A photoelastic investigation of three-dimensional contact stresses
by Douglas Craig Schafer

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
The object of this investigation was to produce an accurate analysis of the contact stress distribution on a three-dimensional body. The case presented in this thesis was a photoelastic study of an elastic body with surface discontinuities, loaded between rigid planes.

A model, shaped similar to a roller bearing, was machined from an epoxy resin and loaded under a constant weight. The "frozen stress" method was then used in an analysis of the strains (stress) in the body and in the contact region in particular. Contact stresses were calculated using a Fortran program of the shear-difference equations and source data obtained from photographs of the photoelastic stress patterns in model slices.

The results of this investigation were compared with theoretical predictions of contact stresses for a two-dimensional body of similar shape. The two-dimensional theory predicted higher contact stresses in the regions of discontinuities than were found actually to occur in the three-dimensional body.

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CHAPTER I

INTRODUCTION AND PROBLEM STATEMENT

A rather special problem facing research and design people today, particularly in the bearing industry, is the determination of contact stresses between elastic bodies. Solutions have been found by both theoretical and experimental means for three-dimensional contact stresses (17,5),[1] but these solutions are currently restricted to a small number of configurations. Unfortunately there is not available a general analytical procedure for solving the contact stress problem. The theory of elasticity does provide a system of differential equations that can be solved for three-dimensional stresses, but because of their complexity a solution is usually possible only for very simple shapes.

In most cases an exact analytical solution to the contact stress problem is unavailable, and so efforts to solve any particular contact problem often turn to the methods of experimental stress analysis. Since there is little chance of obtaining an accurate solution to an unsolved configuration by extrapolating available experimental solutions, each new contact problem will usually require a separate investigation. As might be expected, the number of experimentally-solved problems is small, which makes the need for a relatively simple and general analytical method of determining contact stresses obvious.

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[1] Numbers in parenthesis refer to the reference list at the end of the paper.

The methods of photoelasticity have proven to be the most satisfactory for experimentally determining the contact stress distribution in a three-dimensional body. The stresses are not usually found for the prototype or body of interest but must be analyzed in a model manufactured out of a special plastic. The stress distribution in the model can be shown to be identical to the stresses in the loaded body, subject to certain restrictions explained by Goodier (10).

This approach of using a model to determine the stresses in another loaded body raises important questions on how the model must be constructed to insure a similarity in the stress distributions. Obvious requirements are that the shape and loading of the model be dimensionally similar to that of the prototype. The exact requirements of model similarity have been developed in Appendix (A) by applying the principles of dimensional analysis; the results are expressed in equations which must be satisfied for the model and prototype to have proportional stress distributions. The proportionality of stress is expressed as

$$\frac{Sm}{Sp} = \frac{Em}{Ep}$$

Dimensional analysis provides a justification of photoelasticity, (which will be the experimental method used in this paper), and illustrates the required condition model similarity. In general, an experimental solution for a contact stress problem cannot be extended to other configurations. For instance, experimental solutions for contact stresses between spheres cannot be extended to find the contact stresses between cylinders. What is desired, then, is a general analytical method of finding a solution to the contact stress problem that is convenient and inexpensive.

An analytical technique for two-dimensional elasticity has been developed by Blackketter (1) that frees an analysis of contact stresses from the restraints imposed by the experimental method. Attempts are being made to expand this two-dimensional theory and to develop other analytical tools for accurately predicting three-dimensional contact stress distributions.

For the accuracy of any analytical scheme to be evaluated some reference is needed, some standard for comparison. Though the experimental method of determining stresses by photoelasticity has many shortcomings, it also has the outstanding strength of being quite accurate. Taking advantage of the accuracy of photoelasticity and using the "frozen stress" technique, this paper will present an investigation of contact stress on a three-dimensional body. The results will be used as a reference for an analytical solution using Blackketter's method for the stress distribution on a plane of the three-dimensional body. It is hoped that the comparison might substantiate Blackketter's theory for three-dimensions or at least suggest new approaches to an analytical solution.

CHAPTER II

THREE-DIMENSIONAL STRESS ANALYSIS BY PHOTOELASTICITY

2.1 An Outline of the Frozen Stress Method

In the nineteenth century, Maxwell, who was doing research on torsion, heated an isinglass cylinder and applied a torsional load to it. After permitting the cylinder to cool under load, he found that the strains remained after the load was removed and if the cylinder was placed in polarized light it exhibited a photoelastic effect. This effect was apparently dismissed until recently when M. Hetenyi developed many of the techniques associated with the frozen stress type of analysis.

The procedure of freezing stresses in a body for photoelastic purposes involves heating a plastic model to a certain temperature (referred to as the critical temperature), loading the model at this high temperature, and slowly cooling while maintaining the load. At room temperature the load is removed and the strains are found to be permanently locked or annealed in the model. It is known that these deformations represent an elastic distribution of stress if the yield point of the plastic was not exceeded at the elevated temperature. Slices removed from the model will not significantly distrub the original elastic distribution of stress, and when viewed in a polariscope, produce the same type of information as a two-dimensional photoelastic analysis. The isochromatic fringe patterns represent the difference in maximum and minimum normal stresses in the plane of the slice; these stresses are referred to as secondary principal stresses and are usually different from the principal stresses. The isochromatics result from a relative retardation of light waves passing through the slice and so are an integral effect closely approximating the stresses on the central plane of the slice. The photoelastic fringe pattern more accurately represents the shearing stress on a central plane of a slice as the slice is made thinner. It is particularly desirable to make the slices as thin as possible in regions of high stress gradient.

2.2 Properties of Plastics

The physical behavior of the plastic used in three-dimensional photoelasticity provides the basis for the experimental procedure. The requirements for a material to be used in a "frozen stress" analysis are that the strains produced in the model under load at the high temperature must remain in the plastic after cooling and all strains must represent an elastic stress distribution. The plastic should also be machinable into thin flat plates without disturbing the original stress distribution, and the strains in the model should duplicate strains in the prototype (model similarity).

The behavior of plastics that exhibit this property of locking in the strains is explained by a diphase theory. It is assumed that the plastic is structured of a completely polymerized internal skeleton of molecules and of a surrounding amorphous phase. The strength of the molecular skeleton does not change much with temperature, but at the critical temperature the amorphous phase becomes soft and carries only a minute portion of the total load. When the plastic model in this investigation was heated to its critical temperature of 280°F, the modulus of elasticity dropped from 500,000 psi to 2000 psi.

If the model is slowly cooled from the critical temperature under stress, the soft viscous component will solidify around the primary network holding or locking in the stress and displacements the model underwent when loaded at the high temperature. The cooled model has a stress system in the primary network balanced by stresses in the solidified viscous phase. Because these stresses are in equilibrium on a microscopic scale, sawing of the model will not appreciably disturb the displacements in the plastic. Any planes removed from such a model will exhibit the photoelastic effect, the only difference from the two-dimensional problem being that the birefringence is produced by the secondary principal stresses (principal stresses in the plane of the slice) instead of the principal stresses.

The properties of certain plastics that permit an elastic stress distribution to be frozen into a loaded model and slices to be cut from the model without disturbing the stress equilibrium make a three-dimensional analysis possible. Photoelastic data obtained from certain slices in conjunction with the shear-difference equations (see Chapter 4) make it possible to determine all six components of stress at any point in the body. In the case to be considered in this investigation, all that is required is one normal component -- the contact stress. Because it is not possible to determine this stress directly in the contact area, i.e., on the loaded surface, a somewhat indirect method as explained in Chapter 4 will be used.

2.3 Requirements of a "Frozen Stress" Analysis

Although the details of a photoelastic analysis will vary from problem to problem, the general procedure usually involves the construction of a model out of plastic, determining the optical properties of the plastic, loading and freezing a deformation into the plastic, slicing, recording the isoclinic and isochromatic fringe patterns, and the reduction of the photoelastic data into meaningful graphs. The basic equipment required for carrying out these steps in a three-dimensional analysis includes a polariscope, an annealing oven with programmed temperature control, loading frame, machine tools, polishing wheels and photographic equipment and darkroom.

CHAPTER III

MODEL CONFIGURATION AND FABRICATION

3.1 Model Configuration

The model configuration of a nearly right circular cylinder with end tapers that was used in this analysis is shown in Figure 1. The selection of a configuration took into account the limitations of the two-dimensional analytical solution that the results of this paper were to be compared with, optical and material properties of the plastic, and the difficulty of accurately machining the model.

Restraints on the two-dimensional theoretical development required that the maximum deviation from contact of the surface of the elastic body be of the order of the magnitude of the maximum strain displacement in the body. Also, the surface-defining equations must be continuous functions of position and their derivatives should be small. These limitations require that the slopes of all points on the surface be small but permit such complications as changes in curvature of the surface and transitions from a plane surface to a curved one.

3.2 Selection of a Plastic

Perhaps the most important performance feature of a photoelastic material is its optical sensitivity; this property is expressed as the amount of shearing stress required to produce a fringe. High optical sensitivity in a plastic is shown by a low fringe contrast. For instance, epoxy resin and plexiglass have material fringe con-

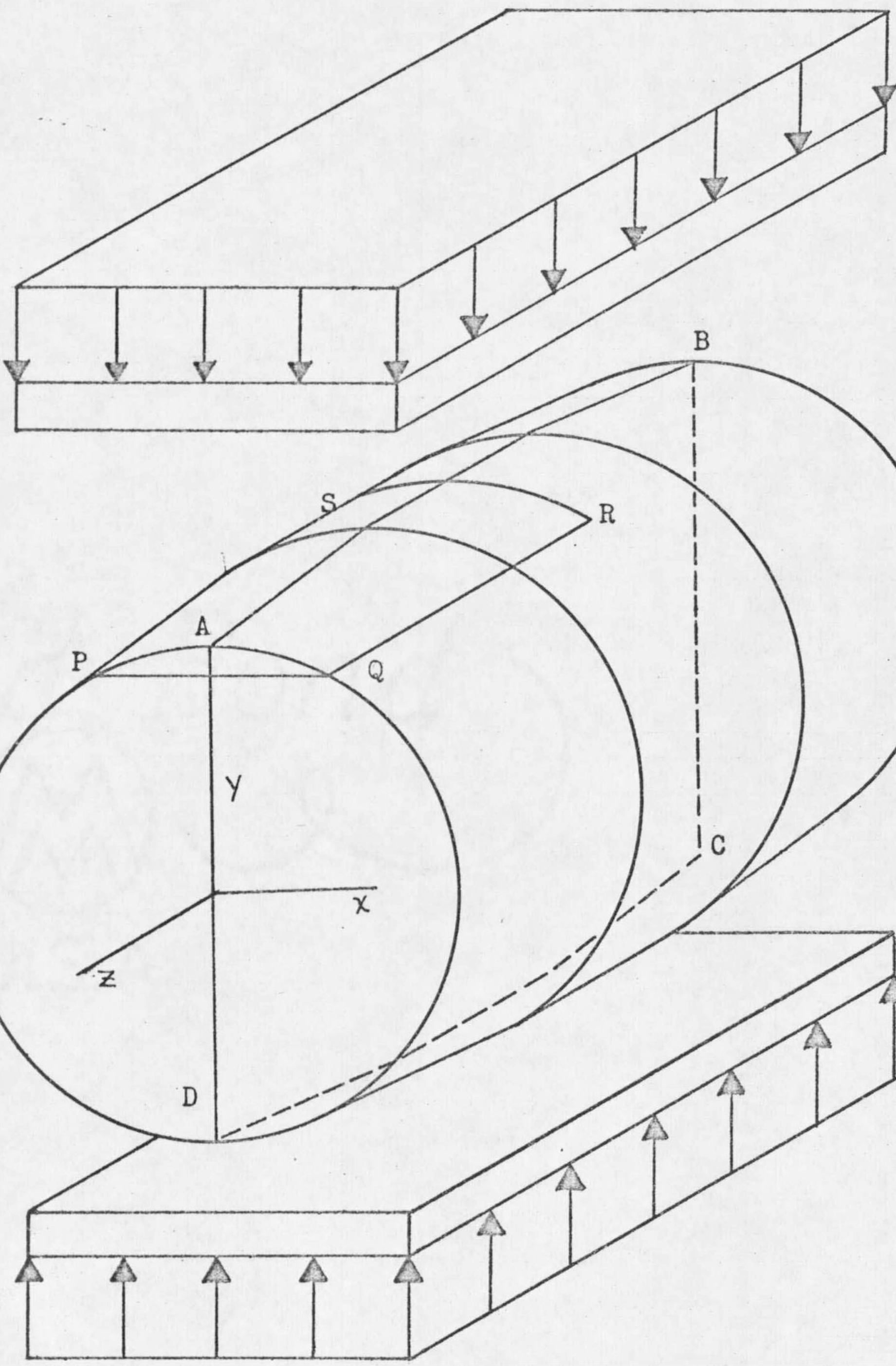

Fig. (1) Model Loading. Contact stresses were calculated by the two-dimensional method of ref. (1) for the central plane ABCD of the model. Plane PQRS is the longitudinal slice removed for the analysis.

stants, f, of 30 psi-in./fringe and 380 psi-in./fringe, respectively, showing that the epoxy is almost 13 times as sensitive to stress as plexiglass. The ratio of the material fringe constant to the modulus of elasticity of the material is a common way of evaluating the desirability of a three-dimensional photoelastic plastic. This ratio gives a term commonly referred to as the figure of merit.

$$Q = E/f$$

Q = Figure of Merit

Other qualities of a plastic being equal, the highest figure of merit indicates the most desirable model material.

An epoxy, Hysol 4290, was finally settled upon and ordered from the Hysol Corporation. The material, ordered in a cast cylinder 4 in. in diameter and 36 in. long, satisfied the above requirements in addition to other important qualities of a photoelastic material such as cost, availability, and a minimum of creep and time edge effects. The resin had a figure of merit of about 2000/.603 = 3320 at the critical temperature, which made it more desirable from the optical standpoint than any other material considered.

3.3 Model Fabrication

The model shape used in this investigation is shown in Figure 2. Ease in machining this model to close tolerances and a straightforward formulation of the analytical computations were deciding factors in choosing this model configuration. The contact