Bubble mechanics by dimensional analysis
by Frederick William Steele

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in MECHANICAL ENGINEERING
Montana State University

Abstract:
Ihe study of bubble mechanics has been of interest to scientific investigators for over a century. However, not until very recently has there been any intensive study of the subject. Ihe increased interest has been caused mainly by two developments: 1.) The problems caused by cavitation in the liquid fuel systems of space vehicles.

2.) The use of liquids for high heat transfer processes in nuclear reactors.

Ihis thesis concentrates on two specific areas in the general field of bubble mechanics. Ihese two areas are bubble motion and bubble inception or nucleation. Because of the complexity of the processes, dimensional analysis was found to be the most effective method for establishing relationships. General equations were derived for both bubble motion and bubble inception. Experimental results obtained by other investigators were then used to verify the general equations for certain special cases. Some predictions were made for the phenomena for which no former experimental evidence was available.

It was concluded that the general equations which were derived would provide a systematic approach to the study of bubble motion and inception.

Ihe limitations of the general equations were stated and some possible experimental methods were suggested.

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Abstract

The study of bubble mechanics has been of interest to scientific investigators for over a century. However, not until very recently has there been any intensive study of the subject. The increased interest has been caused mainly by two developments:

1.) The problems caused by cavitation in the liquid fuel systems of space vehicles.

2.) The use of liquids for high heat transfer processes in nuclear reactors.

This thesis concentrates on two specific areas in the general field of bubble mechanics. These two areas are bubble motion and bubble inception or nucleation. Because of the complexity of the processes, dimensional analysis was found to be the most effective method for establishing relationships. General equations were derived for both bubble motion and bubble inception. Experimental results obtained by other investigators were then used to verify the general equations for certain special cases. Some predictions were made for the phenomena for which no former experimental evidence was available.

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CHAPTER 1

INTRODUCTION

A bubble is defined as a small body of gas within a liquid. More generally, it is the existence of a closed surface which divides the region of concern into two parts, each occupied by a homogeneous fluid (12). Bubble mechanics is thus concerned with the behaviour of bubbles and their interaction with the surrounding fluid.

The study of bubble mechanics has received increasing attention in the past few years. This is due mainly to the importance of bubble mechanics in the design of systems for space applications.

Cryogenic fluids are used as oxidizers in the first stage of the Saturn I, Saturn IB and Saturn V space vehicles. They are used as both oxidizers and fuel in the upper stages of these vehicles. All of these fluids have very small latent heats of vaporization and low boiling points in comparison to water. Because of these properties, they are very susceptible to phase change and boiling. This creates many engineering problems, such as cavitation, vortexing, stratification, two-phase flow in suction lines, pressure losses in lines, the coalescence of bubbles under conditions of extreme vibration, and intermittent firing (3).

In the study of bubble mechanics more and more investigators are turning to the use of dimensional analysis. This is probably due to the many factors present in bubble mechanics which make it too complex for strictly analytical methods. Most of the really significant studies in the past few years have used some method of dimensional analysis.

One of the principal advantages of dimensional analysis is the prediction of similarities which allow small-scale modeling of full-scale operations. In a dimensional analysis the problem being considered is shown to depend on a particular set of dimensionless numbers. These dimensionless numbers can be considered as ratios (usually force ratios). A familiar example of a dimensionless number as a force ratio is the Reynolds number, which is a ratio of two forces in flow: the inertia force, resulting from the motion of the liquid, and the viscous force. Table No. 2 contains a list of dimensionless numbers expressed as force ratios.

Whenever a model has force ratios similar to a prototype, it can be used to simulate the prototype. An example which is illustrative of this principal was presented by Clark (30). When dealing with low gravity systems, the Bond number often becomes important in predicting the phenomena involved. The Bond number is a ratio of body or net bouyant force to surface tension force. At low Bond numbers the surface tension forces predominate over the body forces, which corresponds to the case where low gravity exists. By dipping a slender tube one-millimeter in diameter into water, a situation can be achieved with a Bond number of about 0.036. The surface tension force is the predominate force in the system. In comparison, a container holding liquid hydrogen with a diameter of ten feet will have a Bond number of 0.036 in a gravitational field of about 10^{-7} feet per second squared.

Therefore, the slender tubing can be used as a model to represent the hydrogen container in a simulated low gravity experiment. With this experiment it is possible at terrestial gravity to predict the shape of the liquid interface at low gravities.

Bubble mechanics was first attacked by Euler in the 18th century. He formed the equation of motion for a perfect liquid. In connection with this he recognized that a small reduced pressure localized at some point might create small voids (1).

The collapse or growth of a spherical bubble under adiabatic conditions was first treated for an ideal fluid by Besant in 1859. Lord Rayleigh continued this work on "cavitation bubbles" in 1917 (10). Rayleigh worked with the motion of a single cavity in a liquid. Resurrecting the solution derived by Besant, he attempted to derive relationships which would describe the phenomena occuring when a spherical cavity is suddenly created in a fluid. Equating expressions for the kinetic energy and work involved in the process gave him expressions for the velocity of the fluid and the time for complete collapse. He assumed a constant pressure process at first and later an isothermal process. B. E. Noltinkg and E. A. Neppiras extended Rayleigh's study to include adiabatic processes.

Later experimentation revealed that Rayleigh's work was not in accord with data obtained from real fluids. However, his work established the dynamics of cavities in fluids as a respectable subject for scientific inquiry. His work was also the starting point from which many later investigators began (1).

A few years before Rayleigh's work, Osborne Reynolds investigated the flow of water through constricted tubes, and found that in regions of vanishing pressure small bubbles form and collapse noisily. His study was instigated by his desire to learn why kettles sing. At approximately the same time Sir John Thornycroft and Sidney W. Barnaby proposed the formation of cavities around marine propellers as an explanation for the failure of their destroyer design to meet its predicted speed. Sir Charles Parsons verified their hypothesis by experimentation. Robert C. Froude, then Director of Experimental Research in the British Admiralty, coined the word "cavitation." (1)

Stokes presented one of the earliest bubble mechanics studies in 1880, which proved to be of particular significance. He solved for the velocity of fluid past rigid spheres at low Reynolds number. The expression he obtained is now referred to as "Stokes' Law." Allen and Robinson showed Stokes Law to be restricted to Reynolds number of less than two (8).

Miyagi studies the motion of air bubbles in water. He showed the terminal velocity of bubbles to be independent of size for bubbles with radii of less than 0.01 foot. Byrn classified the motion of bubbles in aqueous solutions and established three distinct types:

1.) Small, spherical bubbles rising in straight lines.

2.) Medium sized, horizontally flattened bubbles rising with an oscillatory motion.

3.) Large, mushroom-shaped bubbles rising relatively straight (1).

O'Brien and Gosline presented one of the first studies which relied entirely on dimensional analysis in 1935. They found the drag coefficient of bubbles to be a function of Reynolds number, Weber number and bubble radius to tube radius ratio. However, they did not present any experimental work to establish the functional relationship between the dimensionless groups (1).

Lapple and Shepherd used dimensional analysis to make calculations of particle trajectories. The equations they developed took into account the effect of fluid friction. Curves were presented for various shaped particles ranging from spherical to disk (10). This study was published in 1940. Many investigators since then have applied the results to bubble mechanics.

Several years later Kaissling used dimensional analysis to study the motion of bubbles in vertical boiler tubes. Functional relationships between the dimensionless groups which he derived were not determined. In 1948 Wigner used dimensional analysis to make some speculations with regard to the velocity of gas bubbles in liquids. The following year Levich applied boundary layer theory, with reference to liquid-gas interfaces, to the computation of total resisting forces acting on a bubble rising in a tube. He obtained an equation for the velocity of the bubble, but was unable to verify his results. The same year Gorodetskaya attempted to verify Levich's equation by experimental methods. His results were 30% off the predicted values (8).

In 1950 several studies were published on bubble mechanics. Verschoor unsuccessfully attempted to establish relationships using the groups obtained by Kaissling. Van Drevelen and Hoftijzer verified the work done by O'Brien and Gosline. Davies and Taylor were the first to use photographic methods to study bubble behaviour. They used it to measure the terminal velocity of bubbles. Rosenberg grouped bubbles into three types similar to those presented by Byrn. He added to Byrn's groups by assigning ranges of Reynolds number to each (1).

In 1951 Rohsenow presented the first of several papers by him on bubble mechanics. In this paper he used data obtained from 0.024 inch platinum wire in distilled water to establish the relationship between several dimensionless numbers. The equation he derived is known as the "Rohsenow correlation."

His relationship was found to agree remarkably well with data for other surface-fluid combinations (7).

Forster and Zuber assumed the movement of the bubble boundary to be of prime importance. Proceeding from this assumption, they obtained an equation known as the "Forster-Zuber correlation." Their results agreed principally with that obtained by Rohsenow. Data obtained by Cichelli, Bonilla and Kazakova agreed well with their equation also (7).

In 1953 Peelbes and Garber presented an extensive study which attempted to verify the work done by earlier investigators. The experimentation consisted of the determination of the steady state velocity of air bubbles in twenty-two liquids. The liquids were chosen so that the variation of liquid density, viscosity, and surface tension were evident. Four distinct regions of bubble motion were noted, each of which could be referred to previous experimental work (8).

Corty and Foust did experimental work in 1955. They found that, if a surface was kept clear of bubbles for a period of time, it was possible to retain free convection at much higher heat transfers than were normally necessary to produce nucleate boiling. This phenomena is known as "aging" or "hysteresis" (7).

With the advent of space travel in the sixties, the volume of work done on bubble mechanics took a sharp increase. The effects of surface roughness on nulceate boiling were studied by Avery. He used boiling mercury in his experiments and found that the nucleate boiling heat transfer was greatly increased by increasing the surface roughness (7).

C. M. Usiskin and R. Siegel made one of the earliest studies of bubble mechanics in reduced gravity fields. They used a pool boiling apparatus which could be dropped a distance of nine feet to achieve reduced gravity. The bubble departure diameters were found to increase as gravity was decreased (31).

J. A. Merte and H. Clark were interested in the effects of increased gravity on pool boiling. By use of a centrifugal motion apparatus they produced system accelerations of from one to 21 times normal gravity. Their results showed that much larger heat fluxes could be obtained at a given surface temperature with increased acceleration. No attempt was made to measure bubble velocity or size (27).

In 1964 R. Siegel and E. G. Keshick completed another study on nucleate boiling bubble mechanics at reduced gravity. In this experiment a 12.5 foot drop tower was used to vary the gravitational field between 1.4 and 100% of earth gravity. In this experiment the departure diameters were found to increase as gravity to the minus 2/7 power. The rise velocity of the bubbles was found to decrease greatly with decreasing gravitational field, due to decreased bouyancy. At very small gravitational fields the bubbles tended to remain close to the surface and act as reservoirs into which newly formed bubbles would pump. This greatly increased the bubble frequency (6).

Recently, F. Numachi did an experimental study on accelerated cavitation induced by ultrasonics. He created clouds of cavitation bubbles on a vibrating cylindrical vessel. These he studied by means of a high speed camera. He found that when a certain frequency was attained, bubbles were generated in a vertical direction. The bubbles were spheroidal in form. The cloud formation could be prevented if the wall thickness was sufficient (16).

Another recent paper was done by T. H. K. Frederking and D. J. Daniels. They studied the kinematics of vapor removal from a sphere during film boiling. A heated glass sphere was submerged in liquid nitrogen. The dynamics of the bubbles formed was studied by means of a high-speed camera. They found that the vapor was removed by gravity regardless of the magnitude of the heat flux. They also established a relationship between bubble departure diameter and frequency of removal (22).

Some of the most recent work with bubble mechanics has been done by C. G. Fritz. His first paper presented a dimensional analysis approach to the study of bubble size and velocity (1). His second papr used an experimental investigation to determine relationships between different dimensionless groups (2). His most recent paper will be published in the coming year in Advances in Cryogenic Engineering. It presents a dimensional analysis of bubble motion in liquid nitrogen. The analysis is followed by experimental data which gives trends of various force ratios (3).

CHAPTER 2

ANALYTICAL PROCEDURE

The field of bubble mechanics is very large and contains almost unlimited possibilities for investigation and experimentation. A paper of this scope, therefore, must be limited to specific areas in the general field.

This paper deals with two aspects of bubble mechanics which are of importance to the space program. The first aspect is the prediction of bubble size and velocity in a fluid under different conditions or environments. The second aspect is the prediction of the rate of inception or nucleation of bubbles under different environments. This aspect is especially vital, since an understanding of the causation of bubbles would allow the more effective use of preventive measures to minimize the engineering problems which they create.

Special consideration is given in this paper to conditions which would be encountered in space vehicles. For this reason the containers are considered to be cylindrical in order to simulate propellant tanks or pipe lines. Also, the fluid is analyzed under both static and dynamic conditions.

One of the most important conditions to be considered with respect to space vehicles is that of weightlessness or reduced gravity. In an orbiting satellite or in a space vehicle which is distant from planetary bodies, the gravity will approzch zero. In a slightly accelerating system in space or on a body such as the moon, the gravity field will be greatly reduced. Since heat transfer processes such as boiling and convection are gravity dependent, they would be expected tovary with reduced gravity. Prime consideration is, therefore, given to the effects of reduced gravity.

The mechanisms of bubble mechanics are so complicated that purely analytical approaches have generally failed to produce significant results. In attempting to simplify the problems into more manageable forms, many investigators have assumed the existence of either ideal fluids or ideal environments. Since their results are contrary to actual experience with all real fluids, they appear to be of academic use only. When the analytical approach fails to yield the solution to a problem, the method of dimensional analysis is often an effective alternative.

Dimensional analysis has proved to be a very useful tool, especially in the field of fluid mechanics. In an equation which expresses a physical relationship, absolute numerical and dimensional equality must exist. In general, all physical relationships can be reduced to the fundamental quantities of force (F), mass (M), length (L), time (t), and heat (Q).

There are three steps in solving a problem by the use of dimensional analysis:

1.) The parameters which will affect the problem must be determined. This is often the most crucial step in the analysis. These parameters must be determined from experiments, by analogy, or from a basic understanding of the processes involved. If all of the parameters are not included, the solution will be faulty. If too many are included, the solution will at least be complicated and unwieldy.

2.) After the parameters are chosen, the relations between the dimensions are established in expressing a physical law. The most common method used for this step is the Buckingham Pi Method. In this paper the ARDA Method discussed in reference 9 is used. This method introduces new procedures which make possible simpler, yet more comprehensive analysis.

3.) Finally, the coefficients and exponents in the relationship must be determined. This is done by empirical methods. Since no appropriate experimental apparatus was available for this problem, the third step had to be foregone. Instead, the paper attempts to correlate previous experiments into regions or domains of applicability. This is done for both the size-velocity analysis and the nucleation rate analysis.

CHAPTER 3

DIMENSIONAL ANALYSIS OF BUBBLE MOTION

The first step in making the dimensional analysis of bubble motion was to choose the parameters which would have a significant effect on the problem. It has been assumed that the liquid is contained in a cylindrical vessel of variable size. The length (L) and the diameter (D_c) of the container should, therefore, be significant parameters. It has also been assumed that the vessel will be subject to variable gravitational field. The gravity (g) acting on the system is consequently included.

Two parameters which have been found important in fluid and bubble mechanics are surface tension (T) and viscosity (u_f). The viscosity of a fluid is a property which measures its resistance to shear stress, and has direct effect on most problems which deal with the flow of real fluids. Surface tension is the force which keeps a bubble from collapsing.

The pressure (P) at the fluid-cylinder interface would probably affect the problem, as would the density (p) of the fluid involved. The amount of heat flux (Q) passing through the cylinder wall would affect the bubble size and velocity also. This is especially evident at high heat fluxes, as the bubbles depart from the surface with large areas of contact.

Two factors which are important in problems dealing with heat transfer, such as boiling, are thermal conductivity (k) and specific heat (c_p). Thermal conductivity is the measure of a material's ability, in this case that of the liquid, to resist the transfer of heat. Specific heat is the ability of a material to store heat.

Since it has been assumed that the fluid could be in motion within the container, as in the case of a pipeline, the relative velocity of the fluid