Wave propagation over a finite ground plane
by Leo Raymond Spogen

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the
degree of Master of Science in Electrical Engineering at Montana State College
Montana State University
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Abstract:
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especially with the increasing use of line of sight transmission. Each time a television broadcasting
antenna is placed at the top of a building a finite ground plane or an irregular ground plane must be
considered. An excellent example of radiation over a finite ground plane is radiation from an antenna
on an airplane.

Although practically all problems pertaining to propagation over a finite ground plane will consider
three dimensional surfaces and not plane surfaces, it is feasible that a thesis of this nature may be the
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disks® The author has attempted to obtain the patterns experimentally and then mathematically
verify the results.

In this thesis, the author has not only considered the effects of the discontinuity caused by the finiteness
of the ground plane but, also, the hole separating the antenna from the ground. The mathematical
results do not have close agreement with experimental results. The author has shown where errors can
exist and gives an explanation of how these errors affect results.
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LEO RAYMOND SPOGEN, JR.

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Leo P. Spogen Jr.
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The consideration of propagation over a finite ground plane is coming into prominence more each day especially with the increasing use of line-of-sight transmission. Each time a television broadcasting antenna is placed at the top of a building a finite ground plane or an irregular ground plane must be considered. An excellent example of radiation over a finite ground plane is radiation from an antenna on an airplane.

Although practically all problems pertaining to propagation over a finite ground plane will consider three dimensional surfaces and not plane surfaces, it is feasible that a thesis of this nature may be the stepping stone to future developments. The ground planes considered in this thesis were circular disks. The author has attempted to obtain the patterns experimentally and then mathematically verify the results.

In this thesis, the author has not only considered the effects of the discontinuity caused by the finiteness of the ground plane but also, the hole separating the antenna from the ground. The mathematical results do not have close agreement with experimental results. The author has shown where errors can exist and gives an explanation of how these errors affect results.
INTRODUCTION

A radiation pattern is a polar curve formed by plotting the electric field intensity at various angles, but at a constant distance from a radiating source. In order to have any meaning, the points must lie in a common plane. Patterns in the horizontal and vertical planes containing the origin are the most common.

Almost exclusively in literature, radiation patterns are obtained by considering propagation over an infinite ground plane. The object of this research was to obtain radiation patterns over a finite plane and attempt to support the experimental results mathematically.

Because of their symmetry, circular disks were used as the finite planes. The use of circular disks will produce circular horizontal patterns. Therefore, the vertical pattern in only one plane containing the origin need be considered.
The Use of Model Antennas

In studying radiation patterns experimentally, it is often difficult to obtain patterns at a required frequency since, in most practical cases, the wavelength will be comparatively large. This means that field measurements should be taken at a great distance, which would require either extremely sensitive detection devices or a rather large power input to the test antenna. Another apparent objection is finding an obstruction-free location for the test work.

The difficulty of test measurements at a required frequency has introduced the use of model antennas. A model test antenna is a miniature reproduction of the actual antenna to be tested. If the size of the model antenna is $A$ times the size of the actual antenna, then the frequency to be used in the testing of the model shall be the reciprocal of $A$ times the operating frequency. All dimensions under investigation shall be $A$ times the dimensions appearing under the actual conditions. The patterns obtained in this particular problem were obtained at only one frequency but, from model antenna reasoning, are applicable to all radio frequencies.

Test-Apparatus

The function of all components, other than the electrical components, are assumed to be apparent and, therefore, shall not be discussed. All components which are not mentioned as being part of Figure 2 or Figure 3 will appear in Figure 1.
FIGURE I
COMPLETE TEST APPARATUS
FIGURE 2
TEST ANTENNA AND ASSOCIATED COMPONENTS
FIGURE 3
RECEIVING ANTENNA AND METERING COMPONENTS
The klystron tube is used as an oscillator to provide the radio frequency. The particular klystron used operated at about 3000 megacycles. The klystron received its power from a power supply which consists of a regulated and a non-regulated supply in series. The maximum voltage obtainable at the output terminals of the power supply is about 1150 volts. The klystron is cooled by a fan. Power from the klystron is fed through a coaxial line to the test antenna and its associated components.

Figure 2 shows the test antenna and its associated components. The coaxial line from the klystron is fed to a stub tuner and then to the test antenna. The purpose of the stub tuner is to match the antenna to the line. All connections are made inside the collar which is detachable. The antenna is connected to the center conductor of the coaxial line by a slip fit and the ground plane to the outer conductor by a screw fitting. The antenna is approximately one-quarter wave length. The ground planes are made of copper and testing was done over planes of 2, 4, 5, 6, 8, and 10 centimeters in diameter. The collar, located below the ground plane, is a frustrum of a right circular cone designed to reflect all waves striking it. These reflected waves, therefore, cannot cause interference in the region in which the measurements were taken.

The receiving antenna and the measuring components are shown in Figure 3. The receiving antenna is supported from the arc by a non-flexible coaxial line. This line is connected to a quarter-wave stub and the stub to a crystal holder. The line from the crystal holder terminates at the parallel combination of a condenser and a direct current, 100
microampere ammeter. The receiving antenna is a half-wave dipole that receives radiation from the test antenna. The quarter-wave stub is in parallel with the series combination composed of the crystal and the parallel combination of meter and condenser. The crystal rectifies the radio frequency. The stub, being a short circuit to direct current, completes the d-c. circuit but has infinite impedance to the radio frequency. The purpose of the condenser is to by-pass the radio frequency components around the meter.

**Test Procedure**

The first step taken, to obtain data for the radiation patterns, was the alignment of the arc. This was done by use of a transit, by suspending a pointer by fine wire from the ninety degree mark to the top of the test antenna, and by adjusting the adjustable legs on the arc's vertical support.

Then the klystron was set into operation and adjusted for maximum output for the power supply voltage available. The stub tuner was adjusted for maximum and, last of all, the antenna height was varied for a maximum reading. For each step the author attempted to keep below the ground plane in order to prevent erroneous peaking due to reflections from his body.

A check on undesired reflections was made by observing changes in meter readings due to the presence of a large sheet of copper in the field. Reflections were checked at all possible locations. The Ballroom of the Student Union Building of Montana State College presented no apparent interference due to reflections.
The receiving antenna was then set in the desired radial position and its plane of polarization made parallel to the vertically polarized signal.

Readings were then taken at five degree increments. To assure that no errors occurred due to fluctuations of line voltage, a reference antenna was placed in the field and readings were taken only at a constant value of reference antenna measurement. The position of the reference antenna was fixed throughout each test run. The readings were taken in the following manner: First the author adjusted the receiving antenna at the desired angle. The author then lowered himself below the ground plane. When the reference antenna indicated its preassigned value, the author signalled his assistant and the assistant took the reading. The readings were taken as close to the antenna as reflections would allow. Even at this, the use of binoculars was found necessary. As soon as the measurements were taken from zero to ninety degrees the author and his assistant interchanged positions and the measurements were repeated. This procedure was then repeated for all six planes.

Since the rectified current through a crystal varies approximately as the square root of the voltage across it (for low current values) the square root of all readings must be taken before the pattern can be plotted. The experimentally obtained patterns appear in Appendix A, along with current readings and square root values for the 6 and 10 centimeter planes.
In a problem of this nature many assumptions must be made. The assumptions that shall be made are assumptions commonly made in the mathematical analysis of radiation patterns. Among these educated guesses are assumptions of the current distribution along the antenna, the assumption of an antenna of infinitesimal diameter, and the assumption that the ground plane is a perfect conductor.

The electric field intensity of a wave passing through any medium must be related to the magnetic field intensity by the intrinsic impedance. If the wave passes from one medium to another, the tangential components of the electric and the magnetic field must be continuous across the boundary. The normal components are not constrained in this manner. In passing across the boundary, the tangential components must still be related to each other by the intrinsic impedance, but the intrinsic impedance of the two mediums are, in general, not equal. Therefore, when a wave passes from one medium to another, in order that boundary conditions be satisfied, not all of the incident wave can continue on into the second medium. This means that reflection must occur. If a perfectly conducting ground plane is considered, the electric field intensity at the boundary must equal zero for all components since a perfectly conducting plane is a short circuit. Since the voltage and the resistance of the ground is equal to zero, the current in the plane is of indeterminant form. In other words, any value of surface magnetic field intensity may exist but, since
The reflected wave cannot be greater than the incident wave, the tangential magnetic field intensity cannot be less than zero or greater than twice the incident magnitude. Since no power can be transmitted into the perfectly conducting plane, the reflected wave must contain the same energy as that existing in the incident wave, i.e., perfect reflection. In order for the energy on the surface of the reflecting plane to equal zero, the incident and reflected components of energy must cancel. This implies, by the Poynting Vector, that the reflected magnetic intensity is equal to the incident magnetic intensity in direction and magnitude.

The derivation of the magnetic field intensity, at a distant point from a current carrying wire, may be obtained by first finding an expression for the vector magnetic potential at any point and then, by vector calculus, taking the curl to obtain an expression for the magnetic field intensity. The direction of the potential vector will be in the same direction as the current flow in the wire. The potential vector, however, will lag the current because of the finite time of propagation. If this current carrying wire extends normally from a perfectly conducting plane, then the vector potential will be normal to the plane and the magnetic field intensity shall be completely tangential. To satisfy the boundary conditions of a perfectly conducting plane simply means that the reflected vector potential be equal to the incident vector potential. If the ground plane is finite, then the vector magnetic potential satisfies the boundary conditions to and at the discontinuity and no reflection occurs past this new boundary.
The general expression for the vector magnetic potential is
\[ A = \int \frac{i dV}{4\pi r} \]  
(1)

where \( i \) is the current density and the differential \( dV \) indicates a volume integration. If the current carrying conductor is straight and of infinitesimal diameter, then the equation for the vector potential becomes
\[ A = \int \frac{I dz}{4\pi r} \]  
(2)

where the current is considered flowing in the \( z \) direction. The current \( I \) will be a space function and in the case of a time varying current will be a function of both space and time. In the distant field, the potential is time retarded because of the finite time of propagation. The time lag at a distant point will be the distance to that point divided by the velocity of the wave. In free space, the velocity of radio waves is practically \( 3 \times 10^8 \) meters per second. The phase shift caused by the time lag will be the product of the time lag and the angular velocity of the wave.

The foregoing discussion has shown how assumptions of a perfectly conducting ground plane and an antenna of infinitesimal diameter simplifies radiation problems. Another very important consideration that must be made before a mathematical solution for the vector magnetic potential can be obtained is the current distribution along the antenna. The assumption commonly made for a wire antenna is that the standing current wave is sinusoidal. The assumption was arrived at from the transmission line theory of an open circuited line.
Solution for Vector Magnetic Potential in Integral Form

The current in the antenna varies sinusoidally along the antenna and shall be considered to vary cosinusoidally with time. The expression for the antenna current of an antenna of height $H$ at any point $l$ from the ground plane, Figure 4, is

$$I = I_m \sin \frac{2\pi}{\lambda} (H-l) \cos \omega t$$

(3)

At a distant point $P$, the vector magnetic potential is time retarded and the expression of current that must be used in solving for the vector potential becomes

$$I = I_m \sin \frac{2\pi}{\lambda} (H-l) \cos \omega (t - \frac{r}{c})$$

(4)

For the direct wave, $r$ is the distance from the point $l$ on the antenna to the point $P$ in space, and for the direct wave this distance shall be designated as $r_d$.

![Diagram](image)

If a cylindrical coordinate system is chosen with the antenna on the $z$ axis and the base of the antenna as the origin then the distance $r_d$ by the Pythagorean Theorem is

$$r_d = \sqrt{(z-1)^2 + \rho^2}$$

(5)
For the reflected wave, the distance \( r \) in Equation (4) is the distance from \( I \) to the ground plane plus the distance from the ground plane to point \( P \), remembering that the angle of incidence must equal the angle of reflection, Figure 5. For the reflected wave this distance shall be designated as \( r_r \).

The reflected wave, as seen from Figure 5, travels through the same distance as though it were passing through the ground plane and to the point \((P, P_z)\). The equation for \( r_r \) is

\[
r_r = \sqrt{(z + l)^2 + \rho^2}
\]  

(6)

The equation for the differential magnetic vector potential of the direct wave, from a differential length, along the antenna, is

\[
d A_{zd} = \frac{I_m \sin \frac{2\pi}{\lambda} (H-l) \cos \omega(t - \sqrt{\frac{(z-l)^2 + \rho^2}{v}})}{4\pi \sqrt{(z-l)^2 + \rho^2}} d l
\]  

(7)
The potential equation for the reflected wave is

\[ d \mathbf{A}_{\text{zR}} = \frac{I_m \sin \frac{2\pi}{\lambda} (H-1) \cos \omega(t - \frac{\sqrt{(z+1)^2 + \rho^2}}{v})}{4\pi \sqrt{(z+1)^2 + \rho^2}} \, d l \] (8)

To find the total vector potential of the direct wave requires integration of the differential vector potential between the limits of zero and \( H \), where \( H \) is the antenna height. The reflected wave, however, will not contain the same limits of integration because of the hole in the ground plane around the antenna and the outer periphery of the ground plane. These limits may be obtained from Figure 6, where \( R \) is the radius of the hole and \( L \) the distance to the outside edge.

![Figure 6](image)

The current flowing from zero to \( q \) will not reflect the vector potential, the current from \( q \) to \( u \) will reflect a vector potential, and from \( u \) to \( H \) will not reflect. The upper limit of integration will be \( u \) and the lower limit \( q \). Using similar triangles, the relationship exist that

\[ \frac{q}{q + z} = \frac{R}{\rho} \]
The equation for $q$ becomes

$$q = \frac{Rz}{\rho - R}$$

(9)

By exactly the same procedure, the upper limit $u$ is found by the equation

$$u = \frac{Lz}{\rho - L}$$

(10)

$u$ and $q$ cannot exceed $H$. The upper limit, when $\frac{Lz}{\rho - L} \geq H$ is $H$ and the lower limit when $\frac{Rz}{\rho - R} \geq H$ is also $H$. This implies that the functions of $q$ and $u$ are continuous but that their derivatives are discontinuous. These derivatives, therefore, will not exist at the point of discontinuity but will exist on either side of that point. The behavior of the derivatives will later explain the existence of the plus and minus points existing in the calculated patterns.

The expression for the direct vector magnetic potential will be, with the substitution that $\frac{w}{v} = \frac{2\pi}{\lambda}$.

$$A_{zd} = \frac{I_m}{4\pi} \int_{0}^{H} \frac{\sin \frac{2\pi}{\lambda} (H-l) \cos \frac{2\pi}{\lambda} (vt - \sqrt{(z-l)^2 + \rho^2})}{\sqrt{(z+1)^2 + \rho^2}} \, dl$$

(11)

The reflected vector magnetic potential will be

$$A_{zr} = \frac{I_m}{4\pi} \int_{u}^{q} \frac{\sin \frac{2\pi}{\lambda} (H-l) \cos \frac{2\pi}{\lambda} (vt - \sqrt{(z+l)^2 + \rho^2})}{\sqrt{(z+1)^2 + \rho^2}} \, dl$$

(12)

The behavior of $u$ and $q$ must be realized at all times.
Method of Integration

The integrals contained in the expressions for the vector potentials of the direct and the reflected waves, require simplification before an easy method of integration can be obtained.

Figure 7 will show the means by which integrands can be changed to an integrable form. A requirement necessary for the simplification is that the distance $r$ is much greater than $H$. Then, $\frac{1}{\sqrt{(z+1)^2 + \rho^2}} \approx \frac{1}{r}$.

$r$ cannot be substituted for $\sqrt{(z+1)^2 + \rho^2}$ in the time varying functions because the addition or subtraction of 1 may cause considerable phase shift.

![Diagram](attachment:image.png)

The distance $r_d$ and $r_r$ were determined previously to be $\sqrt{(z-1)^2 + \rho^2}$ and $\sqrt{(z+1)^2 + \rho^2}$ respectively. From Figure 7 the distance $r_d$ is $r - d_1$ and $r_r$ is $r + d_2$. Since the angles, $\alpha$ and $\beta$ are actually very small $\gamma \approx \varepsilon \approx \theta$. $d_1$ and $d_2$ therefore are equal to $l \cos \theta$.

The equations for the direct and reflected vector magnetic potentials become
\[ A_{zd} = \frac{I_m}{4\pi r} \int_{0}^{H} \sin \frac{2\pi}{\lambda} (H-l) \cos \frac{2\pi}{\lambda} (vt - r + l \cos \theta) \, dl \]  
and
\[ A_{zr} = \frac{I_m}{4\pi r} \int_{q}^{u} \sin \frac{2\pi}{\lambda} (H-l) \cos \frac{2\pi}{\lambda} (vt - r - l \cos \theta) \, dl \]

By use of the trigonometric identity

\[ \sin A \cos B = \frac{1}{2} \left[ \sin (A+B) + \sin (A-B) \right] \]

the equation for the reflected potential becomes

\[ A_{zr} = \frac{I_m}{8\pi r} \int_{q}^{u} \sin \frac{2\pi}{\lambda} \left[ vt - r + H - l(1+\cos \theta) \right] 
+ \sin \frac{2\pi}{\lambda} \left[ (-vt + r) + H-l (1-\cos \theta) \right] \, dl \]  

The integrand now is easily integrated and the vector potential becomes

\[ A_{zr} = -\frac{I_m}{16\pi^2 r^2} \left\{ \cos \frac{2\pi}{\lambda} \left[ (vt-r) + H-l (1+\cos \theta) \right] \right. 
+ \cos \frac{2\pi}{\lambda} \left[ (-vt+r) + H-l (1-\cos \theta) \right] \left. \right\} \int_{q}^{u} \]

By finding a common denominator for \( A_{zr} \) and making the trigonometric substitutions

\[ \cos (A+B) = \cos A \cos B - \sin A \sin B, \cos (-D) = \cos D \]

and

\[ \sin (-D) = -\sin D \]
the equation becomes

\[
A_{zr} = - \frac{\lambda I_m}{16\pi^2 r} \left[ \cos \frac{2\pi}{\lambda} (vt-r) \left\{ \begin{array}{l}
(1-\cos\theta) \cos \frac{2\pi}{\lambda} (H-1) (1+\cos\theta) \\
+ (1+\cos\theta) \cos \frac{2\pi}{\lambda} (H-1) (1-\cos\theta) \end{array} \right\} \\
- \frac{\sin \frac{2\pi}{\lambda} (vt-r)}{\sin^2 \theta} \left\{ \begin{array}{l}
(1-\cos\theta) \sin \frac{2\pi}{\lambda} (H-1) (1+\cos\theta) \\
- (1+\cos\theta) \sin \frac{2\pi}{\lambda} (H-1) (1-\cos\theta) \end{array} \right\} \right] \]
\]

Since

\[
\sin x \pm \sin y = 2 \sin \frac{1}{2} (x \pm y) \cos \frac{1}{2} (x \pm y) \\
\cos x + \cos y = 2 \cos \frac{1}{2} (x + y) \cos \frac{1}{2} (x - y)
\]

and

\[
\cos x - \cos y = -2 \sin \frac{1}{2} (x + y) \sin \frac{1}{2} (x - y)
\]

\(A_{zr}\) can be reduced to

\[
A_{zr} = - \frac{I_m \lambda}{8\pi^2 r} \left[ \cos \frac{2\pi}{\lambda} (vt-r) \left\{ \begin{array}{l}
\cos \frac{2\pi}{\lambda} (H-1) \cos \frac{2\pi}{\lambda} (1+\cos\theta) \\
- \cos\theta \sin \frac{2\pi}{\lambda} (H-1) \sin \frac{2\pi}{\lambda} (1+\cos\theta) \end{array} \right\} \\
+ \frac{\sin \frac{2\pi}{\lambda} (vt-r)}{\sin^2 \theta} \left\{ \begin{array}{l}
\cos \frac{2\pi}{\lambda} (H-1) \sin \frac{2\pi}{\lambda} (1+\cos\theta) \\
+ \cos\theta \sin \frac{2\pi}{\lambda} (H-1) \cos \frac{2\pi}{\lambda} (1+\cos\theta) \end{array} \right\} \right] \]
\]

The equation for the reflected magnetic vector potential, therefore, is
A_{2r} = \frac{I_m \lambda}{8\pi^2 r} \begin{cases} 
\cos \frac{2\pi}{\lambda} (vt-r) \left\{ \cos \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \\
- \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \\
- \cos \theta \sin \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \\
+ \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \right\} \\
+ \sin \frac{2\pi}{\lambda} (vt-r) \left\{ \cos \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \\
- \cos \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \\
+ \cos \theta \sin \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \\
- \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \right\} \end{cases}

By practically the same procedure the equation for the direct magnetic potential becomes

A_{2d} = -\frac{I_m \lambda}{8\pi^2 r} \begin{cases} \cos \frac{2\pi}{\lambda} (vt-r) \left\{ \cos \frac{2\pi}{\lambda} (H \cos \theta) - \cos \frac{2\pi}{\lambda} \right\} \left\{ \frac{\cos \frac{2\pi}{\lambda} (H \cos \theta) - \cos \frac{2\pi}{\lambda} \right\} \sin^2 \theta \\
+ \sin \frac{2\pi}{\lambda} (vt-r) \left\{ -\sin \frac{2\pi}{\lambda} (H \cos \theta) + \cos \theta \sin \frac{2\pi}{\lambda} \right\} \sin^2 \theta \end{cases}

The total magnetic vector potential at a point is the vector sum of the reflected and the direct waves. Since the direct and reflected vectors have identical direction the arithmetic sum is used.

**Solution for the Magnetic Field Intensity**

The curl of the vector magnetic potential is the magnetic field
intensity. The complete vector magnetic potential is

\[
A_z = -\frac{I_m}{8\pi^2 r} \left[ \cos \frac{2\pi}{\lambda} (v t-r) \right] \sin^2 \theta \left\{ \cos \frac{2\pi}{\lambda} (H \cos \theta) \\
- \cos \frac{2\pi}{\lambda} H + \cos \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \\
- \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \\
- \cos \theta \sin \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \\
+ \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \right\} + \sin \frac{2\pi}{\lambda} (v t-r) \left\{ -\sin \frac{2\pi}{\lambda} (H \cos \theta) \\
+ \cos \theta \sin \frac{2\pi}{\lambda} + \cos \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \\
+ \cos \theta \sin \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \\
- \cos \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \\
- \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \right\} \right] \tag{21}
\]

In a spherical coordinate system, the components of the curl of a vector are

\[
\text{Curl}_r A = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta A \phi) - \frac{\partial A_\theta}{\partial \phi} \right] \tag{22}
\]

\[
\text{Curl}_\theta A = \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial r A_\theta}{\partial \phi} \tag{23}
\]

\[
\text{Curl}_\phi A = \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \tag{24}
\]

The vector \( A_z \) will contain components in the \( r \) and the \( \theta \) direction. These components are
\[ A_r = A_z \cos \theta \]  
\[ A_\theta = -A_z \sin \theta \]  

Since \( A_\phi = 0 \) and \( A_r \) and \( A_\theta \) are independent of \( \phi \), the magnetic field intensity is entirely in the \( \phi \) direction. If the \( r \) and the \( \theta \) components of \( A_z \) are substituted into Equation (24), \( H_\phi \) becomes

\[ H_\phi = \frac{1}{r} \left[ -\frac{\partial}{\partial r} (r A_r \sin \theta) - \frac{\partial A_z}{\partial \theta} \right] \tag{27} \]

which can be reduced to

\[ H_\phi = -\left[ \sin \theta \frac{\partial A_z}{\partial r} + \frac{\cos \theta}{r^2} \frac{\partial^2 A_z}{\partial \theta^2} \right] \tag{28} \]

A radiation pattern consists of three different fields. They are the electrostatic field, the induction field, and the distant field. Electrostatic terms vary as the reciprocal of \( r^3 \), the induction field as the reciprocal of \( r^2 \), and the distant field as the reciprocal of \( r \).

Since practically all propagation problems are concerned with the far field, only terms divided by \( r \) shall be retained in the space differentiation.

In polar coordinates, when \( q \leq H \) and \( u \leq H \),

\[ u = \frac{Ir \cos \theta}{r \sin \theta - L} \tag{29} \]

\[ q = \frac{Rr \cos \theta}{r \sin \theta - L} \tag{30} \]

Their derivations, with respect to \( \theta \), are
\[
\frac{du}{d\theta} = -L \frac{r^2 + L^2 \cos \theta}{(r \sin \theta - L)^2} \quad \text{(31)}
\]
\[
\frac{dq}{d\theta} = -R \frac{r^2 + R^2 \cos \theta}{(r \sin \theta - R)^2} \quad \text{(32)}
\]

Since \( r \) is contained in the numerator of both \( u \) and \( q \) and their derivatives, when \( q \leq H \) and \( u \leq H \), the terms, in \( \frac{\cos \theta}{r} \frac{\partial A_z}{\partial \theta} \), multiplied by \( q \) or \( u \) or their derivative will be part of the distant field. The distant field terms obtained from \( \frac{\cos \theta}{r} \frac{\partial A_z}{\partial \theta} \) when \( u \leq H \) and \( q \leq H \) are

\[
\frac{I_m}{4\pi r^2} \left[ \cos \frac{2\pi}{\lambda} (vt-r) \cos \theta \left\{ \frac{du}{d\theta} \sin \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \right. \right.
\]
\[
+ \left[ u \sin \theta - \cos \theta \frac{du}{d\theta} \cos \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \right. \]
\[
\left. - \frac{du}{d\theta} \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \right.
\]
\[
- \left[ q \sin \theta - \cos \theta \frac{du}{d\theta} \cos \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \right. \]
\[
\left. + \frac{du}{d\theta} \cos \theta \cos \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \right. \]
\[
\left. - \left[ u \sin \theta - \cos \theta \frac{du}{d\theta} \cos \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \right. \right. \]
\[
\left. + \left[ q \sin \theta - \cos \theta \frac{du}{d\theta} \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \right. \right. \]
\[
\left. + \frac{du}{d\theta} \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \right. \right. \]
\[
\left. + \sin \frac{2\pi}{\lambda} (vt-r) \cos \theta \left\{ \frac{du}{d\theta} \sin \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \right. \right. \]
\[
\left. - \left[ u \sin \theta - \cos \theta \frac{du}{d\theta} \cos \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \right. \right. \]
\[
\left. - \frac{du}{d\theta} \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \right. \right. \]
\[
+ \left[ q \sin \theta - \cos \theta \frac{du}{d\theta} \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \right. \right. \]
\[
\left. - \frac{du}{d\theta} \cos \theta \cos \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \right. \right. \]
\[
+ \left[ u \sin \theta - \cos \theta \frac{du}{d\theta} \cos \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \right. \right. \]
\[
\left. + \frac{du}{d\theta} \cos \theta \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \right. \right. \]
\[-27\]

\[-\left[ q \sin \theta - \cos \theta \frac{du}{dr} \right] \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \right] \] 

The distant field, when \( q \leq H \) and \( u \leq H \), obtained from 

\[- \sin \theta \frac{\Delta z}{dr} \text{ are} 

\[ \frac{I_m}{4\pi r} \left( \cos \frac{2\pi}{\lambda} (vt-r) \right) \left\{ \frac{du}{dr} \sin \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \right. 

- \frac{du}{dr} \cos^2 \theta \sin \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) 

- \frac{du}{dr} \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) 

+ \frac{dq}{dr} \cos^2 \theta \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) 

- \cos \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) 

- \cos \theta \sin \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) 

+ \cos \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) 

+ \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) 

+ \sin \frac{2\pi}{\lambda} (H \cos \theta) \right. 

\left. - \cos \theta \sin \frac{2\pi}{\lambda} H \right\} \]

\[ + \frac{\sin \frac{2\pi}{\lambda} (vt-r)}{\sin \theta} \left\{ \frac{du}{dr} \sin \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \right. 

- \frac{du}{dr} \cos^2 \theta \sin \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) 

- \frac{du}{dr} \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) 

+ \frac{dq}{dr} \cos^2 \theta \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) 

+ \cos \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) 

- \cos \theta \sin \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) 

- \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) 

+ \cos \theta \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) 

+ \cos \frac{2\pi}{\lambda} (H \cos \theta) \right. 

\left. - \cos \frac{2\pi}{\lambda} H \right\} \]

\[ (34) \]
In both parts of the distant field $H_\phi$, the terms have not been collected so the reader may be able to follow the differentiation.

When $u \leq H$ and $q \leq H$,

\[
H_\phi = \frac{I_m}{4\pi r} \left[ \cos \frac{2\pi}{\lambda} (vt-r) \right] \left\{ -A(L) \sin \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \\
-B(L) \cos \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \\
+A(R) \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \\
+B(R) \cos \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \\
+ \frac{\sin \frac{2\pi}{\lambda} (H \cos \theta)}{\sin \theta} - \frac{\cos \theta \sin \frac{2\pi}{\lambda} H}{\sin \theta} \right\} \\
+ \sin \frac{2\pi}{\lambda} (vt-r) \left\{ -C(L) \sin \frac{2\pi}{\lambda} (H-u) \sin \frac{2\pi}{\lambda} (u \cos \theta) \\
-D(L) \cos \frac{2\pi}{\lambda} (H-u) \cos \frac{2\pi}{\lambda} (u \cos \theta) \\
+C(R) \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) \\
+D(R) \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) \\
+ \frac{\cos \frac{2\pi}{\lambda} (H \cos \theta)}{\sin \theta} - \frac{\cos \frac{2\pi}{\lambda} H}{\sin \theta} \right\} \right\} \right|^{(35)}
\]
When \( q \leq H \) and \( u = H \), \( \frac{du}{d\theta} \) and \( \frac{du}{dr} \) will equal zero.

\[
H \phi = \frac{Im}{4\pi r} \left[ \cos \frac{2\pi}{\lambda} (vt-r) \left\{ A(R) \sin \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) 
\right.
\right.
\]
\[
+ B(R) \cos \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) - \cos \theta \sin \frac{2\pi}{\lambda} H \\
\left. \left( \frac{\sin \theta}{\sin \theta} \right) \right] \\
\left. \sin \theta \right]
\]
\[
+ \sin \frac{2\pi}{\lambda} (vt-r) \left\{ C(R) \sin \frac{2\pi}{\lambda} (H-q) \sin \frac{2\pi}{\lambda} (q \cos \theta) 
\right.
\]
\[
+ D(R) \cos \frac{2\pi}{\lambda} (H-q) \cos \frac{2\pi}{\lambda} (q \cos \theta) 
\right. \\
\left. \frac{2 \cos \frac{2\pi}{\lambda} (H \cos \theta)}{\sin \theta} - \frac{\cos \frac{2\pi}{\lambda} H}{\sin \theta} \right] 
(36)
\]

When \( q = H \) and \( u = H \), \( \frac{du}{d\theta} \), \( \frac{du}{dr} \), \( \frac{dq}{dr} \) and \( \frac{dq}{d\theta} \) will equal zero.

\[
H \phi = \frac{Im}{4\pi r} \left[ \cos \frac{2\pi}{\lambda} (vt-r) \left\{ \sin \frac{2\pi}{\lambda} (H \cos \theta) - \cos \theta \sin \frac{2\pi}{\lambda} H 
\right. 
\right.
\]
\[
+ \frac{\sin \frac{2\pi}{\lambda} (vt-r)}{\cos \theta} \left\{ \cos \frac{2\pi}{\lambda} (H \cos \theta) - \cos \frac{2\pi}{\lambda} H \right\} 
\right] 
(37)
\]

The functions of \( L \) and \( R \), namely \( A(L), B(L), C(L), D(L), A(R), B(R), C(R), \) and \( D(R) \) are obtained merely by adding Equations (33) and (34), collecting terms, and making simplifying trigonometric substitutions.

The functions of \( L \) are exactly the same as the functions of \( R \), except the \( L \) has been substituted for \( R \). These functions, therefore, shall be referred to as functions of \( F \) to eliminate repetition.
These coefficients are defined as

\[
A(F) = \frac{1}{(r \sin \theta - F)^2} \left\{ \frac{r^2 \sin 2 \theta}{2} + F r (1-\cos \theta)^2 + 2 \cot^2 \theta + F^2 \frac{\sin 2 \theta}{2} (\sin^2 \theta - 2 \cos \theta + 1 - \frac{1}{\cos \theta}) \right\}
\]

\[
B(F) = \frac{1}{(r \sin \theta - F)^2} \left\{ r^2 \sin \theta - Fr (2+\cos \theta + \cot \theta) \right\}
\]

\[
C(F) = \frac{1}{(r \sin \theta - F)^2} \left\{ \frac{r^2 \sin 2 \theta}{2} + Fr (\sin^2 \theta - 2 \cos \theta) + F^2 \sin \theta (\cos \theta - 1) + \cot \theta (\cos \theta + 1) \right\}
\]

\[
D(F) = \frac{1}{(r \sin \theta - F)^2} \left\{ r + F \frac{(\cos \theta + 1)}{\sin \theta} \right\}
\]

The Electric Field Intensity

The electric field intensity must be obtained before the radiation pattern can be determined. The resultant electric field intensity is the product of the intrinsic impedance and the resultant magnetic field intensity. Maxwell's first equation states that

\[
\text{Curl} \, H = \Phi = \varepsilon \cdot \dot{E}
\]

where \( \Phi \) is the electric displacement density and the dot represents the first derivative with respect to time. From Equations (22), (23) and (24), it is evident that the electric field intensity can contain components only in the \( r \) and the \( \theta \) direction since the magnetic field
intensity is entirely in the $\phi$ direction. Since the equation for the magnetic field intensity is very complex, it shall be assumed that the $r$ component is negligible. This assumption is based on the fact that for an infinite ground plane the $r$ component is zero. If this assumption is made and is correct then

$$E_\theta = \eta H_\phi$$

(43)

where $\eta$ is the intrinsic impedance which for free space conditions is equal to $120\pi$. The equation for the electric field intensity can now be written as

$$E_\theta = \frac{30\ I_m}{r} \left[ A \cos \frac{2\pi}{\lambda} (vt-r) + B \sin \frac{2\pi}{\lambda} (vt-r) \right]$$

(44)

$$E_\theta = \frac{30\ I_m}{r} \sqrt{A^2 + B^2} \sin \frac{2\pi}{\lambda} (vt-r + \tan^{-1} \frac{A}{B})$$

(45)

The effective value of the electric field intensity is

$$E_{\theta\ eff} = \frac{707 \times 30\ I_m}{r} \sqrt{A^2 + B^2}$$

(46)

The radiation pattern is given by the expression $\sqrt{A^2 + B^2}$. Because of the number of terms contained in $A$ and $B$, it is best to obtain numerical results for $A$ and $B$ before squaring.

The calculated radiation patterns for the 6 and 10 centimeter planes appear in Appendix B along with the calculated values of $q_3$, $u$, $A$, $B$, and $\sqrt{A^2 + B^2}$. The points, because of the complexity of $A$ and $B$, were only calculated for ten degree increments and at the angle at which
discontinuities appear. The angles at which the discontinuities occur are found by setting \( \frac{R \cos \theta}{r \sin \theta - R} \) and \( \frac{L \cos \theta}{r \sin \theta - L} \) equal to \( H \). At \( 0^\circ \) the radiation pattern is in indeterminate form but has zero value.
DISCUSSION OF RESULTS

A comparison of the patterns contained in Appendix A to those contained in Appendix B is fairly good for the 10 centimeter plane. The patterns for the 6 centimeter plane do not show as good a comparison. In searching for the reason for the disagreement, the possible location of errors should be reviewed.

The patterns obtained experimentally contain the following errors, which were not controllable by the author:

1. The coaxial line radiates directly from its termination at the ground plane. This should give errors at small angles.

2. Because of the limited voltage supply, the distance from the test antenna to the receiving antenna was only eight and one-third wave lengths. When the induction field terms are obtained from the vector magnetic potential, some terms will be divided by \( \sin^3 \theta \). At small values of \( \theta \), the terms containing \( \sin^3 \theta \) may be appreciable.

3. A certain amount of undesirable reflection is bound to occur. It is difficult to determine just where these errors will exist.

4. The receiving antenna, to pick up the electric intensity at a point, should be extremely small or the distance from the test antenna extremely large. Neither of these were the case. The appreciable errors will appear where the change
of electric field intensity with respect to $\theta$ is large.

A suggestion for future work in obtaining experimental patterns is to let the test antenna be the receiving antenna receiving radiation from an antenna in front of a reflecting parabola. This is an example of the reciprocal theorem. Most of the errors previously mentioned would be eliminated by this method. Undesirable reflections would be practically eliminated because of the directivity of the reflector. The distance from the test antenna could be made greater, again because of the directivity. Radiation from the coaxial line would not exist because the only power in the line would be power received by the antenna under test.

The mathematical results contain double value points at two angles because of the discontinuous derivatives of $u$ and $q$ with respect to $\theta$. From the values of $u$ and $q$, included in Appendix B, it is clearly evident how the slope varies and how and why the double values appear. Because the curves were plotted at ten degree increments, maximums and minimums may actually exist, but do not appear in the radiation patterns. The pattern for the 10 centimeter plane varies from the pattern obtained experimentally in the range of smaller angles. If the radius of the hole is allowed to approach zero, the pattern becomes a better comparison to the experimental pattern. This indicates that the diameter of the antenna is appreciable or the effective size of the hole is smaller than the actual size.

When the image antenna is considered, the current distribution will be sinusoidal from $u$ to $q$, but the charge existing on the image should
be the negative of that on the antenna. Since the area beneath the current distribution curve is proportional to the effective charge, the image charge is a better comparison to that of the antenna when the ground plane is large. This explains why the results were better for the 10 centimeter plane than for the 6 centimeter plane.

Another assumption that was made was that the electric intensity in the \( r \) direction was negligible. The electric intensity in the \( r \) direction is known to exist and, therefore, may be appreciable. This could produce a definite error because then the \( \theta \) component of electric intensity would not be related to the magnetic field intensity by the intrinsic impedance.
## EXPERIMENTAL RESULTS

Measured Current, Square Root Value, and Radiation Patterns

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<th>L = 10 cm</th>
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Figure 8

Experimental patterns obtained over circular ground planes of (a) 2, (b) 4, (c) 5, (d) 6, (e) 8, and (f) 10 centimeters in diameter.
Figure 3 (Cont'd.)
Figure 3 (Con'td.)
### APPENDIX B

**MATHEMATICAL RESULTS**

<table>
<thead>
<tr>
<th>Degrees</th>
<th>( u )</th>
<th>( q )</th>
<th>( A )</th>
<th>( B )</th>
<th>( \sqrt{A^2 + B^2} )</th>
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<table>
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<tr>
<th>Degrees</th>
<th>( u )</th>
<th>( q )</th>
<th>( A )</th>
<th>( B )</th>
<th>( \sqrt{A^2 + B^2} )</th>
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Mathematically derived patterns for radiation over circular ground planes of (a) 6 and (b) 10 centimeters in diameter.
LITERATURE CITED AND CONSULTED


Ramo, Simon and Winnery, John R., 1944. FIELDS AND WAVES IN MODERN RADIO, John Wiley and Sons, Inc., New York


AUTHOR:
Sporen, Leo Raymond

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