Motional electric fields associated with relative moving charge

by Kyle Aaron Klicker

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
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Abstract:
The concept that the magnetic flux, induced by moving charge or an electrical current, moves with the charge carriers that induce it, is explored. This idea was promoted as late as the 1960's by W.J. Hooper and still remains a contested issue. Hooper claimed to have verified this experimentally and also identified some fundamental qualitative differences between types of electric fields distinguished by their origin. An analytical investigation of these claims has been undertaken.

This author has not been able to disprove Hooper's claims. It is established that there are three types of electric fields. The first due to a distribution of charge known as an electrostatic field. The other two are associated with the two types of electromagnetic induction. The first type of induction is known as flux cutting is due to relative spatial motion with respect to magnetic flux. The electric field resulting from this type of induction is the motional electric field. This type of electric field has unique properties that separate it from the other two. Experimentally, it is confirmed that this electric field is immune to shielding due to the fact that magnetic (not electric) boundary conditions apply to it. Motional electric fields can also exist where the total magnetic field that induces it consists of non-zero components that sum to zero. The other type of induction is due to linking time changing magnetic flux.

Inclusion of the concept of magnetic flux moving with the current or charge carriers that induce it into classical electro-magnetic theory results in a small additional force between relative moving charge that is not predicted by classical EM theory. This difference is due to a motional electric field that surrounds all moving charge if the idea of moving magnetic flux is subscribed to. This term is dependent on the square of the relative velocity and is equivalent to the term generated by special relativity when applied to relative moving charge. Ampere electrodynamics also predicts the existence of this force. Consequently, three incompatible and fundamentally different models of EM effects yield the same results.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

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This author has not been able to disprove Hooper's claims. It is established that there are three types of electric fields. The first due to a distribution of charge known as an electrostatic field. The other two are associated with the two types of electromagnetic induction. The first type of induction is known as flux cutting is due to relative spatial motion with respect to magnetic flux. The electric field resulting from this type of induction is the motional electric field. This type of electric field has unique properties that separate it from the other two. Experimentally, it is confirmed that this electric field is immune to shielding due to the fact that magnetic (not electric) boundary conditions apply to it. Motional electric fields can also exist where the total magnetic field that induces it consists of non-zero components that sum to zero. The other type of induction is due to linking time changing magnetic flux.

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CHAPTER 1

INTRODUCTION

Classical electromagnetic (EM) theory is a composite of pieces developed by such notables in the history of science as Faraday, Maxwell, Hertz, Lorentz and Einstein. The theory itself was originally developed from empirical results and experimental evidence. Even though the foundational development of the theory took place more than 150 years ago, the final chapters of EM theory have still not been written and this body of knowledge can still be an area for new discovery. As technology progresses and instrumentation becomes more sensitive, experimental evidence is obtained that still raises questions and paradoxes concerning the foundational aspects of EM theory. This is evidenced by the many parts of EM theory that alone are brilliant but when combined, do not fit into an easily understandable whole and do not always yield consistent results when applied to problems. Many authors [1,2,3,4,5] have pointed out inconsistencies and paradoxes in the theory and have demonstrated differences in results when applying one approach to a problem as compared to another.

Engineers and physicists have groped for a concise model and package that works for all of the vast EM phenomena. Maxwell's equations are the accepted answer to this need, but even they require the user to have an extensive prior knowledge of the results he is pursuing since they do not lend much physical insight into the mechanics of EM effects and can yield wrong results when applied indiscriminately. In addition, unless one is careful in applying these relationships [6,7], it is easy to exclude the common vXB term of the Lorentz force equation and this term may not be negligible.

One specific area of EM theory that still causes confusion [8,9] is the law of induction - the linking factor between electric and magnetic fields. It is commonly believed that Faraday's law of induction and the corresponding Maxwell equations describe all forms of induction and equates them, but this belief has been pointed out to be incorrect by many experimenters. In fact, there are two types of induction that must be treated separately [5,9]. The first is due to a time varying magnetic field and the second is due to relative spatial motion with respect to a magnetic field.

EM theory has its roots in experimental investigation and some of the cornerstones of EM theory such as Faraday's Law, The Biot-Savart version of Ampere's current Law and the Lorentz Force Equation are based purely on experimental results. Ampere was one of the first to develop a mathematical interpretation of these experiments. His experiments involved measuring the forces between currents and the results have been put into many forms. The most well known of these is the Biot-Savart Law that establishes the force between two currents as a function of their magnitudes, relative position and orientation. Faraday's law states that the induced electromotive force (e.m.f.) in a closed circuit is proportional to the time rate of change of magnetic flux it encloses. The Lorentz equation describes the force on a charge or an induced e.m.f. as a function of the magnitude of the electric field and magnetic field it sees and also the relative velocity with respect to the magnetic field.

One researcher, W.J. Hooper [10], probed deeply into the topic of classical EM theory and, after much experimentation, came to the conclusion that there are three different types of electric fields. One, due to a distribution of electric charge, and the other two are associated with the two types of induction.

His major interest was investigating the physical characteristics of the motional electric field that is associated with relative motion in space with respect to a magnetic field. Hooper distinguished between the electric field due to relative spatial motion with respect to a magnetic field and the electric field due to relative time motion with respect to a magnetic field. Motional electric fields are due to spatial not time motion with respect to a magnetic field. By adapting an empirical approach to his work, Hooper obtained evidence that the three types of electric fields have different physical characteristics and, therefore, should not be equated as is easy to do when using only mathematical models. Hooper demonstrated by experiment that motional electric fields are immune to shielding and can also exist where the total magnetic field is zero. In addition, he argued that magnetic flux is physically real and not just a mathematical model or a convenient way to describe the effects of moving charge.

More significant, Hooper interpreted the work of Cullwick [11] that assigned inertia and momentum to a current as supporting the idea that the magnetic flux (or field) induced by moving charge actually moves with the charge carriers. Hooper's premise that the magnetic flux associated with a current drifts along with the charge carriers composing the current is a simple one, but it has never been thoroughly investigated. Although never proven [3] before instrumentation was sensitive enough to measure it, it has some important ramifications and a new investigation is warranted. Charge itself can even be modeled by moving magnetic flux of spinning magnetic dipoles and a theory exists [12] that charge is moving flux. There seems to be no way to disprove that magnetic flux does not move with its source [12]. The most important ramification of the moving magnetic flux idea is that this assumption yields a motional electric field in the fixed reference frame of the current that induces the magnetic field. This will be true even if it is a dc current in a neutral conductor. Consequently, this idea proposes that a motional electric field is associated with all moving charge. To this author's knowledge, Hooper (with possibly one exception [13]) has been the only one to actually measure this effect. For moderate charge velocities, Special Relativity, when applied to the moving charge composing the current, supports his conclusion [14]. Besides Special Relativity, a field-free, non-relativistic version of the Ampere equation also supports Hooper's claims. It is ironic that these two approaches that avoid the use of magnetic fields give the same mathematical results as Hooper's theory that is centered on the physical reality of magnetic fields. The moving magnetic flux theory may have advantages, though, in its ability to characterize the force between relative moving charge as due to a motional electric field which by definition is a magnetic force (ie: E=vXB).

This author has set out to investigate the claim that magnetic flux moves with the charge that induces it. Originally an experimental approach was attempted. But limitations in available hardware and instrumentation needed to measure this small effect, has forced an analytical investigation of this idea. What will be shown in this paper is that Hooper's theory, with some clarifications the author proposes, is equivalent to a rigorous application of special relativity at moderate charge velocities (or to a second order approximation) and identically equivalent to an alternate field-free version of the Ampere equation [15]. Although the best way to 'get a hold of' an electromagnetic field is through its effects, the analytical investigation presented here does clarify certain issues and raise some significant points.

In Chapter 2, Hooper's experimental work is reviewed and evidence is presented that supports his claims. Chapter 3 analyzes various configurations of moving charge using three different formalisms of classical electrodynamics. The results generated in Chapter 3 are reviewed in Chapter 4 and a set of results is chosen as a baseline for further comparisons. Chapter 5 applies Hooper's, or the 'moving magnetic flux', approach to the same problems looked at in Chapter 3 and compares these results to the chosen baseline. A magnetic drift velocity that matches the baseline is derived. Advantages and disadvantages of the moving magnetic flux approach are then discussed in Chapter 6. In Chapter 7, conclusions about the work are drawn and experiments that would discriminate between a motional electric field effect and special relativity effects are suggested. It is shown that the motional electric field approach to determining the forces between relative moving charge is a valid one from a mathematical standpoint and may yield additional insight into the physical nature of the force between relative moving charge. Some key experiments that may differentiate between a motional electric field effect and a similar effect due to special relativity are suggested and described.

This work appears significant in light of recent work in the detection of magnetic monopoles [16], effects in field free regions as described by quantum gauge theory [17], and momentum possessed by static EM fields [18]. The moving magnetic flux idea may also help explain why dissimilar materials react differently to gravity that has now required the postulation of a small electric effect called 'Hyper-charge' or a fifth force that is a function of atom composition [19,20]. The linear motion of flux down a conductor may also help explain longitudinal propulsion associated with currents;an effect that has yet to be explained in terms of Lorentzian forces [21,13]. In fact, the incorporation of moving magnetic flux into EM field theory may help resolve the differences between Maxwellian field theory and Ampere-Neumann field free electrodynamics of materials.

CHAPTER 2

THE WORK AND THEORIES OF W. J. HOOPER

This investigation of motional electric fields associated with moving charge was stimulated by the work of W. J. Hooper in this same area. His contributions to the topic of EM theory involve the experimental investigation and description of motional electric fields, differentiating them from other types of electric fields, and his claim that a motional electric field is associated with all moving charge. Hooper's claims can be summed up by two premises. The first is that a motional electric field is physically different than an electrostatic field or an induced electric field due to a time changing magnetic field. His second premise is that a motional electric field is associated with all moving charge and this is due to the fact that the magnetic flux induced by moving charge moves with the charge.

Uniqueness of Motional Electric Fields

A regression at this point to define thoroughly motional electric fields is warranted. Simply stated, a motional electric field is the induced electric field due to relative motion with respect to magnetic flux [22]. It is described by the Lorentz force equation

$$E = vXB \qquad \text{E2.1.1}$$

where E is the induced motional electric field and v is the velocity of motion with respect to the magnetic field B.

Hooper stated and claimed to have proved experimentally that this motional electric field is different than the electric field that arises from a distribution of electric charge known as an electrostatic field and also different than the electric field due to time rate of change of a magnetic field.

In the classical sense, a motional electric field is the force per unit charge on a charge moving with respect to a magnetic field. It is considered a magnetic force and acts normal to both the magnetic field and the velocity of the charge. This is different from an electrostatic force that acts in line with the electric field. Another difference is that a magnetic force can not change the energy of the charge, only change its direction. This is even true in the case of a 'moving magnetic field' where there is a motional electric field produced. The magnetic field does not do work, but the source of the magnetic field or prime mover does work [23].

The differences between motional electric fields and those due to a time varying magnetic field are not always clear. Although a non-uniform moving magnetic field is mathematically equivalent to a time changing magnetic field,

$$\frac{dB}{dx} \cdot \frac{dx}{dt} = \frac{dB}{dt} \qquad \text{E2.1.2}$$

the physical characteristics of the effects they generate are different. The emf generated in a physically moving closed circuit can usually be described by Faraday's law since the amount of flux enclosed by the circuit is changing with time. The motional electric field is due to flux cutting while the electric field generated from a time changing magnetic field is due to flux linking. Although they are mathematically equivalent for certain geometries, Hooper claims that they are two different effects and should not be confused. This is in agreement with others who have rigorously investigated induction [8,9].

The work of Moon and Spencer on induction [9] helps to clarify this somewhat confusing issue. Their electrodynamic theory consists of modeling electromagnetic phenomena purely as forces between charges. This avoids the field concept all together. In their work, they show the equivalence of the flux cutting force described in the Lorentz force equation to a force due purely to relative motion between charge. The force due to flux linking is shown to be equivalent to a force between charge that is due purely to relative acceleration. From their work, it can be surmised that the flux cutting and flux linking are different phenomena since they stem from different fundamental physics. Flux cutting is equivalent to relative motion between charge while flux linking is due to relative acceleration between charge.

Another distinguishing characteristic of the motional electric field is its immunity to shielding. Hooper verified this with experiments. They consisted of showing that the motional electric field cannot be electrostaticly shielded by a faraday cage held at a fixed potential enclosing the detection device [10]. His experiments were also extended to magnetostatic shielding and his representative experiments concerning the shielding of motional electric fields have been duplicated and verified at Montana State University [24]. It was concluded that a motional electric field cannot be shielded by any common means. As long as magnetic flux is cut, an emf is produced independent of the material cutting the flux. This agrees with conclusions arrived at by Maxwell [25]. The only way to shield a material from a motional electric field is to use a magnetic shield (high μ material) around the source of the magnetic flux - in effect containing the magnetic flux at its source. When a magnetic shield is not around the source but around an object that is to be shielded, no shielding takes place since all the shield does is redirect the flux and the shielded area still cuts flux. These conclusions are not startling if one remembers that motional electric fields are a magnetic effect.

Another significant finding of Hooper was the measurement of real effects in field free regions. This may seem like a side-light to his work that is of interest here, but this effect is important and directly associated with motional electric fields. Hooper found that an analysis of the sum of the parts does not always equal the results of an analysis on the whole. A simple experiment that verifies this consists of subjecting a conductor to two different magnetic fields that are equal and opposite,

$$B_1 = -B_2 \qquad \text{E2.1.3}$$

and thus sum to zero,

$$B_1+B_2 = B_t = 0 \ . \qquad \text{E2.1.4}$$

If they also have equal and opposite relative velocity to a conductor,

$$v_1 = -v_2 \ , \qquad \text{E2.1.5}$$

the total E field is not equal to zero.

Not apparent at first glance, the correct result is obtained by application of the Lorentz force equation using simple superposition. Thus

$$E_t = v_1 X B_1 + v_2 X B_2 = 2vBa_n \ . \qquad \text{E2.1.6}$$

An incorrect value is obtained by using the total magnetic field,

$$E_t \neq (vXB_t = 0) \ . \qquad \text{E2.1.7}$$

In this case, even though the B fields cancel, the vXB effects add. A motional electric field due to flux cutting is generated even in an area where the total magnetic flux is zero. Hooper incorporated this effect in his experiment that generated a radial motional electric field using no moving parts.

Moving Magnetic Flux

Hooper's research and experimental work led him to draw the conclusion that the magnetic flux associated with a current actually moves with the charge carriers that compose the current. This assumption leads to the conclusion that a force exists between a dc current in a metallic conductor and an external stationary charge. This is equivalent to saying that a radial electric field surrounds a dc current even in a metallic conductor where charge neutrality (a balance between positive and negative charge numbers) is maintained within the conductor. This conclusion points out a interaction or equivalence of electrodynamic and electrostatic forces. This is in variance with the Biot-Savart Law and classical EM theory. In effect, this premise is equivalent to a type of 'dc induction'.

This force on stationary charge in the presence of a dc current (Figure 1) can be described by the Lorentz equation if one assumes that the magnetic flux due to a current,

$$B = I/(2\pi\epsilon c^2 r)\mathbf{a}_\Phi \qquad \text{E2.2.1}$$

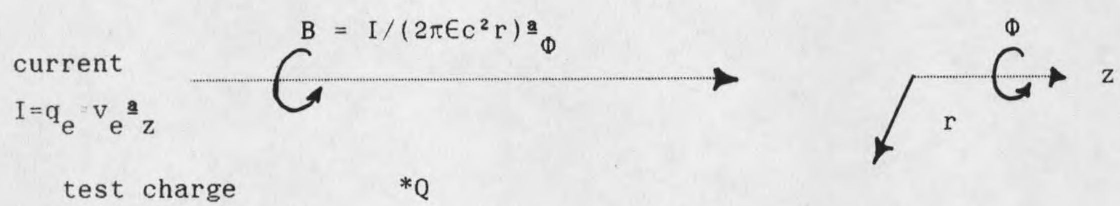
Where:

I is the current magnitude in [amps]
ϵ is the permittivity of free space [farads/meter]
c is the velocity of light in free space [meters/sec.]
r is the radial distance from the center of I [meters]

is drifting or moving with respect to the stationary charge at the drift velocity of the charge carriers that compose the current.

$$v_B = v_e = -v\mathbf{a}_z \qquad \text{E2.2.2}$$

Figure 1. Force on charge adjacent to dc current element.



The force on the charge then becomes

$$F = Q(v_B X B) = -v\mathbf{a}_z X I/(2\pi\epsilon c^2 r)\mathbf{a}_\Phi = Iv/(2\pi\epsilon c^2 r)\mathbf{a}_r \qquad \text{E2.2.3}$$

which is a finite value if Hooper's assumption that the magnetic flux has a finite velocity associated with it is accepted.

This idea that magnetic flux drifts along with its source is still an unresolved issue in EM theory. Although not exactly the same issue, it is worth mentioning that there has been an on-going debate initiated by Faraday with his axially symmetric 'disk' generator of whether the magnetic flux rotates with an axially symmetric source. This debate even intrigued Hertz [12]. Hooper's claim that magnetic flux moves with the charge carriers that comprise a current, has never been addressed with the same zeal as the rotating flux controversy. The most recent thought on the Faraday disk generator controversy is that whether the flux rotates with its source or not cannot be proven and either assumption yields the same results. This idea is suggested by Djuric [12] who has proposed a model of electric charge based on spinning magnetic dipoles. What most of these researchers have failed to realize and investigate thoroughly is one of the properties of the motional electric field, specifically its reaction to shielding. It seems that an experiment can be devised where shielding is used to distinguish between a motional electric field and the electric field of a charge redistribution caused by a motional electric field. Thus the controversy may be resolved once and for all. So far, it appears that most have overlooked the properties of motional electric fields. An example of this oversight is Djuric's model of charge based on spinning magnetic dipoles. It is sound mathematically, but fails to take into account the physical characteristics of the motional electric field. These characteristics would rend the charge of his model totally immune to shielding which is not in agreement with the known properties of electrostatic charge.

Experimental Work of W.J. Hooper

Hooper conducted many experiments that supported his claims concerning the uniqueness of the motional electric field, especially concerning the issue of shielding. His experiments consisted of a detection system that was usually a conductor and an ammeter that would measure the induced current in the conductor when it was passed through a magnetic field. This effect is well understood and is the principle of inducing an emf from cutting lines of magnetic flux. What is unique in Hooper's experiments is his investigation of various types of shielding. He was unable to shield the effects measured in his detection system by any type of electrostatic shielding such as a grounded faraday cage. Additionally, he could not shield his detector by employing any type of magnetic shielding such as high permeability iron. He concluded that the motional electric field due to relative motion between a conductor and a magnetic field was totally immune to shielding of any sort and penetrated all materials equally. These conclusions are in agreement with classical EM theory that defines a motional electric field rigorously as a magnetic force per charge, but his conclusions bring to attention certain characteristics of motional electric fields that are often overlooked.

The premise that there is a motional electric field associated with all moving charge lends itself to a straightforward experimental test. The concept is a simple one and the experiment is also simple in idea, but in practice has proven to be difficult strictly because the magnitude of the effect is so small.

The experiment Hooper used consisted of a source of moving charge and a detection system. The source consisted of a non-inductively (windings arranged so that the individual magnetic fields produced by each winding cancel and sum to zero) axial wound copper coil in a cylindrical configuration (4020 turns) energized by a variable power supply. The detection device consisted of an electrostaticly shielded (grounded faraday cage) cylindrical capacitor around the coil and an electrometer to measure the voltage induced in the capacitor by the motional electric field that surrounds the coil. His experiment and device is well described by his patents and papers [26,27]. A block diagram is shown in Figure 2.

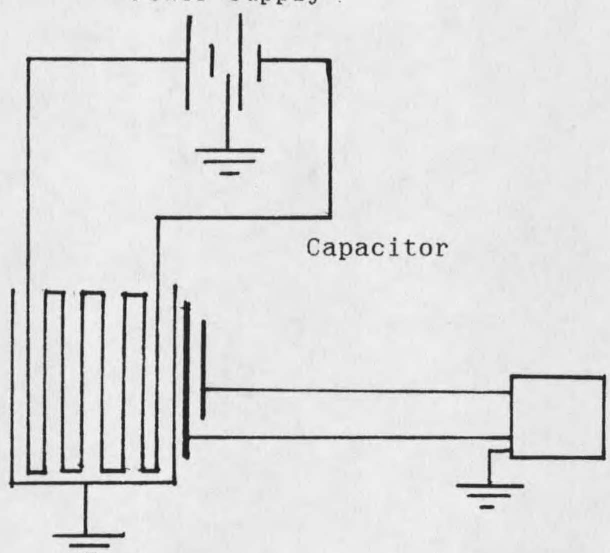


Figure 2. Block Diagram of Hooper's Experiment

Since Hooper's premise is contingent upon experimental proof, it is important to analyze a sample data point. The total magnetic field generated by Hooper's generator is

$$B = \frac{\mu N I}{2\pi r}\,\mathbf{a}_{\Phi} \qquad \text{E2.3.1}$$

Where:

$\mu = 4\pi \cdot 10^{-7}$ permeability of free space [Henrys/meter]

N = 4020 turns

I = current [Amps]

r = radial distance from center of coil [meters]

$\mathbf{a}_{\Phi}$ = unit vector in Φ direction [cylindrical co-ordinates]

The radial electric field surrounding the generator is calculated with the Lorentz equation (Equation 2.3.1). Rewriting this equation with the appropriate values of B and v gives

$$E = v\, a_z \times \frac{\mu NI}{2\pi r}\, a_\Phi = \frac{-v\mu NI}{2\pi r}\, a_r \qquad \text{E2.3.2}$$

To determine the voltage difference that the cylindrical capacitor sees, the electric field must be integrated between the two plates of the capacitor.

$$V = \int_{r1}^{r2} \frac{-v\mu NI}{2\pi r}\, dr = \frac{-v\mu NI}{2\pi} \cdot \ln(r2/r1) \qquad \text{E2.3.3}$$

The distances from the center of the coil to the plates of the capacitors are, from Hooper's laboratory notes [28].

OD of inner cylinder = 0.10265 [Meters]

ID of outer cylinder = 0.10615 [Meters]

The electron drift velocity must also be obtained. This velocity depends on the conducting material, charge carrier mobility, charge carrier density, and the electric field applied to the material. For copper, at room temperature, a velocity of 0.02 meters per second is an accepted value. Hooper derived a value of 0.0176 meters/second [29,30] using Fermi-Dirac statistics and used this as a comparison when measuring drift velocity of the electrons in the copper coil with his generator. Using all of the proper values, equation 2.3.3 gives for a current of 30 Amps

$V = 14\ \mu$Volts

This compares favorably with some of his measured results and helps to confirm that the magnetic flux moves with the charge carriers. The result would be zero if the flux did not move.

Aspects of his experimental results that are significant are the characteristics of his data for a given test. He found that the voltage measured had a parabolic dependence on current for tests run at room temperature. If the electron drift velocity increases with current this relationship would result. This can be explained by assuming a linear relationship between applied voltage to the generator and current flow in the generator (this would be true for a constant temperature where the resistance of the coil did not change with applied voltage) and assuming that the number of charge carriers and their mobility is a constant for a given test and temperature. Since the drift velocity is equal to the electron mobility times the applied electric field, the drift velocity appears to increase linearly with current. The result of this is that the Iv term in equation 2.3.3 actually is qv^2 (i.e. parabolic in v) where v is linearly proportional to the current, I. Hence the ensuing parabolic relationship between measured voltage and applied current is obtained.

One other detail concerning Hooper's results needs to be checked and that is whether the voltage on the capacitor can actually be measured. The nature of this effect requires an electrostatic measurement of the voltage on the capacitor. This requires a high impedance electrometer. Hooper used a device that could measure charge as small as

$$Q = 10^{-16} \text{ coulombs}$$

The amount of charge accumulated on the capacitor must be calculated to determine if it exceeds this value. The capacitance of the detector is given by Hooper as 285 pico-farads. The charge accumulated on the capacitor for a voltage across its plates of 1 μVolt is 2.85 times above the limitations of the electrometer used. So it appears that the measurements made are physically possible.

The results of Hooper's experiments support his theory in that applying his assumption with the Lorentz equation and using a typical electron drift velocity for copper at room temperature a result is obtained that agrees with experiment. His results have a squared dependence on current that makes sense if the number of charge carriers remains a constant for varying currents and for the range of temperatures that this relationship held.

CHAPTER 3

CALCULATIONS OF FORCE BETWEEN MOVING CHARGE

To establish a baseline of results to compare to the moving magnetic flux model, the most accepted and commonly used analytical tools used to determine the force between relative moving charge and or current elements are explored here. Three different methods are employed to calculate the force between two current elements or two systems of moving charge (designated 1 and 2). The goal is to establish confidence in a set of results that the moving magnetic flux approach can be tested against in a later chapter. The three methods explored here include the classical method of the Biot-Savart law analysis of Ampere's experimental results (and it's equivalent, the Lorentz force equation), Special Relativity applied to the moving charge carriers that comprise the current, and a field free interpretation of Ampere's experiments derived by Moon and Spencer. From this point on, the three methods will be referred to as the Biot-Savart law, Special Relativity, and Moon and Spencer.

To cover the entire spectrum of combinations of moving charge and yet retain visibility and a realistic number of combinations of moving charge and calculations, the moving charge will be modeled as parallel in-line current elements of both a metallic and ionic nature, electron beams and stationary point charges. All configurations are chosen as co-linear charge distributions or currents since this eliminates much complexity of geometry, and makes the effects due purely to the motion of charge obvious.

In total, fifteen cases are examined using the three different analysis methods. Table 1 lists the fifteen combinations of moving charge that are investigated and Table 2 details a pictorial representation of the fifteen cases. The cases are designated by Roman numerals I through XV, each one being a unique configuration of moving charge in location 1 with respect to location 2. A metallic conductor is designated by a 'm' in Table 1 while an ionic conductor is designated with a 'i'. An electron beam is designated with an e and a stationary point charge by a '*'. Both situations of current 1 and 2 flowing in the same direction and opposing direction are examined. In Table 1, the opposing current configurations are designated by an 'o'.

Table 1. Combinations of Moving Charge Investigated

Case #	Descriptors: m=metallic, i=ionic, e=e-beam, *=stationary charge, o=opposing direction	
	Charge configuration 1:	Charge configuration 2:
I	m	m
II	m	m,o
III	m	*
IV	i	i
V	i	i,o
VI	i	*
VII	m	i
VIII	m	i,o
IX	m	e
X	m	e,o
XI	i	e
XII	i	e,o
XIII	e	e
XIV	e	e,o
XV	e	*