An economic analysis of nitrogen fertilization of livestock pastureland in a semi-arid region
by Frederick Vincent Linse

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Agricultural Economics
Montana State University

Abstract:
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Date December 6, 1971

AN ECONOMIC ANALYSIS OF NITROGEN FERTILIZATION OF LIVESTOCK PASTURELAND IN A SEMI-ARID REGION

by

FREDERICK VINCENT LINSE

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

Agricultural Economics

Approved:

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MONTANA STATE UNIVERSITY
Bozeman, Montana

December, 1971

ACKNOWLEDGEMENT

The author wishes to express gratitude to Dr. M. S. Stauber for services ably rendered in his capacity as major advisor. He desires to acknowledge the invaluable manuscript criticisms contributed by Drs. O. R. Burt, D. H. Myrick and R. Stroup. Finally the author is deeply indebted to his parents, Vincent and Evelyn Linse, for their moral support throughout the course of his graduate study.

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ABSTRACT

Physical experimentation has shown nitrogen carryover in livestock pastureland fertilization to be of considerable importance. Little attempt has been made to evaluate the economic significance of nitrogen carryover. The present paper utilizes an implicit method of estimating nitrogen carryover to provide quantitative information for a dynamic programming analysis of the economic decision-making problem. The results of the study demonstrate the potential for increases in economic efficiency through use of non-zero nitrogen fertilization policies.

INTRODUCTION

Returns to the agricultural production activity are low relative to some other forms of productive endeavor in our society. In response, ambitious agricultural research programs have been initiated to search out potential return-increasing technology. One such area of research interest has been nitrogen fertilization of livestock pasture lands in semi-arid areas as a means of increasing livestock output.

Physical research has indicated that grasses growing in the northern great plans usually respond to applications of nitrogen fertilizer [1,2,3]. In addition, such grasses have shown a residual response to nitrogen applied in previous years [3,4,5]. The latter result implies that nitrogen fertilizer in the soil is carried over from production period to production period in a plant-available form. The observed possibility of nitrogen carryover brings forth significant implications for economic decision-making.

If the carryover effect is important, optimal decisions pertaining to management of plant available nitrogen in the soil will depend heavily upon levels of nitrogen carried over from past production periods. Also, when consideration is given to initiating a nitrogen fertilization program, knowledge of expected nitrogen carryover levels are required to weigh the total discounted value marginal products against the total current costs for each alternative rate of nitrogen application.

A complicating factor to the decision process is the stochastic character of nitrogen carried over into succeeding production years. This stems from the established fact that nitrogen fertilizer recovery and plant growth are directly related to precipitation levels [2].

Prior economic analyses have failed to evaluate the stochastic and dynamic propensities of the problem. Black [1] emphasized comparisons of expected returns under alternative rates of nitrogen application, but did not consider the carryover effect of nitrogen fertilization. Stauber and Burt [6] provided a more sophisticated approach to the problem using an implicit estimate of nitrogen carryover in a deterministic framework to determine an optimal level of nitrogen to maintain in the soil.

It was the intent of this study to provide a meaningful economic analysis of the nitrogen fertilization of pasture land. The approach taken incorporates the stochastic and dynamic characteristics of the problem into an appropriately formulated economic decision model. The major objective is to provide a set of policy guidelines which livestock producers in the general study area can utilize in determining the proper decisions for their individual situation.

OPTIMAL APPLICATION OF NITROGEN FERTILIZER WHEN CARRYOVER IS SIGNIFICANT

Dynamic programming possesses features which lend themselves readily to problems similar to the one currently under consideration. Typically, dynamic programming is concerned with multistage decision processes where a given set of decisions recurrs at each stage of the process. The objective is to find the sequence of decisions which maximizes (or minimizes) some appropriately defined criterion function for all possible combinations of states and stages. The selected decision sequence is then termed the optimal policy for the multistage process.

The optimal policy depends upon two primary factors: 1) the process length and 2) the initial state of the system. In regard to the length of the process, a grassland livestock-producing firm with a one stage or a single production year planning horizon would likely pursue a different fertilization policy than if the expected process length were infinite. This conclusion follows from the apparent inability of the firm, under a one stage planning constraint, to capture all of the expected benefits attributable to a decision optimal under an infinite planning horizon. In addition, the optimal policy is dependent upon the initial state of the system, where a state is defined as the sum total of all relevant information pertaining to the process of concern. For example, decisions relative to the amount of nitrogen to apply will likely depend upon the current levels of plant-available nitrogen in the soil.

A decision made at a given stage will generally change the state of the process in the succeeding stage. The nature of the change brought about by a specific decision will either be deterministic or stochastic depending upon the inherent characteristics of the process considered. If decisions made at a given initial state enable one to predict exactly the subsequent new state, the nature of the process change is deterministic. Obviously, the inclusion of weather, as a relevant variable in the determination of nitrogen carryover requires a stochastic approach to the problem at hand.

In view of the foregoing discussion, it would seem appropriate to formally define the essential dynamic programming components in terms of the decision problem currently under consideration. Accordingly, define the state of the process to be the current level of plant-available nitrogen in the present stage, the decision variable to be the alternative rates of nitrogen which can be applied, and each stage in the process to be equivalent to one production year.

Note that our state variable definition must satisfy the Markovian requirement of dynamic programming [7]. Specifically, it requires the optimal policy beginning in a particular state to be dependent on that state and not upon how the state was attained. The state variable as defined above, complies quite readily with the requirements.

In order to construct the appropriate dynamic programming model, or recurrence relation, some justification is required with regard to linkage of the n-stage process to the (n-1) stage process of a given multistage activity. To determine the optimal policy for an n-stage process, we consider the sum of the immediate expected return in the nth stage at some initial state "i" and the optimal expected return from the (n-1) stage process, which begins at some new state which may or may not be different from "i". The above procedure is firmly supported by the dynamic programming principle of optimality. Said principle states, "an optimal policy has the property that whatever the initial state and the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision [7].

The discrete stochastic recurrence relation, below, (1) was selected for use in the dynamic programming analysis of the nitrogen fertilization decision problem.

$$f_n(Y) = \max_k \left[R_Y^X + \beta \sum_{j=1}^{m} P_{YV}^X f_{n-1}(V)\right], \; Y = 1, 2, \ldots, m. \qquad (1)$$

where

$f_n(Y)$ = total discounted expected returns under an optimal policy from an n-stage process when the initial state if "Y".

R_Y^X = expected immediate return under decision X when the initial state is "Y".

β = the appropriate discount rate.

R_{YV}^{X} = the probability of moving from state "Y" in stage n to state "V" in stage n-1, given that decision X is selected; this is commonly termed a transition probability.

$f_{n-1}(V)$ = total discounted expected return under an optimal policy from an (n-1) stage process when the initial state is "V".

Clearly, (1) has been formulated in a manner consistent with the principle of optimality.

It will be noted that n, in (1), denotes the number of stages remaining in the decision process. This is the result of the unique dynamic programming computational technique of driving optimal policies by beginning at the last stage of the process and, thence, working back, stage by stage, to the present. Note further that optimal policies, resulting from (1), tend to converge to a constant decision rule for large n. Although a systematic proof of convergence will not be offered here, reference can be made to several published sources for detailed presentations of the proof [8,9].

Physical Data

As indicated within the recurrence relation, two primary types of information are required to determine the optimal policies of dynamic programming. Specifically, knowledge is necessary relative to the expected immediate returns and the transition probabilities.

The data requirements for computation of these vital components include determination of a production-response relationship which adequately describes grass hay yield as a function of plant available nitrogen and quantitative information pertaining to the stochastic carryover effect of nitrogen fertilizer.

The yield and carryover relationships used in this paper were estimated by Stauber and Burt [6]. Basically, the estimated yield response relationship attempts to explain the relationship between grass yield and the two independent variables, plant-available nitrogen and weather. Equally as important, the model provides an implicit estimate of carryover nitrogen. The data set used in their analysis was derived from an experiment site in western South Dakota near Newell, approximately 30 miles southeast of the Montana border [10]. The study evaluated the interrelationships of nitrogen, phosphorus and seasonal precipitation and their relative influences on the yield and composition of bromegrass-crested wheatgrass hay. The effects of residual nitrogen were also examined.

The yield model was hypothesized to be

$Y_t = f(U_t, W_t)$

where

Y_t = hay yield in pounds per acre in year "t". (2)

$U_t = N_t + R_t$ = total plant available nitrogen in year "t".

N_t = pounds of nitrogen applied in year "t".

R_t = residual nitrogen, where R_t is assumed to be a measure of nitrogen carried over from applications in previous years.

R_t was estimated in first order difference equation form as

$$R_t = p(N_{t-1} + R_{t-1})/W_{t-1}, \quad t = 1, 2, 3, \ldots \tag{3}$$

where

p = the proportion of nitrogen available in year t carried over to year t+1.

W_j = a standardized measure of seasonal precipitation adjusted for temperature in the jth year. Or,

$$W_j = \left[\frac{P_j}{T_j}\right]/W^*$$

where

P_j = total precipitation in the jth growing season.

T_j = mean daily maximum temperature in the jth growing season.

W^* = the sample mean of the ratio of growing season precipitation to growing season mean daily maximum temperature.

The logic of the weather variable's formulation is based on the adverse effect of high temperatures on plant growth. Division of adjusted seasonal precipitation by the sample mean of adjusted seasonal precipitation results in a standardized weather variable which reflects weather deviations from the sample mean. Specifically, W_j values greater than unity depict situations of above average adjusted seasonal precipitation while values less than unity are indicative of the converse.

Returning to residual nitrogen as hypothesized earlier (3), assuming $R_o = N_o = 0$ and iterating (3), the following sequence of equations results

$R_1 = 0$

$R_2 = pN_1/W_1$

$R_3 = p[N_2 + R_2]W_2 = pN_2/W_2 + p^2N_1/W_1W_2$

$R_4 = p[N_3 + R_3]W = pN_3/W_3 + p^2N_2/W_2W_3 + p^3N_1/W_1W_2W_3$

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$$R_t = p[N_{t-1} + R_{t-1}] = pN_{t-1}/W_{t-1} + p^2N_{t=2}/W_{t-1}W_{t-2} + p^3N_{t-3}/W_{t-1}W_{t-2}W_{t-3} + \cdots$$

It follows that total plant available nitrogen can be expressed as

$$U_t = N_t + R_t = N_t + pN_{t-1}/W_{t-1} + p^2N_{t-2}/W_{t-1}W_{t-2} + p^3N_{t-3}/W_{t-1}W_{t-2}W_{t-3} + \cdots \quad (4)$$

or total plant available nitrogen is the sum of nitrogen applied in year t and a weighted sum of nitrogen applied in years prior to "t".

Previously, in (2) yield (Y_t) was defined as a function of total plant available nitrogen (U_t) and adjusted seasonal precipitation (W_t). In explicit form, yield was hypothesized to be a polynomial considering both the individual and interactive influences of nitrogen and weather. Expressed mathematically

$$Y_t = b_1U_t + b_2W_t + b_3U_t^2 + b_4W_t^2 + b_5U_tW_t + b_6U_t^2W_t + b_7U_tW_t^2 + \varepsilon_t \qquad (5)$$

ε_t = random error term assumed to be normally distributed.

b_i = the contribution of the ith variable to yield.

Estimates of the forementioned parameter, including p, were derived by nonlinear regression techniques. These coefficients plus the values of other statistical indicators are listed in Table I.

As emphasized earlier, p (the proportion of U_t carried over to year t+1 under average weather conditions) is of particular interest from a decisionmaker's standpoint. The estimated value of p, approximately 62 percent, points out the important influence of residual nitrogen on hay yields in years subsequent to the year of application.

The R^2 coefficient of .906 indicates the model is capable of fairly accurate hay yield predictions. Also, it will be noted the model possesses diminishing marginal returns of hay yields with respect to additions of either precipitation or nitrogen. This is evidenced by the second order mixed derivative of the yield equation.

$$\frac{\partial^2 Y_t}{\partial U_t \partial W_t} = 18.04 - .0364U_t - 5.70W_t$$

To quote Stauber and Burt [6], "the marginal effect of precipitation on the marginal yield response of nitrogen or the marginal effect of nitrogen on the marginal yield response of precipitation is seen to diminish with increases in usage of either variable."

TABLE I. STATISTICAL RESULTS OF THE REGRESSION ANALYSIS.

Parameter	Variable	Estimate	Standard Error
P		0.6194	0.0389
b_1	U_t	-6.3136	2.2124
*b_3	U_t^2	0.0029	.0084
*b_4	W_t^2	-19.7000	126.3052
b_5	$U_t W_t$	17.0422	3.3473
b_6	$U_t^2 W_t$	-0.0173	0.0068
*b_7	$U_t W_t^2$	-2.8495	1.6496

Standard error of the estimate 214.43

Coefficient of determination 0.906

* Variable insignificant at the 5% probability level.

DERIVATION OF IMMEDIATE EXPECTED RETURNS AND TRANSITIONAL PROBABILITIES

To derive the expected immediate returns, it was necessary to reduce the yield equation to a function of plant available nitrogen independent of the stochastic weather effects. Taking the expectation of the yield equation with respect to weather gave the desired result. In functional form, we have

$$E(Y_t|U_t) = -6.3136\ U_t + 614.0656\ E(W_t) + .0029\ U_t^2 - 19.700\ E(W_t^2) +$$

$$17.0422\ U_tE(W_t) - .0173\ U_t^2E(W_t) - 2.8495\ U_tE(W_t^2)$$

Estimates for $E(W_t)$ and $E(W_t^2)$ were obtained through use of available weather history data for Newell, South Dakota. These values, 1.1418 and 1.5761 for $E(W_t)$ and $E(W_t^2)$, respectively, were then substituted into the yield equation giving the expected yield response function

$$E(Y_t\ U_t) = 670 + 8.65\ U_t - 0.0167\ U_t^2$$

As noted earlier, U_t is the sum of nitrogen applied in production year t (N_t) and nitrogen carried over into year t from applications made in prior productionmyears (R_t). Similarly, these components of U_t correspond to "Y" and "X" within the dynamic programming framework.

Computation of the expected immediate returns, in a manner consistent with the discrete dynamic programming approach, required the assignment of incremental values for "Y" and "X". In order to provide increased clarity and interpretative value in the dynamic programming results, the state variable was divided into five-pound intervals while the decision variable was assumed to take on discrete values of five pounds per acre. Consideration was given to a 0 to 300 pound range of total nitrogen (U_t). This range was felt to be more than adequate in view of the fact that the yield equation under expected weather conditions attains a maximum at 259 pounds of total nitrogen per acre. Consequently, 61 possible decisions (X = 0, 5, 10 ... , 300) and 60 potential initial states (Y = 0-5, 5-10, 10-15, ... , 295-300) were defined.

In equation form the expected immediate returns are expressed as

$$R_X^Y = P_H \cdot E(Y_t | U_t) - P_N \cdot X_k - \phi$$

$$= P_H[670 + 8.65(y_i+x_k) - 0.0167(y_i+x_k)^2 - P_N \cdot x_k - \phi$$

where "Y" and "X" are defined above and,

y_i = Midpoint corresponding to Yth initial state for i = 1, 2, ... , 60.

x_k = kth amount of nitrogen applied for k = 1, 2, ..., 61.

P_H = Price per pound for hay

P_N = Price per pound for nitrogen

ϕ = Costs of spreading fertilizer

A set of immediate expected returns was subsequently determined for all meaningful combinations of states and decisions using a range of cost and price parameters.

The primary function of transition probabilities is to explain changes in the level of plant available soil-nitrogen as movement occurs from production stage to production stage. Specifically, their basic concern is definition of the probability distribution associated with future levels of nitrogen carried over from the current production period. Since it is within the decisionmaker's power to influence the amount of nitrogen carried over by varying the quantities of nitrogen fertilizer applied in the present production stage, the probabilistic statement is necessarily conditional in form. In terms of the current problem, transition probabilities can, therefore, be represented as

$$P^{X}_{YV} = \text{Prob}[v_j < V^{n-1} \leq v_{j+1} \mid X^n = x_k \; Y^n = y_i] \qquad (5)$$

Where

V^{n-1} = Carryover nitrogen in stage n-1.

Y^n = Carryover nitrogen in stage n.

X^n = Amount of nitrogen applied in stage n.

v_j = an endpoint corresponding to the Vth state interval for stage n-1, (j = 1, 2, ..., 61).

y_i = ith midpoint corresponding to state interval for stage n, (i=1, 2, ..., 61).

x_k = kth level of nitrogen applied in stage n (k=1, 2, ..., 61).

Quantitative estimates of the transition probabilities were derived through use of the carryover equation (3). Specifically, (3) was reformulated in terms of R_{t+1} so as to reflect nitrogen carryover one production year into the future. Restatement of $R_t + 1$ within the context of dynamic programming gave

$$R_{n-1} = P\left(\frac{N_n + R_n}{W_n}\right) \tag{6}$$

Redefining (6) according to the notation set forth in (5) and substituting the resulting expression into (5) yields

$$P^X_{YV} = \text{Prob}\left[v_j < P\left(\frac{x_k + y_i}{W_n}\right) < v_{j+1}\right] \text{ for } X^n = x_k \text{ and} \tag{7}$$

$Y^n = y_i$, where k = 1 corresponds to zero pounds of nitrogen applied per acre, j = 1 is equivalent to zero pounds per acre of carryover nitrogen and i = 1 is associated with 2.5 pounds per acre of carryover nitrogen. The remaining integer-state and decision variable correspondences follow in ascending numerical order.

Computation of the transition probabilities was achieved through use of a range of W_n values derived from a 32-year weather record for Newell, South Dakota. Coefficients for the individual probability elements were computed by determining the proportion of years in which W_n satisfied the above inequality for every meaningful combination of initial state, decision and state of carryover nitrogen in stage n-1.

To clarify the above discussion, let us consider a numerical example. Assume we entered stage n with plant available nitrogen between 0 and 5 pounds per acre. Also, assume 20 pounds of nitrogen fertilizer was immediately applied in response to this low level of soil nitrogen. It is our desire to determine the probability of carryover nitrogen being between 5 and 10 pounds per acre in the next succeeding production year.

To begin, we approximate the level of nitrogen carried over from past years by selecting the midpoint of our initial state interval, or 2.5 pounds per acre. Substituting this value plus the given magnitudes of the decision and new state variable into (7) yields

P^X_{YV} = Prob $[5 < p(20 + 2.5) \leq 10]$ for $X = x_5 = 20$ and

$Y^n = y_1 = 2.5$. Utilizing our estimate for p(.6194) and solving the inequality for W_n gives P^X_{YV} = Prob $[2.7873 > W_n \geq 1.3937]$.

Scanning Table II, we see that in ten out of the past thirty-two years, weather would have been favorable enough to induce the stated level of nitrogen carryover. Hence, on the basis of past weather experience, we say that the probability of nitrogen carryover falling within the interval of 5 and 10 pounds per acre is 10/32, given we have an initial state of zero to five pounds of nitrogen per acre and that we apply 20 pounds per acre in the current production year.

In several cases, non-zero probability elements were associated with nitrogen carryover levels of greater than 100 percent. This peculiarity occurred whenever the computed W_t values were less than the estimate for the carryover parameter "p". Since, for every positive value an even smaller W_t is possible, greater than 100 percent carryover predictions could occur regardless of our estimate for "p".

To prevent carryover estimates greater than 100 percent, all W_t values less than "p" were assigned magnitudes equal to "p". In Table II,

TABLE II. COMPUTED W_n VALUES FOR THE PERIOD 1938-1969.

Year	W_n	Year	W_n
1938	.7745	1954	1.1461
1939	.5986	1955	.6151
1940	1.0334	1956	.7669
1941	2.1381	1957	1.2724
1942	1.6932	1958	1.4245
1943	.7292	1959	.6202
1944	1.4303	1960	.4426
1945	1.2570	1961	.6567
1946	2.4922	1962	2.0834
1947	1.2019	1963	1.4273
1948	1.2593	1964	1.3479
1949	.6339	1965	1.7780
1950	.7422	1966	.5476
1951	.9458	1967	1.5362
1952	.4540	1968	.7764
1953	1.7321	1969	.9791

we see that five years fall into this category hence, it was necessary to adjust these values to .6194 (the estimate for the carryover parameter "p").

Numerical Results

To begin our discussion of the dynamic programming solutions, let us consider the optimal policies pertaining to the parameter set where the price of hay equals $15 per ton, the price of nitrogen is $.10 per pound and the interest rate appropriate for discounting is ten percent. Within this basic situation two variations related to spreading costs are examined. In the first case the fertilizer is assumed to be custom spread at a cost of $1.50 per acre. In the second case it is assumed the operator owns the necessary equipment to spread the fertilizer and a charge of $.25 is made which represents the operating costs for a typical farm situation. These policies and their associated expected returns are listed in Table III for stages prior to policy convergence and a fifty stage planning horizon.

From Table III, the optimal application rates and the lowest state level at which nitrogen is applied are noted to increase as the length of the planning horizon increases. This result can be attributed to the enhanced ability of the process to capture a greater share of the distributed value marginal products associated with each application decision. In addition, the lower ownership costs of spreading encourage nitrogen applications earlier in the process than custom spreading costs.

To illustrate the decision rule, examine the optimal solutions corresponding to stage fifty under spreading costs of $1.50 per acre. Here, we see that the level of total nitrogen (nitrogen applied plus the initial state of nitrogen) is maintained between 105 and 110 pounds per acre for the five lowest initial states of nitrogen. For higher initial state levels, the optimal fertilization policies recommend that nitrogen not be applied. The decision rule implied by this behavior indicates that if the level of plant available nitrogen in the soil falls below 25 pounds, the optimal response is to apply enough nitrogen to bring soil nitrogen up to at least 105 pounds per acre. On the other hand, if carryover nitrogen is greater than 25 pounds, expected returns will be maximized by not applying any nitrogen.

The decision rule described is identical to the structure of (S,s) policies of inventory systems analysis [15].

Using inventory theory terminology, s is defined as the reorder level of the item currently in stock where as "S" represents the reorder level of said item. The decision rule proposed by (s,S) policies states that if the amount on hand of the given item falls below a specified level (s), the appropriate policy response is to restock the inventory of that item until a previously determined upper level is reached (S). On the other hand, if the inventory level falls somewhere between s and "S", the proper optimizing decision is to make no further additions to the inventory in question. In terms of current problem, s represents