Wave propagation over a finite ground plane
by Leo Raymond Spogen

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering at Montana State College
Montana State University

Abstract:
The consideration of propagation over a finite ground plane is coming into prominence more each day especially with the increasing use of line of sight transmission. Each time a television broadcasting antenna is placed at the top of a building a finite ground plane or an irregular ground plane must be considered. An excellent example of radiation over a finite ground plane is radiation from an antenna on an airplane.

Although practically all problems pertaining to propagation over a finite ground plane will consider three dimensional surfaces and not plane surfaces, it is feasible that a thesis of this nature may be the stepping stone to future developments. The ground planes considered, in this thesis were circular disks® The author has attempted to obtain the patterns ex= perimentally and then mathematically verify the results.

In this thesis, the author has not only considered the effects of the discontinuity caused by the finiteness of the ground plane but, also, the hole separating the antenna from the ground. The mathematical results do not have close agreement with experimental results. The author has shown where errors can exist and gives an explanation of how these errors affect results.

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Leo R. Spogen Jr.

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ABSTRACT

The consideration of propagation over a finite ground plane is coming into prominence more each day especially with the increasing use of line-of-sight transmission. Each time a television broadcasting antenna is placed at the top of a building a finite ground plane or an irregular ground plane must be considered. An excellent example of radiation over a finite ground plane is radiation from an antenna on an airplane.

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INTRODUCTION

A radiation pattern is a polar curve formed by plotting the electric field intensity at various angles, but at a constant distance from a radiating source. In order to have any meaning, the points must lie in a common plane. Patterns in the horizontal and vertical planes containing the origin are the most common.

Almost exclusively in literature, radiation patterns are obtained by considering propagation over an infinite ground plane. The object of this research was to obtain radiation patterns over a finite plane and attempt to support the experimental results mathematically.

Because of their symmetry, circular disks were used as the finite planes. The use of circular disks will produce circular horizontal patterns. Therefore, the vertical pattern in only one plane containing the origin need be considered.

OBTAINING EXPERIMENTAL PATTERNS

The Use of Model Antennas

In studying radiation patterns experimentally, it is often difficult to obtain patterns at a required frequency since, in most practical cases, the wave length will be comparatively large. This means that field measurements should be taken at a great distance, which would require either extremely sensitive detection devices or a rather large power input to the test antenna. Another apparent objection is finding an obstruction-free location for the test work.

The difficulty of test measurements at a required frequency has introduced the use of model antennas. A model test antenna is a miniature reproduction of the actual antenna to be tested. If the size of the model antenna is A times the size of the actual antenna, then the frequency to be used in the testing of the model shall be the reciprocal of A times the operating frequency. All dimensions under investigation shall be A times the dimensions appearing under the actual conditions. The patterns obtained in this particular problem were obtained at only one frequency but, from model antenna reasoning, are applicable to all radio frequencies.

Test Apparatus

The function of all components, other than the electrical components, are assumed to be apparent and, therefore, shall not be discussed. All components which are not mentioned as being part of Figure 2 or Figure 3 will appear in Figure 1.

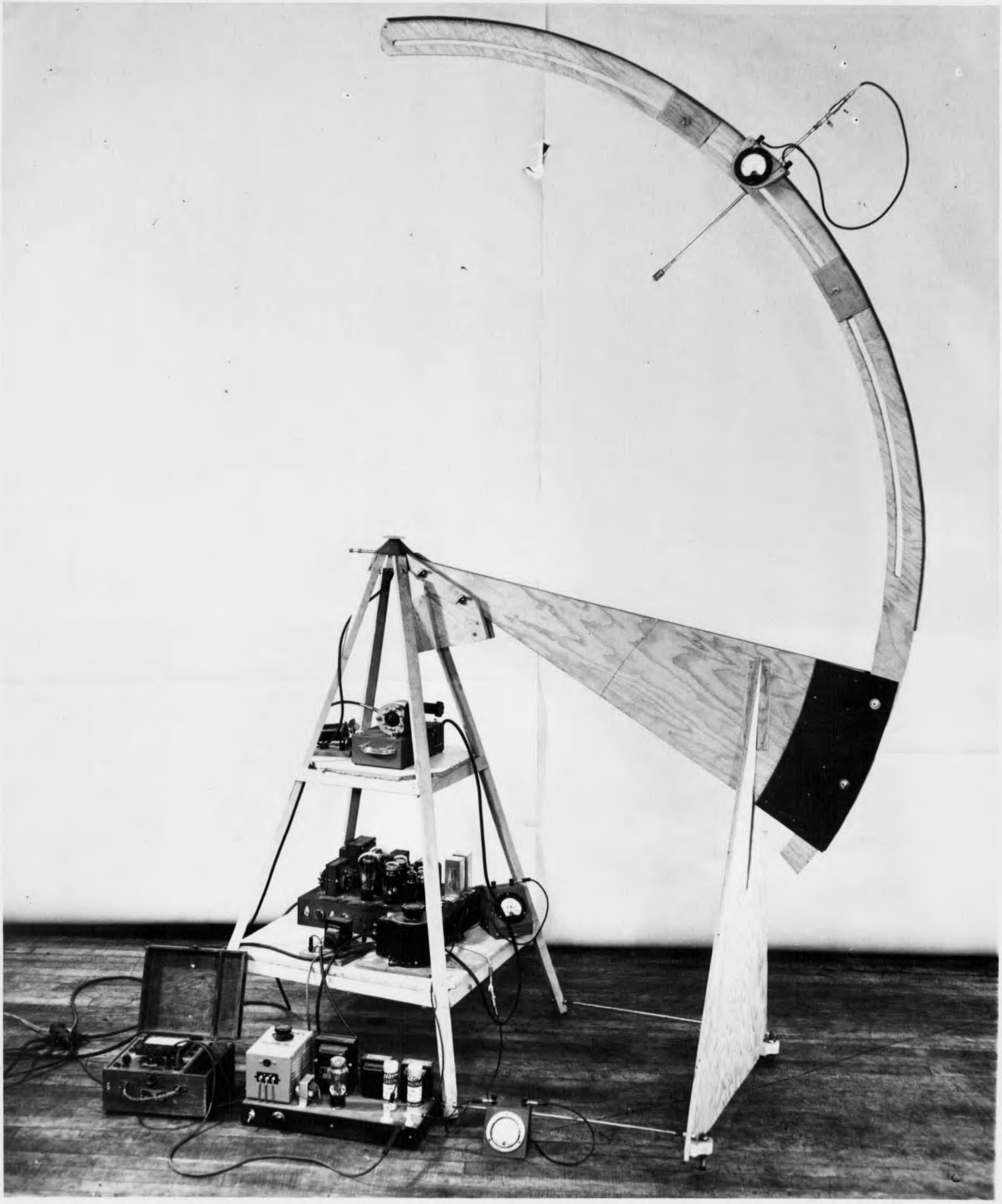

FIGURE I
COMPLETE TEST APPARATUS

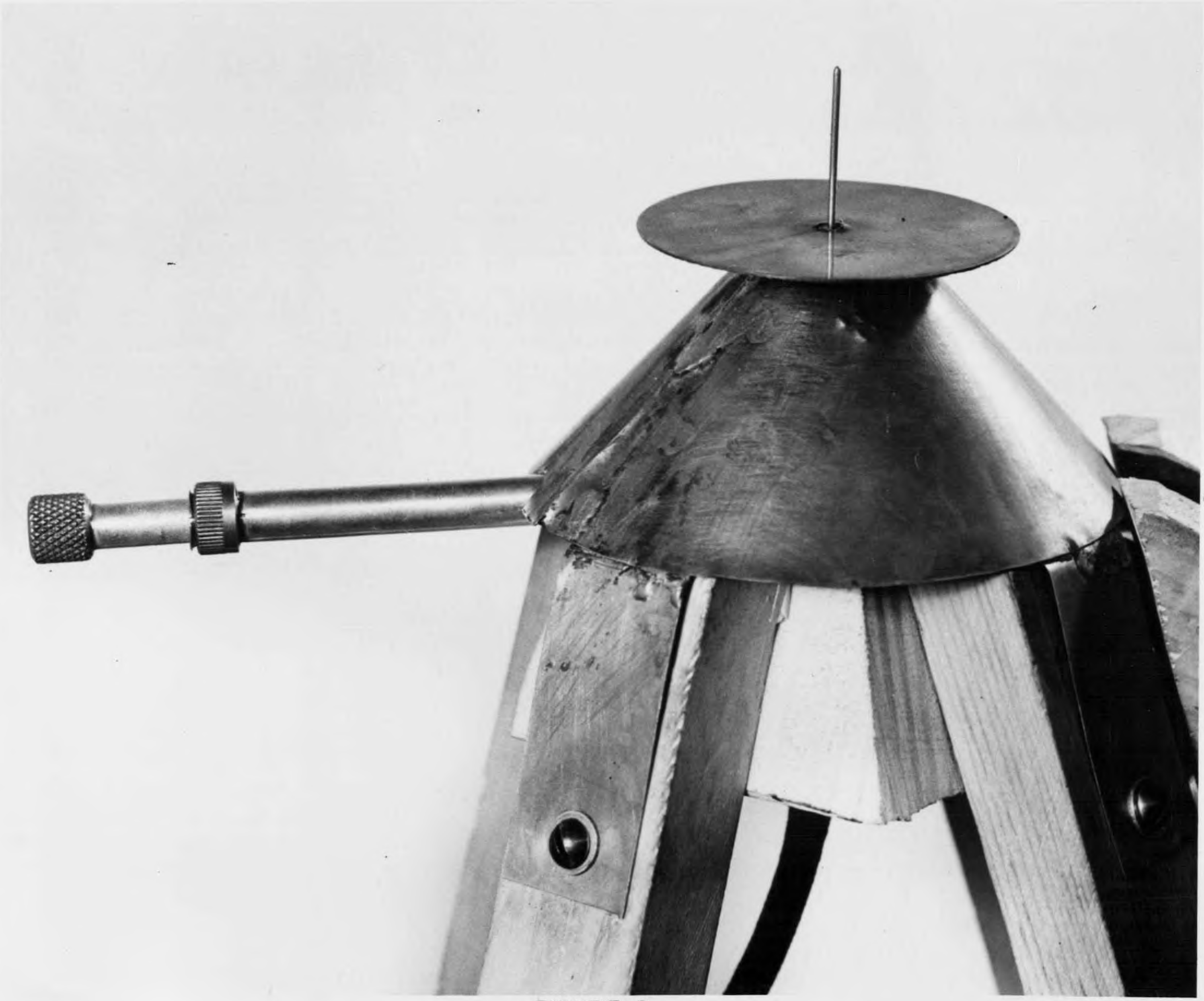

FIGURE 2
TEST ANTENNA AND ASSOCIATED COMPONENTS

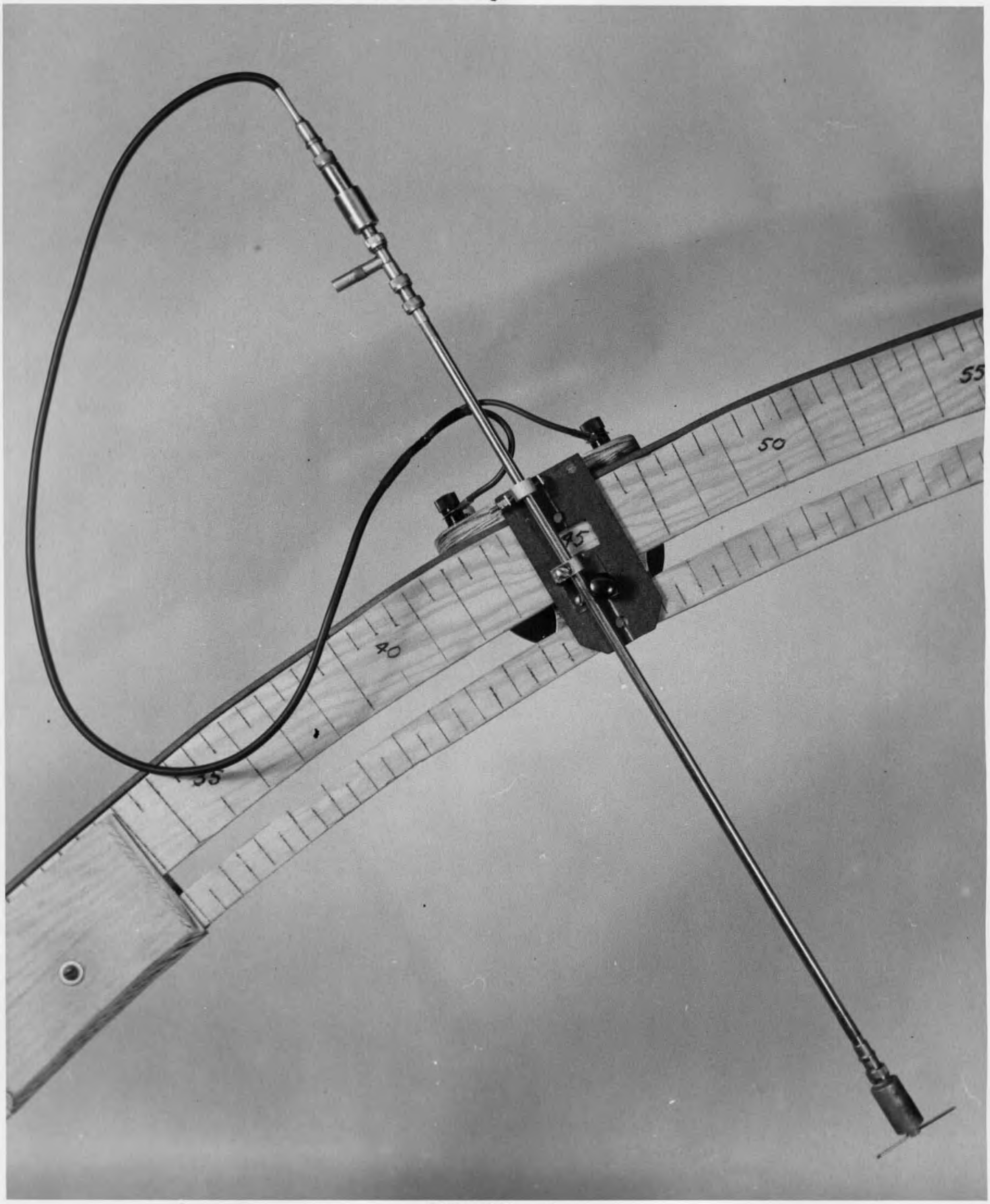

FIGURE 3
RECEIVING ANTENNA AND METERING COMPONENTS

The klystron tube is used as an oscillator to provide the radio frequency. The particular klystron used operated at about 3000 megacycles. The klystron received its power from a power supply which consists of a regulated and a non-regulated supply in series. The maximum voltage obtainable at the output terminals of the power supply is about 1150 volts. The klystron is cooled by a fan. Power from the klystron is fed through a coaxial line to the test antenna and its associated components.

Figure 2 shows the test antenna and its associated components. The coaxial line from the klystron is fed to a stub tuner and then to the test antenna. The purpose of the stub tuner is to match the antenna to the line. All connections are made inside the collar which is detachable. The antenna is connected to the center conductor of the coaxial line by a slip fit and the ground plane to the outer conductor by a screw fitting. The antenna is approximately one-quarter wave length. The ground planes are made of copper and testing was done over planes of 2, 4, 5, 6, 8, and 10 centimeters in diameter. The collar, located below the ground plane, is a frustrum of a right circular cone designed to reflect all waves striking it. These reflected waves, therefore, cannot cause interference in the region in which the measurements were taken.

The receiving antenna and the measuring components are shown in Figure 3. The receiving antenna is supported from the arc by a non-flexible coaxial line. This line is connected to a quarter-wave stub and the stub to a crystal holder. The line from the crystal holder terminates at the parallel combination of a condenser and a direct current, 100 microampere ammeter. The receiving antenna is a half-wave dipole that receives radiation from the test antenna. The quarter-wave stub is in parallel with the series combination composed of the crystal and the parallel combination of meter and condensor. The crystal rectifies the radio frequency. The stub, being a short circuit to direct current, completes the d-c. circuit but has infinite impedance to the radio frequency. The purpose of the condenser is to by-pass the radio frequency components around the meter.

Test Procedure

The first step taken, to obtain data for the radiation patterns, was the alignment of the arc. This was done by use of a transit, by suspending a pointer by fine wire from the ninety degree mark to the top of the test antenna, and by adjusting the adjustable legs on the arc's vertical support.

Then the klystron was set into operation and adjusted for maximum output for the power supply voltage available. The stub tuner was adjusted for maximum and, last of all, the antenna height was varied for a maximum reading. For each step the author attempted to keep below the ground plane in order to prevent erroneous peaking due to reflections from his body.

A check on undesired reflections was made by observing changes in meter readings due to the presence of a large sheet of copper in the field. Reflections were checked at all possible locations. The Ballroom of the Student Union Building of Montana State College presented no apparent interference due to reflections.

The receiving antenna was then set in the desired radial position and its plane of polarization made parallel to the vertically polarized signal.

Readings were then taken at five degree increments. To assure that no errors occurred due to fluctuations of line voltage, a reference antenna was placed in the field and readings were taken only at a constant value of reference antenna measurement. The position of the reference antenna was fixed throughout each test run. The readings were taken in the following manner: First the author adjusted the receiving antenna at the desired angle. The author then lowered himself below the ground plane. When the reference antenna indicated its preassigned value, the author signalled his assistant and the assistant took the reading. The readings were taken as close to the antenna as reflections would allow. Even at this, the use of binoculars was found necessary. As soon as the measurements were taken from zero to ninety degrees the author and his assistant interchanged positions and the measurements were repeated. This procedure was then repeated for all six planes.

Since the rectified current through a crystal varies approximately as the square root of the voltage across it (for low current values) the square root of all readings must be taken before the pattern can be plotted. The experimentally obtained patterns appear in Appendix A, along with current readings and square root values for the 6 and 10 centimeter planes.

MATHEMATICAL ANALYSIS

Preliminary Considerations and Basic Assumptions

In a problem of this nature many assumptions must be made. The assumptions that shall be made are assumptions commonly made in the mathematical analysis of radiation patterns. Among these educated guesses are assumptions of the current distribution along the antenna, the assumption of an antenna of infinitesimal diameter, and the assumption that the ground plane is a perfect conductor.

The electric field intensity of a wave passing through any medium must be related to the magnetic field intensity by the intrinsic impedance. If the wave passes from one medium to another, the tangential components of the electric and the magnetic field must be continuous across the boundary. The normal components are not constrained in this manner. In passing across the boundary, the tangential components must still be related to each other by the intrinsic impedance, but the intrinsic impedance of the two mediums are, in general, not equal. Therefore, when a wave passes from one medium to another, in order that boundary conditions be satisfied, not all of the incident wave can continue on into the second medium. This means that reflection must occur. If a perfectly conducting ground plane is considered, the electric field intensity at the boundary must equal zero for all components since a perfectly conducting plane is a short circuit. Since the voltage and the resistance of the ground is equal to zero, the current in the plane is of indeterminant form. In other words, any value of surface magnetic field intensity may exist but, since the reflected wave cannot be greater than the incident wave, the tangential magnetic field intensity cannot be less than zero or greater than twice the incident magnitude. Since no power can be transmitted into the perfectly conducting plane, the reflected wave must contain the same energy as that existing in the incident wave, i.e. perfect reflection. In order for the energy on the surface of the reflecting plane to equal zero, the incident and reflected components of energy must cancel. This implies, by the Poynting Vector, that the reflected magnetic intensity is equal to the incident magnetic intensity in direction and magnitude.

The derivation of the magnetic field intensity, at a distant point from a current carrying wire, may be obtained by first finding an expression for the vector magnetic potential at any point and then, by vector calculus, taking the curl to obtain an expression for the magnetic field intensity. The direction of the potential vector will be in the same direction as the current flow in the wire. The potential vector, however, will lag the current because of the finite time of propagation. If this current carrying wire extends normally from a perfectly conducting plane, then the vector potential will be normal to the plane and the magnetic field intensity shall be completely tangential. To satisfy, the boundary conditions of a perfectly conducting plane simply means that the reflected vector potential be equal to the incident vector potential. If the ground plane is finite, then the vector magnetic potential satisfies the boundary conditions to and at the discontinuity and no reflection occurs past this new boundary.

The general expression for the vector magnetic potential is

$$A = \int_V \frac{i\,dV}{4\pi r} \tag{1}$$

where i is the current density and the differential dV indicates a volume integration. If the current carrying conductor is straight and of infinitesimal diameter, then the equation for the vector potential becomes

$$A = \int \frac{I\,dz}{4\pi r} \tag{2}$$

where the current is considered flowing in the z direction. The current I will be a space function and in the case of a time varying current will be a function of both space and time. In the distant field, the potential is time retarded because of the finite time of propagation. The time lag at a distant point will be the distance to that point divided by the velocity of the wave. In free space, the velocity of radio waves is practically 3×10^8 meters per second. The phase shift caused by the time lag will be the product of the time lag and the angular velocity of the wave.

The foregoing discussion has shown how assumptions of a perfectly conducting ground plane and an antenna of infinitesimal diameter simplifies radiation problems. Another very important consideration that must be made before a mathematical solution for the vector magnetic potential can be obtained is the current distribution along the antenna. The assumption commonly made for a wire antenna is that the standing current wave is sinusoidal. The assumption was arrived at from the transmission line theory of an open circuited line.

Solution for Vector Magnetic Potential in Integral Form

The current in the antenna varies sinusoidally along the antenna and shall be considered to vary cosinusoidally with time. The expression for the antenna current of an antenna of height H at any point l from the ground plane, Figure 4, is

$$I = I_m \sin \frac{2\pi}{\lambda} (H-l) \cos \omega t \quad (3)$$

At a distant point P, the vector magnetic potential is time retarded and the expression of current that must be used in solving for the vector potential becomes

$$I = I_m \sin \frac{2\pi}{\lambda} (H-l) \cos \omega (t - \frac{r}{v}) \quad (4)$$

For the direct wave, r is the distance from the point l on the antenna to the point P in space, and for the direct wave this distance shall be designated as r_d.

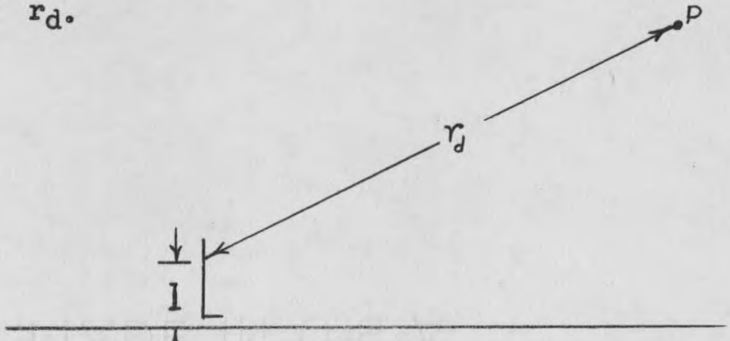

Figure 4

If a cylindrical coordinate system is chosen with the antenna on the z axis and the base of the antenna as the origin then the distance r_d by the Pythagorean Theorm is

$$r_d = \sqrt{(z - l)^2 + \rho^2} \quad (5)$$

For the reflected wave, the distance r in Equation (4) is the distance from 1 to the ground plane plus the distance from the ground plane to point P, remembering that the angle of incidence must equal the angle of reflection, Figure 5. For the reflected wave this distance shall be designated as r_r.

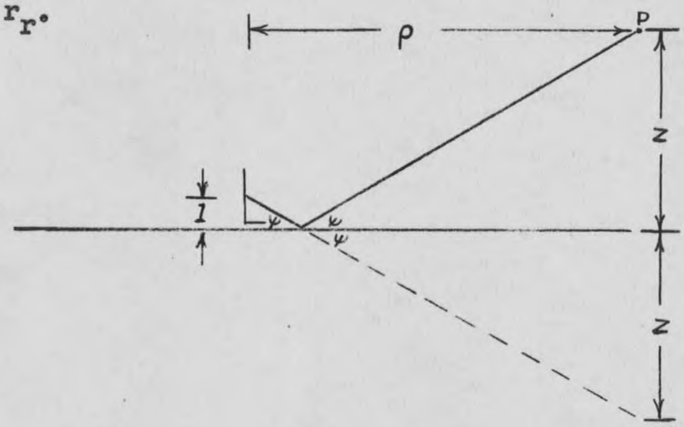

Figure 5

The reflected wave, as seen from Figure 5, travels through the same distance as though it were passing through the ground plane and to the point $(P\rho, -P_z)$. The equation for r_r is

$$r_r = \sqrt{(z+1)^2 + \rho^2} \tag{6}$$

The equation for the differential magnetic vector potential of the direct wave, from a differential length, along the antenna, is

$$d\,A_{zd} = \frac{I_m \sin \frac{2\pi}{\lambda}(H-l) \cos \omega\left(t - \frac{\sqrt{(z-l)^2 + \rho^2}}{v}\right) d\,l}{4\pi \sqrt{(z-l)^2 + \rho^2}} \tag{7}$$