Determination of the temperature coefficient of reactivity of the Montana State College subcritical assembly
by Gary James Russell

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Physics at Montana State College
Montana State University

Abstract:
This thesis is a report of the research in. which the temperature coefficient of reactivity of the Montana State College subcritical assembly has been calculated. This assembly is a light water moderated, natural uranium system. In order to calculate the temperature coefficient of reactivity, the temperature effect on the diffusion length and buckling has been determined. The diffusion length has been measured at 10°, 27°, 50°, and 75° C. Experimentally, the variation of the diffusion length with temperature, $\Delta L/\Delta T$, is 0.00778 cm./°C; whereas the accepted variation is 0.006 cm./°C. The method used for measuring the diffusion length of thermal neutrons in light water utilizing a relatively high energy Pu-Be (α,n.) source is outlined. The vertical as well as the radial buckling has been measured at 13°, 23°, 35°, and 45° C. The temperature coefficient of the material buckling, B^am, is found to be -2.87x10^-3/°C , The net temperature coefficient of reactivity (assuming $k\infty$ is independ- ent of temperature) is -16.2 x 10^-5/°C. The contribution to the temperature coefficient due to the change of $k\infty$ with temperature $[1/\Delta T\ (\delta k\infty/k\infty)]$ is -10.6 x 10^-5/°C. Experimentally, the infinite multiplication factor at 23°C is 0.965• The calculated value of -$k\infty$ is O.960. Using the experimental value of $k\infty$, the effective multiplication factor is 0.869 and the suhcritical multiplication is 7.64. The ratio of the volume of uranium to the volume of water is 0.097° The average coefficient of linear expansion, αT, for the natural uranium-water system is 8.4 x 10^-5/°C. All measurements at both high and low temperatures have been made with a thermal neutron scintillation counter.

THE DETERMINATION OF THE TEMPERATURE COEFFICIENT OF REACTIVITY OF THE MONTANA STATE COLLEGE SUBCRITICAL ASSEMBLY

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ABSTRACT

This thesis is a report of the research in which the temperature coefficient of reactivity of the Montana State College subcritical assembly has been calculated. This assembly is a light water moderated, natural uranium system. In order to calculate the temperature coefficient of reactivity, the temperature effect on the diffusion length and buckling has been determined. The diffusion length has been measured at 10°, 27°, 50°, and 75° C. Experimentally, the variation of the diffusion length with temperature, $\Delta L/\Delta T$, is 0.00778 cm./°C; whereas the accepted variation is 0.006 cm./°C. The method used for measuring the diffusion length of thermal neutrons in light water utilizing a relatively high energy Pu-Be (α,n) source is outlined. The vertical as well as the radial buckling has been measured at 13°, 23°, 35°, and 45° C. The temperature coefficient of the material buckling, B^2_m, is found to be -2.87×10^{-3} cm^{-2}/°C. The net temperature coefficient of reactivity (assuming f_{∞} is independent of temperature) is -16.2×10^{-5}/°C. The contribution to the temperature coefficient due to the change of f_{∞} with temperature $[1/\Delta T\ (\delta k_{\infty}/f_{\infty})]$ is -10.6×10^{-5}/°C. Experimentally, the infinite multiplication factor at 23°C is 0.965. The calculated value of f_{∞} is 0.960. Using the experimental value of f_{∞}, the effective multiplication factor is 0.869 and the subcritical multiplication is 7.64. The ratio of the volume of uranium to the volume of water is 0.097. The average coefficient of linear expansion, α_t, for the natural uranium-water system is 8.4×10^{-5}/°C. All measurements at both high and low temperatures have been made with a thermal neutron scintillation counter.

INTRODUCTION

A problem of considerable importance to the control of nuclear reactors is the evaluation of the temperature coefficient of the reactor system. The temperature coefficient is defined as the change in the reactivity of the system due to a change in the operating temperature of the reactor. Therefore, the temperature coefficient is a measure of the inherent stability of the reactor. If the temperature of the reactor increases (this could be due to a change in the power demands of the reactor system) and if the temperature coefficient is positive, the multiplication constant and the reactivity will increase; this will in turn cause the rate of production of energy to increase and further raise the temperature of the system. A system of this nature would be unstable and would require a positive and continuous influence of a control system in order to maintain a steady state system. However, if the temperature coefficient is negative, the effect of a temperature rise on the neutron population (and therefore on the power production) would be to decrease the number of neutrons (and power), so that the temperature would then decrease. Thus, in this type of system, temperature "disturbances" would eventually disappear and the reactor would return to a steady state even in the absence of external control orders.

Ordinary reactors are designed to have negative temperature coefficients of reactivity to ensure adequate stability. These ordinarily lie in the range 10^{-3} to 10^{-2} % reactivity per degree centigrade.

In a subcritical assembly the reactivity, being negative and not

necessarily close to one, is not of too great interest. However, a subcritical system still has a temperature coefficient and this coefficient for the Montana State College subcritical assembly was calculated. This assembly is a light water moderated, heterogeneous, natural uranium system.

For the theoretical treatment, to be outlined later, it is convenient to separate the effect of temperature on reactivity into two main parts: the nuclear temperature coefficient, which is determined by the effect on the nuclear cross sections, and the density temperature coefficient, due to changes in density (and volume) of the system. The discussion which follows applies particularly to a homogeneous reactor system, but the general conclusions will hold, to a good approximation, for most heterogeneous reactors and were applied in the calculations of the temperature coefficients in this report.

DESCRIPTION OF THE HEATING AND COOLING MECHANISM

The aluminum tank that contains the moderator[1], reflector[1], and fuel[1] was covered with three inches of fiber glass insulation to reduce heat losses. A thin aluminum cover was then placed over the insulation in order to preserve the appearance of the subcritical assembly.

The complete heating and cooling system consisted of a pipe extending from the bottom of the tank, a series of valves, a pump (capacity $\approx$10 gal./min.), a heat exchanger and a pipe connecting the heat exchanger with the top of the subcritical assembly. In order to heat the assembly, steam (at about 40 psi) was introduced into the heat exchanger. Cooling was obtained by running cold ground water through the heat exchanger.

A diagram of the system is given below: (not to scale)

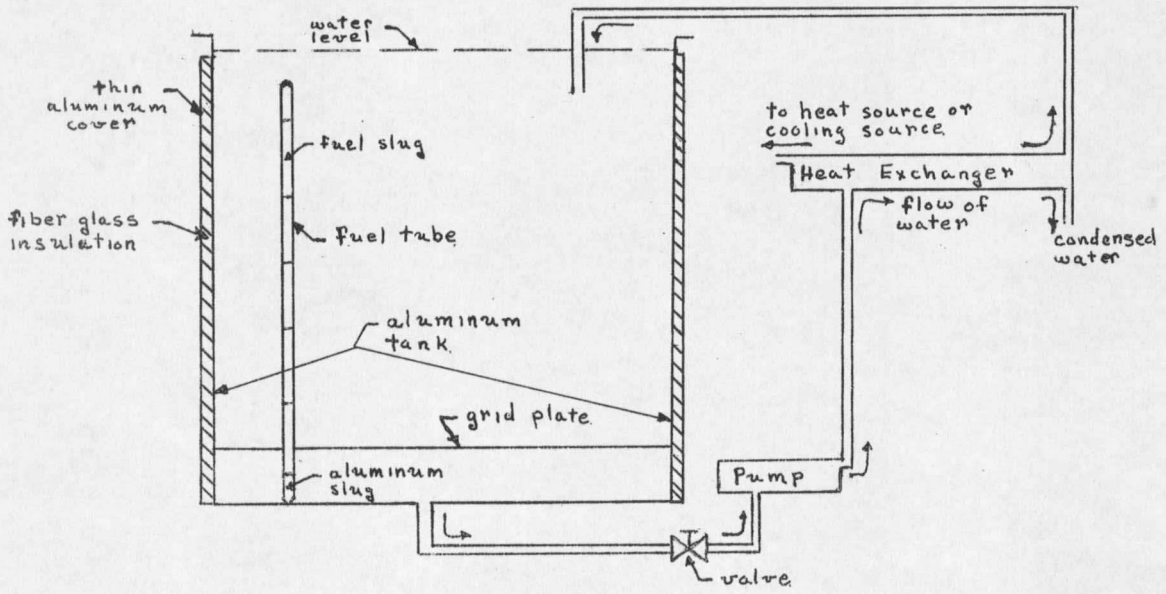

Figure 1. Heating and Cooling Mechanism.

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1. There were 1.19×10^6 cm.^{3} (approximately 315 gal.) of water and 1.15×10^5 cm.^{3} of natural uranium.

THEORY

THE DIFFUSION LENGTH

Before a description of the diffusion length, L, is attempted, one has to understand what is meant by a thermal neutron. Imagine a fast neutron undergoing elastic scattering with molecules of hydrogen. After a number of scattering collisions, the velocity of the neutron is reduced to such an extent that it has approximately the same average kinetic energy as the molecules of the hydrogen medium in which it is undergoing scattering. The energy depends on the temperature of the medium and is called thermal energy. Neutrons whose energies have been reduced to values in this region are designated as thermal neutrons. At 25°C the energy of thermal neutrons is approximately 0.025 ev.[2]

In reactor physics, the diffusion length, L, of thermal neutrons in a medium is defined as $L \equiv \sqrt{\frac{D}{\Sigma_a}}$, where D is the diffusion coefficient and Σ_a is the macroscopic absorption cross section of the particular scattering medium. The diffusion coefficient, in general, will be a function of the nuclear properties of the medium and of the neutron speed.

As seen from the defining equation of the diffusion length, it varies inversely as $\sqrt{\Sigma_a}$; hence, if the scattering medium has a relatively large absorption cross section (for light water and 0.025 ev neutrons, $\Sigma_a = 0.022$

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2. In atomic studies, energies are frequently expressed in electron volt units. The electron volt, that is, 1 ev, is the energy acquired by any charged particle carrying a unit electric charge when it passes, without resistance, through a potential difference of 1 volt. It is equivalent to 1.6×10^{-12} ergs. The million electron volt (or Mev) unit is 1×10^{6} ev or 1.6×10^{-6} erg.

$cm.^{-1})^3$ the diffusion length will be small. It should be noted that L has the dimensions of length. For light water L=2.85 cm. at 25°C.[4]

In order to obtain a physical picture of just what the diffusion length is, consider a point source of thermal neutrons and let ϕ be the thermal neutron flux of neutrons per $cm.^2$ per sec. at a distance r. The rate at which neutrons are absorbed is then equal to $\Sigma_a \phi$ per $cm.^3$ per sec., where Σ_a is the macroscopic absorption cross section of the medium. In a spherical shell element of radius r and thickness dr, that is, volume $4\pi r^2 dr$, surrounding the point source, the number of neutrons captured per second will be $4\pi r^2 dr \Sigma_a \phi$. This is a measure of the probability that a neutron will be absorbed within the element dr at a distance r from the source. It is preferable now to consider the second spatial moment of r; therefore, the mean square distance, $\overline{r^2}$, which a neutron travels from its source to where it is absorbed is given by

$$\overline{r^2} = \frac{\int_0^\infty r^2 (4\pi r^2 \Sigma_a \phi)\, dr}{\int_0^\infty 4\pi r^2 \Sigma_a \phi\, dr} \tag{1}$$

It can be shown[5] that for a single point source in an infinite medium $\phi = \frac{e^{-Kr}}{4\pi D r}$, where K is defined as $\sqrt{\frac{\Sigma_a}{D}}$.

Therefore, upon substituting ϕ into the equation for $\overline{r^2}$ one obtains

$$\overline{r^2} = \frac{4\pi \Sigma_a \int_0^\infty \frac{r^2 e^{-Kr}}{4\pi D r}\, dr}{4\pi \Sigma_a \int_0^\infty \frac{r^2 e^{-Kr}}{4\pi D r}\, dr} = \frac{6/K^4}{1/K^2} = \frac{6}{K^2} \tag{2}$$

then,

$$K^2 = 6/\overline{r^2} \tag{3}$$

3. Etherington, Nuclear Engineering Handbook (New York: McGraw-Hill Inc., 1958), p. 12-70.
4. Ibid., p. 1-20.
5. Meghreblian and Holmes, Reactor Analysis (McGraw-Hill Inc., 1960), p. 183.

However, by definition, $L \equiv \sqrt{\frac{D}{\Sigma_a}}$ cm. or $L^2 = \frac{1}{\kappa^2}$ and after substituting for κ^2 it is seen that

$$L^2 = \frac{\overline{r^2}}{6} \tag{4}$$

so that the diffusion length squared is one-sixth of the mean square distance, as the crow flies, that a thermal neutron travels from the point at which it just becomes thermal to the point of capture.

For a right circular cylinder it can be shown[6] that

$$\varphi(z) = C e^{-\gamma z}\left\{1 - e^{-2\gamma(\tilde{H}-z)}\right\} \tag{5}$$

where z is measured from the plane where neutrons are thermalized and γ is defined by

$$\gamma^2 = \left(\frac{1}{L}\right)^2 + \left(\frac{2.405}{\tilde{R}_0}\right)^2 \tag{6}$$

For distances not too close to the source and not too close to the top of the cylinder, the term in brackets is not very different from unity, then

$$\varphi(z) = C e^{-\gamma z} \tag{7}$$

Therefore, the slope of the straight line portion of $\ln \varphi(z)$ versus z yields γ. It should be noted that $\tilde{R}_0$ is the extrapolated radius, that is, the radius of the moderator configuration plus $0.71\,\lambda_{tr}$, and 2.405 is the first zero of the Bessel function $J_0(x)$. Also, λ_{tr} is the transport mean free path.

In the thermal energy region the absorption cross sections may be assumed to follow the 1/v law. Since v is proportional to $\sqrt{T}$ and if σ_a is the microscopic absorption cross section at temperature T and σ_{a_0} is the value at T_0, then

$$\sigma_a = \sigma_{a_0}\left(\frac{T_0}{T}\right)^{1/2} \tag{8}$$

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6. Glasstone and Edlund, The Elements of Nuclear Reactor Theory (Van Nostrand Inc., 1952), p. 287.

Scattering cross sections do not change greatly with temperature and through the remainder of this report it will be assumed that they remain constant.

Now, $L = \sqrt{\frac{D}{\Sigma_a}}$ and $D = \frac{\lambda_{tr}}{3}$, $\Sigma_a = \frac{1}{\lambda_a}$ (9)

then, $L = \sqrt{\frac{1}{3}\lambda_{tr}\lambda_a}$ or $L^2 \approx \frac{1}{3\Sigma_a\Sigma_s}$ (10)

This holds for substances of high atomic mass provided the medium does not absorb too strongly and scattering is isotropic in the laboratory systems.

Then, $L^2 = L_o^2\left(\frac{T}{T_o}\right)^{1/2}$ (11)

if the effect of density changes on the number of nuclei per $cm.^3$, which is involved in the macroscopic cross section, is ignored.

In contrast to charged particles which have a definable range, neutrons and γ-rays are stopped approximately exponentially. In this exponential approximation, use is made of the relaxation length, which is the thickness of material that causes a drop in intensity by a factor of e. For thermal neutrons the relaxation length is equal to the thermal diffusion length, which can be made quite small in absorbing materials.

In order to gain an additional physical significance of the diffusion length concept, consider a plane thermal neutron source in an infinite medium. It can be shown[7] that the flux distribution in a nonmultiplying medium has an exponential behavior given by the factor exp (-x/L). Then,

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7. Meghreblian and Holmes, Reactor Analysis (McGraw-Hill Inc., 1960) p. 182.

in this form one can identify the diffusion length L as an attenuation or relaxation length. Therefore, L is the distance from the source plane at which the neutron density is reduced to 1/e of its value at the source. One can think of the diffusion length, then, as a measure of the average depth of penetration into a medium by a source neutron. It should be noted that a proper measure of the penetration is the "crow-flight" distance from the source to the point of capture as was explained previously.

The thermal neutron diffusion length in a hydrogenous medium can be simply determined by measuring the variation of the thermal neutron density at relatively large distances from the center of the neutron source. The necessary requirement is that the slowing-down length of the source neutrons in the medium be less than or equal to the diffusion length, so that the diffusion length dominates at large distances from the source.

The square root of the Fermi age is called the slowing-down length. The Fermi age is related to the mean square distance traveled while slowing down.[8] For thermal neutrons, of age τ_{th}, $\sqrt{\tau_{th}}$ is a measure of the net vector (crow-flight) distance traveled from their formation as fission neutrons to their attainment of thermal energy.

If the slowing-down length of the source neutrons is greater than the diffusion length of thermal neutrons in the medium, then, the thermal neutron flux or density will decay by means of something other than an exponential.

For an Ra-Be (γ,n) source the maximum neutron energy emitted is only

8. Glasstone, Principles of Nuclear Reactor Engineering (Van Nostrand Inc., 1955), p. 165.

0.7 Mev and the slowing-down length of 0.7 Mev neutrons is 2.45 cm.[9] This distance of 2.45 cm. is less than the diffusion length of thermal neutrons in light water at 25°C which is 2.85 cm.. Thus, a direct measurement of the diffusion length in light water using a Ra-Be (γ,n) source is possible. However, a Pu-Be (α,n) source emits neutrons of fairly high energies ranging from 5 to 12 Mev or more. The slowing-down length for fast neutrons of this energy in light water would be greater than 5.74 cm.[10]. Therefore, a direct measurement of the diffusion length in light water, utilizing a Pu-Be (α,n) source, is impossible owing to the fact that the slowing-down length of the source neutrons is greater than the diffusion length. A typical result for the diffusion length (obtained by making a direct measurement of the decay of Pu-Be neutrons in light water) is 5.08 cm.[11]. The experimental procedure used and results found for measurements of L in light water, using a Pu-Be neutron source, are given later.

BUCKLING

One of the most important quantities measured in an exponential experiment is the material buckling, B_m^2. The material buckling, B_m^2, is the value of B^2 that satisfies the critical transcendental equation

$$\frac{k e^{-B^2\tau}}{1 + L^2 B^2} = 1 \qquad (12)$$

provided the continuous slowing-down (Fermi Age) model is applicable.

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9. DeJuren and Rosenwasser, "Diffusion Length of Thermal Neutrons in Water," Journal of Research of National Bureau of Standards, October 1953, p. 203.
10. Glasstone, Principles of Nuclear Reactor Engineering (Van Nostrand Inc., 1955), p. 166.
11. Curtiss-Wright Research Division, "Laboratory Manual for the Subcritical Facility," p. 63.

Now, k_∞ is the infinite multiplication factor of the medium, that is, k_∞ (or k_{∞}) is the ratio of the average number of neutrons produced in each generation to the average number of corresponding neutrons in the preceeding generation. In other words k_∞ is essentially the value the effective multiplication factor would have if the system were infinitely large and there were no loss of neutrons by leakage. Again, τ is the Fermi age or the average (net vector) distance a fission neutron travels while slowing down to thermal energies.

B_m^2 is the parameter of a given fuel lattice which determines the critical size of the reactor. In the critical reactor, the spatial flux is given by

$$\nabla^2 \phi(\vec{r}) + B_m^2 \phi(\vec{r}) = 0 \qquad (13)$$

The geometrical buckling, B_g^2, is defined as the lowest eigenvalue that results from solving the wave equation

$$\nabla^2 \phi(\vec{r}) + B_g^2 \phi(\vec{r}) = 0 \qquad (14)$$

with the boundary condition that $\phi(\vec{r})$ shall be zero at the extrapolated boundary of the system.

The critical condition for a reactor system can be written as

$$B_m^2 = B_g^2 \qquad (15)$$

In other words, the material buckling, B_m^2, for the given multiplying medium is equal to the geometrical buckling, B_g^2, of the critical system of a specified shape. Therefore, if B_g^2 is less than B_m^2, the reactor will be larger than the critical size and the system will be supercritical, and if B_g^2 is greater than B_m^2, the system will be subcritical.

In the case of the Montana State College subcritical assembly, which

is essentially cylindrical in form, B_m^2 is given by the expression

$$B_m^2 = \left(\frac{2.405}{\widetilde{R}_o}\right)^2 - \gamma^2 \tag{16}$$

where $\widetilde{R}_o$ is the extrapolated radius and γ is the reciprocal of the relaxation length. The first term on the right of equation (16) constitutes the radial buckling while γ^2 is referred to as the vertical buckling.

The geometric buckling for a finite, cylindrical reactor, in terms of its radius and height is

$$B_g^2 = \left(\frac{2.405}{\widetilde{R}_o}\right)^2 + \left(\frac{\pi}{\widetilde{H}_o}\right)^2 \tag{17}$$

where $\widetilde{H}_o$ is the extrapolated height of the system.

$$\widetilde{H}_o = H_o + 2(0.7104\,\lambda_{tr}) \tag{18}$$

where H_o is the actual height of the reactor and λ_{tr} is the transport mean free path of a thermal neutron in water. For light water the value of λ_{tr} is 0.48 centimeters.[12]

The flux distribution in a subcritical assembly does not satisfy the wave equation for a critical reactor, but for a relatively large assembly the thermal neutron flux variation at a distance from boundaries and the extraneous neutron source can be represented by

$$\nabla^2 \Phi(r,z) + B_m^2 \Phi(r,z) = 0 \tag{19}$$

where Φ is the thermal flux and B_m^2, the material buckling, a constant for the particular assembly. The solution of equation (19) can be obtained for a cylindrical reactor by setting

$$\Phi(r,z) = \Theta(r) Z(z) \tag{20}$$

This is then substituted into equation (19) and the terms in $\Theta(r)$ and

12. Etherington, Nuclear Engineering Handbook (McGraw-Hill Inc., 1958) p. 6-86.

$Z(z)$ are set equal to the constants α^2 and γ^2 so that

$$B_m^2 = \alpha^2 - \gamma^2 \tag{21}$$

The general solution of equation (19) using the correct Laplacian operator is

$$\Phi(r,z) = \sum_{n=1}^{\infty} A_n J_o(\alpha_n r) \sinh \gamma_n(\tilde{H}-z) \tag{22}$$

Previous experimentation has shown that for flux measurements not too near the source the fundamental (n=1) term is sufficient for good accuracy.

Thus, the radial flux distribution becomes

$$\Theta(r) = Q(r) = A_1 J_o(\alpha r) \tag{23}$$

The constant α can be evaluated from the boundary condition that the flux falls to zero at the extrapolated radius $\tilde{R}_o$ and is found to be $\frac{2.405}{\tilde{R}_o}$.

Then, the radial flux distribution becomes

$$Q(r) = A_1 J_o\left(\frac{2.405}{\tilde{R}_o} r\right) \tag{24}$$

It can be seen from equation (16) that α^2 is the radial buckling which is equal to $\left(\frac{2.405}{\tilde{R}_o}\right)^2$.

Therefore, radial flux measurements at a constant height above the source should provide data from which a graph can be made which will yield the extrapolated radius. The extrapolated radius can be gotten from the graph of relative thermal neutron flux versus radial distance either by visual extrapolation or by fitting the graph to a Bessel curve. Using only the fundamental term (n=1) of equation (22), the flux variation along the z-axis may be represented as

$$Z(z) = Q(z) = A_2 \sinh \gamma(\tilde{H}-z) \tag{25}$$

or

$$\phi(z) = A_3 e^{-\gamma z}\left\{1 - e^{-2\gamma(\tilde{H}-z)}\right\} \tag{26}$$

For measurements taken not too near the top of the assembly

$$\phi(z) = A_3 e^{-\gamma z} \tag{27}$$

Therefore, if relative flux measurements are made in the core along lines parallel to the cylinder's axis γ can be determined from the semilogarithmic plot of activity versus vertical distance .

TEMPERATURE COEFFICIENTS

A temperature increase in a nuclear reactor will alter the reactivity[13] for at least two reasons: first, the thermal neutron mean energy changes and, therefore, their absorption is affected since nuclear cross sections vary with energy; and, second, the mean free paths and the non-leakage probabilities are temperature dependent.

Nuclear Temperature Coefficients-- Previously, ignoring the effect of density changes on the number of nuclei per cm.^{3}, it was shown that

$$L^2 = L_o^2 \left(\frac{T}{T_o}\right)^{1/2} \tag{28}$$

This means that the diffusion length is temperature dependent.

The Fermi age, τ, defined by[14]

$$\tau(E) = \int_E^{E_o} \frac{D}{\xi \Sigma_s E}\, dE \tag{29}$$

will be temperature independent. This will be so because the temperature variation of τ will be determined by the effect on $\frac{D}{\Sigma_s}$ or $\frac{1}{3\Sigma_s^2}$ and Σ_s is regarded as being temperature independent.

The infinite multiplication, k_∞, is independent of temperature. The factor k_∞ is defined through the four factor formula[15]

$$k_\infty = \eta \epsilon p f \tag{30}$$

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13. The reactivity, ρ, is defined as $\rho = \frac{k_{eff} - 1}{k_{eff}}$, where k_{eff}, the effective multiplication factor, is the ratio of the average number of neutrons produced by fission in each generation to the total number of corresponding neutrons absorbed or leaking out, on the average.
14. Glasstone, Principles of Nuclear Reactor Engineering (Van Nostrand Inc., 1955), p. 162.
15. Ibid., p. 298.

where η is the average number of neutrons liberated for each neutron absorbed, ϵ is the fast fission factor, p is the resonance escape probability and f is the thermal utilization. If all the absorbers present obey the $1/v$ law, then the thermal utilization will not be affected by temperature very much. The resonance escape probability may decrease somewhat as the temperature is raised while f increases slightly. However, as a reasonable approximation, since η and ϵ remain unchanged, it may be assumed that k_∞ is independent of temperature.

For a large reactor, the effective multiplication factor is given by[16]

$$k_{eff} = \frac{k_\infty}{1 + M^2 B_g^2} \tag{31}$$

but, $$\rho = \frac{k_{eff} - 1}{k_{eff}} \tag{32}$$

then, $$\rho = \frac{k_\infty - 1 - M^2 B_g^2}{k_\infty} \tag{33}$$

but, $$M^2 = L^2 + \tau \tag{34}$$

then, $$\rho = \frac{k_\infty - 1}{k_\infty} - \frac{B_g^2}{k_\infty}\left\{L_o^2\left(\frac{T}{T_o}\right)^{1/2} + \tau\right\} \tag{35}$$

The nuclear temperature coefficient, at constant density, that is, the temperature coefficient of ρ due to changes in the cross sections is given by

$$\left(\frac{\partial \rho}{\partial T}\right)_d = -\frac{B_g^2 L_o^2}{2 k_\infty} \cdot \frac{1}{(T T_o)^{1/2}} \tag{36}$$

The subscript d indicates constant density and it is assumed that the $1/v$ law is applicable. The temperature coefficient at the temperature T is

16. Glasstone and Edlund, The Elements of Nuclear Reactor Theory (Van Nostrand Inc., 1952), p. 341.

obtained by setting $T = T_0$, so that

$$\left(\frac{\partial \rho}{\partial T}\right)_d = -\frac{B_g^2 L^2}{k_\infty T} \tag{37}$$

Density Temperature Coefficients-- An increase in temperature causes expansion of the reactor materials, and this affects the reactivity in two ways: first, by changing the mean free paths for absorption and scattering and, second, by an over-all change in the size of the system.

In a homogeneous system where density changes have the same effect on all components, it is possible to write

$$L^2 + \tau = L_0^2\left(\frac{d_0}{d}\right)^2 + \tau_0\left(\frac{d_0}{d}\right)^2 \tag{38}$$

or
$$M^2 = M_0^2\left(\frac{d_0}{d}\right)^2 \tag{39}$$

then substituting into equation (33) it is seen that

$$\rho = \frac{k_\infty - 1}{k_\infty} - \frac{B_g^2 M_0^2}{k_\infty}\left(\frac{d_0}{d}\right)^2 \tag{40}$$

If the volume is held constant, B_g^2 is constant, and if it is assumed that the microscopic cross sections are constant, then

$$\left(\frac{\partial \rho}{\partial T}\right)_{B^2,\sigma_a,\sigma_s} = \frac{2 B_g^2 M_0^2}{k_\infty} \cdot \frac{d_0^2}{d^3} \cdot \frac{\partial d}{\partial T} \tag{41}$$

If α is the coefficient of linear expansion of the material, that is, $l = l_0[1+\alpha(T-T_0)]$, then $V = V_0[1+\alpha(T-T_0)]^3$, so that

$$d = \frac{d_0}{[1+\alpha(T-T_0)]^3} \tag{42}$$

then
$$\frac{\partial d}{\partial T} = \frac{-3\alpha d_0}{[1+\alpha(T-T_0)]^4} \tag{43}$$

substituting for $\frac{\partial d}{\partial T}$ from (43) into (41) gives

$$\left(\frac{\partial \rho}{\partial T}\right)_{B^2,\sigma_a,\sigma_s} = -\frac{6 B_g^2 M_0^2 \alpha}{k_\infty} \cdot \frac{d_0^3}{d^3} \cdot \frac{1}{[1+\alpha(T-T_0)]^4} \tag{44}$$

At $T = T_0$ this becomes

$$\left(\frac{\partial \rho}{\partial T}\right)_{B^2,\sigma_a,\sigma_s} = -\frac{6 B_g^2 M_0^2 \alpha}{k_\infty} \tag{45}$$