MONTANA STATE UNIVERSITY

Natural convection from a body to its spherical enclosure

by Charles Timothy McCoy

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
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Abstract:
An analysis of the heat transfer results and temperature profiles obtained experiment ally in a finite natural convection environment are presented and discussed. Overall heat transfer correlations for a cylinder and a cube located concentrically within a spherical enclosure were formulated using several test fluids yielding a wide range of the important parameters. These parameters include the Prandtl number, Rayleigh number, and a ratio of characteristic lengths describing the test geometry under investigation.

A Prandtl number effect on the convective heat transfer was noted for the cylinder study. Increasing viscosity and low ratios of overall cylinder length to cylinder diameter appeared to damp out the geometric effect inherent to the cylindrical body compared to a sphere.

The cube study indicated a very slight Prandtl number effect.

For this geometry, the heat transfer results could be correlated in terms of a Nusselt number as a function of a Rayleigh number and a geometric parameter only.

Possible flow patterns existing in the gap were deduced from the temperature profiles for both the cylinder and cube inner bodies.

Five characteristic regions were generally inherent to all temperature profiles. A multicellular flow was postulated to exist to explain exceptions to the general shape of the temperature profiles for both the cube and cylinder investigations.

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ABSTRACT

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NOMENCLATURE

Symbol	Description
a, b	Characteristic lengths
A	Area
A_s	Surface area of inner body
C_p	Specific heat
C_1, C_2, C_3, C_4, C_5	Empirical constants
F	Denotes function
Gr_a	Grashof number, $g\beta\rho^2 a^3 \Delta T/\mu^2$
g	Acceleration of gravity
h	Length of vertical section of cylinder
H	Overall length of cylinder, $h + 2r_i$
$\overline{h}$	Average heat transfer coefficient for inner body, $q_c/A_s \Delta T$
I	Current through heaters located in inner body
k	Thermal conductivity
k_{eff}	Effective thermal conductivity
l_c	Length of cube side
L	Gap width, $r_o - r_i$
$\overline{L}$	Dimensionless length ratio
Nu_a	Nusselt number, $\overline{h}\, a/k$
Nu^*	Modified Nusselt number defined by equation (1.5)
Pr	Prandtl number, $\mu C_p/k$

Symbol	Description
q_c	Heat transferred by convection
q_{cond}	Heat transferred by conduction down supporting stem
q_L	Heat loss by system
r_a	Average radius, $(r_o + r_i)/2$
r_i	Inner body radius or $l_c/2$
r_o	Outer body radius
r_θ	Distance from geometric center to a selected location on surface of inner body
$\overline{R}$	Dimensionless radius ratio, $(r - r_\theta)/(r_o - r_\theta)$
Ra_a	Rayleigh number, Gr_a Pr
Ra*	Modified Rayleigh number, Ra_L (L/r_i)
T	Temperature
$\overline{T}$	Dimensionless temperature, $(T - T_o)/(T_i - T_o)$
T_{am}	Arithmetic-mean temperature, $(T_i + T_o)/2$
T_i	Inner body temperature
T_m	Volumetric-mean temperature defined by equation (1.3)
T_o	Outer body temperature
V	Voltage across heaters located in inner body
β	Thermal expansion coefficient
ΔT	Temperature difference, $(T_i - T_o)$
ΔX	Distance

Symbol	Description
μ	Dynamic viscosity
ρ	Density
ϕ	Angular position measured from upward vertical axis.

CHAPTER I

INTRODUCTION

The analytical study of natural convection heat transfer occurring within finite enclosures is very difficult even when working with relatively simple geometries. This results from the governing equations describing the energy and flow fields being coupled and nonlinear. The boundary conditions inherent to each heat transfer system adds to the complexity of an analytical solution in many cases. Since many engineering systems involving natural convection heat transfer within a finite boundary do not lend themselves to an analytical analysis, other methods must be used for the prediction of the heat transfer phenomena occurring. One such method is an experimental investigation.

The objective of this investigation is to experimentally study natural convection heat transfer between either a cylinder or a cube and its spherical enclosure. This study is an extension of past work.

As shown in many publications discussing natural convection within finite enclosures, non-dimensionalizing the governing equations show that the heat transfer can be formulated in the following manner:

$$Nu_a = F\,[Gr_a, Pr, \overline{L}]\ , \tag{1.1}$$

where

$$Nu_a = \bar{h}\, a/k \ ,$$

$$Gr_a = g\rho^2\beta\ \Delta T\ a^3/\mu^2 \ ,$$

$$Pr = \mu C_p/k \ ,$$

$$\bar{L} = a/b \ .$$

The dimensionless parameters given above are the Nusselt number, Nu_a, Grashof number, Gr_a, Prandtl number, Pr, and a ratio of lengths, $\bar{L}$. These parameters adequately characterize the geometry under investigation.

At the present time there is only a limited amount of information available concerning natural convection heat transfer within finite enclosures. The case of an inner body within a spherical enclosure was first studied experimentally in 1964 by Bishop [1]. In his investigation, a concentrically located inner sphere was used for the inner body and air was employed as the gap fluid. Both spheres were maintained isothermal. Four different inner spheres were used yielding diameter ratios of 1.25, 1.67, 2.00 and 2.50. An empirical equation for the heat transfer was presented for each inner sphere used. The following equation was determined for all of the experimental data.

$$Nu_L = 0.332\ Gr_L^{0.270}\ (L/2r_i)^{0.520} \ . \qquad (1.2)$$

The above relationship fitted the data to within -12.7 per cent and +14.6 per cent, subject to the conditions:

$$Pr = 0.7 ,$$

$$0.333 \leq L/2r_i \leq 0.750,$$

$$2.0x10^5 \leq Gr_L \leq 3.6x10^5 .$$

All fluid properties were evaluated at a volumetric-mean temperature defined as

$$T_m = [(r_a^3 - r_i^3)\ T_i + (r_o^3 - r_a^3)\ T_o]/(r_o^3 - r_i^3) . \tag{1.3}$$

Bishop also obtained data describing the temperature profiles existing at selected locations within the test gap. These were presented in conjunction with photographic studies in order to depict qualitative results with respect to the flow field which existed in the natural convective environment under investigation.

Beckmann [3] first utilized an effective conductivity in correlating natural convection heat transfer. Physically the effective conductivity represents the thermal conductivity a fluid would have to have in order to transfer the same amount of energy, by pure conduction, as actually is transferred by conduction and convection combined. Its definition -- obtained from the conduction solution of the particular geometry under investigation -- for concentric spheres is

$$k_{eff} = q_c\ (r_o - r_i)/4\pi\ \Delta T\ r_o r_i\ . \qquad (1.4)$$

Obviously the ratio k_{eff}/k has a lower limit of 1.0 which would correspond to a system transferring heat by conduction only.

Utilizing the conduction solution between concentric spheres, Bishop, Mack, and Scanlan [2] showed that a modified Nusselt number could be related to the ratio of k_{eff}/k. This relationship is given by

$$Nu^* = Nu_L\ (r_i/r_o) = k_{eff}/k\ . \qquad (1.5)$$

By employing the ratio k_{eff}/k in correlating the data, a geometric effect is included in the definition of the modified Nusselt number in an implicit manner.

Referencing the data obtained by Bishop [1], Bishop, et al [2] obtained a simplified empirical correlation using k_{eff}/k as the dependent variable. The equation,

$$k_{eff}/k = 0.106\ Gr_L^{0.276}\ , \qquad (1.6)$$

correlated the data within the limits of -13.4 per cent and +15.5 per cent. The conditions specified for the range of applicability of equation (1.2) are imposed upon this expression as well.

Scanlan, Bishop, and Powe [4] extended the work of Bishop [1] by utilizing water and two silicone oils as test fluids to yield the following ranges of the important parameters:

$$0.7 \leq Pr \leq 4148. \ ,$$

$$0.09 \leq L/r_i \leq 1.81 \ ,$$

$$1.2x10^2 \leq Ra_L \leq 1.1x10^9 \ .$$

Correlating in terms of the modified Nusselt number, the empirical relation obtained subject to the above conditions was

$$Nu^* = 0.202 \ (Ra_L)^{0.228} \ (L/r_i)^{0.252} \ Pr^{0.029} \ . \qquad (1.7)$$

The average per cent deviation of the data was 13.7 per cent.

The power of the Prandtl number appearing in equation (1.6) indicates only a slight effect of this parameter on the heat transfer results. Also, since the exponents on Ra_L and L/r_i are very nearly equal, Scanlan, et al [4] defined an adjusted Rayleigh number as

$$Ra^* = Ra_L \ (L/r_i) \ . \qquad (1.8)$$

The data were then correlated in the following form:

$$k_{eff}/k = 0.228 \ (Ra^*)^{0.226} \ . \qquad (1.9)$$

The average per cent deviation of the data from the prediction of this expression was 15.6 per cent. Although this is larger then that of equation (1.6), this form is more desirable in that it contains fewer parameters and empirical constants.

Temperature profiles were obtained for selected temperature differences impressed across the gap for each test fluid. The same trends as noted by Bishop, et al [2] were observed. In addition, the Prandtl number effect on the temperature profiles was analyzed.

The most recent investigation is that of Weber [5]. The effect of eccentricity of the two spheres was investigated, as well as the effect of changing the geometry of the inner body by using a vertical cylinder with hemispherical ends. Eccentricity is defined as a vertical displacement of the geometric center of the inner body relative to the geometric center of the enclosing sphere.

Eccentricities of $\pm$ 0.25L, $\pm$ 0.50L, and $\pm$ 0.75L, where L represents the gap width defined by $(r_o - r_i)$, were used in the investigation for each inner sphere tested. A negative eccentricity was found to enhance the convective activity and a positive eccentricity was found to have just the opposite effect.

The heat transfer data for the eccentric case were correlated using a conformal mapping technique [6] involving a geometric transformation. By utilizing this technique, a relationship was developed which also predicted the concentric data obtained by Scanlan, et al [4]. The resulting expression was

$$k_{eff}/k = 0.231\ (Ra^*)^{0.225}\ , \qquad (1.10)$$

subject to the conditions

$$0.7 \leq Pr \leq 4148. ,$$

$$1.5x10^2 \leq Ra^* \leq 9.0x10^8 ,$$

$$0.09 \leq L/r_i \leq 1.18 .$$

The inner radius corresponding to the mapped concentric sphere was used in defining all pertinent heat transfer parameters.

In working with the cylinder-sphere geometry, data were obtained using water as the test fluid. The height of the vertical section and the radius of the cylinder were varied in order to describe actual geometric effects on the convective heat transfer.

To maintain a certain degree of continuity in defining the pertinent heat transfer parameters, the radius of the cylinder was used in place of an inner sphere radius. Correlation of the data yielded the following equation:

$$Nu^* = 0.234 \ (Ra_L)^{0.261} \ (H/2r_i)^{-0.209} \ (L/r_i)^{0.466} , \qquad (1.11)$$

for

$$1.14 \leq H/2r_i \leq 2.0 ,$$

$$3.2x10^4 \leq Ra_L \leq 2.7x10^8 .$$

In equation (1.11), H represents the total height of the cylinder. Thus the ratio $H/2r_i$ has a lower limit of 1.0 corresponding to the sphere-sphere case.

As in the previous studies, temperature profiles were obtained and these enabled deductions regarding the flow field to be made.

Since water was the only test fluid employed for the cylinder investigation, there exists a need to extend these results by using additional fluids. In the current study, two silicone oils will be used in conjunction with the cylindrical inner body to investigate the effect, if any, the Prandtl number has on the convective heat transfer. Sufficient data were obtained by Weber [5] to depict the geometric effects.

As noted in the previous discussion, the cylindrical inner body produced results which varied significantly from the sphere data. This is what would be physically expected. However, there is a certain degree of similarity between the two geometries. Therefore, to more adequately describe the natural convection phenomena occurring within a spherical enclosure, the effects of a cube located concentrically within the enclosing sphere for several test fluids will also be investigated.

CHAPTER II

EXPERIMENTAL APPARATUS AND PROCEDURE

EXPERIMENTAL APPARATUS

The experimental apparatus, shown in Figure 2.1, consisted of an inner body located concentrically within a sphere having an inner diameter of 9.828 inches and a wall thickness of 0.125 inches. The sphere was formed from two stainless steel hemispheres joined together by an external flange to permit disassembly and removal of the inner body. An O-ring was placed in the parting plane for sealing purposes. The lower hemisphere was mounted to an enclosing spherical water jacket by using a metal spacer.

The spherical water jacket consisted of two steel hemispheres, the lower being attached to a tubular support frame mounted on a metal table. The hemispheres were united using a flexible band surrounding external flanges attached to the hemispheres. The inner diameter of this sphere was approximately 14 inches.

A stainless steel tube of 0.25 inch inside diameter was installed through the apparatus, extending from the inner surface of the enclosing sphere to the outside of the water jacket. The tube, which was located near the lower extreme of the apparatus, served as a fill port for injecting the test fluid into the gap.

The inner bodies were of cylindrical and cubical geometry. The

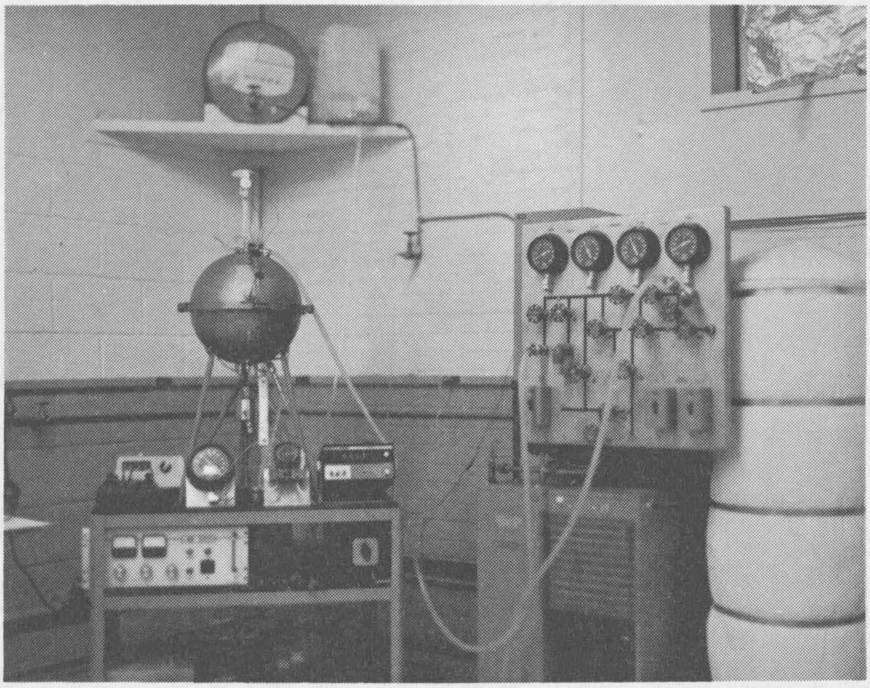

Figure 2.1. Heat transfer apparatus.

cylindrical bodies, made of 0.065 inch thick copper, were formed from a cylinder having hemispherical ends. The cubes were fabricated out of an aluminum alloy suitable for welding. Each inner body was mounted on a stainless steel stem having an inside diameter of 0.37 inches and an outer diameter of 0.50 inches. The outer surface of the stem was insulated to reduce the amount of heat loss from the system.

A uniform inner body temperature was maintained by condensing Freon-11 on the inner surface of the test geometry. Heaters mounted to the stem extended into the inner body. Electrical disk heaters were used with the cylindrical geometry and cartridge heaters for the cubical geometry. The size and number of heaters used was dependent upon inner body size. Figures 2.2 and 2.3 illustrate typical arrangements of the heaters within the bodies prior to their final assembly. Saturation conditions of the Freon-11 could be varied by regulating the power supplied to the heaters. Thus, the inner body temperature could be varied over a desired range of values.

Copper-constantan thermocouples were used to monitor all temperatures. In the case of the cylinders they were located in the joining seams. For the cubical geometries a small hole was drilled through the cube wall at a selected location, and a Shell Epon resin was used to affix the thermocouple to the outer surface. All power and thermocouple leads were passed down through the supporting stem.

The support stem passed through the enclosing sphere and water