Energy dissipation in prestressed concrete beams
by Khimji N Vira

A thesis Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Civil Engineering
Montana State University

Abstract:
An apparatus was designed and fabricated to investigate the damping properties of prestressed concrete beams.

Ten prestressed concrete beams 6” wide by 3" deep by 78" long, having prestress cables in the center, were tested under repeated reversible static loading. The equivalent viscous, damping ratio, determined from the area within the resulting hysteresis loops, did not exceed 4% critical even when the beams were close to failure.

The equivalent viscous damping ratio, determined from steady state vibrations and also from the decay of free vibrations, did not exceed 5% critical. After the beams had been loaded past the point of deterioration the equivalent viscous damping ratio was as high as

ENERGY DISSIPATION IN PRESTRESSED CONCRETE BEAMS

by

KHIMJI N. VIRA

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

Civil Engineering

Approved:

Head, Major Department

Chairman, Examining Committee

Dean, Graduate Division

MONTANA STATE COLLEGE
Bozeman, Montana

March, 1964

ACKNOWLEDGEMENT

The author wishes to express his appreciation and sincere thanks to all those individuals who have assisted in this study. He would like especially to thank Dr. W. O. Keightley for his guidance and many helpful suggestions. He is also thankful to Professor Martin Moss for his guidance and advice.

The facilities and the funds for the materials for this work were provided by the Civil Engineering and Mechanics Department. Laboratory facilities were also provided by the Mechanical Engineering Department. Thanks are extended to the Ideal Cement Plant at Trident, Montana, for donating the cement.

Finally, thanks are due to Mr. and Mrs. Robert Romanowski for their assistance in typing this thesis.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont.)

LIST OF TABLES

LIST OF FIGURES

Figure

LIST OF FIGURES (Cont.)

ABSTRACT

An apparatus was designed and fabricated to investigate the damping properties of prestressed concrete beams.

Ten prestressed concrete beams 6" wide by 3" deep by 78" long, having prestress cables in the center, were tested under repeated reversible static loading. The equivalent viscous damping ratio, determined from the area within the resulting hysteresis loops, did not exceed 4% critical even when the beams were close to failure. The equivalent viscous damping ratio, determined from steady state vibrations and also from the decay of free vibrations, did not exceed 5% critical. After the beams had been loaded past the point of deterioration the equivalent viscous damping ratio was as high as 7%.

CHAPTER 1

INTRODUCTION

In recent years the behavior of structures subjected to various types of dynamic loading conditions has become the subject of much investigation. These dynamic loading conditions may be produced by earthquake loading, blast loading, wind loading, impact loading due to traffic, or by the operation of unbalanced machinery. To predict the response of structures to dynamic excitation it is necessary to know the dynamic characteristics of the structural elements.

The primary objective of this investigation was to determine the damping characteristics of prestressed concrete beams and to compare the damping, determined from static measurements, with the damping determined from dynamic measurements. An apparatus was designed and fabricated to test simply supported prestressed concrete beams under repeated reversible static loads. The energy dissipation was determined by measuring the area within the resulting hysteresis loops. The dynamic damping was determined from the resonance curve produced by steady state excitation of the beams and by the rate of decay of the amplitude of free vibrations.

The primary emphasis was placed on the measurement of solid (static) energy dissipation and the apparatus was specially built for this purpose. The dynamic tests were complicated by the fact that portions of the support system were observed to be vibrating at amplitudes as high as one-half the amplitude at the center of the beam. Only a few beams were tested dynamically to attempt to find some correlation with the static test.

The experimental investigation reported herein was undertaken with the primary objectives of determining:

1. If the mix design strength of concrete affects damping.
2. If air entrained in the concrete affects damping.
3. Whether badly cracked beams dissipate more energy than uncracked beams undergoing the same displacement cycle.
4. If a relationship can be found between static damping and dynamic damping.
5. If grouting of the prestressed cables in the beam affects the damping.

CHAPTER 2

REVIEW OF THE LITERATURE

2.1 HISTORICAL BACKGROUND

Research on the damping properties of materials and their engineering significance was started almost two hundred years ago. One of the early pioneers in this field was Coulomb. In 1784, he hypothesized a micro-structure mechanism of damping and also conducted research and concluded that the damping of torsional oscillations is not caused by air friction, but it is due to internal losses in the material (1)*.

Intensive work on hysteresis damping was not started until the end of the 19th century. Work on the hysteretic effect under cyclic stress was done by Ewing (1) in 1889. Work on hysteresis damping under cyclic bending was started by Voight (1) in 1892.

More recent studies on the behavior of structural elements under impulsive loading were undertaken at the Massachusetts Institute of Technology in 1951. It was established there that conventional concrete beams absorb 30% more energy under dynamic loading (impulsive loading) than under reversible static loading (2).

Work done at the United States Naval Civil Engineering Laboratory at Port Hueneme, California, revealed that the damping ratio decreased with the decrease of load duration. The load duration was defined as

---

*Numbers in brackets refer to references on page 74. (LITERATURE CITED) the ratio of actual (effective) load duration to the natural period of the beam (3).

In 1962, Penzien (4) concluded from dynamic tests that damping in prestressed concrete beams increases with the development of large cracks, from 2% at low stresses to 6% at stresses close to failure, and so the magnitude and type of prestress in the concrete member has an indirect effect on the damping.

2.2 TYPES OF DAMPING

Damping is a complex mechanism. Many investigators have tried to develop different types of simple equations and simple mechanisms to represent damping. Damping can be classified as:

1. Damping dependent on velocity.
2. Damping independent of velocity.

2.2.1 DAMPING DEPENDENT ON VELOCITY

Viscous damping is included in this classification. The concept of viscous damping has been known to the world for hundreds of years. Viscous damping is represented by a simple mechanism, based on the concept that damping forces are proportional to the velocity of the vibrating system.

Figure 1 represents a system that is subjected to harmonic force $F_o \sin \omega t$. The mass, m, is suspended on a linear spring of stiffness, k, in parallel to the dashpot that provides viscous damping with a factor c. This dashpot opposes the motion of the mass with a force,

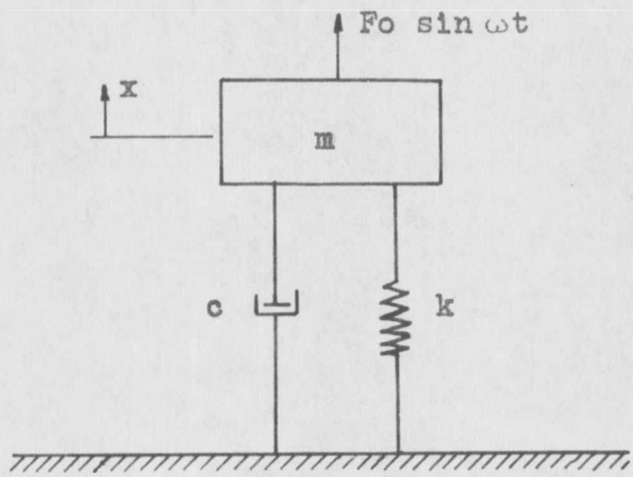

Figure 1: One-degree-of-freedom system with viscous damper.

$c\dot{x}$, where x is measured from the equilibrium position of the mass (5).

Summing the forces on the mass the following equation is derived:

$$m\ddot{x} + c\dot{x} + kx = F_o \sin \omega t \quad \ldots \quad 1$$

If the steady state response is

$$x = a \sin (\omega t - \phi)$$

where a is the amplitude of vibration and ϕ is the phase angle, the net work input per cycle, ΔE, at steady state is

$$\begin{aligned} \Delta E &= \int (\text{force})\, dx \\ &= \int_0^{\tau} (F_o \sin \omega t)\, \dot{x}\, dt \\ &= \omega F_o a \int_0^{\tau} \sin \omega t \cos (\omega t - \phi)\, dt \\ &= F_o a \int_0^{2\pi} \sin \omega t \cos (\omega t - \phi)\, d(\omega t) \\ &= F_o a \pi \sin \phi \quad \ldots \quad 2 \end{aligned}$$

where τ is the period of vibration.

When the force is in phase or 180° out of phase with the displacement, the work done per cycle is zero. The work done per cycle is maximum when the phase angle is 90°. Thus the harmonic force $F_o \sin \omega t$ can be separated into two parts, one in phase or 180° out of phase with displacement, x, and other 90° out of phase with the displacement.

The energy dissipation per cycle, ΔE, due to the damping force is

$$\begin{aligned} \Delta E &= \int (\text{damping force})\, dx \\ &= \int_0^{\tau} (\text{damping force})\, \dot{x}\, dt \end{aligned}$$

In a system with viscous damping this energy dissipation is

$$\Delta E = \int_0^{\tau} (c\dot{x})\, \dot{x}\, dt$$
$$= \int_0^{\tau} c\,\omega^2 a^2 \cos^2(\omega t - \phi)\, dt$$
$$= c\,\omega \pi a^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

By equating the net work input to the energy dissipation per cycle

$$Fo\, a\, \pi \sin\phi = c\omega\pi a^2$$
$$a = \frac{Fo \sin\phi}{c\omega}$$

At natural frequency of the system, ω_n, $\sin\phi = 1$ and

$$a = \frac{Fo}{c\,\omega_n}$$

Thus the amplitude of the system is inversely proportional to the damping constant.

A strong reason for the popularity of the viscous damping concept is that it does not destroy the linearity of the differential equation describing the motion.

2.2.2 DAMPING INDEPENDENT OF VELOCITY

Damping which is independent of velocity is also referred to as static damping. The concept of viscous damping, in which the energy absorption is proportional to the velocity, fails in the case of a system subjected to static loading irrespective of velocity. Several investigators have tried to develop mathematical concepts which permit energy dissipation independent of velocity.

Myklestad (6), in 1952, suggested that damping can be described mathematically by multiplying the modulus of elasticity of the material by complex number, e^{2bi}, where 2b is called the complex damping factor and i is $\sqrt{-1}$. Thus the equation of motion becomes

$$m\ddot{x} + k\,(1 + e^{2bi})\;x = Fo\;e^{i\omega t}$$

The damping force is always at 90° to the displacement, but it is independent of velocity.

Jennings (8), in 1963, developed a general nonlinear hysteretic force deflection relation for a one degree-of-freedom structure. He represented the force deflection relation by equation

$$\frac{x}{x_y} = \frac{p}{p_y} + \alpha\left(\frac{p}{p_y}\right)^r$$

where x is the displacement of the structure, x_y is a characteristic displacement, p is the restoring force, p_y is a characteristic force, α is a positive constant, and r is a positive integer greater than one. This formulation yields a hysteresis loop, the shapes of which depend on α and r, but which are independent of velocity.

Coulomb damping is independent of velocity. Coulomb damping, or dry friction damping, is the dissipation of energy that occurs when a particle in a vibrating system is resisted by a force whose magnitude is constant, independent of displacement and velocity and whose direction is opposite to the direction of the velocity of the particle (7).

2.2.3 EQUIVALENT VISCOUS DAMPING

The assumption here is that the actual nonlinear system may be approximated by a linear system with viscous damping. This concept of damping is frequently used to simplify the analysis of actual structural systems.

Nonviscous damping may be approximated by viscous damping by equating the energy dissipated in a nonviscous system to the energy dissipated in a viscous system.

$$\Delta E = c_{eq} \omega \pi a^2$$

where c_{eq} is the equivalent viscous damping coefficient. Since

$$c_{eq} = \frac{\Delta E}{\omega_n \pi a^2} \qquad \text{and} \qquad c_c = \frac{2k}{\omega_n}$$

where a system is vibrating at its natural frequency, ω_n, then

$$\zeta_{eq} = \frac{c_{eq}}{c_c} = \frac{\Delta E}{2\pi k a^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4$$

where ζ_{eq} is the equivalent viscous damping ratio and c_c is the critical damping constant*.

Equation No. 4 defines the equivalent damping ratio regardless of the type of damping mechanism involved.

2.3 HYSTERESIS LOOPS

Materials do not behave perfectly elastically even at very low

---

*Critical damping is defined physically as the minimum viscous damping that will restrain a displaced system to return to its equilibrium position without oscillation (7).

stresses. When a member is subjected to repeated loads, the stress-strain or load-deflection diagram forms a hysteresis loop as shown in Figure 2. For any type of cyclic stress this loop will be present, but in some cases it is too narrow to be noticed by conventional tests. The shape of this loop varies with the type of material and with the stress level reached (1).

Static damping depends on a stress-strain law which is independent of time, not sensitive to stress rate. In such a case the shape of the loop, and as a result the damping constant, is independent of frequency.

Table I shows the simplest representative mechanical models for each of the classified behaviors. In these models S is a spring having linear elasticity, D is a dashpot which produces a resisting force proportional to velocity and C is a Coulomb friction unit which produces a constant force whenever slip occurs within the units, the direction of force being opposite to the direction of motion (1).

TABLE I

CLASSIFICATION OF VARIOUS TYPES OF HYSTERETIC DAMPING OF MATERIALS

	TYPES OF DAMPING	
NAME USED HERE	DYNAMIC HYSTERESIS	STATIC HYSTERESIS
Other Names	Viscoelastic, rheological, & rate-dependent hysteresis	Plastic, plastic flow, plastic strain and rate-independent hysteresis
Nature of Stress-Strain Laws	Essentially linear. Differential equation involving stress, strain, and their time derivatives	Essentially nonlinear, but excludes time derivatives of stress or strain