Deflection of a circular plate subjected to blast loading

by Larry Eugene May

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
The object of this investigation was to develop an analytical method for predicting the permanent deformation of a simply supported circular plate subjected to a blast loading. This development is based upon using the circular plate equations in conjunction with a hyperbolic tangent stress-strain relationship. The Modified Galerkin Method was used in obtaining solutions. The linear stress-strain relationship (Hooke's Law) was included to allow a comparison of results. The effects of strain rate sensitivity and time duration of blast loads upon deflection were also investigated.

By using the results of an experimental bomb blast test, a comparison of results was performed. From this comparison, it was found that by using the method developed, the plate deflections are greatly overestimated. This error results from using the plate equations which neglect the stretching effect upon the plate. Evidently, the energy absorbed by stretching is very significant when compared to the energy absorbed by bending.

1098-13
18-

DEFLECTION OF A CIRCULAR PLATE SUBJECTED TO BLAST LOADING

by

LARRY EUGENE MAY

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

Mechanical Engineering

Approved:

Irvin L. Reis
Head, Major Department

Dennis O. Blackketter
Chairman, Examining Committee

K. Goering
Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

December, 1968

ACKNOWLEDGMENT

The author is indebted to the U. S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, for sponsoring this research project which provided the material for this thesis and financial aid.

The assistance of the following persons is gratefully acknowledged: Dr. Dennis O. Blackketter who directed this research project and provided guidance in the preparation of this thesis, Prof. George Frank for providing help in using the analog computer, and the author's wife, Carol, for typing this thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The object of this investigation was to develop an analytical method for predicting the permanent deformation of a simply supported circular plate subjected to a blast loading. This development is based upon using the circular plate equations in conjunction with a hyperbolic tangent stress-strain relationship. The Modified Galerkin Method was used in obtaining solutions. The linear stress-strain relationship (Hooke's Law) was included to allow a comparison of results. The effects of strain rate sensitivity and time duration of blast loads upon deflection were also investigated.

By using the results of an experimental bomb blast test, a comparison of results was performed. From this comparison, it was found that by using the method developed, the plate deflections are greatly overestimated. This error results from using the plate equations which neglect the stretching effect upon the plate. Evidently, the energy absorbed by stretching is very significant when compared to the energy absorbed by bending.

CHAPTER I

INTRODUCTION AND STATEMENT OF PROBLEM

At the present there are no analytical methods available to accurately predict the permanent deformation of a simply supported plate subjected to blast loading. This blast loading may result in large deflections being experienced by the plate. In general, analytical solutions do not produce valid results since the equations of motion of the plate are based on small elastic deflection theory. This blast produces an impulse type loading when the time duration of the blast is much shorter than the fundamental mode of vibration of the plate.

When a plate undergoes large deflections resulting in the plate being stressed beyond the material yield point, the plate passes through the elastic deformation region into the plastic region. Thus, a complete solution of the plate deflection problem takes into account the deformation during the elastic region and the plastic region. Wang (1)* has derived the theoretical permanent deformation of a simply supported circular plate subjected to air blasts, which is based upon the "Plastic Rigid" theory he developed. In Wang's theory, a perfectly plastic-rigid material cannot deform when the stress is below the yield point, thus the elastic region is ignored. Some aluminum alloys approach this ideal plastic rigid condition.

---

* Numbers within a parenthesis refer to references.

Hoffman (2), in his thesis presented to the University of Delaware, used Wang's Plastic Rigid theory to calculate the permanent deformation of a 24 in. diameter simply supported 61S-T6 aluminum plate subjected to blast loading. He also performed blast tests to obtain the actual deformation, thus allowing a comparision. The stress-strain relationship for 61S-T6 aluminum can be interpreted as being plastic rigid. However, it was found by comparison of the theoretical results with experimental that the theoretical greatly overestimates the permanent deformation. This was true for the three different plate thicknesses used in his experiment; 1/8, 1/4, and 3/8 in.

This paper presents the development of an analytical method for calculating the deflection of a simply supported circular plate subjected to a bomb blast. This development consists of using the fundamental plate equation given by Timoshenko and Woinowsky-Krieger (3) in conjunction with a hyperbolic tangent stress-strain relationship. The Modified Galerkin Method was used to reduce the governing partial differential equation to a differential equation in time. By solving this differential equation, the plate deflection was obtained as a function of time. The development using the linear stress-strain relationship (Hooke's Law) was also performed to allow a comparison. It was assumed that the blast load was symmetric about the origin, thus reducing the problem to one dimension, and that all energy imparted to the plate was absorbed by the bending of the plate. A method for obtaining the permanent deformation from the deflection was not developed.

CHAPTER II

THEORETICAL ANALYSIS

2.1 Equilibrium Equation

The bending moment equilibrium equation for the symmetrical bending of a circular plate is given by Timoshenko and Woinowsky-Krieger (3) as

$$(1) \qquad \left[M_r + \frac{dM_r}{dr} r - M_t + Qr \right] dr\, d\theta = 0$$

Where M_r and M_t denote the radial and tangential bending moments per unit length, respectively. Q is the shearing force per unit length along the periphery of a cylindrical section. The equation for Q, developed by using the concept of equilibrium, includes both the surface load and the inertia effect (Figure 1).

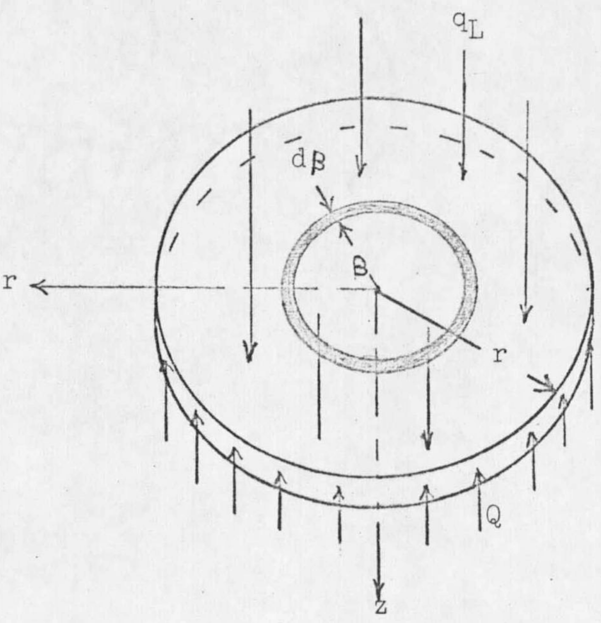

Fig 1. Cylindrical section of plate.

Thus, the resulting shear force per unit length is

$$(2)\quad Q = \frac{q_L r}{2} - \frac{\rho h}{rg} \int_0^r \left[\frac{\partial^2 w(\beta)}{\partial t^2}\right] \beta \, d\beta$$

with

β = distance to elemental section from origin.

q_L = uniform load on plate, psi.

ρ = density, lb/in^3.

h = plate thickness, in.

g = gravitational acceleration, in/sec^2.

$\frac{\partial^2 w}{\partial t^2}$ = second derivative of assumed deflection equation with respect to time, in/sec^2.

Timoshenko and Woinowsky-Krieger (3) developed the radial and tangential bending moments using the rectangular coordinate system and then converting to the polar coordinate system for the circular plate. However, in the development of the bending moments presented in this paper polar coordinates are used. The radial and tangential bending moments per unit length on the element are developed by taking the sum of the moments about the neutral axis (Figure 2). These bending moments per unit length are

$$(3)\quad M_r \, r d\theta = \int_{-h/2}^{h/2} z \, \sigma_r \, dz \, r d\theta$$