The influence of plastic flow upon the stresses developed in short span reinforced concrete beams
by Dimitri Lambropulos

A THESIS Submited to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University

Abstract:
In this thesis, it has been attempted to determine the actual stress variation In the concrete on the compression -side of short-span reinforced concrete beams* and to prove that the "Straight-Line Theory", adopted up to now, gives only a portion of the stresses developed,since, in the deri-vation of its formulae, the effects of plastic flow and shrinkage on the final stresses set up in the concrete are not considered.

Data and results were obtained from' tests conducted on two short beams; one with web reinforcement designed by the plastic flow method, and one assuming straight-line stress-strain variation.

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THE INFLUENCE OF PLASTIC FLOW UPON THE STRESSES DEVELOPED IN SHORT SPAN REINFORCED CONCRETE BEAMS

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DIMITRI LAMBROPULOS

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Submitted to the Graduate Committee

in

partial fulfillment of the requirements

for the degree of

Master of Science in Civil Engineering

at

Montana State College

Approved:

In Charge of Major Work

Chairman, Examining Committee

Dean, Graduate Division

Bozeman, Montana
September, 1948

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ABSTRACT

In this thesis, it has been attempted to determine the actual stress variation in the concrete on the compression side of short-span reinforced concrete beams, and to prove that the "Straight-Line Theory", adopted up to now, gives only a portion of the stresses developed, since, in the derivation of its formulae, the effects of plastic flow and shrinkage on the final stresses set up in the concrete are not considered.

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The actual stress distribution on the compression side of the reinforced concrete beams tested, as obtained from the results shown in Tables IV and V, are shown in Figures 15 and 16. A line representing the distribution of stress in the concrete is curved, somewhat close to the shape of a parabola, but far from a straight line.

The capacity of the test machine was not sufficient to break the beams; therefore, a comparison between the two web reinforcement design methods was not obtained.

NOMENCLATURE

fs = Compressive stress in steel.

s = The shortening of the concrete block due to shrinkage if the block was not reinforced.

fc = Tensile stress in concrete.

Es = Modulus of elasticity of steel.

Ec = Modulus of elasticity of concrete.

Ac = Effective area of concrete.

p = Steel ratio (As/bd).

A = Overall area of concrete in a column.

$n = \frac{Es}{Ec}$

Δe = Elastic shortening of column

c_p = The plastic flow of a unit length of plain concrete under unit stress from the time of loading to any given time under consideration.

dc = The plastic flow of a unit length of plain concrete for a given infinitesimal time interval dt due to a unit stress intensity.

dfc = The change of stress in concrete.

dfs = The change of stress in steel.

fs_1 = Actual unit steel stress combining elastic and plastic effects.

Δfc = Total change in concrete stress due to plastic flow.

Δfs = Total change in steel strength due to plastic flow.

fc_1 = The intensity of stress in the concrete at the time interval dt due both to the applied load and the effects of plastic flow up to that time, and assumed constant through the small interval dt.

fco = The original concrete stress due to elastic action only.

fso = The original steel stress due to elastic action only.

y = The plastic flow in millionths of an inch per inch for a unit stress of one pound per square inch.

x = Time after loading, in days.

a_1 and c_1 = Constants varying with the material.

Ecx = The effective modulus of elasticity.

Ec = Instantaneous modulus of elasticity.

fc' = Compressive strength of a concrete cylinder.

C = The total compression.

a = Depth of equivalent stress diagram.

kd = The depth from the compressive side of a beam to the neutral axis.

b = Width of beam.

c = Arm of resisting couple (plastic theory method).

M = Moment of resistance.

T = Total tension.

d' = The distance from the compression steel to the tension steel.

As' = Area of the compression steel.

As = Area of tension steel.

P = Applied load.

ds = A short element of distance.

V = Shearing force.

v = Intensity of shear.

ΔC = Increment in compressive force.

ΔT = Increment in tensile force.

u = Intensity of bond stress.

Σo = The sum of the bar perimeters.

t = Average Shearing intensity.

ϵ_s = Unit shear deformation.

G = Shearing modulus of elasticity for concrete.

INTRODUCTION

1. Statement and scope of problem

From several investigations on the behavior of concrete, it has been found that the elastic strains due to application of a load give only a portion of the actual strains at any time in the concrete and steel. In addition to the above stated strains, there are also strains produced even before the application of the load, due to shrinkage. A long-continuing load, such as dead load, will give increasing plastic strains as time passes. Another factor that might give an increase in the above strains is the change in temperature.

In any case, the relation between steel and concrete strains will be very different from that given by the equations derived by the standard method, due to loads only. Recently, certain designers have advocated designing for failure, the method of analysis being based upon tests of beams to failure. By this approach, the ultimate strains will include shrinkage, load and some plastic flow strains.

These additional strains due to shrinkage and flow in members in bending, result in producing a non linear stress variation in the concrete on the compression side. An assumption of a parabolic variation is frequently made which the writer feels is not justified.

In this thesis, the author has attempted to find the actual stress variation in the concrete on the compression side by tests conducted on full size beams in the laboratory at Montana State College, and to determine the shearing stress at failure.

2. Acknowledgement

Grateful acknowledgement is made to Professor R. C. DeHart for his kind assistance, interest and suggestions during and before the experimental work. Acknowledgement is also due to the Mechanical Engineering Department for the use of their equipment.

PLASTIC FLOW AND VOLUME CHANGES OF CONCRETE

It is only comparatively recently that the attention of the engineering profession has been called to the plastic action of concrete under sustained constant loads. [5] *

In the technical literature up to about twenty years ago, there was frequent reference to the strength and elasticity of concrete, but only an occasional reference to shrinkage due to drying and almost no reference to the gradual deformation under the action of sustained load, which has been variously called "creep", "time yield," and "plastic flow".

As a result of the research of recent years, there has been developed a general conception of the effect of shrinkage and plastic flow upon the behavior of concrete structures. Under constant load, the deformation of the concrete increases progressively, due to plastic flow, which may be of appreciable magnitude and cause large changes of stress from those set up initially, thus leading to marked changes in design and construction practices.

It is now generally believed that shrinkage and plastic flow are closely related phenomena, each being primarily due to changes in the amount of absorbed water in the cement gel and being but little directly influenced by the free water occupying the pore spaces within the concrete mass.

Plastic flow of concrete has been found to depend upon such factors as the magnitude of stress, the strength of the concrete, the duration

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* Numbers refer to references given in Bibliography.

of the loading period, the humidity of the atmosphere, the age of the concrete, the characteristics of the aggregates and the quantity of cement paste. [5]

In spite of the rather extensive researches that have been made in this field, it is not yet possible quantitatively to state with any degree of certainty what is likely to be the magnitude of either plastic flow or shrinkage under the conditions which surround any given concrete structure, nor is it possible quantitatively to state with accuracy what is the effect of plastic flow and shrinkage upon the magnitude and distribution of stresses. [1]

The property of shrinkage of concrete due to drying is altogether undesirable, but while probably shrinkage can never be eliminated, there seems to be the possibility, through the proper selection of materials and methods, of reducing shrinkage to a point where it will not be a factor for serious consideration. On the whole, plastic flow does not seem to be an undesirable property. In certain reinforced concrete members, it tends to make possible more efficient use of steel, and in thin structures subjected to drying, as well as in mass structures subjected to thermal changes due to the hydration of cement, it tends to promote a more favorable distribution of stresses than would otherwise exist.

Recent findings of investigations carried on at the University of California are as follows. [1]

Plastic Flow Under Long-Time Loading

In tests under long-time loading, it was found that even after 10 years some plastic flow was still discernible in plain concretes under sustained stresses of several hundred pounds per square inch; however, over 95 percent of the probable total flow took place within 5 years. In a series of tests on reinforced concrete columns under load for more than 5 years, it was found that, in terms of stresses in the steel or concrete, the movements due to drying shrinkage or to plastic flow are of practical importance during the first year.

The plastic flow of plain concrete cylinders, which have been under load for 10 years, are shown in Figure 1. One group of specimens was loaded at the age of 28 days, and the other at the age of 3 months; prior to loading, the concrete was moist cured, and after loading, it was stored in air at 70° F. The sustained compressive stresses were 300, 600, 900 and 1200 pounds per square inch.

In table I are summarized the results of tests upon reinforced concrete columns under load for about 5½ years. There are shown the stresses in the longitudinal steel and in the concrete due to the combined effect of elastic and plastic strains produced by the sustained load and of length changes due to causes other than applied load. As flow progressed, more and more load was transferred from the concrete to the steel, and the rate of flow decreased in greater proportion than would be the case for plain concrete sustaining the same initial stress.

It will be noted that while under conditions of wet storage, the stress in the steel has not greatly increased during the period of sustained load; under dry conditions for the columns with the smaller percentage of reinforcement the stress in the steel has increased more than five times and for the columns with the larger percentage of reinforcement, the concrete is actually in tension.

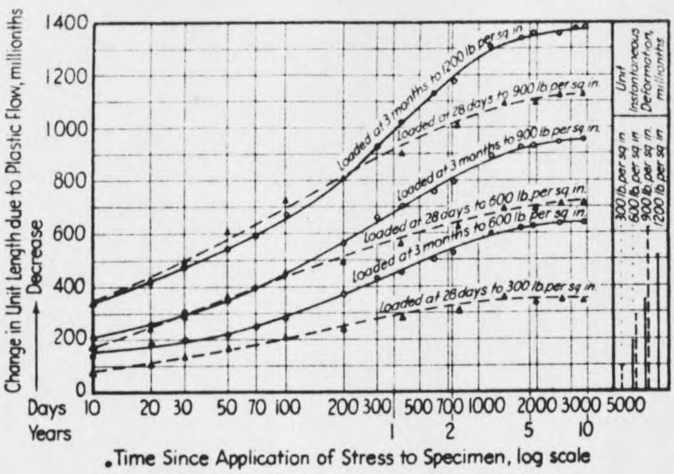

FIG. 1.—Flow Under Long-Sustained Stress

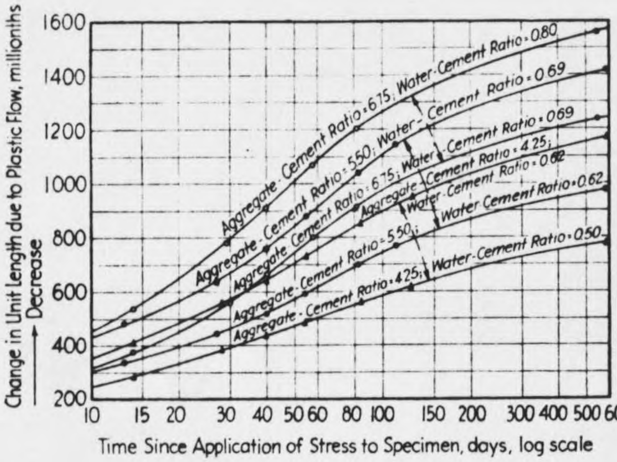

FIG. 2.—Effect of Aggregate-Cement Ratio and Water-Cement Ratio Upon Flow

Effect of Aggregate-Cement Ratio and Water-Cement Ratio upon Plastic Flow.

Tests to determine the effect of water-cement ratio and aggregate-cement ratio upon plastic flow indicated that the more the cement paste, the less the flow and that this variable was very important. Within the range of normal concretes tested, it was observed that with pastes with the same water cement ratio, the flow was practically proportional to the paste content of the concrete.

For a more thorough understanding of the effects of the water-cement and aggregate-cement ratios upon the plastic flow, refer to Figure 2.

Flow in Axial Tension and Compression

In tests to compare the flow of concrete in axial tension and compression, it was observed that, at least during the early ages, the flow under tensile stress was greater than that under compressive stress. At the later ages, the rate of flow was less under tensile than under compressive stress.

The results of a series of tests to study the flow of concrete under sustained tension and compression are shown in Figures 3 and 4.[1]

Fiber Strains Under Sustained Flexural Load

Similar results as those obtained in determining the flow in axial tension and compression, were found in tests to determine the fiber strains in plain concrete beams under constant sustained bending moment.

Effect of Fineness and Composition of Cement upon Plastic Flow.

In tests to determine the effect of fineness and composition of cement upon plastic flow, it was found that a low-heat type * of portland cement

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* A low-heat cement has a low percentage of tricalcium aluminate, and a

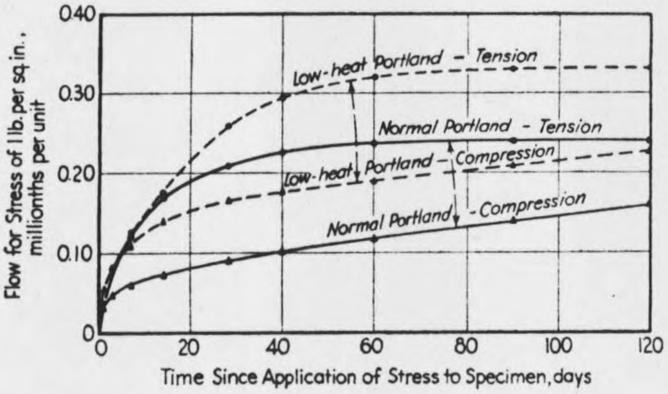

FIG. 3.—Flow in Compression and Tension—Mass-Cured Concretes.

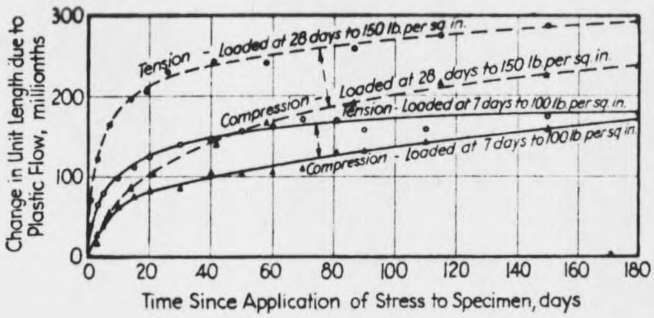

FIG. 4.—Flow in Compression and Tension—Standard-Cured Concrete.