Analysis of an open spandrel arch including superstructure
by William Lorell

A THESIS Submitted to the Graduate Committee In partial fulfillment of the requirements for the degree of Master of Science In Civil Engineering
Montana State University

Abstract:
A procedure for the analysis of an open spandrel arch including the effects of the superstructure is developed by neons of a special adaptation of the method of moment distribution and the virtual Trork equations. The arch is considered cut at the center line, and the action of one half of the structure upon the other is replaced by six unknown forces (one horizontal force, one vertical force and one moment applied at both deck and arch at the center section). All joints of the remaining arch structure are then held against translation by sets of restraining forces. The moments resulting from separate application of the real loads and each one of the six above mentioned unknown forces are determined. Additional joint moments are introduced, obtained from the solution of so-called equilibrium equations, which remove all restraints and restore equilibrium of external and internal forces. The joint moments, expressed in terms of actual loads and the six unknowns, are then utilized to determine rotation and deflections in horizontal and vertical directions of both deck and arch at the center section. The fact that rotations and deflections at that section must be identical for the two halves of the structure, provides six equations, permitting solution for the six unknowns. A model arch, tested by the University of Illinois experiment Station in 1928, is analyzed by this procedure, and a comparison between calculated and measured stresses is made. Close agreement is obtained, indicating decided superiority of this approach over the conventional one of neglecting the effects of interaction between deck and arch rib.

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ABSTRACT

A procedure for the analysis of an open spandrel arch including the effects of the superstructure is developed by means of a special adaptation of the method of moment distribution and the virtual work equations. The arch is considered cut at the center line, and the action of one half of the structure upon the other is replaced by six unknown forces (one horizontal force, one vertical force and one moment applied at both deck and arch at the center section). All joints of the remaining arch structure are then held against translation by sets of restraining forces. The moments resulting from separate application of the real loads and each one of the six above mentioned unknown forces are determined. Additional joint moments are introduced, obtained from the solution of so-called equilibrium equations, which remove all restraints and restore equilibrium of external and internal forces. The joint moments, expressed in terms of actual loads and the six unknowns, are then utilized to determine rotation and deflections in horizontal and vertical directions of both deck and arch at the center section. The fact that rotations and deflections at that section must be identical for the two halves of the structure, provides six equations, permitting solution for the six unknowns. A model arch, tested by the University of Illinois Experiment Station in 1928, is analyzed by this procedure, and a comparison between calculated and measured stresses is made. Close agreement is obtained, indicating decided superiority of this approach over the conventional one of neglecting the effects of interaction between deck and arch rib.

INTRODUCTION

The conventional analysis of an open spandrel arch bridge as a rib without restraint due to the superstructure is clearly incomplete. The argument that bridges designed by a procedure neglecting deck participation have apparently given good service is not considered to be a particularly valid reason for neglecting this effect. Since it is quite obvious that the superstructure tends to stiffen the rib, it is not particularly astounding that ribs designed on a basis neglecting this stiffening effect have stood up well under design loads. A comparison between the weights of open spandrel arch structures designed by conventional means (neglecting the effects of the superstructure) on one hand, and those designed with due consideration to interaction on the other, would come closer to the point.

In contrast to the customary American approach, European designs of open spandrel arches generally take into consideration the interaction of rib and superstructure. Perhaps the most outstanding result of this type of analysis is the 260'-span Grandfey Viaduct in Switzerland, a structure of startlingly slender proportions. The European method of analyzing these bridges, however, is mainly based on model tests, and involves no definite mathematical method.[1]

Current literature on the problem of arch bridge analysis generally restricts itself to stating the fact of interaction and its most apparent effects, but skips lightly over possible mathematical solutions. In spite of a most thorough search, the author has succeeded in locating only one

1. Elastic Arch Bridges, by McCollough and Thayer, John Wiley and Sons, Inc., N. Y., 1931, pp. 313.

outline of an exact solution,[2] which, however, involves no less than 64 equations with 64 unknowns. It is fairly clear that a method of this type would add little toward the primary purpose of this thesis: to find a workable, comparatively short approach to the analysis of open spandrel arches including the effects of the superstructure. For the above mentioned reasons, references to existing books and publications will be extremely scarce throughout this thesis.

The method subsequently to be developed is used in the final part of this thesis to analyze mathematically a model arch which had been experimentally tested to failure at the University of Illinois Experiment Station in 1928.[3] Since stresses for various loadings had been recorded during these tests, a comparison of analytic and experimental results is possible. Whereas stresses calculated by the elastic theory neglecting deck participation show deviations up to 150% from observed stresses, much closer agreement is obtained by the method of analysis developed in this thesis. No mistaken conclusions as to the accuracy of this method, however, should be drawn. The uncertainties involved in the determination of quantities such as the modulus of elasticity of concrete, the moment of inertia of reinforced concrete sections, etc., make closer agreement than, say 50% purely accidental.

It has been demonstrated conclusively by the University of Illinois Experiment Station in the above mentioned experiments that the decks of the arch models tested increased the strength of the structures to 40%. Any consideration for economy of design cannot overlook a factor of that

2. Ibid. pp. 309-310.
3. Laboratory Tests of Reinforced Concrete Arches With Decks, Bulletin 226, University of Illinois Experiment Station, Urbana, Ill., 1929.

magnitude. The author therefore believes that any and all attempts to develop a procedure for the analysis of integral arch action are well justified.

SCOPE

A procedure will be developed for the analysis of the conventional type of open spandrel arch, such as the one shown in Fig. 1. The effects of interaction between deck and superstructure will be given due consideration.

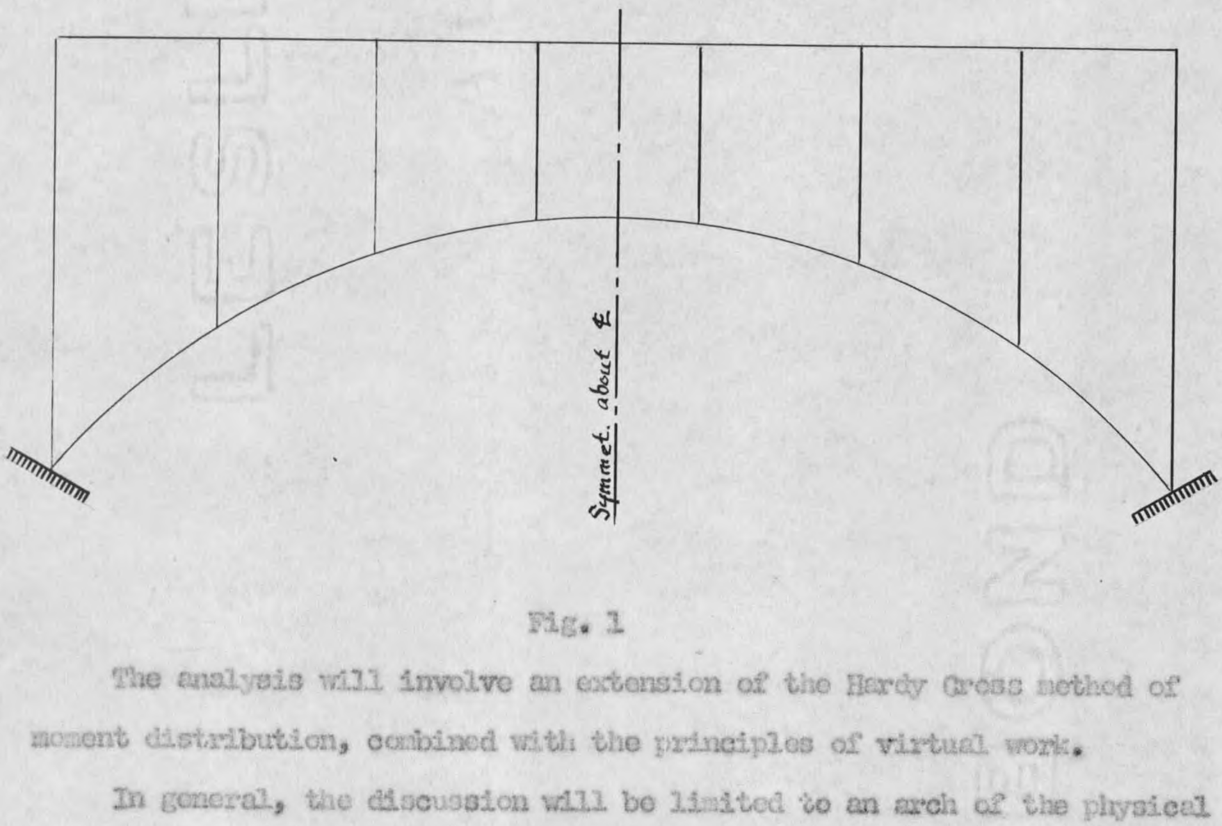

Fig. 1

The analysis will involve an extension of the Hardy Cross method of moment distribution, combined with the principles of virtual work.

In general, the discussion will be limited to an arch of the physical characteristics of the symmetrical 7-panel arch analyzed in the last part of this thesis. An extension to any other type of arch, however, can be accomplished quickly and without difficulty.