td>
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<tr>
<td>( W )</td>
<td>( \frac{w}{w_0} )</td>
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<td>( w_e )</td>
<td>Fin tip thickness</td>
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<td>( w_0 )</td>
<td>Fin base thickness</td>
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<td>Dimension</td>
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<tr>
<td>$\beta$</td>
<td>Ratio of orifice diameter to inside pipe diameter</td>
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<td>$(1-\epsilon)$</td>
<td>Particle fraction</td>
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<td>$n_f$</td>
<td>Efficiency of fin surface</td>
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</tr>
<tr>
<td>$n_t$</td>
<td>Efficiency of entire finned tube surface</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\bar{t}$</td>
<td>Average particle-surface contact time</td>
<td>hr</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>Viscosity of fluidizing gas at $t_f$</td>
<td>lbs/hr-ft</td>
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<tr>
<td>$\rho_s$</td>
<td>Density of solid particles</td>
<td>lb/ft³</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>Density of air at upstream pressure</td>
<td>lb/ft³</td>
</tr>
</tbody>
</table>


