An analytical approach to finite slope stability analysis
by William Arthur Vischer

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Civil Engineering
Montana State University

Abstract:
The development of an analytic approach to the finite slope stability problem is initialized in this paper. This method is similar to the "method of slices," in terms of the static analysis; however, exact integration is used for determining the actuating and resisting forces.

An equation was derived expressing the safety factor of a homogeneous, finite slope in terms of the slope geometry, geometry of a circular failure arc, and soil parameters. Safety factors obtained from the derived equation, were compared with those obtained by methods currently in use. Differences of up to five percent were noted in the comparison. The equation for the safety factor was then differentiated, with respect to the radius of the failure arc, in a futile attempt to derive an analytical expression for the radius that yields the minimum factor of safety for any given center. Results of the differentiated expression and the basic expression were compared. This comparison showed that when the differentiated expression was nearly satisfied, the center yielding the minimum safety factors was normally defined.

Further extensive studies are required before any definite conclusion can be made concerning the characteristics of the differentiated expression.

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Signature William Arthur Vischer

Date December 5, 1969

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Chairman, Examining Committee

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ABSTRACT

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An equation was derived expressing the safety factor of a homogeneous, finite slope in terms of the slope geometry, geometry of a circular failure arc, and soil parameters. Safety factors obtained from the derived equation, were compared with those obtained by methods currently in use. Differences of up to five percent were noted in the comparison. The equation for the safety factor was then differentiated, with respect to the radius of the failure arc, in a futile attempt to derive an analytical expression for the radius that yields the minimum factor of safety for any given center. Results of the differentiated expression and the basic expression were compared. This comparison showed that when the differentiated expression was nearly satisfied, the center yielding the minimum safety factors was normally defined.

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CHAPTER I

INTRODUCTION

Probably the most widely used method of analysis for finite slopes is the "method of slices" based on circular failure surfaces. According to Taylor (1948), K. E. Petterson is thought to be the first to use such a method in the study of a quay wall in 1915. Further investigations and studies revealed that actual failure surfaces do not deviate greatly from this assumed circular failure surface.

The method of slices basically consists of dividing the soil mass into vertical slices and performing a static analysis on the soil above the assumed failure surface. As there are many possible circular arcs for a given cross-section, a trial and error procedure must be used to locate both the location of the center of the critical arc and the radius of the critical arc. The time and labor involved in the graphical trial and error solutions is excessive. With the advent of the digital computer, the problems associated with this trial and error analysis were alleviated.

The above remarks indicate the benefit of a direct analytical analysis to the engineering profession. An analytical analysis would not only eliminate the repetitive calculations, but would also allow more efficient use of the computer in slope stability problems.

CHAPTER II

ANALYTICAL DEVELOPMENT

The initial portion of the derivation of the analytic expression is similar to the analysis used in the "method of slices;" the only difference being that exact integration is used to obtain the actuating and resisting forces. After performing the required exact integrations, the actuating and resisting forces can be expressed in terms of the radius, R, and the known geometric and soil parameters. The factor of safety can similarly be expressed as it is a function of these actuating and resisting forces. Differentiating the factor of safety with respect to the radius, R, and setting the resultant equal to zero, maximizes or minimizes the factor of safety with respect to the radius. Solving the expression for R should then yield the radius, for a given slope geometry and arc center, that produces the minimum factor of safety.

For a simple finite slope, three geometric failure possibilities, as shown in Figures 1, 2, and 3, must be investigated. The failure geometry shown in Figure 1 is probably the most common failure noted in soft, cohesive materials, whereas the failure geometry shown in Figure 2 is most common in "mixed" soils. The failure geometry of Figure 1 is the general case and degenerates to the cases shown in Figures 2 and 3 when the appropriate geometric substitutions are made.

Definition of Geometry

The initial step of the derivation consists of defining the intercepts of the slopes and failure arc.

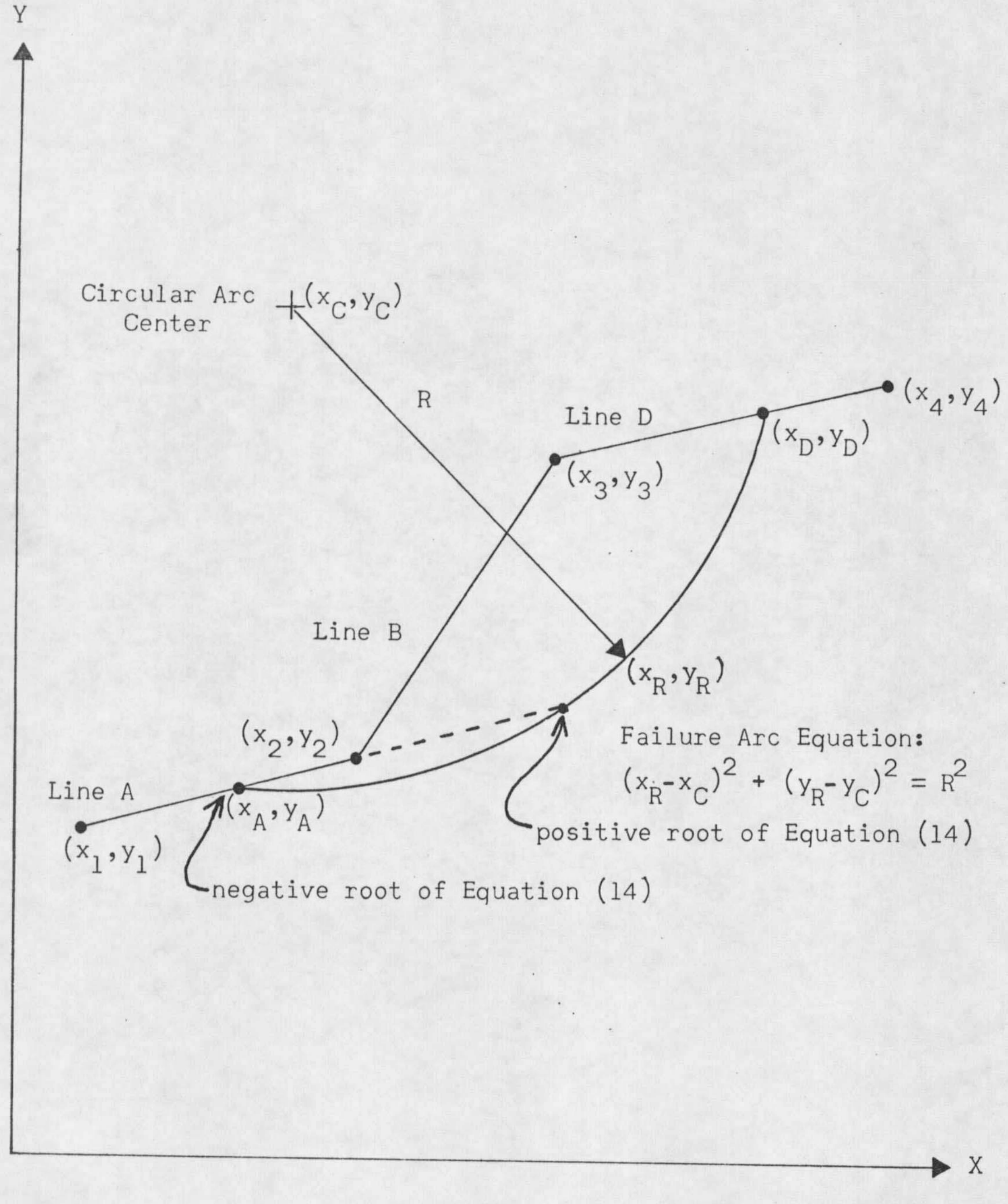

Figure 1. Possible failure geometry of a simple, finite slope, Case 1.

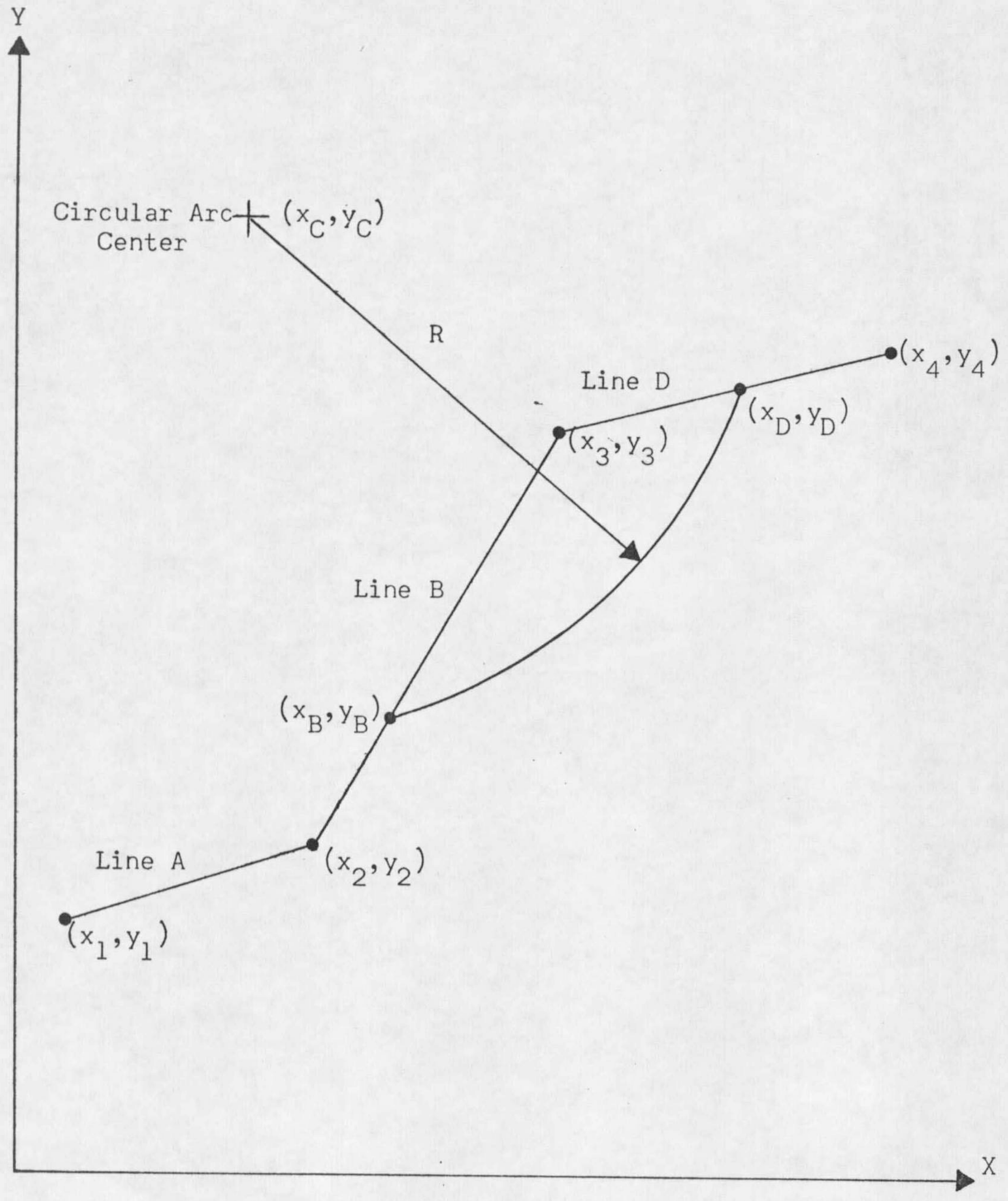

Figure 2. Possible failure geometry of a simple, finite slope, Case 2.