Instrumentation requirements and design of a facility for turbulent natural convection studies
by David Allyn Pracht

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering
Montana State University

Abstract:
A facility was designed and constructed which is to be used for turbulent natural convection boundary layer studies in air adjacent to a heated vertical thin wire. The apparatus was shown experimentally, by the use of anemometer bridge voltage output fluctuations, to be of a suitable size to achieve fully developed turbulent flow. From these fluctuations transition to turbulence has occurred by Rax. = 1.2x10^10 and by Rax = 7.8x10^11 the flow has become fully turbulent. Measurements of the mean velocity and temperature profiles were made and the qualitative results obtained compare very favorably with experimental data of other research efforts. As the distance along the wire is increased the mean velocity profiles indicate a decrease in maximum velocity and a widening of the velocity boundary layer. The temperature profiles tend to retain the same basic shape over the entire length of the wire. A slight increase in the thermal boundary layer thickness was observed as measurements were made further up the heated vertical wire.

INSTRUMENTATION REQUIREMENTS AND DESIGN OF A FACILITY FOR TURBULENT NATURAL CONVECTION STUDIES

by

David Allyn Pracht

A thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Mechanical Engineering

MONTANA STATE UNIVERSITY
Bozeman, Montana

March 1983

APPROVAL

of a thesis submitted by

David Allyn Pracht

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

3/1/83 — Date

William Martindale — Chairperson, Graduate Committee

Approved for the Major Department

3/1/83 — Date

Dennis O. Blackketter — Head, Major Department

Approved for the College of Graduate Studies

3-3-83 — Date

Michael Malone — Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a master's degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Director of Libraries when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my written permission.

Signature David Pracht

Date 3/1/83

ACKNOWLEDGEMENTS

The present author wishes to express his thanks and appreciation to the following for their contribution to this investigation.

His advisor, Bill Martindale for his guidance and support throughout this investigation.

Bob Warrington, Ron Mussulman and Anthony Demetriades, for serving as committee members and reviewing this thesis.

Gordon Williamson, Pat Vowell and Luther Hartz for their helpful assistance in the construction and maintenance of the experimental apparatus.

And the Mechanical Engineering Department of Montana State University, for financial assistance and funding of this investigation.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

Symbol	Description
c	Specific heat at constant pressure, BTU/lb°F (J/kg°C).
CF	Correction factor used to correct bridge voltage output for variations in fluid temperature.
e	Local kinetic energy flux, $e = G^{*}(\nu^{2}/gx^{3})^{2/15}$.
E_b	Anemometer bridge voltage output with variable fluid temperature, volts.
E_c	Anemometer bridge voltage output with variable overheat resistance, volts.
g	Gravitational acceleration, 32.17 ft/sec^2 (9.81 m/sec^2).
G^{*}	$5(Gr_x^{*}/5)^{1/5}$.
Gr_x	Grashof number, $Gr_x = g\beta x^{3}\Delta T_w/\nu^{2}$.
Gr_x^{*}	Modified Grashof number for uniform heat flux surface, $Gr_x^{*} = g\beta x^{4}q'/k\nu^{2}$.
h	Heat transfer coefficient, BTU/hrftPt2Pt°F (W/m^2°C).
k	Thermal conductivity, BTU/hr ft°F (W/m°C).
Nu_x	Nusselt number $Nu_x = hx/k$.
Pr	Prandtl number, $Pr = \mu c/k$.
q'	Uniform heat flux, BTU/hr ft^2 (W/m^2).
r	distance from the heated vertical wire, in. (cm).
R	Change in overheat resistance to simulate fluid temperature variations, ohms.
R_h	Overheat probe resistance, ohms.

Symbol	Description
Ra_x	Rayleigh number, $Ra_x = Gr_x Pr$.
Ra_x^*	Modified Rayleigh number for uniform heat flux surface, $Ra_x^* = Gr_x^* Pr$.
T	Mean temperature of the fluid, °F (°C).
T_c	Calibration temperature of the hot-wire sensor, °F (°C).
T_o	Reference temperature, °F (°C).
T_s	Hot-wire sensor temperature, °F (°C).
T_w	Temperature of the heated surface, °F (°C).
T_∞	Ambient temperature, °F (°C).
ΔT	Local temperature difference, $\Delta T = T - T_\infty$, °F (°C).
ΔT_w	Surface temperature difference $\Delta T_w = T_w - T_\infty$, °F (°C).
U	Meanvelocityinthexdirection,ft/sec (m/sec).
x	Distance from the leading edge of the plate/ wire, ft (m).
y	Horizontal distance from the flat plate, in. (cm).
α	Thermal diffusivity $\alpha = k/\rho c$.
β	Coefficient of thermal expansion, 1/°F (1/°C).
δ_v	Velocity boundary layer thickness, in. (cm).
η	Inner length scale for constant temperature surface, $\eta = [\alpha^2/g\beta(T_w - T_\infty)]^{1/3}$.
λ	Resistance coefficient of temperature for hot-wire sensor, 1/°F (1/°C).
μ	viscosity, lb/sec ft (kg/sec m).
ν	Kinematic viscosity evaluated at the mean temperature, ft^2/sec (m^2/sec).

Symbol	Description
ν_w	Kinematic viscosity evaluated at the surface temperature, ft^2/sec (m^2/sec).
ν_∞	Kinematic viscosity evaluated at the ambient temperature, ft^2/sec (m^2/sec).
ρ	Density, lb/ft^3 (kg/m^3).

ABSTRACT

A facility was designed and constructed which is to be used for turbulent natural convection boundary layer studies in air adjacent to a heated vertical thin wire. The apparatus was shown experimentally, by the use of anemometer bridge voltage output fluctuations, to be of a suitable size to achieve fully developed turbulent flow. From these fluctuations transition to turbulence has occurred by $Ra_x = 1.2 \times 10^{10}$ and by $Ra_x = 7.8 \times 10^{11}$ the flow has become fully turbulent. Measurements of the mean velocity and temperature profiles were made and the qualitative results obtained compare very favorably with experimental data of other research efforts. As the distance along the wire is increased the mean velocity profiles indicate a decrease in maximum velocity and a widening of the velocity boundary layer. The temperature profiles tend to retain the same basic shape over the entire length of the wire. A slight increase in the thermal boundary layer thickness was observed as measurements were made further up the heated vertical wire.

CHAPTER 1.

INTRODUCTION AND STATEMENT OF THE PROBLEM

A thorough understanding of the mechanics of transition and turbulence is essential to the continued development and advancement of aerospace, nuclear reactor, and turbomachinery technology. To date, many studies of forced and free convective turbulence have provided Nusselt number relationships and general temperature and velocity profiles in the turbulent boundary layer, both within enclosures and in an infinite atmosphere. Generally the onset of free convective turbulence is considered similar in nature to forced convective turbulence. Beyond the initial stages of transition, however, the mechanics of free turbulence are still fairly unclear, with simple and accurate mathematical models an elusive goal. The actual mechanics of the transition stage are even less well defined.

In this study, the distributions of the mean velocities, mean temperatures, and turbulent velocity fluctuations have been investigated in order to provide further definition of the turbulent free convection process. Measurements were taken at four locations in the range of $10^{10} < Ra_x < 10^{12}$ in the buoyancy driven flow field around a 24.0 ft. (7.32 m) long heated vertical wire. Air was used as the fluid medium.

STATEMENT OF THE PROBLEM

The objectives of the investigation may be summarized as:

(1) To show that the facility is of suitable size to give turbulent flow.

(2) To show that the instrumentation used is adequate to make temperature and velocity measurements.

(3) To experimentally determine profiles of mean velocity and mean temperature in the boundary layer over a range of values of the Rayleigh number.

(4) To estimate the boundary layer size at various locations.

It was realized, after several problems were encountered, that attainment of the second objective was not possible with the time and resources available. This will be discussed further in Chapters 4. and 5.

CHAPTER 2.

LITERATURE REVIEW

The problem of turbulent natural convection boundary layer flow next to a heated vertical surface has been the subject of numerous investigations. Almost all theoretical efforts to date have depended on analogies with the dynamics of the forced flow boundary layer. Experimental studies have included heat transfer relations, velocity and temperature profiles and fluctuations, and investigations of the mechanisms involved in the transition to turbulence, with typical geometries being the flat plate as well as curved surfaces.

The buoyancy induced flow of a fluid along a surface can be divided into three regions: laminar, transition to turbulence, and turbulent. The laminar region has been analyzed by many investigators in the past and will not be discussed further. The remainder of this chapter is intended to provide a useful background for this particular investigation and is not a complete survey of natural convection research. The following discussion will include the transition to turbulent and turbulent studies done in the past. A discussion of velocity measurement techniques used by previous investigators of natural convection heat transfer is also included.

NATURAL CONVECTION - TRANSITION

The most extensive work in the field of natural convection transition has been carried out by B. Gebhart and his colleagues [4], [13], [14], and [15]. Jaluria and Gebhart [13] have postulated that transition events can be correlated by the ratio of the modified Grashof number to the characteristic length, x, raised to an empirical power, n: G^*/x^n: where x is the distance from the lower edge of a vertical flat plate and n is of the order 1/2. At the first transition event, from laminar to transition flow, the ratio is proportional to a parameter defined as the local kinetic energy flux, $e = G^*(\nu^2/gx^3)^{2/15}$. In studies with water the velocity and thermal transitions occur separately at e = 13.6 and 15.2, respectively. In air, where the Prandtl number is on the order of 0.7, these values should nearly coincide. A value of e for the development of fully turbulent flow was not identified. In terms of the conventional Rayleigh number modified for constant heat flux on the wall surface, Ra_x^*, Qureshi and Gebhart [4] have identified transition points in the range of $1.2 \times 10^{13} < Ra_x^* < 4 \times 10^{13}$ as the beginning of transition and $5 \times 10^{13} < Ra_x^* < 10^{14}$ for fully developed turbulence. These values agree fairly well with the ranges obtained by Vliet and Liu [6] in earlier studies. Vliet and Liu [6] define the initial transition event as the point where the measured surface temperature begins to decrease. Cheesewright [10] has defined the beginning of turbulence as the point where significant temperature fluctuations in the boundary layer begin to occur. Both studies defined the end of transition as that point where the respective temperature fluctuations decreased.

NATURAL CONVECTION - TURBULENCE

The earliest attempt to analyze the turbulent natural convection boundary layer on vertical surfaces was due to Eckert and Jackson [7] whose empirical approach was based on the assumed similarity between forced and free convection. This analysis provided reasonable predictions for the rate of heat transfer, but has not been found to be consistent with experimental data for the mean velocity profile obtained by Warner and Arpaci [1] or Cheesewright [10]. Eckert and Jackson [7] found the heat transfer-Rayleigh number relation to be:

$$Nu_x = (.021)Ra_x^{2/5}. \qquad (2.1)$$

More recently there have been a number of partially successful attempts to apply turbulence computational models to the calculation of buoyant flows next to vertical surfaces. Kato et al [12] applied the integral method by assuming an eddy diffusivity relationship and a distribution of the heat flux across the boundary layer and so derived the profiles of velocity and temperature rather than assuming them. They then found a local Nusselt number relation to be:

$$Nu_x = .149[(\nu/\alpha)^{.175} - .55](Gr_x)^{.36}. \qquad (2.2)$$

The results are not very different from those of Eckert and Jackson [7]. Cebeci and Khattab [8] have integrated the continuity, momentum, and energy equations numerically. In their study they consider the eddy viscosity formulation developed for forced convection flows and apply it to free convection flows to predict heat transfer rates, velocity profiles, and temperature profiles on vertical flat plates. They report that their Nusselt number versus Rayleigh number curves agree with the experimental data of Warner and Arpaci [1] but disagree with the predictions of Eckert and Jackson.

Mason and Seban [9] used a modification of a program of the Patankar-Spalding type to numerically predict heat transfer coefficients. After a substantial exploration of empirical constants adjusted to fit the experimental data, they were able to obtain results that agreed fairly well with available measured data. Their report also used turbulence parameters which successfully predicted forced convection flows modified for free convection flows from vertical surfaces.

A more recent analysis has been done by George and Capp [2] in which classical scaling arguments were used to predict the existence of a two layer turbulent natural convection boundary layer for flows adjacent to vertical heated surfaces. The two layers include: an outer region consisting of most of the boundary layer in which viscous and conduction terms are negligible, and an inner region in which the mean convection terms are negligible. The inner layer is identified as a constant heat flux layer. The results of their analysis compared well with experimental data of several authors.

Many different measurements of the heat transfer and temperature and velocity profiles have been made for turbulent natural convection past a vertical surface. The most common geometry used has been the flat plate. One of the earliest and most recognized of these works was conducted by Cheesewright [10], who measured temperature profiles next to a constant temperature wall in air. Both distance along the plate and wall temperature were varied. The experiments were carried out over a range of Grashof numbers from 10^4 to 1.5×10^{11}. For $Gr_x > 2 \times 10^{10}$ the boundary layer is reported as being turbulent. One problem encountered was due to the side screens of the apparatus causing a lack of two-dimensionality, but Cheesewright felt that the plate was sufficiently wide so that measurements down the center-line were not significantly affected. Temperature profiles are plotted as $\Delta T/\Delta T_w$ versus $(y/\eta)^{-1/3}$ so that the buoyant subrange, as predicted by George and Capp [2], appears as a straight line. The data not only collapsed to a single curve, but also exhibited a well defined inverse cube root region where the buoyant sublayer would appear, as well as the linear region next to the wall. The data are shown to be in reasonable agreement with the empirical results of Eckert and Jackson [7] also.

More recently Cheesewright and Doan [21] have measured spacial correlations normal to a heated vertical flat plate and longitudinal space-time correlations of temperature fluctuations in the transition and fully developed turbulent regions. The flow in the viscous sublayer was shown to be relatively independent from that outside. Also a fairly uniform turbulence structure outside the sublayer was found, as noted by the slow variation of the dissipation length and correlation length scales of turbulence with distance normal to the wall.

Another investigation of the flat plate geometry was undertaken by Warner and Arpaci [1], using a single aluminum flat plate. They showed that for a single constant wall temperature in air, most of the temperature profile data could be collapsed when plotted as $\Delta T/\Delta T_w$ versus y, the dimensional distance from the wall. An experimental study done by Fujii [11] with a vertical cylinder in ethyleneglycol provides the following Nusselt number relationships based on temperature distributions in the boundary layer:

$$\begin{aligned} Nu_x &= .49(Ra_x)^{1/4} \qquad Ra_x < 8.5\ x10^9 \\ Nu_x &= .87(Ra_x)^{1/4} \qquad 8.5x10^9 < Ra_x < 8x10^{10} \end{aligned} \tag{2.3}$$

Comparison of these results with those of Warner and Arpaci [1] at similar Rayleigh numbers shows a poor agreement. This seems to indicate that the relations of Fujii are useful only in the Prandtl number range, (Pr=55), used in his experiment. It should be noted however that because of physical dimensions, length / diameter ratio = 4.34, Fujii's cylindrical apparatus may have produced a significant curvature effect. There were no velocity profiles measured for this geometry.

Fujii et al [3] conducted an extensive series of experiments using vertical cylinders at constant wall temperature. Since the heat transfer at the wall is entirely governed by the wall layer, it can be expected that the heat transfer relation will be the same as that for flat plates as long as the radius of curvature is much greater than the wall layer thickness. They found that for a narrow range of Prandtl numbers, the temperature profiles measured over a wide range of wall conditions collapsed onto a single curve over most of the boundary layer. The profiles for substantially different Prandtl numbers were different as expected. Their results for water, spindle oil, and Mobiltherm oil could be shown to correlate as:

$$(Nu_x)(\nu_w/\nu_\infty)^{.21} = .13(Ra_x)^{1/3} \qquad (2.4)$$

$$10^{10} \leq Ra_x$$

These results are in excellent agreement with George and Capp [2]. It was also determined by mirage method, a method based on the principle that the refractive index of the fluid in the boundary layer decreases with temperature rise, that turbulence began at $Ra_x = 2 \times 10^{11}$ for spindle oil. Again, there were no velocity profiles given.

VELOCITY MEASUREMENT TECHNIQUES

Although the main interest of the work was in the turbulent flow region, it was none the less intended to make some measurements in laminar flow, particularly for calibration purposes. It was therefore desirable that the measuring technique should be such as to permit the measurement of mean velocities both in laminar and turbulent flows. For mean velocity measurements in the latter, the output of any suitable measuring instrument had to be capable of being averaged over a period of one minute or longer, because of the expected low frequencies of the turbulence. Many techniques for the measurement of fluid velocities have been developed and several are described below, along with a discussion of three calibration procedures that have been used by Hollasch and Gebhart [16], Cheesewright [10] and Aydin and Leutheusser [22].

One of the most commonly used techniques for measuring flow velocities is that involving the measurement of the difference between the local static pressure and the local stagnation pressure at a point in the fluid, using a pitot-static tube and some form of manometer. It was immediately obvious that this technique could not be used in the present problem because the maximum pressure difference involved would be of the order of $9x10^{-4}$ in. ($2x10^{-3}$ cm) of water, much too small to be measured with any degree of accuracy, particularly in view of the need for long time period averaging.

A straight forward optical technique of photographing tracer particles, most often helium filled bubbles, and measuring track lengths has been used with limited success. This method would require a large amount of work involving the analysis of the photographs in turbulent flow. It would not be sufficient to make velocity and velocity fluctuation measurements of the type needed in this investigation, due to the random movement of the air especially near the heated vertical wire where good accuracy is essential.

Another commonly used technique, hot-wire anemometry, is based on the relationship between the rate of heat transfer from a heated wire and the velocity of the fluid flowing past it. Since this was the method that was eventually used to make velocity measurements, its use in previous natural convection heat transfer investigations and the respective calibration procedures, will be discussed in the remainder of this chapter.

Although hot-wire calibration at high velocites, greater than 10.0 ft/sec (3.0 m/sec), is a relatively simple task using a pitot-static tube and thermocouple to determine a known velocity and temperature, calibration at low velocities is far more complicated. One reason is due to the lack of an instrument comparable to the pitot-static tube that can accurately determine low velocities with which an anemometer output signal can be matched. Another problem encountered is due to the importance of the natural convection cooling of the hot-wire sensor combined with the forced convection flows that are to be measured. This makes it necessary that the direction of the flow in the calibration facility is the same as that in the actual experimental apparatus.

Since the anemometer output bridge voltage is governed by the temperature as well as the velocity of the fluid being measured, calibration must be made at the exact temperature encountered in the flow or a temperature correction factor becomes necessary.