



An economic analysis of grain cropping sequences in Montana  
by James Howard Nybo

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of  
MASTER OF SCIENCE in Agricultural Economics  
Montana State University  
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**Abstract:**

The purpose of this study is to provide a summary and an application, of a relatively new methodology for performing an economic analysis of grain cropping sequences. The primary objective is to analyze viable grain cropping sequences in Montana using a dynamic programming framework.

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Some historical factors affecting grain cropping decisions are discussed, as are the mathematical applications used in the study. The primary model utilizes stochastic Dynamic Programming. Multiple regression is used to explain the grain production relationships.

Three optimal policies were developed based on different sets of assumptions. Two of these optimal policies were compared with fixed decision rules. It was demonstrated that the optimal policies would, on the average, provide higher expected returns than either a rigid wheat-fallow or continuous barley alternative policy.

A qualitative discussion of several environmental factors is included.

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Date

Dec. 3, 1971

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by

JAMES HOWARD NYBO

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

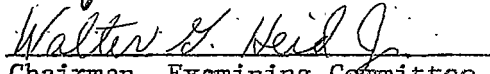
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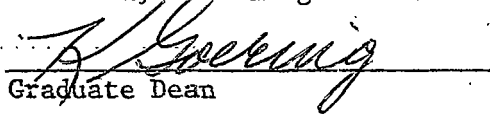
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## TABLE OF CONTENTS

	Page
LIST OF TABLES.....	v
LIST OF FIGURES.....	vii
 Chapter	
1. INTRODUCTION.....	1
Problem.....	1
Purpose.....	2
Objectives.....	2
Sources.....	4
2. HISTORICAL PERSPECTIVE.....	5
Historical Summary.....	5
Current Cropping System.....	7
Plant Disease.....	14
3. THE MODEL.....	15
Dynamic Programming and This Application.....	15
Production Functions.....	22
Costs of Production.....	23
Transition Probabilities.....	23

## TABLE OF CONTENTS (continued)

Chapter	Page
4. DATA PREPARATION.....	27
Production Functions.....	27
Costs of Production.....	38
Expected Immediate Returns.....	40
5. GENERATION OF AN OPTIMAL CROPPING DECISION RULE USING DYNAMIC PROGRAMMING.....	42
Specific Results of the Three Runs.....	44
Comparing the Optimal Policies With Fixed Decision Rules.....	52
6. OUR ENVIRONMENT.....	55
Externalities.....	55
Fallow.....	56
The Use of Chemicals.....	57
7. SUMMARY.....	61
Conclusions.....	62
Recommendations.....	63
BIBLIOGRAPHY.....	65
APPENDIX.....	69

## LIST OF TABLES

Table	Page
1. Harvested Acres of Dryland Barley, Winter Wheat, Spring Wheat: Montana 1948 - 1969.....	9
2. Historical Cropping Sequences - Norris Hanford Ranch, Fort Benton, Montana.....	12
3. Array of Cropping Decisions made on Norris Hanford Ranch, Fort Benton, Montana, 1955 - 1971.....	13
4. Descriptive Matrix of States in the Sixteen State Model.....	20
5. Descriptive Matrix of States in the Twenty-four State Model.....	21
6. Regression values for Production Functions Continuous Wheat Experiment.....	29
7. Regression values for Production Functions Continuous Barley Experiment.....	30
8. Regression values for Production Functions Continuous Barley-Fallow.....	31
9. Regression values for Production Functions Continuous Wheat-Fallow Experiment.....	32
10. Schedule of Yields for Four Treatment Combinations and Eight Moisture Levels.....	34
11. Yields and Moisture Levels for Wheat and Barley, as Used in the Dynamic Programming Model.....	37
12. Variable Costs of Production.....	39
13. Prices Used in Analysis.....	41
14. Expected Immediate Returns, Runs 1, 2 and 3.....	42

## LIST OF TABLES (continued)

Table	Page
15. Optimal Decision Rule in Stage $n = 1, \dots, 5$ ; Run Number One.....	47
16. Optimal Decision Rule in Stages $n = 1, \dots, 6$ , and 10; Run Number Two.....	49
17. Optimal Decision Rule in Stages $n = 1, \dots, 5$ ; Run Number Three.....	51
18. Fixed or Optimal Policies; A Comparison of Present Value of Expected Returns at stage $n = 20$ .....	53
19. Fixed or Optimal Policies; A Comparison of Present Value of Expected Returns at stage $n = 20$ , $i = 1$ and $i = 8$ .....	54

## LIST OF FIGURES

Figure	Page
1. Hypothetical Regression of Moisture in Time $t$ over Moisture in Time $t-1$ .....	25
2. Yield Schedules for Wheat, Based on Regression Analysis of Experimental Data.....	36
3. Yield Schedules for Barley, Based on Regression Analysis of Experimental Data.....	36



ABSTRACT

The purpose of this study is to provide a summary and an application of a relatively new methodology for performing an economic analysis of grain cropping sequences. The primary objective is to analyze viable grain cropping sequences in Montana using a dynamic programming framework.

A model is formulated using long-term crop experiment data and the results of a recent Cost>Returns Survey to generate an optimal cropping decision rule. This rule, which is conditional on land use the preceding year and available soil moisture at planting time, tells the individual farm decision-maker whether he should plant spring wheat, plant barley, or fallow.

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Some historical factors affecting grain cropping decisions are discussed, as are the mathematical applications used in the study. The primary model utilizes stochastic Dynamic Programming. Multiple regression is used to explain the grain production relationships.

Three optimal policies were developed based on different sets of assumptions. Two of these optimal policies were compared with fixed decision rules. It was demonstrated that the optimal policies would, on the average, provide higher expected returns than either a rigid wheat-fallow or continuous barley alternative policy.

A qualitative discussion of several environmental factors is included.

## CHAPTER I

### INTRODUCTION

#### THE PROBLEM

More and more, in Montana and elsewhere, the dryland grain farmer is questioning the determinants of his cropping decisions, and considering alternatives other than the now-traditional wheat-fallow-wheat-fallow-...system. As in all forms of competitive productive activity there are many pressures exerted on the dryland grain farmer to induce him to be more efficient in the totality of his operations.

One of these forces, a problem of large proportions and far-reaching implications, is that of the saline seep areas now appearing and growing in many of Montana's grain producing regions. The magnitude and recent growth of the saline seep problem facing the dryland grain farmer in parts of Montana and other parts of the U.S. and Canada appears to be directly related to the way he uses soil moisture as reflected in the cropping sequences to which he subjects his land.<sup>1</sup>

Recent developments in technology including better tractors, bigger and more efficient implements, pesticides (to include fungicides, herbicides, and insecticides), chemical fertilizers, and new

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<sup>1</sup>c.f. Proceedings, Saline Seep-Fallow Workshop, Feb. 22-23, 1971, Great Falls, Montana, Published by Montana Cooperative Extension Service.

plant varieties all contribute to the relevance of the investigation.

With uncertainty in grain markets, in government programs, in actual production activities, and in environmental implications, it is both timely and necessary to better understand the economics of cropping decisions.

#### THE PURPOSE

The purpose of this study is to provide a summary and an application of a relatively new methodology for performing an economic analysis of grain cropping sequences. By including available soil moisture as a decision variable, a change in the cropping system can be suggested which will more fully utilize soil moisture and at the same time maintain or improve the economic position of the farm firm. The use of dynamic programming permits the problem to be considered over a time horizon of any length, and data needs for future analysis can be estimated.

#### THE OBJECTIVES

The primary objective of this study is to analyze viable grain cropping sequences in Montana using a dynamic programming framework in a manner similar to that presented by Burt and Allison

in 1963<sup>2</sup>. By using the dynamic programming model to look at costs of production and production functions it is possible to generate a conditional decision rule which will tell the individual operator what cropping sequence he should follow in order to maximize the present value of his net returns for an infinite time horizon. The term "viable" restricts analysis to those alternatives which are deemed realistic by the producer for the time period and location studied. Crops other than grain could conceivably help alleviate the saline seep problem more rapidly, but would create other management problems with regard to enterprise combinations, machinery investment, and managerial knowhow. "Grain cropping sequences" refers to the cropping pattern of the individual cash-grain farmer, i.e. what crops or fallow periods (which for purposes of analysis are treated as crops) he subjects his land to over time. Data limitations have restricted the analysis to a consideration of the cropping alternatives of spring wheat, barley, and fallow. It is immediately clear that these three alternative uses of land are not all-inclusive. Certainly purely livestock-supporting alternatives such as range, pasture, or hay are possibilities, as is winter wheat. Unavailability of data and the

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<sup>2</sup>Oscar R. Burt and John R. Allison, "Farm Management Decisions with Dynamic Programming", Journal of Farm Economics, Vol. XLV, NO. 1, February, 1963

complexities of the model have served to constrain the study to preclude these alternatives. They should be considered if and when the model is put to practical use.

#### SOURCES OF DATA

Production data and transition probabilities were generated from experiments done at the Havre branch of the Montana Agricultural Experiment Station, from 1917 to 1947. Cost data are taken from a study done by Heid.<sup>3</sup>

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<sup>3</sup>Dr. Walter G. Heid, Jr., Farm Production Economics Division, Economic Research Service, U.S.D.A., Stationed at Montana State University, Bozeman, Montana

## CHAPTER II

### HISTORICAL PERSPECTIVE

#### HISTORICAL SUMMARY

To gain perspective on the problem, it is helpful to look at the historical determinants of cropping decisions in Montana dryland grain producing regions. Farming in Montana before the 1900's was for the most part practiced on a small scale adequate enough to provide for the needs of the placer mining communities and the fur traders.<sup>1</sup> The real expansion of agriculture into Montana, however, did not take place until the early 1900's. At that time farmers came from the humid areas of the east; first in small numbers, and then after the passage of the Enlarged Homestead Act of 1909, they came in great numbers.<sup>2</sup> Although the weed-reducing benefits of occasional summer fallow had been recognized by earlier farmers in some of the valleys of the region, most of the new farmers brought with them the agricultural capital and techniques of their former homes. They came to Montana prepared to farm as they had before, and for a number of years many did. The rains in the first years after the passage of the

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<sup>1</sup>Mary Wilma M. Hargreaves, Dry Farming in the Northern Great Plains (Cambridge: Harvard University Press, 1957), p. 29.

<sup>2</sup>Merrill G. Burlingame, K. Ross Toole, Robert G. Dunbar, A History of Montana (New York: Lewis Historical Publishing Co., 1957)

Homestead Act were very good. Then things changed, and a five-year drought began in 1917; 1918 was drier, and 1919 even drier still. Many were disillusioned and left; those who stayed adapted to the semi-arid nature of the region and changed their farming techniques.

The technique which was adopted came to be known as "dry farming". Burlingame characterizes dry farming as "the culture of drought-resistant plants by means of moisture-conserving tillage practices."<sup>3</sup> The major element in dry farming was the inclusion of the practice of summer fallow. With the establishment of the Montana Agricultural Station and many railroad field stations, new techniques were speedily developed, tested, and disseminated.<sup>4</sup> The first half of the twentieth century has been characterized by the use of many different tillage techniques and many varieties of equipment. Basic to virtually all of them since the early twenties has been the practice of summer fallow. Through time, agronomic research has shown that summer fallow not only conserves soil moisture, it permits soil nitrification (thus replenishing the nitrogen removed by cropping), it controls weeds, it controls certain soil-borne plant diseases, and in the case of some operations it allows the operator to take care of much more acreage than he could

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<sup>3</sup> Ibid

<sup>4</sup> Ibid

if he were to crop it every year. An increasingly important factor favoring an alternate crop-fallow sequence has been the advent of the U.S. Government wheat and barley programs, which require the setting aside of certain percentages of one's wheat or barley "allotment." By taking land out of production and subsidizing the farmer, the government can both limit the supply of grain and help to provide the farmer a better return. When land is set aside it is commonly left fallow, and cropped on alternate years.

#### CURRENT CROPPING SYSTEM

The choice of whether to plant wheat or barley has been and is one of economics, i.e. the most profitable alternative is generally taken. The broad acceptance of the summer fallow technique has already been mentioned. While there are available data from several long-term cropping experiments in various parts of the state, which provide meaningful information relating to production relationships, there is a great shortage of information dealing with what decisions have been made by individual farmers on a specific plot of land. These decisions are not explained or recorded in the Montana Agricultural Statistics, as they have been lost in the aggregation process. The aggregate figures of land in wheat and barley are of value to reflect the magnitude of the dryland grain industry in Montana. Table 1 shows the total harvested dryland acres of barley, spring wheat, and winter



wheat for the state of Montana in the years 1948 to 1969.<sup>5</sup>

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<sup>5</sup> Although the cropping decision is an economic one, it is generally not made in a purely competitive free market environment. One very significant "non-market" factor which has served to limit alternatives and to change the profitability of alternatives has been the U.S. Government Farm Program.

TABLE 1: Harvested Acres of Dryland Barley, Winter Wheat, Spring Wheat\*; Montana, 1948-1969<sup>6</sup>

Year	Barley	Winter Wheat	Spring Wheat
48	750,100	1,505,000	3,146,700
49	409,800	1,382,300	3,699,500
50	723,600	1,111,300	3,645,600
51	357,600	1,303,400	4,379,800
52	367,600	1,610,700	3,959,000
53	441,800	1,487,100	4,292,800
54	1,153,500	1,641,600	2,911,500
55	1,270,800	1,989,100	2,254,900
56	965,600	1,183,100	2,496,900
57	1,626,500	1,811,200	1,748,800
58	1,489,600	2,310,000	1,903,200
59	1,770,600	1,702,500	2,058,300
60	1,628,200	1,958,200	1,687,600
61	1,374,000	2,022,500	1,449,300
62	1,699,000	1,653,000	1,474,300
63	1,420,500	1,854,400	1,820,800
64	1,443,000	1,797,000	1,804,900

\*Durum included in Spring Wheat from 1948-1954.

<sup>6</sup>Table from various years of Montana Agricultural Statistics.

Table 1 (continued)

Year	Barley	Winter Wheat	Spring Wheat
65	1,214,400	2,288,600	1,645,500
66	1,548,700	2,110,800	1,398,700
67	1,161,600	2,776,700	1,641,000
68	1,062,000	2,720,000	1,395,000
69	1,510,000	2,282,000	1,072,000

In the absence of published information as to historical time-patterns of dryland agricultural land use, Mr. Norris Hanford, an established dryland farmer of Fort Benton, Montana, was consulted regarding those practices which he has followed in the past several years. The following table shows his cropping practices for four pieces of land for the years 1955 to 1971. Although this table is not meant to be a picture of what all dryland grain farmers are doing in Montana, it does provide an indication of what is being done in the absence of broader published information.

TABLE 2: Historical Cropping Sequences - Norris Hanford Ranch, Fort Benton, Montana

Year	Field A (102.1 Acres)	Field B (97.8 Acres)	Field C (151.5 Acres)	Field D (119.5 Acres)
1971	Barley	Barley	Barley	Barley
1970	Barley	Barley	Wheat	Barley
1969	Fallow	Wheat	Fallow	Fallow
1968	Fallow	Barley	Wheat	Wheat
1967	Fallow	Barley	Wheat	Fallow
1966	Barley	Fallow	Fallow	Wheat
1965	Wheat	Wheat	Wheat	Fallow
1964	Fallow	Fallow	Fallow	Wheat
1963	Wheat	Barley	Wheat	Fallow
1962	Fallow	Fallow	Fallow	Barley
1961	Wheat	Wheat	Barley	Fallow
1960	Fallow	Fallow	Fallow	Wheat
1959	Barley	Wheat	Wheat	Fallow
1958	Wheat	Fallow	Fallow	Barley
1957	Fallow	Barley	Wheat	Barley
1956	Barley	Fallow	Fallow	Wheat
1955	Wheat	Wheat	Barley	Fallow

While no attempt is made to analyze the above set of cropping decisions, Table 3 shows the frequency of choosing each of the three alternatives in the period 1955 through 1971.

TABLE 3: Array of Cropping Decisions made on Norris Hanford Ranch, Fort Benton, Montana, 1955-1971.

Cropping Decision	Number	Percentage of Total
Barley	19	28.0%
Wheat	22	32.4%
Fallow	27	39.6%
Total	68	100.0%

Plant Disease

The mathematical portion of this analysis is so structured as to rule out the consideration of any alternatives other than spring wheat, barley, and summer fallow. Although data and technical constraints were the primary determinants of such restriction of the analysis, another factor is important to winter wheat growers. In areas of high moisture a wheat fungus, known as Cephalosporium Stripe, preys on winter wheat. According to Mathre<sup>7</sup>, winter wheat, in a wheat-fallow-... sequence, is very susceptible to the fungus, which is formally known as Cephalosporium Gramineum. In affected areas, botanists are recommending that spring crops can be grown for 5 or 6 years until the fungus is killed off. This conveniently supports the exclusion of winter wheat as an alternative. Because of plant pathogenic problems associated with the continuous cropping of wheat, this analysis includes a consideration of the cropping system when wheat following wheat is not permitted.

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<sup>7</sup>From a personal discussion with Dr. Don Mathre, Botany-Microbiology Department, Montana State University, Bozeman, Montana.

## CHAPTER III

### THE MODEL

#### DYNAMIC PROGRAMMING AND THIS APPLICATION

This study is concerned with the management decision facing the individual farmer who for any reason has made the decision to raise either dryland spring wheat, dryland barley, or let the land lie fallow. The analytical framework used is an adaptation of the methodology first presented by Burt and Allison.<sup>1</sup> The methodology can be called "Dynamic Programming," and is consistent with the definition presented by Bellman.<sup>2</sup>

In order for a problem to be validly considered in a dynamic programming framework it must meet certain requirements. One of these is that it must be a multi-stage process. In the case of the cropping decision problem, it is clearly multi-stage. The stage is the time interval into which the process is divided. At each stage a decision must be made. Since this analysis has been restricted to the three mentioned alternatives, there is only one time each year--planting time in the spring--that a decision must be made regarding what to

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<sup>1</sup>Oscar R. Burt and John R. Allison, "Farm Management Decisions with Dynamic Programming," Journal of Farm Economics, Vol. XLV, No. 1, February, 1963

<sup>2</sup>Richard Bellman, Dynamic Programming, (Princeton University Press, 1957)



plant. The interval between stages, then, is taken to be one year, beginning and ending at planting time in the spring.

If a system is to be considered using dynamic programming, then it must fit the definitional limitations of a Markov process. Fundamental to Markov processes are the concepts of the "state" of a system, and "state transition." A system is said to occupy a state when it is completely described by the values of variables that define the state. When those state variables change to values describing another state, the system makes a state transition.<sup>3</sup> Changes in state variables can be taken to be a continuous process, or a discrete-time process. In this analysis, although conceptually it is realized that the physical system is continuous, it shall be treated as a discrete-time process. A process can be treated as either deterministic or stochastic. The stochastic nature of the soil moisture and crop response has necessitated that this analysis be carried out in a stochastic framework.

Evaluation of the dynamic programming problem requires a precise statement and definition of the problem including all relevant variables. It has been shown above that the cropping decision problem

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<sup>3</sup>Ronald A. Howard, Dynamic Programming and Markov Processes, (The M.I.T. Press, Cambridge, Massachusetts, 1960)

is a multi-stage process. If the Markov requirement is met, then each state is fully defined by its state variables, and is independent of any state variables at any other stage. The problem is one of solving a sequential decision process. It can be evaluated using what Howard calls value iteration.<sup>4</sup>

After defining the recurrence relation and the relevant variables, the value iteration process can be better conceptualized. The decision process requires solution of the recurrence relation

$$V_i(n) = \text{Max}_k [q_i^k(n) + \beta \sum_{j=i}^m P_{ij}^k(n) V_j(n-1)]$$

where

- $V_i(n)$  is the present value of the total expected net return in  $n$  stages, starting from state  $i$ , if an optimal policy is followed.
- $k$  is the decision alternative variable; in this model,  $k = 1, 2, 3$ , where  $k = 1 =$  fallow;  $k = 2 =$  plant barley;  $k = 3 =$  plant wheat.
- $q_i^k(n)$  is the term for expected immediate returns, given the  $i^{\text{th}}$  state, the  $k^{\text{th}}$  decision alternative, and the  $n^{\text{th}}$  stage.
- $i = 1, 2, \dots, m$  is the index for the state occurring in stage  $n$ .

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<sup>4</sup>Ibid.

$j$  = 1, 2, ...,  $m$  is the index for the states occurring in stage  $N-1$ .

$P_{ij}^k(n)$  is the probability of making the transition from state  $i$  in stage  $N$  to state  $j$  in stage  $N-1$  given the  $k^{\text{th}}$  decision alternative.

$\beta$  is the discount factor;  $\beta = 1/(1+r)$ , where  $r$  is the relevant periodic interest rate.

The value iteration process uses a technique of iteration of the recurrence relation to generate a policy, which defines the decision to be made for a given state, at each stage for all possible combinations of stages and states. Dynamic programming yields the optimal policy for decision processes of any length. In this case, that optimal policy is defined as one which maximizes the present value of net returns over the entire planning horizon. An optimal policy has the property that whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

In the value iteration process,  $n$ , the stage variable, represents the number of stages remaining in the planning horizon. If a planning horizon of 20 years is being considered, at the beginning  $n$  would assume a value of 20, and at the beginning of the 20th year would assume a value of 1. From a conceptual viewpoint this is the reverse of the normal  $t, t+1, t+2, \dots, t+n$  convention of treating a time variable. The value iterative process, however, begins at

the end of the planning horizon, and works back to the present. In so doing, an optimal policy is generated for all time periods up to the total number considered in the iteration.

Prior comments have emphasized the potential pathogenic problems associated with the inclusion of the continuous wheat alternative. This problem has been considered in two frameworks: (1) A sixteen-state ( $i = 1, 2, \dots, 16$ ) model where the continuous wheat alternative is allowed, and (2) a twenty-four state ( $i = 1, 2, \dots, 24$ ) model where the alternative of planting wheat after wheat is not permitted. The flexibility of the model is one of its pleasing qualities. As new information becomes available, or as institutional structures such as government programs change, the model can be adapted to handle a wide range of changes.

The foundation of the dynamic programming analysis lies in the recurrence relation, and the economic foundation of the recurrence relation lies in the  $q_i^k(n)$  term, the expected immediate returns in stage  $n$  given the  $i^{\text{th}}$  state and the  $k^{\text{th}}$  decision alternative. The model as structured for this application has either sixteen or twenty-four states. This analysis assumes that a state is completely defined by the cropping decision in the preceding stage, and the level of available moisture in the soil profile. The sixteen state model assumes that the decision in the preceding state could have been one

of two alternatives: (1) fallow, or (2) crop. Wheat or barley are treated the same in this case. In the sixteen state model this is sufficient as it describes the transition with respect to soil moisture and the previous crop. With eight levels of available soil moisture, it is possible to completely describe all possible states with sixteen states, as table 4 shows.

TABLE 4 : Descriptive Matrix of States in the Sixteen State Model

		Land Use in the Preceding Stage	
		Fallow	Crop (wheat or barley)
Moisture Level	1	1	9
	2	2	10
	3	3	11
	4	4	12
	5	5	13
	6	6	14
	7	7	15
	8	8	16

The twenty-four state model does not allow the wheat-wheat alternative, and requires an additional eight states beyond the 16 state model in order to do this. In the twenty-four state model there are again eight levels of available soil moisture, but the state is also described by the decision in the preceding stage, to include fallow, wheat, or barley as the decision, rather than just crop or fallow. The following matrix shows the 24 states in this model.

TABLE 5: Descriptive Matrix of States in the Twenty-four State Model.

		Land use in the preceding stage		
		Fallow	Barley	Wheat
Moisture Level	1	1	9	17
	2	2	10	18
	3	3	11	19
	4	4	12	20
	5	5	13	21
	6	6	14	22
	7	7	15	23
	8	8	16	24

### Production Functions

Expected immediate returns are assumed to be unrelated to the stage of the process, and hence are a function of the state and the decision alternative at any stage. Expected immediate returns is the difference between gross returns and variable costs. Once a commodity price has been determined, price times yield determines gross returns. The problem is to determine yield for each state. In order to determine yield in this study it was necessary to analyze production data from the Havre Branch of the Montana Agricultural Experiment Station, and generate production functions which would provide this information. The production data was analyzed in a multiple regression model, and was finally fitted into a logarithmic function where yield was estimated as a function of the logarithm of available soil moisture and the logarithm of five different precipitation variables. The state variable is soil moisture, so it was necessary to take the expectation with respect to precipitation in order to isolate the relationship between yield and soil moisture. The soil moisture variable was included as the state variable, and physically measured each year at planting/decision time. The regression values and matrix of expected immediate returns can be found in the next chapter.

### Costs of Production

The second important element in the determination of the expected immediate returns is the costs of production. In the case of the production functions, it was only necessary to estimate the functions for wheat and barley. However, because the fallow alternative does carry with it real out-of-pocket costs it is necessary to estimate costs of production for all three alternatives.

The analysis is carried out using variable costs only to meet the short run assumption of economic analysis. The determination of costs of production is explicitly internal to the individual farm firm. Chapter 6 contains a qualitative discussion of some potential external costs. Chapter 4 contains a table showing the complete breakdown of costs of production.

### Transition Probabilities

The recurrence relation contains the term  $P_{ij}^k$ , which is the probability of making the transition from state  $i$  to state  $j$  given decision alternative  $k$ . This term is essential to the model, as it represents our knowledge of the historical relationship between soil



moisture level and cropping decisions<sup>5</sup>. It has been pointed out that the definition of a Markov process requires that a state be completely defined by its state variables. The transition probabilities provide a means to estimate those state variables given an initial state. States  $i$  and  $j$  depend on moisture level and on cropping decision in the preceding stage, we are really concerned with the changes in moisture levels associated with states  $i$  and  $j$ . With the stated assumptions regarding homogeneity of crop water use, it is necessary to analyze the four cropping combinations to determine the probabilities of changing moisture levels. Considering time  $t-1$  and time  $t$ , and assuming that wheat and barley have identical soil moisture consumption patterns, there are four possible crop sequences that can occur. They are crop-crop, crop-fallow, fallow-crop, and fallow-fallow.

Experimental information was available for the combinations of crop-crop, crop-fallow, and fallow-crop. Since the experiment which has provided the data for this study did not include the fallow-fallow alternative, it was assumed that the fallow-crop experiment would provide the same information as the fallow-fallow experiment. The

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<sup>5</sup>It should be mentioned that the state variable is inches of available water at different levels in the soil broken down by feet. To a degree, then, this discrete breakdown by feet ignores the continuous distribution of the water in the soil profile.

soil moisture reading is taken at seeding time, before crop consumption of moisture. Hence, only three separate linear regressions over available soil moisture variables in different times were required. These regressions, for each experiment, gave the relationship

$$M_t = f(m_{t-1})$$

where

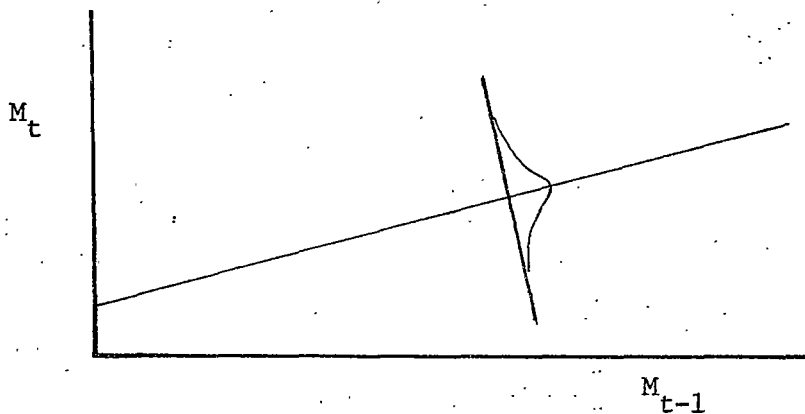
$M_t$  = available soil moisture in time period  $t$

$M_{t-1}$  = available soil moisture in the preceding time period.

In a linear regression, the function takes the form

$M_t = a + b(M_{t-1}) + e$ , where  $a$  is the  $y$  intercept,  $b$  is the regression coefficient, and  $e$  is an error term. Figure 1 shows a hypothetical plotting of this regression line.

FIGURE 1 : Hypothetical Regression of Moisture in Time  $t$  over Moisture in Time  $t-1$ .



One of the assumptions of linear regression is that the population being estimated is normally distributed around the regression line. The regression line is, then, the locus of the means of these normal distributions. The normal distribution around one point of the hypothetical regression line is represented by the familiar bell-shaped curve in figure 1.

Once the mean and standard deviation of a normal distribution are known, the distribution can be transformed into standard form using the transformation equation

$$Z = \frac{M_t - \mu}{\sigma}$$

where

$\mu$  = the mean of the  $M_t$  observations.

$\sigma$  = the standard deviation of the  $M_t$ 's.

$Z$  = the standardized value for  $M_t$ .

In a standardized normal distribution the mean is equal to zero, the standard deviation is equal to one, and the area under the curve is equal to one. By standardizing the distribution of  $M_t$ 's, it is possible to measure the area under the distribution curve, and therefore approximate the probability of observing some  $M_t$  given  $M_t-1$ .

The regression parameters are tabulated in the following chapter.

## CHAPTER IV

### DATA PREPARATION

The preceding chapter dealt with the model without discussing the analysis of actual data. The present chapter is concerned with the steps involving data preparation and analysis which precede the running of the dynamic programming model. The theory and assumptions behind these steps have been discussed.

### Production Functions

Using multiple regression, data were analyzed from four experiments extending over the 31 years from 1917 to 1947. Those experiments were continuous spring wheat, continuous barley, barley-fallow, and spring wheat-fallow. Regressions were structured so as to give

$$Y = f(ASM, P_0, P_1, P_2, P_3, P_4),$$

where

Y = Yield in bushels

ASM = Adjusted soil moisture (observed soil moisture minus four inches)

$P_0$  = Precipitation in inches from seeding to emergence

$P_1$  = Precipitation in inches from emergence to tillering

$P_2$  = Precipitation in inches from tillering to heading

$P_3$  = Precipitation in inches from heading to soft dough

$P_4$  = Precipitation in inches from soft dough to harvest

Soil moisture is read at planting time in the spring, to a depth of four feet. Because all observed values of soil moisture were greater than four inches the value used in the analysis is known as adjusted soil moisture (ASM), and is equal to actual soil moisture minus four inches.

The relationship used in estimating crop yields for the matrix of expected immediate returns was in the form

$$Y = f[\ln(\text{ASM}+1), \ln(P_0+1), \ln(P_1+1), \ln(P_2+1), \\ \ln(P_3+1), \ln(P_4+1)]$$

By taking the natural logarithm of the variable plus one  $[\ln(\text{variable} + 1)]$ , the possibility of having to take the logarithm of zero was ruled out. This was done because the logarithm of zero is not defined.

Table 6, 7, 8, and 9 are the regression values for the four experiments analyzed.

TABLE 6: Regression values for Production Functions  
Continuous Wheat Experiment

Variable	Mean	Standard Deviation	Correlation X VS Y	Regression Coefficient	Standard Error of Reg. Coefficient	Computed T Value
$\ln(ASM+1)$	1.185	0.3023	0.4147	8.572	2.394	3.581
$\ln(P_0+1)$	0.6667	0.3502	0.3413	2.625	2.121	1.238
$\ln(P_1+1)$	0.8458	0.3804	0.7331	7.717	2.026	3.809
$\ln(P_2+1)$	1.031	0.4203	0.5363	8.145	1.737	4.689
$\ln(P_3+1)$	0.6617	0.3361	0.2335	3.885	1.916	2.027
$\ln(P_4+1)$	0.5370	0.2948	0.2664	0.8246	2.214	0.3725
Dependent Yield	7.833	7.037				
Intercept		-22.02	R Squared	0.8164	Std Error - $SY \cdot X$	3.371

TABLE 7: Regression values for Production Functions  
Continuous Barley Experiment

Variable	Mean	Standard Deviation	Correlation X VS Y	Regression Coefficient	Standard Error of Reg. Coefficient	Computed T Value
ln(ASM+1)	1.185	0.3023	0.2153	5.584	3.977	1.404
ln(P <sub>0</sub> +1)	0.6667	0.3502	0.2104	1.665	3.523	0.4727
ln(P <sub>1</sub> +1)	0.8458	0.3804	0.6848	10.79	3.366	3.207
ln(P <sub>2</sub> +1)	1.031	0.4203	0.5797	10.05	2.886	3.481
ln(P <sub>3</sub> +1)	0.6617	0.3361	0.8959E-01	0.9027	3.184	0.2835
ln(P <sub>4</sub> +1)	0.5370	0.2948	0.7705E-01	-3.467	3.678	-0.9427
Dependent Yield	8.780	8.609				
Intercept		-17.18	R Squared	0.6615	STD Error - SY·X	5.601

TABLE 8: Regression values for Production Functions  
Continuous Barley-Fallow

Variable	Mean	Standard Deviation	Correlation X VS Y	Regression Coefficient	Standard Error of Reg. Coefficient	Computed T Value
$\ln(\text{ASM}+1)$	1.775	0.2088	0.3441	24.00	10.56	2.273
$\ln(\text{P}_0+1)$	0.6667	0.3502	0.1163	-0.6539	6.242	-0.1048
$\ln(\text{P}_1+1)$	0.8458	0.3804	0.6262	15.16	6.306	2.404
$\ln(\text{P}_2+1)$	1.031	0.4203	0.5062	15.46	5.302	2.916
$\ln(\text{P}_3+1)$	0.6617	0.3361	-0.2565E-02	1.648	5.988	0.2752
$\ln(\text{P}_4+1)$	0.5370	0.2948	0.7441E-01	-5.515	6.565	-0.8401
Dependent Yield	18.49	14.28				
Intercept		-50.56	R Squared	0.5937	STD Error--SY·X	10.17



























































































































