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by Dennis Nottingham

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EXPERIMENTAL TESTING
OF A
SADDLE TYPE HYPERBOLIC PARABOLOID
USING THREE DIFFERENT LOAD CONDITIONS

by

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ABSTRACT

Results of tests performed on a saddle type ten foot square hyperbolic paraboloid with rigid supports and a two inch shell are shown in the following text in the form of tables and graphs. Three types of loads were employed - uniform load, concentrated center load, and uniform load over half the surface. These are probably the most common loads used and were chosen for that reason.

The basic design and problems encountered in the construction of forms and placing the concrete are also covered. Photographs and illustrations give a good picture of the procedure and apparatus used.

SR-4 strain gages were used to determine strains in the concrete. Methods employed in attaching gages to concrete and the nature of the readings are discussed.

INTRODUCTION

HISTORY

Shell forms which are becoming increasingly popular today are not a new type of construction. These forms found their beginning in vaults, barrels, and domes in Middle Eastern and Byzantine building. Structures of this type were early attempts at spanning space beyond the capacity of post and lintel construction. Economically, these early shells were also justified. Brick and stone provided the materials for craftsmen with no formal mathematical training, yet many of these ancient buildings still stand.[1] The dome of St. Peter's spans 131 feet and weighs 10,000 tons. This may seem ponderous when compared with concrete shell domes in the market hall in Leipzig which span 240 feet and weigh 2,160 tons, but the latter was built in 1929.[2]

Even before early building began, nature was at work carving shells from stone. These shells are usually doubly curved, thus avoiding the tendency for bending moments to occur. This shows that shapes of this type can be utilized when building with materials which have a low tensile

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1. Chermayeff, Serge, "History of Thin Concrete Shells," Proceedings of a Conference on Thin Concrete Shells, June 21 to 23, 1954, MIT, p.2.
2. "Shell Concrete for Spanning Large Areas," Architectural Forum, December 1949, p.101.

strength.[3]

The appearance of modern shells evolved from two independent sources, namely, a development in mathematical analysis and a perfection in structural materials. Lame and Clapeyron formulated some mathematical expressions for stress analysis of shells in the early nineteenth century. Results of these findings led to further studies by Love in 1892, which involved the way that shells support loads. By this time, the advancement in analytical methods was sufficient for practical application to the design of actual structures.

Construction materials, such as concrete, were available in the late nineteenth century, but the greatest contribution in this field was the introduction of steel reinforced concrete. This discovery may be partly attributed to J. Monier, a French gardener, who built a reinforced concrete flower pot in 1867.

The combination of concrete and steel plus the proper methods of analysis set the stage for a period of increasing shell construction.

Antoni Gaudi was probably one of the first men to use reinforced concrete in imaginative fashions foreign

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3. Candela, Felix, "The Shell As a Space Encloser," <u>Proceedings of a Conference on Thin Shells</u>, June 21 to 23, 1954, MIT, p.5.

to existing styles of construction. In 1909 he designed a parochial school in Barcelona which may have been the first modern shell roof ever built. Gaudi probably didn't analyze his shell mathematically. An example of a mathematically designed shell was the Zeiss dome built in 1924. The formal analysis was based on mathematical expressions by Love. In countries where material was scarce, the popularity of shells began to increase. This growth in interest accounts for the improved design practices of the present time.[4]

One great advantage of concrete shell structures is their resistance to fire. Severe fires in a shell concrete textile plant in Buenos Aires and a shell concrete hanger in the U. S. both failed to collapse the structures. A similar building with a steel framework would have surely fallen down. Shell type buildings have withstood impact due to bombs during wartime. The shell roof of the Fronton Ricaleto's in Madrid was hit by a shell which knocked a six foot hole in it, but the roof remained standing. These examples refer to cylindrical shells which are surfaces of single curvature and are more exposed to failure than shells of double curvature.[5]

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4. Levy, Matthys P., "Thin Shells: Some Basic References for Architects and Engineers," Architectural Record, June 1959, p.224.
5. "Shell Concrete for Spanning Large Areas," Architectural Forum, December 1949, p.103.

Although shell technique is now at home in the United States, many more advanced ideas are being produced outside our borders. The United States was slow to start building shells because of the high cost of labor and forming. New construction methods, movable forms, and high strength materials are gradually overcoming costs. The customer is just beginning to realize that shell roofs are not just a novelty, but relatively inexpensive handsome structures. The men who have done much to start this trend in the United States began more than a decade ago working in South America and Mexico. Two of these men are Felix Candela and Guillermo Gonzalez.[6]

The one doubly curved shell that cuts costs through easier forming is the hyperbolic paraboloid. The use of reinforced concrete in the hyperbolic paraboloid offers the same advantages inherent to all shells of this material --lightness, incombustibility, economy of materials, security against impact, and little sensitiveness to foundation settlement. Felix Candela is probably greatly responsible for the present interest in hyperbolic paraboloids. He has built many of these shells in Mexico where the cost of labor is relatively low.

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6. Candela, Felix, "Market Project, Bandshell, "Architectural Forum, January 1957, p.132.
7. Candela, Felix, "Structural Applications of Hyperbolic Paraboloidical Shells," Journal of the American Concrete Institute, January 1955, p.397.

Shells of this type have been used for entrance canopies, churches, footings, warehouse roofs, gas stations, dwellings, factories, bowling lanes, and many other buildings.

Actual controlled tests on concrete hyperbolic paraboloids have been run recently by the Portland Cement Association. Investigations of this nature are rapidly increasing with the coming interest in shells.

The simple beauty and many advantages of the hyperbolic paraboloid mark it as a structure which will be progessively utilized in the future.

THEORY AND DESIGN

ANALYSIS AND PROOF

The following is an analysis of a hyperbolic paraboloid loaded with a uniform load.

A description of the shell surface is obtained in the following manner. Referring to Figure 1:

$c/h = x/a$ and $z/c = y/b$ from similar triangles.

Therefore

$$c = hx/a = bz/y; z = xyh/ab.$$

If $k = h/ab$, then

$$z = kxy.$$

For convenience an axis rotated an angle theta from the original axis is chosen. See Figure 1. Using this new axis

$$x = x'\cos\Theta + y'\sin\Theta \text{ and}$$

$$y = x'\sin\Theta - y'\cos\Theta$$

If theta is equal to 45° then

$$x = 0.707(x' + y') \text{ and}$$

$$y = 0.707(x' - y').$$

Substituting the values of x and y,

$$z = 0.5k(x'^2 - y'^2).$$

When $x' = 0$ then $z = 0.5k(-y'^2)$ and when $y' = 0$ then $z = 0.5k(x'^2)$;

these are both equations for parabolas.

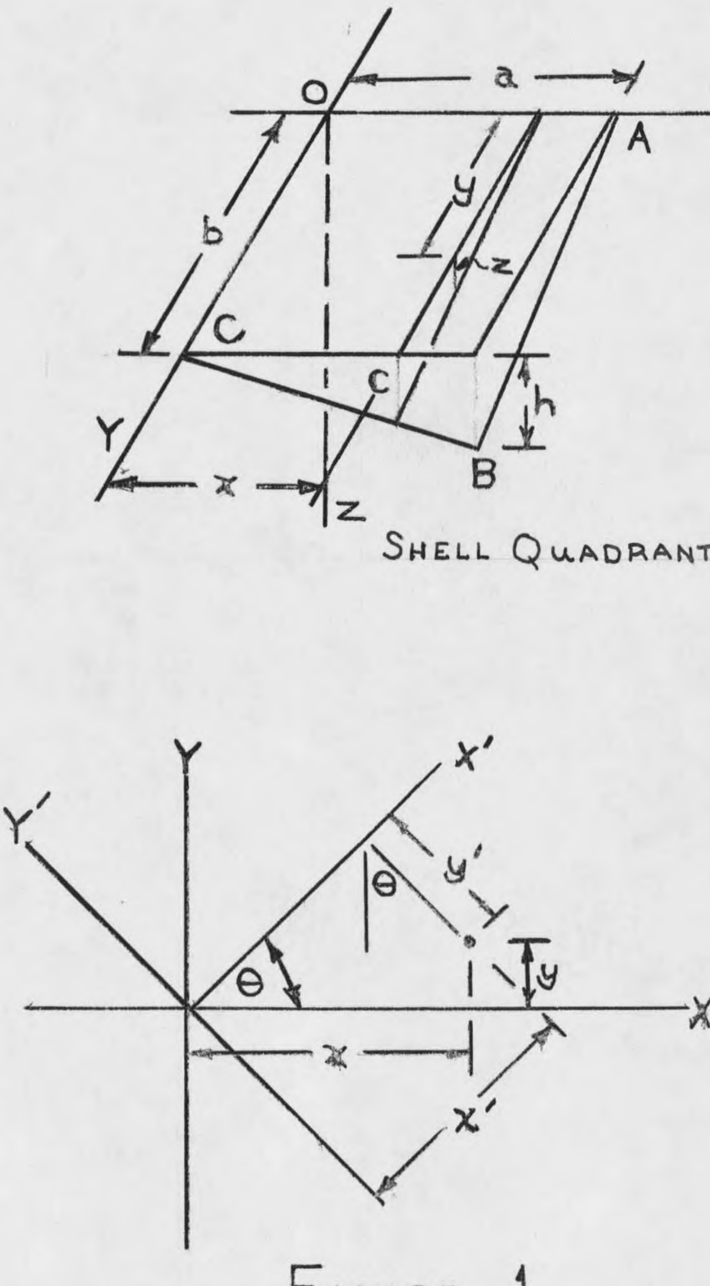


FIGURE 1.

DERIVATION FIGURES.

A parabolic arch under uniform load has zero bending moment throughout. Using this fact, the horizontal thrusts can be found. Note Figure 1a. The sum of the moments about the center of the span is equal to zero or $H(-h') = (wL/4)(L/2) - (w/2)(L/2)(L/4) = wL^2/16$. $w/2$ is used as the uniform load since it is assumed that half the load goes to the tension parabola and half to the compression parabola.

Therefore

$H = -wL^2/16h'$.

$z = -0.5ky'^2$. If $y' = L/2$ and $z = h'$ then $h' = -0.5kL^2/4$.

Therefore $H = (-wL^2/16)(-4/0.5kL^2) = w/2k$ or

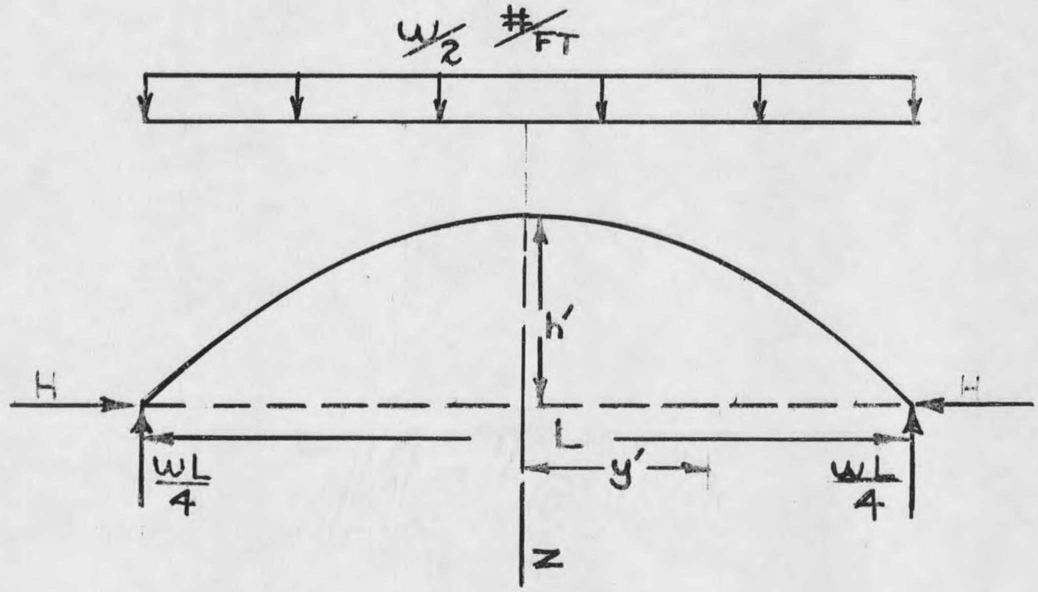
$H = wab/2h$

Up to this point, it has been assumed that the edges of the structure are rigidly restrained. It will now be shown that this assumption is valid. From Figure 1a it can be shown that the thrusts perpendicular to the edge beam are equal in magnitude and opposite in direction. Therefore, the edge beam is restrained from moving laterally. A force exists along the beam horizontally. This force $(S) = 2H \sin \Theta \, ds/dx = 2H \sin \Theta \cos \Theta$.

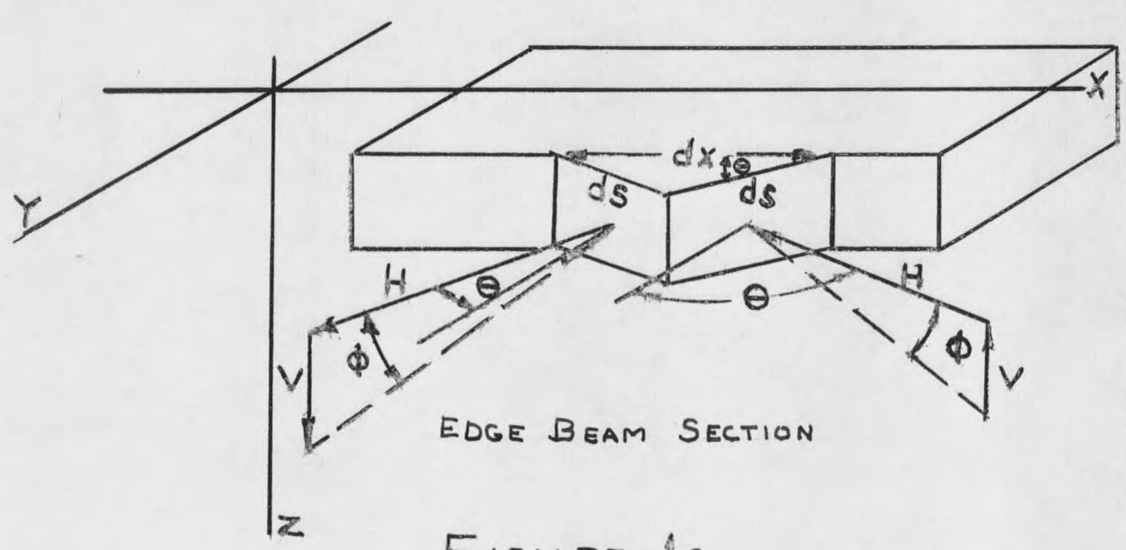
When theta equals 45° and $H = wab/2h$ then

$S = wab/2h$ per unit length of beam.

The vertical forces at the edge beam are equal to



TYPICAL PARABOLIC ARCH



EDGE BEAM SECTION

FIGURE 1a.

DERIVATION FIGURES.

$V = \sum H \tan \phi = H\, dz/dy' + H\, dz/dx'$.

Since $z = -0.5k(x'^2 - y'^2)$, then $dz/dy' = -ky'$ and $dz/dx' = kx'$. At any point on the horizontal edge $x' = y'$ and the vertical components cancel each other. Along the sloping edge AB, y' will not equal x', but instead equals $x' - a\sqrt{2}$. Now $dz/dy' = -k(x' - a\sqrt{2})$ and $dz/dx' = kx'$.

Substituting these values, $V = \sum H \tan \phi = H[-k(x' - a\sqrt{2})] + Hkx'$ or

$$V = a\sqrt{2}kH = a\sqrt{2}\, H(h/ab) = Hh\sqrt{2}/b.$$

V in this case is applied to a length ds. V' per unit length of beam is equal to $Vds/dx = Hh\sqrt{2} \cos \theta/b$. For angles equal to 45°, $V' = Hh/b$ along an edge parallel to edge b. Similarly, along an edge parallel to edge a, $V' = Hh/a$. This shows that the vertical force in the edge beam is then $H \tan \gamma$ where γ is the angle of rise of the edge beam. Therefore, the edge beam is in direct tension or compression.

The preceeding proof and analysis was taken mainly from <u>Elementary Analysis of Hyperbolic Paraboloid Concrete Shells</u>, an edition of the Portland Cement Association.

<u>DESIGN</u>

Available laboratory space limited the size of the hyperbolic paraboloid to ten feet square in plan. A rise of four feet was chosen as a maximum height. This was to keep the fresh concrete from moving during placing and vibration. A shell thickness of two inches was selected as being a minimum for good concrete workability and ease of placing. The following design loads were used:

two inch shell	-	25 psf,
edge beam	-	1 psf,
live load	-	40 psf,
for a total of		66 psf.

The horizontal component of the force in the edge beam equals

$S = wab/2h = 66(5)(5)/2(2) = 413$ pounds per foot of edge beam.

Total edge beam compression equals

$413\sqrt{116} = 4450$ pounds.

Vertical reactions at the supports equal

$2(4450) \sin 21.8^{\circ} = 3300$ pounds. Where 21.8° is the slope of the beam. Horizontal thrusts at the supports equal

$2(4450) \cos 21.8^{\circ} \cos 45^{\circ} = 5840$ pounds.

Most edge beams are designed as columns, but here they were designed as being in direct bearing. With a $f'_c =$ 3000psi, then $f_c = .25f'_c = 750$ psi. Total edge beam cross sectional area equals

$4450/750 = 5.94$ square inches.

Since eccentric loads were to be used, the assigned edge beam dimensions were three inches wide and four inches deep on the outside edge. A number three reinforcing rod was run along each beam, two inches up from the bottom and continued into the footings. Shell tension and compression along the parabolic arches was equal to

$413/24 = 17.2$ psi.

This was reinforced with six by six number ten welded wire fabric. The steel in the shell and beams was used primarily to prevent shrinkage or temperature cracks. Footing dimensions were 5 ft x $3\frac{1}{2}$ ft x $5\frac{1}{2}$ in reinforced two ways with number five rods on six inch centers. The footings were greatly over designed to insure no footing failure under eccentric loads. The length of connection between the shell and footing was arbitrarily chosen. The tie rod had a 7/8 in diameter with a turnbuckle in the center. Six inch lengths of $3\frac{1}{2}$ in x $3\frac{1}{2}$ in x $\frac{1}{2}$ in angle were welded to the ends of the tie rod as bearing plates.

The preceeding shell design was based on the assumption that the structure was uniformly loaded. In the following text a comparison will be made between stresses and strains predicted by the design and those that are created by several types of loading.

CONSTRUCTION AND PROCEDURE

CONSTRUCTION

To begin forming, the bottom edge two by four beams were fitted together in the proper position. These beams constituted all that was necessary to generate the required surface. The beams were braced and the interior two by four beams were added parallel to one set of parallel sides. Stringers were run under the beams at the one-third points and shored at the end and one-third points. The entire group of shoring was strengthened with diagonal one by four bracing. Sheeting was placed parallel to the other set of parallel edge beams. Six inch shiplap had to be ripped into three inch strips for easy handling, since the boards assumed a warped form. By placing the covering in this manner, small cracks were developed near the edges of the forms. These cracks were eliminated when the entire surface was covered with roofing paper. Edge forms consisting of two by six lumber were placed next and braced to the existing structure. The bottom of the trough was closed off by one by three boards which gave the finished concrete edge beam the required three inch width. Completion of the footing and abutment forms finished the forming. See Figure 4.

Next the reinforcing steel and tie rod were placed and held in position with wires and blocks.

Concrete was brought to the laboratory in a ready-mix truck which was located as close as possible to the forms. However, it was still necessary to use a wheelbarrow. Slump of the one and one half cubic yards of concrete was about one to two inches. This presented some problems in placing, most of which were eliminated when a vibrator was put to use. Surface finishing was done by troweling followed by a light brooming. See Figure 3.

The structure was kept damp with a wet canvass for a five day curing period. Stripping the forms began after five days and was accomplished with little difficulty. Test cylinders broke at a five day strength of 1950 psi and 3910 psi.

Many curious and interested people viewed the completed hyperbolic paraboloid during the annual High School Week festivities. See Figure 5.

GAGE PLACEMENT

A-1 and AR-1 were the two types of SR-4 strain gage chosen for use on the hyperbolic paraboloid. The A-1 type measures strain in one direction while the AR-1 type is a rosette made up of three gages similar to the A-1 gage.

Twelve positions were selected for gage placement. Their exact location can be seen on Figure 2, Figure 9, and Figure 2a. By symmetric placement of the gages at the

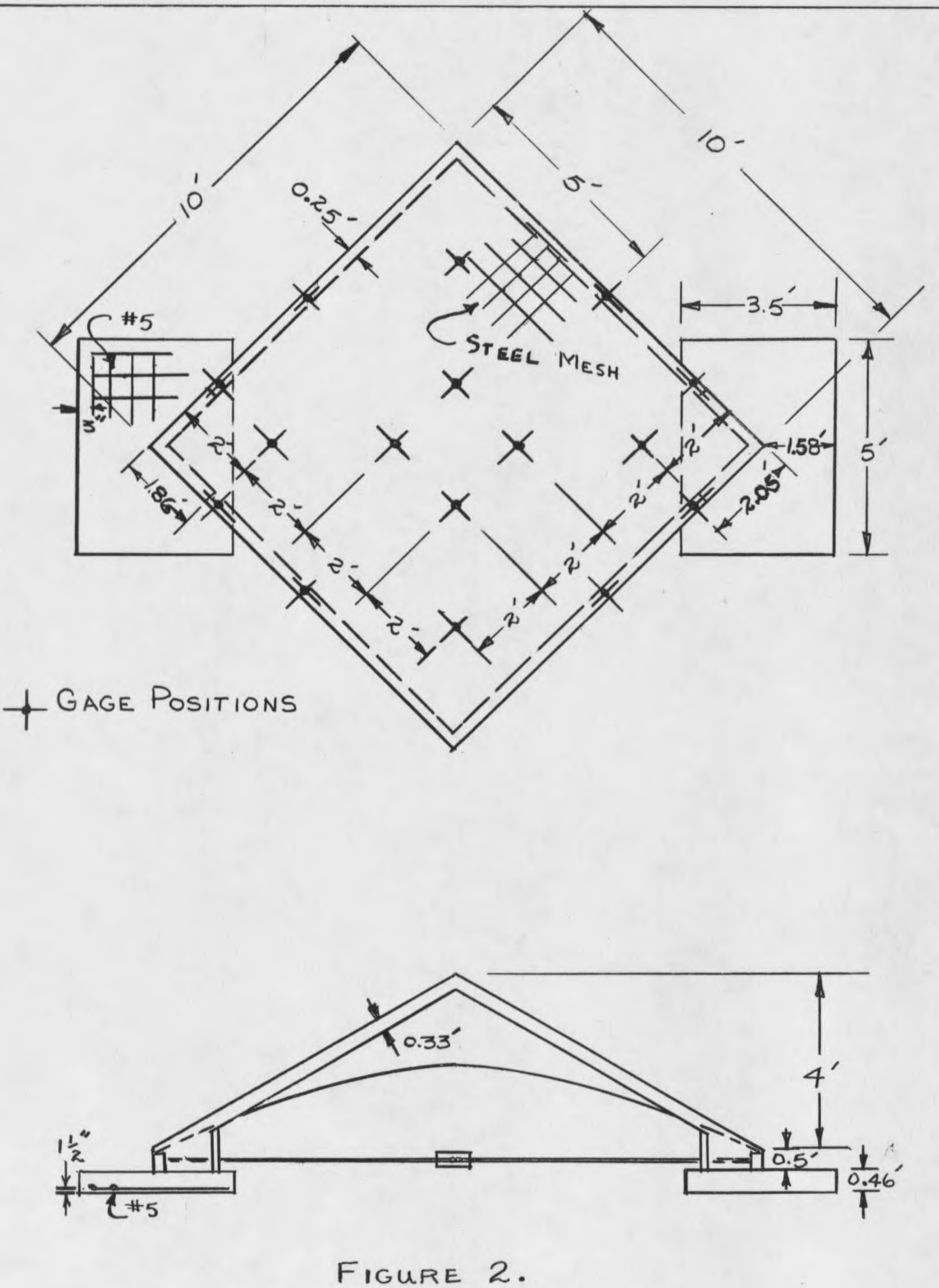


FIGURE 2.
DIMENSIONS AND GAGE POSITIONS

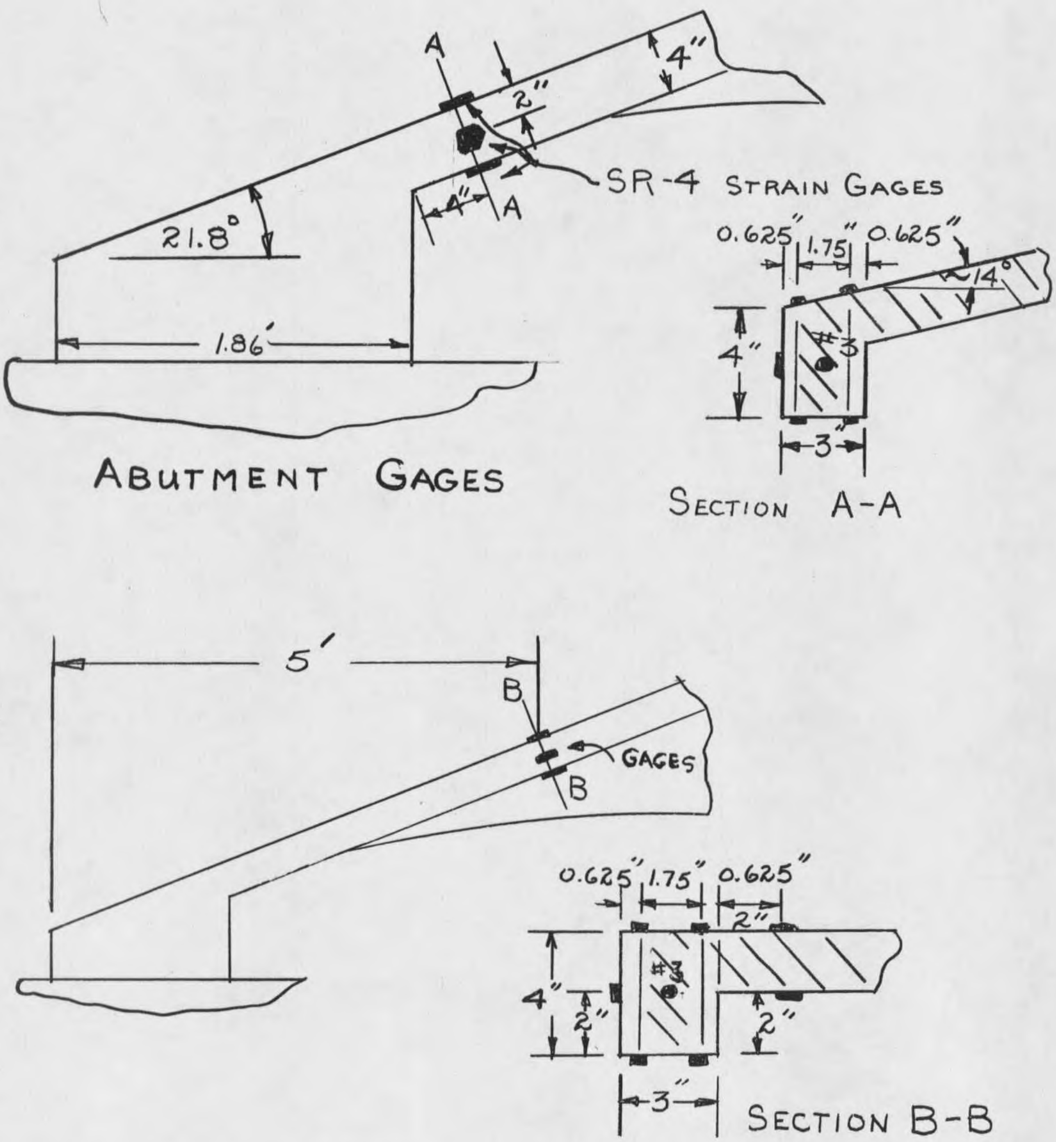


FIGURE 2a.
GAGE LOCATION.