Productivity of resources used in grade A milk production in Ravalli County, Montana
by Lloyd C Rixe

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Masters of Science in Agricultural Economics
Montana State University

Abstract:
This thesis has several basic objectives concerning resource use in Ravalli County, Montana. It is designed: (1) to determine the economic efficiency of farm resources as they are now combined on farms; (2) to indicate or project what income changes might take place if there were a change in resource organization; and (3) to derive estimates of the economy of scale on Grade A dairy farms in Ravalli County, Montana.

The general hypothesis was that income on Grade A dairy farms in Ravalli County could be increased by a reorganization of the resources employed on these farms.

The data were collected by personal interview with Grade A dairy farmers in the County. A "Cobb-Douglas" type function is used in the study. This is a mathematical function linear in the logarithms. The data are fitted to the function by the least-squares method, with the computed regression coefficients indicating the rate of change in the dependent variable as an independent variable is changed.

Five different models were developed with gross farm income as the dependent variable in all models, but with the independent variables differing from model to model. The elasticities and marginal value productivities were then computed and they serve as the basis for the . economic interpretations.

Several implications can be drawn from the aggregate functions derived. The study indicates that there would be a high return to additional funds spent on cash operating expenses. This indicates a need for additional operating capital. Feed is another input category which the study suggests should be expanded to improve resource use in the area. Labor seems to be employed in larger amounts than profitable in relation to other inputs. There is also an indication that land should be used for cash crops whenever possible, and forage crops reduced on land highly suitable for cash crops. All estimates in the study indicate constant returns to scale.

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Head, Major Department

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ABSTRACT

This thesis has several basic objectives concerning resource use in Ravalli County, Montana. It is designed: (1) to determine the economic efficiency of farm resources as they are now combined on farms; (2) to indicate or project what income changes might take place if there were a change in resource organization; and (3) to derive estimates of the economy of scale on Grade A dairy farms in Ravalli County, Montana. The general hypothesis was that income on Grade A dairy farms in Ravalli County could be increased by a reorganization of the resources employed on these farms.

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CHAPTER I

INTRODUCTION

Farmers in many areas are faced with the problems of improving the management of their farms to meet changing economic conditions. If changes are not made many are faced with incomes at a subsistence level or lower. A problem of utmost importance to farmers is the basic management problem of establishing and maintaining a proper balance in the amounts of resources used in their given farm organization. Both level of resource use and the combination of resources to employ are of extreme important. Whether a farmer can reorganize his inputs or resources in a different manner and improve his income is a question for which many farmers are seeking answers. The study is specifically concerned with calculating estimates which can be used to aid in determining the optimum level and pattern of resource use.

The Research Problem

The research problem is concerned with the resource productivity of Grade A dairies in Ravalli County, Montana. A Farm and Home Development Program was recently initiated in Ravalli County by the Montana Extension Service and the United States Department of Agriculture. They are directly concerned with aiding the economic development of agriculture and other segments of the economy in the area. One of the first problems which they felt farmers needed help with was in determining the level of resource use in the area. They expressed a need for measures indicating the economic efficiency with which farm resources are now combined and for measures to indicate income changes that would or could be expected if a reorganization of available resources was made.

The study is concerned with relating total factor input to total product. The production function, will be used as a framework for analysis in the study of this problem. The general framework or method within which this study will be conducted is referred to as the "Cobb-Douglas" technique.[1/] The Cobb-Douglas technique yields a function linear in logarithms which will be explained in detail in later sections.

The study is concerned with how resources are now being employed and how the allocation of these resources could be changed so as to increase economic efficiency in Ravalli Grade A dairies. The study is concerned with the possibility of alternative combinations of resources in the production process. The study considers the present combination of resources to determine the relative economic productivities of these resources as they are now combined. The derived production function allows us to study changes in income which would result from changes in resource allocation.

Limitations to the Study

There are many more variable inputs within a given farm organization than it is feasible to study. Certain variables, such as climate, weather, and soil types, are beyond direct control of the operator. In this study, these variables are, in effect, controlled with respect to their influence as between farms in the study area. The sampling

1/ Charles W. Cobb, and Paul H. Douglas, "A Theory of Production," American Economic Review, Volume 18:2, 1928, pp. 138-166.

procedure was such that sample farms were drawn from a relatively small geographic area, thus minimizing weather and climate variations between farms. There is considerable soil variation due to geographic position even though the land is all in a small geographic area of the state. The main variation occurs because of some of the land on many of the farms is above the irrigated bottom land in the Bitteroot Valley. Thus, a wide variation in soil type exists between land in the valley and land out of the valley on either side.

There is also a wide variation in farm organization and size in the area covered in this study. There is also a wide range in the way resources are now employed on the various farm units within the area.

Another limitation of this study is that it does not include the operator's management capabilities as an identifiable input in the input categories studied. This is true because there is no measureable base to which we can accurately or scientifically measure the management ability of a farm operator. Nevertheless, it is a limitation to the study and most other studies of this type study.

Objectives of the Study

The objectives of the study are:

1. Determine the economic efficiency of farm resources as they are now combined on the farm units within the scope of this study.

2. Indicate or project what net income changes would take place if there was a change in resource organization.

3. To derive estimates of the economy of scale on the Grade A dairy farms in Ravalli County, Montana.

Due to the sampling procedure, as it is explained in Chapter III, this study will be aimed to a substantial degree toward describing the production relationships on farms in the study. Likewise, the implications and conclusions drawn will be mainly for use of the farmer interviewed rather than for prediction or generalization to the entire population.

Hypothesis

The general hypothesis for the study would be that the income of Grade A dairy farms in Ravalli County, Montana can be increased by a reorganization of the resources or input factors employed on these farms.

Procedure

This study used a prepared questionnaire to collect data from a sample of farms in Ravalli County. The questionnaire[1/] completed by personal interview and information was collected regarding gross farm income, enterprise organization and quantities of various resources used in 1958.[2/]

The general framework within which the study is conducted considers several estimates of production functions which combine inputs into categories in alternative ways. Economic interpretation of these functions will be explained in detail in later sections.

1/ Refer to Appendix I for a copy of the questionnaire.

2/ The first section of Chapter III deals with how the questionnaire was used in the study and how it was completed and edited.

CHAPTER II

HISTORICAL BACKGROUND OF AND ECONOMIC INTERPRETATION FROM A PRODUCTION FUNCTION ANALYSIS

Historical Background

As far back as one wises to look in economic thought it is possible to find problems of production being discussed. The body of production theory that is known today has been growing or developing over a long period of time.

During early periods, up through the time of Adam Smith, there was considerable discussion as to what resources might be considered productive and what resources unproductive. For example, the Physiocrats tended to divide labor into two parts, that which was "productive" and that which was "sterile."[1/] They considered productive labor as that portion of the total labor force which was capable of creating a surplus; all other labor was sterile. This surplus was something over the amount of wealth that they consumed in order to produce, in other words the surplus was the difference between goods produced and goods consumed in production.

For purposes of this particular study it is not necessary to examine this early period in detail. All factors of production will be considered productive including land, labor, capital and management in all their different forms. The background of production function analysis will now be traced through with special emphasis on the Cobb-Douglas technique.

1/ Eric Roll, A History of Economic Thought, Revised and Enlarged, New York, Prentice Hall Inc., 1942, pp. 132-142.

In the production of any product there are two types of problems which the firm faces. They are concerned with the actual physical processes of production and with the way of using or combining resources most efficiently in a given production process.1/ As stated by J. D. Black,

> "To secure this combination is the goal of production economics."2/

With given technical conditions of production, the amount of goods produced depends uniquely on the amounts of the variable factors (resources) used. If (Y) is the amount of goods produced and (x_1, x_2, x_3, . . . x_m) are the resources used then we can write the production function.3/

$$Y = f(x_1\ x_2\ x_3 . . . x_m)$$

This type of production function is very difficult to find in the real world as it is difficult to find a firm with controlled variables. An interfirm type function is often used in its place which is an aggregate for a group of firms. Each individual firm is then at a point on the derived equation. When this type of production function is used it

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1/ John D. Black, Production Economics, George G. Harrap & Co., Ltd., London, England, 1926, Chapter 2.

2/ Ibid., p. 59.

3/ R. G. D. Allen, Mathematical Analysis for Economists, MacMillan and Co., Ltd., London, England, 1956, Chapter 11, Section 11.8.

supplies, "diagnostic benchmarks," or measures which can be used from the overall point of view by policy makers, farm managers, and other, but it does not supply a specific answer to the individual firm.1/

Many attempts have been made at production function research and many different functions have been used.

The Spillman Function.-- The function assumes that the elasticity of response is less than one for a factor over all ranges. This would likely be true in nearly all situations but not at all times. It further assumes that the rate of use of an input, such as fertilizer, never becomes so great as to cause a negative marginal product.2/ This is a disadvantage as negative marginal productivities may appear but they are not significant.3/ The general form of the equation is:

$$Y = A(1-R^{n+a})\ (1-R^{p+b})\ (1-R^{k+c}).$$

The significance of the quantities involved is as follows:

Y = yield per acre

A = limit approached by Y as a, b . . . c increases

R = Ratio of the series of increments in yield for successive unit increments in a,b, or c, the size of the unit in each case making $R<1$.

1/ Earl O. Heady, Glenn L. Johnson, Lowell S. Hardin, Resource Productivity, Returns to Scale, and Farm Size, The Iowa State College Press, Ames, Iowa, 1956, Chapter I.

2/ E. L. Baum, Earl O. Heady, and John Blackmore, Economic Analysis of Fertilizer Use Data, the Iowa State College Press, Ames, Iowa, 1956, Chapter I.

3/ Heady, Johnson, and Hardin, op cit., Chapter I.

n, p, and k are the quantities of the input employed.[1/]

Much work was done by Spillman and others with this function in fertilizer experiments.

The Logistic Function.--The logistic function is of the form:

$$Y_t = \frac{K}{1 + bc^{-at}}$$

The significance of the quantities involved is as follows:

a, b, and K are constant parameters.

Y_t is the value of the growth character studied at a point in time.

This function has a lower and upper asymptote at O and K respectively. It is thus an S shaped curve and the inflection point is at K/2 which is half way between the two asymptotes.[2/]

Polynomial Functions.--Any degree of polynomial may be fit from a linear first degree to a general polynomial of the nth degree. They are fit in the sense of least squares and the problem becomes more and more involved as the degree of the polynomial increases.[3/] The polynomial

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1/ W. J. Spillman, Use of the Exponential Yield Curve in Fertilizer Experiments, United States Department of Agriculture Technical Bulletin No. 348, Washington D. C., United States Government Printing Office, April 1933.

2/ K. R. Nair, The Fitting of Growth Curves, Statistics and Mathematics in Biology, Edited by Kempthorne et al., Iowa State College Press, Ames, Iowa, 1954, pp. 119-132.

3/ R. L. Anderson and T. A. Bancroft, Statistical Theory in Research, McGraw-Hill Book Company, Inc., New York, 1952, Chapter 16.

is a function which has considerably more flexibility than many of the others. Higher order polynomials can show all areas of returns to scale in one function.

The Cobb-Douglas Function

The Cobb-Douglas function is explained in detail in later pages as it is the one chosen for use in this study, recognizing that it has certain limitations. It assumes that the percentage increase in yield is constant and equal for all increments of input added. It does not assume that the ratios of marginal yields are equal. The Cobb-Douglas function can indicate either constant, increasing, or decreasing returns to scale, but it indicates only one of these, not a combination of them.[1/] Further limitations will be discussed on pages 19 and 20.

The Cobb-Douglas type of analysis first appeared in published form in 1928 in an article by Charles W. Cobb of Amherst College and Paul H. Douglas of the University of Chicago.[2/] They studied production input-output relationships, as they saw much progressive work being done on measurement of physical production in manufacturing. In this original study, they set out to discover or explore what relationship existed between production and two general inputs, labor and capital. In the process of this analysis they outlined a method by which this relationship might be expressed or formulated and in so doing they

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1/ Baum, Heady and Blackmore, op cit.

2/ Cobb and Douglas, op cit.

opened up the field of the derivation or estimation of production functions to many more economists who followed their work and used this method.

Their study set out to secure answers or approximations to the following questions: (1) Can one estimate within certain limits or with certain degrees of confidence what causes production to increase or decrease? (2) Again, within certain limits can one compare the differences in magnitude of the influence of one input, labor, versus the influence of another input, capital, on production? (3) As one changes the proportion of one input to another over time, can we examine total product and determine or estimate what each adds to production, and (4) can one measure the probable slopes of the curves of incremental product which are imputed to the different factors namely labor and capital? In other words, with a positive or negative change in the value of an independent variable, (labor or capital) what is the corresponding change in the dependent variable or production?

The first step in their original work was to acquire some measures to work with from the past records which were available. They drew up indices for production, labor, and capital for the period 1899 to 1922. The following general type function was chosen: $P = (b)\ L^{(K)}C^{(1-K)}$ where (L) is labor in man-years, (C) is capital in dollars, (P) is total physical production and (b) as well as (K) are derived values. With this type function we have the properties that if either (L) or (C) approached zero then this forces (P) to approach zero. The function is then a first degree homogenous function of labor and capital. If labor and capital are multiplied by some constant factor then (P) is also multiplied by that factor. The numerical values for (b) and (K) are found in the sense of the theory of least squares, so that (P) best approximates the actual production.

For ease in computation this general function may be converted to a function which is linear in logarithms as the following two functions are equal.

$$(P) = (b)L^{(K)}C^{(1-K)}$$

$$\log (P) = \log (b) + (K) \log (L) + (1-K) \log (C)$$

Using the indices that they had computed their final equation took the form of:

$$P' = 1.01 (L)^{3/4} (C)^{1/4}$$

The coefficients (K) and (L) are derived so that their sum is always equal to one. This would say that we have constant returns to scale.

It can then be shown that the marginal productivity of labor is 3/4 P/L, since $\frac{\partial P}{\partial L} = 3/4 \left(\frac{P}{L}\right)$. Likewise, the marginal productivity of capital is $\left(\frac{1}{4}\right)\left(\frac{P}{C}\right)$. This is marginal value productivity if (P) is in value terms. The productivity of total labor is 3/4 P and the productivity of total capital is 1/4 P. The elasticity of the product to small changes in labor is 3/4 and in capital it is 1/4. In other words a one-percent change in labor would cause a .75 percent change in production, and a one-percent change in capital would change production by .25 percent.

It was made clear in the article that the figure derived here and the data used is not meant to be the most important contribution of the study. The item of greatest importance was meant to be the method of attack which they used in analyzing a production problem.

Douglas, since his original use of this type of analysis in 1928, has used it extensively in several subsequent studies. He fitted a production function of this same general form to manufacturing in 1914.[1/] He conducted several other subsequent studies in the manufacturing industry but they are similar and this original one is sufficient to show us the general method of the analysis.[2/]

Advantages of Using the Cobb-Douglas Analysis

Many economists have used this technique in the past. Why have they used this type of approach rather than some other? Several people who have used this type analysis have given reasons for or advantages of using it. One of the major advantages is that the function, because of its general form, can be transformed into logarithmic form and the parameters are thus estimated by the very well known method of least squares.[3/]

1/ Grace T. Gunn and Paul H. Douglas, "The Production Function for American Manufacturing for 1914," Journal of Political Economy, Volume 50, 1942, pp. 595-602.

2/ Grace T. Gunn and Paul H. Douglas, "The Production Function for American Manufacturing for 1919," Journal of Political Economy, Volume 31, 1947.

3/ H. O. Carter, "Modification of the Cobb-Douglas Function to Destroy Constant Elasticity and Symmetry," Resource Productivity, Returns to Scale and Farm Size, Editors, Heady, Johnson and Hardin, The Iowa State College Press, Ames Iowa, 1956, Chapter 19.

Gerhard Tintner, who has done a considerable amount of work with production functions of various types, lists the following reasons for using the Cobb-Douglas function in his study of Iowa Farms in 1942.1/ First of all, the elasticities of the product with respect to the factors of production can be readily seen because the elasticities are the regression coefficients themselves. In other words, they illustrate by what percent the product will increase if a given factor of production is increased by one percent. These elasticities are also dimensionless numbers which says that they are completely independent of any specific unit of measurement which might be used. This form of function permits the fitting of an equation with fewer degrees of freedom. This can be an item of extreme importance in certain studies due to small sample size. In a small sample it is necessary to conserve degrees of freedom. A quadratic function would require about twice as many degrees of freedom. If it can be assumed that the errors are small and normally distributed in the original data when the data are transformed to logarithms, it preserves this normality to a substantial degree. Tintner further points out that even though Douglas and his associates have received a considerable amount of criticism, they have nevertheless clearly demonstrated the usefulness of this type of approach.2/

1/ Gerhard Tintner, "A Note on the Derivation of Production Functions From Farm Records," Econometrica, Volume 12, January 1944, pp. 26-35.

2/ Ibid.

Earl O. Heady lists his reasons for using this approach which parallel the reasons given by Tintner very closely.[1/] Heady, in addition to the reasons listed by Tintner, points out that comparisons can be made between studies when this approach is used. Several other authors have discussed desirable features of using the Cobb-Douglas type analysis and their reasons parallel those discussed above.[2/]

Input and Output Classification

The Cobb-Douglas technique for estimating a production function has several advantages and is relatively easy to fit statistically, as was pointed out above. Setting up the model and grouping the inputs so that there is a finite workable number of input categories is of major importance and is, in most cases, a difficult task.

At any point of time it is evident that given farms employ a large number of resources, and these resources are employed in an infinite number of ways and combinations. These inputs must be combined or grouped into definite input categories which are reasonable in nature and finite in number. As stated by Glenn L. Johnson:

1/ Earl O. Heady, "Production Function From a Random Sample of Farms," Journal of Farm Economics, Volume 28, 1946, pp. 989-1004.

2/ W. J. Forman, Resource Returns and Productivity Coefficients on Owner-Operated Farms in the Piedmont of Georgia, Georgia Agricultural Experiment Stations, University of Georgia, College of Agriculture, Technical Bulletin N.S. 9, December 1956. Also see: H. R. Jensen, and W. B. Sundquist, Resource Productivity and Income for a Sample of West Kentucky Farms, Kentucky Agricultural Experiment Station, University of Kentucky, Lexington, Kentucky, Bulletin No. 630, June 1955.

"The number of inputs and, less often, the number of products involved in a farm business analysis ordinarily exceeds the analytical facilities of the human mind. This makes it necessary to group inputs, and often times, outputs into a limited number of categories before analysis can be carried out."1/

Johnson further points out that from an economic standpoint inputs must be studied carefully before they are grouped or aggregated into input categories. If inputs are perfect complements then there will not be much variation in the proportion in which they are used from farm to farm. They are, therefore, often times grouped together into input categories. An example of this might be machinery. For certain items of equipment, such as tractors, there are certain machines that are used with them in relatively fixed proportions depending on the type of farm organization. If, on the other hand, when inputs are good substitutes for each other, the proportions in which they are used will vary widely from farm to farm. They can logically be grouped together in most cases by converting them to a common base measure and use this to combine them. For example, hired labor and family labor are good substitutes, but they can logically be combined into a single input category by converting each one to man-months and adding them with the aid of this base denominator. Johnson, then would imply that either good substitutes or good complements should be grouped together as explained above.2/ Likewise, input categories

1/ Heady, Johnson, and Hardin, <u>op cit</u>., Chapter 9.

2/ <u>Ibid</u>.

should be neither good substitutes nor good complements for each other. James S. Plaxico agrees very closely to the ideas of Glenn Johnson on grouping complements and substitutes into aggregate input categories.[1/] He points out that if they are not grouped it will bias the estimate of the function. This is true because the Cobb-Douglas model assumes diminishing marginal rates of input substitution and thus assumes that the level of each input affects the productivity of others.

It can be seen that aggregating inputs into input categories or groups is a very important part of any study of this type. In their study on 145 West Kentucky Farms, H. R. Jensen and W. B. Sundquist grouped the inputs into five categories. These categories were as follows: x_1 = estimated gross income and was the dependent variable; x_2 = land measured in acres; x_3 = labor measured in months; x_4 = crop services in value terms; and x_5 = livestock services in value terms.[2/] Darrel F. Fienup, in his study on Montana dryland crop farms, fitted two different functions.[3/] In his livestock function he set up the following input groups: Y_L = the value of total livestock output which was the dependent variable; x_1 = value of total feed fed;

1/ James S. Plaxico, "Problems of Factor-Product Aggregation in Cobb-Douglas Value Productivity Analysis," Journal of Farm Economics, Volume 37, No. 4, November 1955, pp. 664-675.

2/ Jensen and Sundquist, op cit.

3/ Darrel F. Fienup, Resource Productivity on Montana Dryland Crop Farms, Montana State College, Agricultural Experiment Station, Bozeman, Montana, Mimeograph Circular 66, 1952.

x_2 = man-months of labor expended on livestock; x_3 = total livestock input; and x_4 = value of all other inputs to livestock. Earl O. Heady in his study, previously mentioned, grouped the inputs into land measured in acres; labor measured in months; farm improvements such as buildings etc; liquid assets; working assets and cash operating expenses.[1/]

The input of management is excluded in these studies. Weather condition is another variable that is usually omitted as there would be a great deal of error in estimating measures for them.

Output classification is also a problem not to be overlooked when analyzing survey data. Glenn Johnson feels that the less interrelated two enterprises are the more reason you have for placing them in different categories.[2/]

James Plaxico, in general, agrees with the idea of separating output or enterprises on the basis of how much interrelation there is between them.[3/] He also implies that as one tends to consider a separate treatment of output by source to keep from introducing error or bias into the analysis, it soon becomes practically impossible to perform the analysis. The amount of bias introduced may be reduced, however, by aggregating some of the output and then deriving separate functions according to type-of-farm. For example, Earl Heady in a study of this

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1/ Heady, op cit.

2/ Heady, Johnson and Hardin, op cit, Chapter 9.

3/ Plaxico, op cit.

type of 738 Iowa farms divided his sample up into three different groups. First, in accordance with location in the state, he divided the group into Northwest dairy, cash grain, Western meat, Southern pastures and Eastern meat farms.[1/] On a second breakdown in accordance with type of farm, he divided the sample into crop, hay, dairy and dual purpose, general and specific farms. On a third division he broke them down as to size or scale of operation. A specific function was fitted to each one of these divisions as well as an all farm function which was fitted to the entire sample. W. J. Forman in his study of owner-operated farms in Georgia divided the sample into four sub-classes as to type of farm and derived a separate equation for each one.[2/] Gerhard Tintner and O. H. Brownlee in their study of this kind of 468 Iowa farms divided the sample into five type-of-farming groups which were hogs, beef feeder, dairy, crops, and general type farms.[3/] It becomes quite obvious that most studies which are conducted for a general group of farms are further subdivided into groups as to output.

Most of the material which is written concerning this type analysis makes it clear that some difficult problems must be considered in the area of aggregating inputs and outputs in deriving a Cobb-Douglas function. This does not say that these problems are peculiar to the Cobb-Douglas technique as this same type problem is present in any

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1/ Heady, op cit.

2/ Forman, op cit.

3/ Brownlee and Tintner, "Production Functions Derived From Farm Records," Journal of Farm Economics, Volume 26, 1944, pp. 566-571.

analytical method of fitting a production function to farm records and accounts.

Economic Interpretation from a Production Function

This outlines how and why this type of analysis has been used. It now becomes necessary to ask how the Cobb-Douglas technique aids in answering economic questions. Proper economic interpretation of these equations is very important in any study of this kind. A production function analysis regardless of the specific method used in fitting it has within it the possibility of answering many economic questions facing the firm. It can aid most in answering questions in two very basic areas. First, it provides the framework necessary to determine optimum use of input factors in the production process and the optimum level of production. Secondly, it can give indications as to how resources are being used on farms in certain areas and how these resources might be employed more efficiently.

There has been some criticism by various people on economic interpretation from a Cobb-Douglas function. M. W. Reder has criticized this approach from the standpoint of using an interfirm production function to make intrafirm decisions.[1/] The Cobb-Douglas function is an interfirm function and the production function commonly taked about in production theory is an intrafirm function. As Reder points out these two functions are different in several ways. The production function in

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1/ Melvin W. Reder, "An Alternative Interpretation of the Cobb-Douglas Function," Econometrics, Volume 11, July-October, pp. 259-264.

theory describes a relationship between the quantities of inputs employed and the physical quantity of the firms output. Then, given the demand curves for the firms products and the supply curves of the factors to the firm, the point on the production surface where the firm's profits are maximized can be found. All the other pointson the surface tell what the output would have been if this combination of input factors had been employed, but they are not proft maximization points. The Cobb-Douglas function, however, describes the relationship between the output of firms in the study and the combination of factors they employ. The Cobb-Douglas function is a focus of output and factor quantities of all the firms included in the study. The second main difference between the two functions, as pointed out by Reder, is that the function in theory relates physical quantities of output to physical quantities of input of the factors. In his functions, Douglas related the value added in manufacturing (defined as products) to physical quantities of factors.

Because of these differences in the two types of functions, Reder feels there are problems in drawing intrafirm decisions from an inter-firm function. Bronfenbrenner and others also discuss the matter of drawing intrafirm decisions from an interfirm function but they do not feel that this is as limiting as Reder suggests.[1/]

1/ M. Bronfenbrenner, "Production Functions: Cobb-Douglas Interfirm, Intrafirm," Econometrics, Volume 11, January 1944, pp. 35-44. Also see: Plaxico, op cit., p. 672.

How is an actual Cobb-Douglas function interpreted once it has been fitted? Cecil B. Haver outlines how this is done.[1] First of all, assume that a function has been fitted with the Cobb-Douglas technique, and it is of the form:

$$\log x_1 = .200 + .060 \log x_2 + .500 \log x_3 + .430 \log x_4$$

Standard errors of $x_2 = .05$

$x_3 = .08$

$x_4 = .10$

Adjusted $R^2 = .83$.

First, the coefficient of multiple determination (R^2) indicates the percent of variation in output, or the dependent variable, that is explained by the input factors included in the equation. Here, for example, one would say that 83 percent of the variation in x_1, on the average, is explained by x_2, x_3, and x_4. As mentioned previously, in discussing the advantages of using this type analysis, it was pointed out that the regression coefficients themselves are the elasticities of the product with respect to the factors of production. In other words, they indicate the expected percentage change in output if an input factor is increased by one-percent. In this example, if x_3 was increased by one-percent and x_2 and x_4 fixed, it would result in approximately .50 percent increase in output. If any one of these elasticities is greater than one, then there are increasing marginal returns to this factor. If they are less than one this indicates decreasing marginal returns to

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[1] Heady, Johnson, and Hardin, op cit., Chapter 16.

each factor. In this example, as all but one factor is held constant, the marginal return to the factor varied will decrease the more of the factor that is used. The sum of these elasticities is also important in proper economic interpretation of a function of this type. If the sum of these elasticities is equal to one, this indicates constant returns to scale. In other words, if all inputs were increased by one-percent this would also increase output by one-percent. If the sum of the elasticities is significantly greater than one, then this indicates increasing retursn to scale. If all inputs were increased by one-percent then output would increase by something greater than one percent. Likewise, if the sum of the elasticities is significantly less than one, this indicates decreasing returns to scale.

It is also necessary to compute standard errors of the regression coefficients which enables one to show the relative reliability of the regression coefficients. In this example, if x_4 is increased by one-percent it will, on the average, increase output by (.43 $\pm$ .10) percent or from (33 to 53) percent.

The estimated marginal value productivities are also computed at the geometric mean of the input factors. These are computed in the common production function in economic theory by taking the partial derivative of physical output with respect to the input in question. The partial derivative is then multiplied by the price of the product to arrive at marginal value product. In the Cobb-Douglas function the output is in value terms so the marginal value product is the partial derivative of output with respect to the factor in question. Bradford and Johnson lay out a formula for computing the marginal value productivity which has been used in subsequent studies.[1/] Forman used this formula in his study of Georgia farms. The formula reads as follows:

$$MVP_{x_i} = b_i \left(\frac{Y_e}{X_i} \right)$$

where (MVP_{x_i}) is the marginal value productivity for the input x_i, b_i is the elasticity or regression coefficient of the input x_i and Y_e is the estimated gross farm income which is established using the computed constants in the regression equation and the geometric means of the variables. The geometric mean of the variable is X_i. For example, in the study on Coweta County, Georgia, x_5 was current operating expenses in dollar terms and the geometric mean of this input was $1,282. The estimated gross farm income computed with the regression equation was $3,126. The elasticity of this input category was found to be .7464, and the marginal value productivity of x_5 can be computed by the method outlined above:

$$MVP_{x_5} = (.7464) \left(\frac{3.49499}{3.10789} \right) = \$1.82.$$

These marginal productivities indicate the returns which one would expect, on the average, from adding one more unit of the productive factor. The marginal productivities just computed would indicate that for every additional dollar spent on current operating expenses (x_5) would yield an approximate return of $1.82 in increased output. The other input factors are all considered held constant when marginal value productivities are computed.

1/ Lawrence A. Bradford and Glenn L. Johnson, Farm Management Analysis, New York, John Wiley and Sons, 1953, p. 149.

CHAPTER III

THE DATA AND ANALYSIS

Source of Data

As outlined in Chapter I this study was requested by the Ravalli Farm and Home Improvement Association. The population was then all Grade A dairy farms in the bounds of Ravalli County who are on Grade A milk routes of three different distributors located in Missoula, Montana. S. J. Tietema compiled a list of these 68 producers and sent them a one-page questionnaire which was aimed at answering such basic questions as number of cows, number of acres, and percentage of farm income that was from the dairy itself.[1/] A letter also accompanied this questionnaire which explained the intent of conducting a study such as they requested through the Ravalli Farm and Home Improvement Association.

The sample drawn from the population was then made up of all the producers who returned their original questionnaire. There were 37 producers in the sample from which a personal interview schedule was taken. Due to numerous reasons, only 31 of the 37 schedules were considered usable and included in the study. This makes it clear that by such a method of sampling there is no assurance of having a random sample. It is very possibly a bias sample. The sample does, however, include approximately half of the population.

The interview schedule was quite detailed. Complete information was obtained as to total output from the farm regardless of the source from which it comes. The remainder of the schedule was aimed at

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1/ S. J. Tietema, County Agent-at-Large in Ravalli County, Montana.

obtaining information as to inputs or resources employed on the farm and more specifically in what quantity these inputs were employed.

A copy of the schedule is contained in Appendix A. The only value figures that were taken from the farmers was the value of gardens and fruit trees. The other value figures were equal for all farmers as it was believed that very little price differential would exist from farm to farm in such a small geographic area. An equal price is used for all farmers.1/

The prices for all cash grain crops were obtained from P. J. Creer in Helena, Montana.2/ The prices used were average prices received in Ravalli County for specific crops in 1958. Hay prices were also computed with the aid of the Montana Agricultural Statistics of 1958. There were many estimates on the schedules of farmers in the area purchasing alfalfa hay and these followed a nearly uniform price of $19.50 per ton. This was quite comparable to the price in the above mentioned statistics. This price was used for alfalfa hay and all other hay was priced at $11.50 per ton as found in the Montana Agricultural Statistics for 1958. Silage was converted to a dry matter basis as compared to alfalfa hay and its value was $6.50 per ton for all silage.3/

1/ See Appendix B for a list of the prices used.

2/ P. J. Creer, Agricultural Marketing Service in Helena, Montana. This office publishes the Montana Agricultural Statistics for Montana.

3/ Conversion ratios for the silage to a dry matter basis was obtained from Morrison, Feeds and Feeding, 22nd Edition, The Morrison Publishing Company, 1957.

Animal unit months of grazing were calculated from the number and age of livestock on pasture and the time span which they were on the pasture. An animal unit month of grazing is then defined as a month of grazing for one animal unit.[1] Each animal unit month of grazing was valued at $3.25. This figure was arrived at from estimates given regarding rental arrangements on the interview schedules and also on recommendation from William Thomas.[2]

The state average price of $.37 was used for the value of eggs both used in household and sold. If the individual farmer had a specific market for eggs then they were valued at that specific price. It was assumed that all farmers met their quota of Grade A milk and thus any milk used in the household or fed to calves was valued according to Class II or at the manufacturing milk price. This varied slightly as to the distributor to whom their milk was sold. Value of all milk sold, both manufacturing and Grade A, was obtained for each rpdocuer from the distributor handling his milk.

Man-equivalent months was not answered as such by the farmer but was computed from the data they gave as to labor employed. In converting to man-months the following assumptions were made. Day labor and hours of labor were converted to man-month by two ratios: 26 days equal one man-month and 260 hours equal one man-month. Hours or days spend by

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1/ The table for animal unit conversion is in Appendix B.

2/ William Thomas is County Extension Agent in Ravalli County and he arrived at $3.25 per animal unit month of grazing from his experience in the area.

children under 14 was valued at half-time and for children 15 or over were full-time. The labor of the wife on farm work was valued as full-time. These conversions were used as there are many jobs on a dairy farm that it would take a man to do if the children and wife did not take care of them.

General Procedure

As discussed previously, a Cobb-Douglas type function was used in this study. Several different models were drawn up but the same general type function, linear in logarithms, was used for all models. The dependent variable was gross farm income for all models. The type and number of independent variables differed from function to function. Several models were considered which combined the inputs in different ways.

The functions all followed the general form:

$$Y = a(x_1)^{b_i}(x_2)^{b_2}(x_3)^{b_3}(x_4)^{b_4} \ldots (x_m)^{b_m}$$

where (Y) is gross farm income measured in dollar terms and (m) is the number of variables. Gross farm income is the value of all products produced on the farm. Inventory changes are also taken into account when computing the value of gross farm production or gross farm income which we define as (Y). As mentioned previously, when the Cobb-Douglas type analysis is used, the general form of the equation is linear in logarithms which reads as follows:

$$\log (Y) = a+(b_1) \log (x_1) + (b_2) \log (x_2) +(b_3) \log (x_3) + (b_4) \log (x_4)+ \ldots +(b_m) \log (x_m).$$

With the data converted to logarithms and the function fitted by the method of least squares, the (b_i) are the regression coefficients and (a) is the Y-axis intercept of the equation. The (x_i) are measured at the geometric means of the input category. The geometric mean of the input group number one would be (X_1). This form of equation, linear in logarithms, is the form in which the results of this problem will be presented.

An Added Limitation to Economic Interpretations

In making economic interpretations it is assumed that the prices used in computing gross farm income remain the same as they were in the derivation of the function. This assumption is not necessarily a realistic one on Grade A dairy farms. In many areas of agricultural production this assumption is realistic, but because of base milk quotas it is not realistic on dairy farms. A milk producer has a base quota of milk which he is under obligation to supply and most producers thus produce some amount over their quota at most times during the year. All of the milk produce does not go into the fluid milk market and thus it does not command a fluid milk price. This problem was outlined in setting up the Montana Milk Control Board as follows:

> "It is hereby declared that fluid milk is a perishable commodity, which is easily contaminated with harmful bacteria, which cannot be stored for any great length of time, which must be produced and distributed fresh daily, and the supply of which cannot be regulated from day to day, but due to natural and seasonal conditions, must be produced on a constantly uniform and even basis."[1/]

1/ Rev. Codes, Montana 1947, Replacement 2, pp. 1082-1094, 27-401.

"It is hereby declared that the demand for this perishable commodity fluctuates from day to day and from time to time making it necessary that the producers and distributors shall produce and carry on hand a surplus of milk in order to guarantee and insure to the consuming public an adequate supply at all times, which surplus must of necessity be converted into by-products of milk at great expense and ofttimes at a loss to the producer and distributor."1/

"It is hereby declared that this surplus of milk, though necessary and unavoidable, unless regulated, tends to undermine and destroy the fluid milk industry, which causes producers to relax their diligence in complying with the provisions of the health authorities and ofttimes to produce milk of an inferior and unsanitary quality."2/

The Montana Milk Control Board sets a minimum price which distributors may pay producers for milk that is sold as fluid milk and cream for consumption as such.3/ They do not set a price for surplus milk, referred to as manufacturing milk in the text of this thesis. The Board does not set up or regulate base quotas or lay down laws or suggestions for formulating base quotas. This is a matter which is handled between the producer and distributor involved in the transaction.

Wallace A. Rehberg made the following statements concerning manufacturing milk and milk produced over base quotas.

"The Montana Milk Control Board also has no provisions for base-excess plans of pricing. The Montana dairy producer has little assurance of a just price for production in excess of his base. Each producer must take what the distributor will

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1/ Ibid., "27-401 (i)

2/ Ibid., "27-401 (h)

3/ There are several specific laws pretaining to how the price is to be set and what steps must be followed in setting it in the laws of the Milk Control Board which we have just quoted from above.

pay or keep his excess at home. If a distributor purchases enough milk in the months of low production he usually has an excess of milk during the period of high production that must be utilized in products of lower value."1/

The blend price, the average price received per unit of milk or fat sold by a milk producer, varies from producer to producer. It is dependent on the individual producers base quota, on plant usage of the milk, on retail consumption of fluid milk, the product prices for different classes of milk and also the amount he produces in relation to his base quota.

Let us now further assume that less than 100 percent of the base quota is used as fluid milk, and that most farmers are producing over their base quota. Empirical observation would support this assumption to a substantial degree. On the basis of these assumptions it is clear that additional production is directly related to the farmers blend price. As the dairyman produces more, his blend price decreases. There is little if any evidence to say how much it will decrease. The amount of the base quota itself and the price received for manufacturing milk are both determined mainly by a bargaining process between the producer and the distributor. It is then difficult to forecast accurately what the price of milk over the base quota would be. One can, with some degree of certainty, indicate that as more milk is produced it will command a lower price.

1/ Wallace A. Rehberg, Effectiveness of Selected Methods for Increasing the Consumption of Dairy Products in Montana, Master's Thesis, Department of Agricultural Economics and Rural Sociology, Montana Agricultural Experiment Station, Bozeman, Montana, 1959.

This leads one to question the assumption that as inputs are increased the prices used in computing gross farm income remain the same as in the derivation of the function. The prices for milk would be lower, as pointed out above, but those for the other protions of gross farm income would likely be nearly the same. Thus, when we interpret a function we must examine our results carefully and recognize a substantial degree of uncertainty. For example, if there is an elasticity of .60 and a marginal value productivity $1.32 for an input category, certain economic interpretations can be made. If the input category was increased by one-percent and others held constant it would increase gross value of output by .60 percent. Likewise, if one increased the input by an additional unit it would increase output by ($1.32). In a dairy function, such as the ones in this study, one would expect the elasticity and the marginal value productivity to be somewhat lower due to the market structure which the firm faces.

In the following pages the regression equations will be presented and they will be interpreted as though the prices, in computing gross farm income, did remain the same. These economic interpretations must, in turn, be considered with a certain degree of caution by the reader.

Model I.

The first model presented is a general farm model where the input categories are very general in nature: When one asks what inputs are likely to be very influential in determining the level of gross farm income, the following groupings seem logical:

(Y) = gross farm income;

(x_1) is the value of all land and buildings excluding the house;

(x_2) is labor measured in man-months;

(x_3) is the dollar value of all livestock on the farm including all other livestock and poultry as well as dairy stock;

(x_4) is the value of all feed fed livestock which includes both roughage and concentrates;

(x_5) is the value of all farm machinery; and

(x_6) is cash operating expenses.

Land was measured in value terms rather than in acres as there is a very wide variation in productivity of land in the area. Land in the bottom of the valley compared to land up out of the valley on either side is far from equal in productivity acre per acre. Thus, value was used in all models as a common denominator for the land input, which assumes the market prices of land reflect differences in productivities. The regression equation expressed in logarithms was found to be:[1/]

$$\text{Log } (Y) = (.005209) - (.003738) \log (x_1) - (.122509) \log (x_2) + (.194859) \log (x_3) + (.349692) \log (x_4) + (.011038) \log (x_5) + (.598586) \log (x_6).$$

The coefficient of multiple determination is significant at the one-percent level. This indicated that there is less than a one-in-a-hundred chance that a relationship such as this could arise in a

1/ Appendix C gives additional statistical data concerning Model I.

population where there is no relationship. The corrected coefficient of multiple determination is (.8242) and the standard error of the regression equation is (.0711). This would indicate that approximately 82 percent of the variations in gross farm income is explained by the independent variables. The standard error thus sets an upper and lower limit for the geometric mean of gross farm income which in logarithms will range from 4.2873 - .0711 to 4.2874 + .0711 where 4.2874 is the calculated geometric mean of gross farm income. In dollar terms this would say that gross farm income varies from a low of $16,457.00 to $22,830.00 as the upper limit.

The regression coefficients are elasticities, and their standard errors and "t" values are shown in Table I. As show in that table, four of the six coefficients are significant at the 20 percent level or above. Cash operating expenses and all feed show the greatest significance and have the elasticities of the largest magnitude. If cash operating expenses were increased by one-percent while other inputs were held constant, one would expect an approximate increase of .60 percent in gross farm income. If all feed were to be increased by one-percent in the same manner one would expect a .35 percent increase in gross farm income. Land and buildings, as well as all farm machinery failed to show significance above the 20 percent level. Labor shows a negative elasticity which would indicate that gross farm income would lower as labor was increased. This is very difficult to visualize, but it tends to support the hypothesis that these farms are overemployed in labor in

TABLE I. RANGES IN QUANTITIES OF THE INPUT CATEGORIES EMPLOYED, GEOMETRIC MEANS, ELASTICITIES, STANDARD ERRORS OF THE ELASTICITIES, THE SUM OF THE ELASTICITIES AND THE CALCULATED "t" VALUES ON RAVALLI COUNTY GRADE A DAIRY FARMS.

Input Categories	Units	Range Lowest	Range Highest	Geometric Mean	Elasticity	Standard Error	"t" Value
Land & Buildings	dollars	$10,000	$69,000	28,839	-.003738	.053386	.053386
Total Labor	man-mo's	9.50	37.7	23.19	-.122509	.076824	1.59467 c/
All Livestock	dollars	6,110	29,130	14,110	.194859	.126134	1.54486 c/
All Feed	dollars	3,620	12,830	6,776	.349692	.101394	3.44884 b/
All farm Machinery	dollars	4,345	22,630	9,082	.011038	.276154	.039970
Cash operating Expense	dollars	2,977	12,830	6,300	.598586	.062761	9.53754 a/
Total					1.027928		

a/ Significant at the .1 percent leve of probability.

b/ Significant at the 1. percent level of probability.

c/ Significant at the 20 percent level of probability.

relation to other input categories. Some error may have been introduced when labor was converted to man-months as outlined earlier in this Chapter.

The sum of the elasticities as presented in Table I was 1.03 which would indicate constant returns to scale as the sum of the elasticities is not significantly different from one. In the returns to scale problem it is necessary to be careful due to the milk pricing situation as mentioned above and it would seem logical that any returns to scale estimates would tend to be high.

The marginal value productivities as presented in Table II are computed at the geometric means of the input categories in question. They are computed by the formula:

$$MVP_{(x_i)} = b_i \left(\frac{Y_e}{X_i}\right)$$

where (b_i) is the elasticity, (Y_e) is expected value of gross farm income as explained in the function, and (X_i) is the geometric mean of the input category.[1/] All marginal value productivity estimates are in dollar terms per unit increase of the input category in question.

TABLE II. MARGINAL VALUE PRODUCTIVITIES MEASURED AT THE GEOMETRIC MEAN OF THE INPUT CATEGORIES. THE EXPECTED GROSS FARM INCOME FROM THE FUNCTION EQUALS ($19,383)

Input Category	Geometric Mean	Marginal Value Productivity	Unit of Input
Land & buildings	28,839	$- .003	dollars
Total labor	23.19	-102.39	man-months
All livestock	14,110	.27	dollars
All feed	6,776	1.00	dollars
All farm machinery	9,082	.02	dollars
Cash operating expenses	6,300	1.84	dollars

Marginal value productivities are of little or no meaning if the elasticities are not significant. Here one must be extremely careful in interpretation due to the market structure for fluid milk which was

1/ Bradford and Johnson, op. cit., p. 149.

mentioned above. Cash operating expense is the only variable which yields a net return in the group of the variable considered in the function. According to the above table it would yield a net return of $0.84 per dollar invested but it would likely be lower than this due to the decreasing price for additional milk as explained above.

Model II.

A second function was computed which again considered (Y) as gross farm income and the dependent variable. The inputs were aggregated into five categories which were mainly oriented to the dairy operation as it is the major enterprise on all farm interviewed. The inputs were grouped as follows:

(x_1) is the value of dairy cows and two-year old heifers;

(x_2) is the value of farm buildings plus dairy equipment;

(x_3) is total labor measured in man-months;

(x_4) is cash operating expenses; and

(x_5) is the value of all feed fed.

A value figure was placed on the dairy cattle input because there is a considerable variation in quality of dairy cattle and a value figure should take this into consideration while mere animal numbers would not. Value of farm buildings and dairy equipment were grouped together, with the house not considered as a farm building in this input group. This was done on the basis of an assumption that nearly all of the buildings on these Grade A Dairies would be oriented to the dairy operations. Also permanent type equipment such as stalls and special feeding equipment should actually be considered dairy equipment but they may be included in the value of a building. The other three variables, labor, cash operating expenses, and feed fed are the same as they were in Model I.

The regression equation[1/] expressed in logarithms was found to be:

$$\log (Y) = -.015551 + .192915 \log (x_1) - .095634 \log (x_2)$$
$$- .113900 \log (x_3) + .677990 \log (x_4) + .387089 \log (x_5).$$

The coefficient of multiple determination is significant at the .1 percent level. The corrected coefficient of multiple determination is (.8351) which indicates that over 83 percent of the variation of gross farm income is explained by the regression equation. The standard error of this estimating equation is (.0689).

The range of the input categories as well as their geometric means, elasticities, standard error and "t" values are presented in Table III. Total labor in man-months again had a negative sign and it is not significant at the 20 percent level or higher. This leads to the same type of conclusion as was made in the analysis of Model I concerning labor. The coefficient for buildings and dairy equipment is also not significant. Cash operating expenses and value of all feed fed seem to be the variables in this model which are of the greatest significance as they are significant at the .1 percent and 5 percent level respectively.

The sum of the elasticities is 1.05 which is not significantly different than 1. This indicates constant returns to scale as in Model I and it should be viewed with the same caution.

1/ Appendix D gives additional statistical data concerning Model II.

TABLE III. THE RANGE IN QUANTITIES OF INPUT CATEGORIES EMPLOYED ALONG WITH THE GEOMETRIC MEANS, ELASTICITIES, THE STANDARD ERROR OF THE ELASTICITIES, THE SUM OF THE ELASTICITIES AND THE CALCULATED "t" VALUES

Input Categories	Units	Range Lowest	Range Highest	Geometric Mean	Elasticity	Standard Error	"t" Value
x_1	dollars	3,400	23,000	10,875	.192915	.119640	1.61246[c]
x_2	dollars	3,400	28,220	9,200	-.095634	.070140	1.307547
x_3	man-mo's	9.50	37.7	23.19	-.113900	.106172	1.07278
x_4	dollars	2,977	12,830	6,300	.677990	.152727	4.43923[a]
x_5	dollars	3,620	12,830	6,776	.387089	.184410	2.09907[b]
Total					1.048460		

a/ Significant at the .1 percent level of probability.

b/ Significant at the 5 percent level of probability.

c/ Significant at the 20 percent level of probability.

The marginal value producitivites, which are computed at the geometric mean of the input categories, are presented in Table IV. Here again, labor shows a marginal value productivity of -$95.20 per man-month of labor added as compared to -$102.39 in Model I. Labor and value of buildings and dairy equipment are not significant variables in the regression equation, therefore, their marginal value productivities cannot be considered with confidence. The marginal value productivity of cash operating expenses and total feed fed are both relatively high in this model, as they were in Model I. The marginal value productivity.

TABLE IV. MARGINAL VALUE PRODUCTIVITIES AT THE GEOMETRIC MEANS OF THE INPUT CATEGORIES. THE EXPECTED GROSS FARM INCOME FROM THE FUNCTION EQUALS ($19,383).

Input Category	Geometric Means	Marginal Value Productivities	Unit of Input
Dairy cows and two-year old heifers	10,875	$ .34	dollars
Buildings and dairy equipment	9,200	-.20	dollars
Total labor	23.19	-95.20	man-months
Cash operating expenses	6,300	2.08	dollars
Feed fed	6,776	1.11	dollars

of dairy cows and two-year old heifers is $0.34 in this model. It shows up higher in the following two models which will be discussed. It may have been higher if only producing cows had been included in this category.

Model III.

An additional model very similar to Model II was also analyzed. The major difference in the two models is that buildings and dairy equipment were not considered as an input category and forage acres were included as a separate input category. The results were nearly the same for the inputs which were considered in both functions. The forage acre input did not prove to be a significant variable. It's elasticity was negative and of very small magnitude.[1/] This function also indicated constant returns to scale as the sum of the elasticities was not significantly different than one.

1/ Appendix E gives additional statistical data concerning Model III.

Model IV and V.

Two models were considered which were set up with the specific intent of trying to get some ideas as to resource use within the farm organization. To accomplish this purpose several ratios or percentages were included as input groups in the two models. The results from these two models, due to their similariyt, will be presented together. The models referred to above were all computed using 31 observations. There are two of these observations which include a second major livestock enterprise, beef, within the farm organization. In models IV and V, these two obser- ations were excluded from the analysis as they were not of the same general class with the other 29 units studied.

The dependent variable in both models was gross farm income, (Y). Both models consider six independent variables, or input categories:

(x_1) is land measured in value terms;

(x_2) is farm buildings and dairy equipment;

(x_3) is the percentage of dairy labor to total labor;

(x_4) is the value of all dairy cows and two-year old heifers;

(x_5) is the percentage of forage acres to total acres.

These five variables were the same for both models. In Model V (x_6) is the value of total feed fed, and in Model IV it is the ratio of roughages fed to the concentrates fed.

The labor variable was divided into dairy labor and non-dairy labor, in hope of being able to give some indication as to where to concentrate available labor in the farm organization. Likewise, the percentage figure used in relation to land was introduced to give an indication as to land-use within the given farm organization. The roughage-concentrate ratio was also considered in the hope that it might give some indication as to where additional capital should be employed if it is spent on feed.

Due to the ratio or percentage groups being used, the sum of the elasticities indication of returns to scale will not be considered. These ratio figures would tend to destroy that part of the economic interpretation in Models IV and V.

The regression equations expressed in logarithmic form were found to be as follows:

Model IV[1/]

$$\log (Y) = 3.533052 + .047389 \log (x_1) + .043092 \log (x_2) - .141276 \log (x_3) + .692307 \log (x_4) - 1.131111 \log (x_5) + .008213 \log (x_6)$$

Model V[2/]

$$\log (Y) = 2.465871 + .059455 \log (x_1) + .028234 \log (x_2) - .035950 \log (x_3) + .428462 \log (x_4) - .875064 \log (x_5) + .384177 \log (x_6).$$

The corrected coefficients of multiple determination are .7635 and .8158 for Model IV and V, respectively. This indicates that approximately 76 percent of the variation in gross farm income (Y) is explained by the inputs in Model IV, and approximately 82 percent in Model V. The standard errors for the two equations are (.0788) and (.0694) for IV and V respectively.

1/ Appendix F gives additional statistical data concerning Model IV.

2/ Appendix G gives additional statistical data concerning Model V.