Hanle effect in mercury vapor
by Anne Green Romer

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in Physics
Montana State University
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Abstract:
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illustrating the Hanle Effect (zero-field level crossing method) in Hg vapor.
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ABSTRACT

A description is given of an apparatus suitable for an advanced undergraduate laboratory experiment illustrating the Hanle Effect (zero-field level crossing method) in Hg vapor.
I. INTRODUCTION

The Hanle effect provides a simple illustration of the phenomenon of resonance radiation and a method of measuring the lifetimes of the energy level transitions involved. It can be demonstrated in an effective way by an experiment suitable for an advanced undergraduate laboratory. The equipment required has been purposely kept to a minimum, and of such nature as to be found in a standard physics laboratory. Once built, the experiment can also be used as a classroom demonstration. The Hanle effect can be explained classically as well as quantum mechanically, and thus lends itself to an introductory experiment in atomic physics.

II. HISTORY

The term resonance radiation is given to the process whereby an atom absorbs light of a certain frequency and emits radiation of the same frequency. The atom absorbs a quantum of light which causes a transition from the ground state of the atom to a higher energy level. It then returns to the original ground state by radiating a quantum of light of the same energy as that absorbed. Thus resonance radiation must be of such frequency that it connects the ground state with a higher, excited state.
It has been observed that if the exciting radiation is polarized, the emitted resonance radiation of an atom is also polarized with its electric vector in the same direction as that of the exciting light. In 1922, Rayleigh discovered this effect using the 2537\(^\AA\) line of mercury.

Wood and Ellett in 1924 investigated the effect of a weak magnetic field at various orientations to the incoming, exciting light on the polarization of emitted resonance radiation in mercury. The outgoing beam was examined along the line of the incoming beam and in two directions perpendicular to it. They found that the radiation observed perpendicular to both the incoming beam and the direction of its polarization was polarized with its electric vector in the same direction as the incoming beam. Applying a field of 2 gauss decreased the polarization of the emitted radiation.

Hanle also investigated the polarization of the 2537 line of mercury and the effect of applied magnetic fields. He shone resonance light on a cell containing mercury vapor and observed the outgoing radiation from the cell in a direction perpendicular to the incoming beam (Fig. 1). He observed the direction of polarization of the emitted light for various orientations of the electric vector of the polarization of the incoming beam. He also placed the resonance cell in a magnetic field at various orientations to the incoming and outgoing beams and observed the
degree and direction of the polarization of the emitted radiation.

Depending on the orientation of the magnetic field and the direction of polarization of the exciting light, Hanle found that for fields around one gauss, the degree of polarization of the observed outgoing beam was decreased and its direction of polarization rotated. This effect can be explained by the classical model of an excited atom behaving as a damped oscillator precessing with Larmor frequency about the applied field. Hanle also gave a quantum mechanical explanation of the effect.

If unpolarized light is used, and the outgoing beam observed perpendicular to the exciting light, then the application of a weak field perpendicular to both the incoming and outgoing beams will increase the intensity of the emitted radiation. This effect has been used to measure the lifetime of the atomic transition producing the resonance radiation. The experiment described in this paper demonstrates this effect of magnetic fields on unpolarized resonance radiation.

III. THEORY

Classical Explanation

The zero-field level crossing method or Hanle effect using unpolarized light can be explained by the classical model of an
excited atom acting as a damped oscillating electric dipole precessing with Larmor frequency about a magnetic field.

Consider an atom located at the center of a coordinate system exposed to polarized light in the Y direction. The atom will act as an oscillating dipole oriented in the direction of the electric vector of the incoming light. If a magnetic field is applied in the Z direction, the dipole will precess about the direction of the field with a frequency \( \omega = \gamma H \), where \( \gamma \) is the gyromagnetic ratio. The oscillations are damped by a damping factor \( \Gamma = 1/\tau \), where \( \tau \) is the lifetime of the atom in the particular excited state. The intensity of the emitted light is given by

\[
\frac{dI}{dt} \propto \left( \mathbf{p} \times \mathbf{n} \right) e^{-\Gamma t}.
\]  

(1)

where \( \mathbf{p} \) is a unit vector in the direction of the electric dipole and \( \mathbf{n} \) is a unit vector in the direction of observation of the outgoing beam. After a time \( t \), the coordinates of \( \mathbf{p} \) are

\[
\mathbf{p} = (\sin \theta \cos \omega t, \sin \theta \sin \omega t, \cos \theta).
\]  

(2)

The direction of observation is chosen along the X axis perpendicular to the incoming beam and the magnetic field.

\[
\mathbf{n} = (1,0,0).
\]
Then \( \mathbf{p} \times \mathbf{n} = (0, \cos \theta, -\sin \theta \sin \omega t) \), \( \mathbf{3} \)
and \( dI \propto (\cos^2 \theta + \sin^2 \theta \sin^2 \omega t)e^{-\Gamma t} \).
\( \mathbf{4} \)

To find the total intensity of emitted radiation it is necessary to average over \( \theta \) for unpolarized exciting radiation, and integrate over times greater than \( \tau \), the lifetime of the transition.

\[
I \propto \int_0^\infty (\cos^2 \theta + \sin^2 \theta \sin^2 \omega t)e^{-\Gamma t} \, dt
= \int_0^\infty \left( \frac{1}{2} \frac{\sin^2 \omega t}{2} \right) e^{-\Gamma t} \, dt.
\]
\( \mathbf{5} \)

Thus

\[
I \propto 1 + \frac{1}{2} \frac{(2 \omega \tau)^2}{1 + (2 \omega \tau)^2}
\]

where

\[
\omega = \frac{geH}{2mc}
\]

The intensity of the emitted light, as seen at right angles to the incoming unpolarized beam, is a function of the applied magnetic field, the lifetime of the transition producing the resonance radiation, and the \( g \) value of the excited state. Thus a plot of intensity as a function of magnetic field produces an inverted Lorentzian line shape (Fig. 2). The lifetime can be
determined from the value of the field at half maximum on the intensity curve. At that point

\[ \tau = \frac{1}{2 \omega} = \frac{m_0}{g \hbar \mu} \]  

\( g \) has been measured for the \( ^3P_1 \) state for even isotopes of mercury by Brossel and Bitter \(^6\) as \( 1.4838 \pm 0.0004 \).

The Hanle effect can also be explained pictorially (Fig. 3). The incoming unpolarized light is resolved into two components, the electric vector of one component vibrating in the \( X \) direction, the other in the \( Z \) direction. In the presence of zero magnetic field, the outgoing light will contain a polarized component with electric vector vibrating along the direction of observation (\( E_\perp \) of the incoming beam), and one component polarized in the \( Z \) direction perpendicular to the direction of observation (\( E_\parallel \)). Since the component vibrating along the line of observation cannot be seen, the outgoing light will appear polarized in the \( Z \) direction.

Consider a magnetic field applied in the \( Z \) direction. The incoming light polarized parallel to the field will not be affected. However, the component of light with electric vector vibrating perpendicular to the field will excite an atom whose electric dipole precesses in the \( XY \) plane about the field. The outgoing light will contain an unchanged component in the \( Z \).
direction ($E_{\parallel}$ of the incoming beam), and a component in the Y direction due to the rotation of $E_{\perp}$ through an angle $\omega t$.

Since $E_{\parallel}$ remains unaffected by the application of a magnetic field in the Z direction, it can be considered as a background contribution. The analysis of the Hanle effect is concerned only with $E_{\perp}$, and the same result is obtained by polarizing the incoming light along the direction of observation. With zero field no light will be observed.

When a magnetic field is applied in the Z direction, the vector $E_{\perp}$ will precess about the Z axis with a frequency $\omega$. At any time $t$, it will form an angle $\omega t$ with the direction of observation along the X axis. This vector can be resolved into two components, $E_{\perp} \sin \omega t$ perpendicular to the line of sight, and $E_{x} \cos \omega t$ along the line of sight. The observed light will be $E_{\perp} \sin \omega t$ polarized in the Y direction. Since $\omega = \gamma H$, the magnitude of $E_{\perp} \sin \omega t$ will increase as the value of the field is increased from zero.

Quantum Mechanical Explanation

According to a quantum mechanical treatment given by Briet\(^7,8\), the observed increase in light intensity with increased field is produced by the interference of the $\sigma$ components of the emitted radiation. The dipole matrix element giving the transition between
levels is given by \( f_{\mu m} \), where \( \mu \) is the magnetic quantum number of the excited state and \( m \) is the quantum number of the ground state.

\[
f_{\mu m} = \langle \mu | \hat{e} \cdot \mathbf{r} | m \rangle,
\]

where \( \hat{e} \) is a unit vector in the direction of polarization. In zero magnetic field, the \( m = \pm 1 \) levels give a contribution

\[
\left| f_{01} + f_{0-1} \right|^2 = f_{01}^2 + f_{0-1}^2 - \frac{2f_{01} f_{0-1}^*}{1 + (\omega \tau)^2}.
\]

In the presence of a magnetic field, the \( m = \pm 1 \) contributions appear as \( f_{01}^2 + f_{0-1}^2 \). An evaluation of the Breit formula for the simple case of a \( J = 0 \) to \( J = 1 \) transition has been given by Anderson.9

Consider an incoming beam polarized along the direction of observation. The emitted radiation is composed of \( \sigma^+ \) and \( \sigma^- \) components produced by the transition \( J = 1, m = \pm 1 \) to \( J = 0, m = 0 \) (Fig. 4). In the presence of zero field these \( \sigma \) components (circularly polarized in the XY plane) are degenerate in energy and combine so as to produce linearly polarized light vibrating in the direction of observation. Thus no light is observed.

In a magnetic field the Zeeman levels of the excited state are split and the energies of the \( \sigma^+ \) and \( \sigma^- \) components are
no longer degenerate. These components combine to produce light polarized at an angle to the direction of observation and thus will not exactly cancel each other as they did in the presence of zero field. As the field increases in magnitude, the magnetic sub-levels will become further split and there will be a greater contribution to the emitted light. The intensity of the outgoing light as a function of applied magnetic field will form an inverted Lorentzian curve.

IV. EXPERIMENTAL PROCEDURE

The lamp was focused on a resonance cell containing mercury vapor and the emitted radiation observed at $90^\circ$ to the incoming beam (Fig. 5). A pair of Helmholtz coils was oriented around the resonance cell with its axis perpendicular to both the incoming and outgoing beams. The intensity of the emitted resonance radiation was then measured as a function of the applied magnetic field. A second set of Helmholtz coils was used to eliminate the horizontal component of the earth's magnetic field.
V. APPARATUS

The exciting light was produced by a Blak-Ray and Mineral-light Short Wave UVS 11 lamp operated by a dc power supply. The lamp was connected in series to a variable resistor and a 2h choke in order to minimize the plasma fluctuations in the lamp. Suitable intensity and stability were obtained with the lamp operated at 100 to 150 volts and 60 ma.

The incoming light was focused approximately on the center of the resonance cell by two converging quartz lenses 40 mm in diameter.

The emitted radiation was detected by a lucite rod whose tip was coated with a solution of sodium salicylate which fluoresces under ultraviolet light. A smooth surface on the outside of the rod was produced by fire polishing. The tip was shaped so as to reflect down the light pipe all light produced by the sodium salicylate on the tip (Fig. 6). The condition for total reflection along the pipe is given by:

\[ d = 2R \sin(42^\circ), \]  \hspace{1cm} (8)

where \( R \) is the radius of the rod, \( d \) is the length of the shaped tip, and \( 42^\circ \) is the critical angle for total reflection inside lucite. \(^{11}\)
A quartz window Dumont K 1306 photomultiplier tube with a Mumetal magnetic shield was used as a detector. In order to detect the small change in current from the photomultiplier tube with changes in applied magnetic field, a negative current with respect to the output was applied to the output of the photomultiplier tube to reduce the background current (Fig. 7). The light intensity was measured by a General Radio Company 1230-A electrometer. A type 601 Keithley electrometer was alternatively used.

The resonance cell was constructed from a quartz tube one-inch in diameter, closed at one end with a graded seal on the open end. This tube was joined to a glass tube long enough to extend below the Helmholtz coils placed around it with a side arm extension. A drop of mercury liquid was introduced into the tube through the side arm, and then the whole tube was evacuated to a pressure of $10^{-4}$ mm of mercury, and sealed off. It was found that cooling the lower portion of the tube containing the drop of mercury to $0^\circ$C increased the signal to noise ratio.

The Helmholtz coils were constructed with a 5 cm average radius to produce an average field of 24 gauss/amp over the volume of the resonance cell. The coils were designed to have a resistance of approximately 25 ohms in order to match the emitter-follower circuit.
The ratio of the magnetic field to current through the coils was calculated from the dimensions of the Helmholtz coils, and their resistance measured using a 5 ohm precision resistor and a Fluke 881A differential voltmeter. The coils were calibrated at 1.1 gauss/volt to within 1.4%. The magnetic field was constant to within 0.1% along the axis of the coils within the volume of the resonance cell.

An inverter emitter-follower circuit was designed to supply the Helmholtz coils with a variable voltage with a range of 3.5 volts positive and negative (Figs. 8 and 9). With such a circuit it was possible to employ the 150 volt saw-tooth sweep from a type 545A Tectronics oscilloscope to drive the coils. The output intensity from the electrometer was then fed into the scope input to obtain an intensity vs voltage curve. Photographs were obtained of the scope trace (Fig. 10).

The intensity curve was also recorded on a Hewlett-Packard Moseley 7035A X-Y recorder. The output of the electrometer was plotted on the ordinate and the voltage through the coils on the abscissa (Fig. 2). Such a curve could also be obtained with a chart recorder whose time scale was adjusted to the time sweep of the oscilloscope. The X-Y recorder was also used to obtain a curve for the voltage sweep output of the emitter-follower circuit (Fig. 9).

The entire apparatus was oriented in a north-south direction.
so as to allign the horizontal component of the earth's magnetic field of 0.17 gauss along the direction of observation. A second set of Helmholtz coils was placed about the resonance cell with its axis along the earth's horizontal field in order to nullify it. The vertical component of the earth's field produced a bias on the applied field in the Z direction.

VI. RESULTS

The intensity vs magnetic field curves obtained were of the predicted Lorentzian shape. It was found that the amount of asymmetry in the curve was very sensitive to the positioning of the detecting light pipe. Slight deviations from 90° produced noticeable asymmetry in the wings of the curve. The curve was also sensitive to the position of the lamp with respect to the focusing lenses.

The lifetime was determined from the value of the field at half maximum (see equation 6) as $1.3 \times 10^{-7}$ sec for a mercury vapor pressure inside the cell corresponding to 24°C, and $1.2 \times 10^{-7}$ sec at 0°C. The greater value of the lifetime at higher pressures is due to coherence narrowing. The atom density inside the cell is sufficient to produce a finite probability for the incoming light to be absorbed more than once before leaving the cell. Thus the
observed lifetime is greater than the true lifetime and is a function of the pressure inside the cell. This effect has been calculated theoretically by Barrat\textsuperscript{13} for single isotopes of mercury. A quantitative comparison with his results cannot be made for this experiment since naturally abundant mercury was used in the resonance cell. However, the broadening of the curve with increased pressure can be shown qualitatively. Barrat has calculated the true lifetime for all isotopes of mercury to be $1.18 \times 10^{-7}$ sec.

The mercury vapor in the lamp and in the resonance cell contained the natural abundance distribution of mercury isotopes (Table 1).

**TABLE 1**

NATURAL ABUNDANCE OF ISOTOPES OF Hg

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>%</th>
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<tr>
<td>Even Isotopes</td>
<td>69.88%</td>
</tr>
<tr>
<td>$\text{Hg}_{204}$</td>
<td>6.85</td>
</tr>
<tr>
<td>$\text{Hg}_{202}$</td>
<td>29.27</td>
</tr>
<tr>
<td>$\text{Hg}_{200}$</td>
<td>23.77</td>
</tr>
<tr>
<td>$\text{Hg}_{198}$</td>
<td>9.89</td>
</tr>
<tr>
<td>$\text{Hg}_{196}$</td>
<td>0.10</td>
</tr>
<tr>
<td>Odd Isotopes</td>
<td>30.12%</td>
</tr>
<tr>
<td>$\text{Hg}_{201}$</td>
<td>13.67</td>
</tr>
<tr>
<td>$\text{Hg}_{199}$</td>
<td>16.45</td>
</tr>
</tbody>
</table>
The main purpose of the experiment was to design an operating demonstration of the Hanle effect that could be practically constructed as an advanced undergraduate experiment using a minimum of equipment. The apparatus lends itself to such a project because of its simple techniques and use of basic physical concepts.
COORDINATE SYSTEM FOR HANLE'S EXPERIMENT

FIG 1
RELATIVE INTENSITY vs. MAGNETIC FIELD

FIG 2
EFFECT OF MAGNETIC FIELD ON EMITTED RADIATION

FIG 3
ZEEMAN DIAGRAM

FIG 4
Diagram of apparatus

Fig 5
TIP OF LIGHT PIPE

FIG 6
CIRCUIT TO REMOVE BACKGROUND CURRENT FROM PHOTOMULTIPLIER OUTPUT

FIG 7
INVERTER EMITTER-FOLLOWER CIRCUIT

FIG 8
VOLTAGE OUTPUT vs OSCILLOSCOPE SWEEP

FIG 9
SCOPE TRACE OF ELECTROMETER OUTPUT

FIG 10
LITERATURE CITED
LITERATURE CITED


7. C. Breit, Rev. Mod. Phys. 5, 91 (1933).


11. I am indebted to D. K. Anderson for his derivation of the dimensions of the tip of the light pipe for total internal reflection.


Romer, A. G.
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