An empirical comparison of alternative functional forms of systems of consumer demand equations
by Cathy Anne Roheim

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Applied Economics
Montana State University

Abstract:
This study considers five functional forms of consumer demand models applied to the same set of data to estimate systems of demand equations: the Rotterdam model, the Indirect Addilog model, the Linear Expenditure System, the Almost Ideal Demand System, and the double logarithmic model. These are compared to each other and the "best" one is determined on the basis of its adherence to existing theoretical restrictions on its respective parameters and on its ability to statistically summarize observed consumer behavior accurately.

Theoretical restrictions assumed to hold include weak separability of the utility function and perfect aggregation of the utility function over individuals. Homogeneity of degree zero in prices and income was imposed on all estimated demand equations, while Slutsky symmetry was tested for explicitly in each model. The Rotterdam model, Indirect AddiIog model, and the Almost Ideal Demand System accepted the symmetry conditions, whereas the remaining models did not.

To statistically compare the performance of the models, the information inaccuracy measure was used in which the actual budget shares of each commodity are compared to the budget shares predicted by the model. The functional form corresponding to the information inaccuracy closest to zero is considered best. In this study, the Rotterdam model placed first with the Almost Ideal Demand System and the Indirect Addilog in second and third places, respectively.

Data used to estimate the model are annual observations on U.S. real personal income, real prices and per capita consumption of fed and non-fed beef, pork, and chicken from 1962 through 1980. The deflator used to adjust for inflation over the observation period was the Implicit Price Deflator for Personal Consumption Expenditures.

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ABSTRACT

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INTRODUCTION

An understanding of theoretical implications of consumer demand models is as important to the accurate estimation of demand equations as the correct use of econometric techniques. Not only is obtaining unbiased and efficient estimators of the parameters of concern, but so is determining whether the model in question obeys properties set forth by basic utility theory. An empirical demand equation with an excellent statistical fit is of little use to economists if it has no theoretical basis and cannot be used for instructional or predictive purposes. This study considers five functional forms applied to the same set of data to estimate systems of consumer demand equations: the Rotterdam model (Theil 1978), the Indirect Addilog model (Houthakker 1960), the Linear Expenditure System (Stone 1954), the Almost Ideal Demand System (Deaton and Muellbauer 1980), and the double logarithmic model. These will be compared to each other and the "best" one will be determined on the basis of their adherence to existing theoretical restrictions on their respective parameters and on their ability to statistically summarize observed consumer behavior accurately.

Data used to estimate the models are annual observations on U.S. real personal income, real prices, and per capita consumption of fed and non-fed beef, pork, and chicken from 1962 through 1980. The deflator used to adjust for inflation over the observation period was the Implicit Price Deflator for Personal Consumption Expenditures.[1]

Other studies have also attempted to determine the best functional form specification for demand equations. Parks (1969) applied the information inaccuracy measure, outlined in Chapter VII, to compare the Rotterdam model, the Indirect Addilog model, and the

[1] U.S. Department of Agriculture (see Appendix).

Linear Expenditure System. Data used were a time-series of prices, quantities, and total consumption in Sweden for the period 1861-1955. Eight sectors of the economy, including agriculture, manufacturing, transportation and communications, commerce and insurance, domestic services, housing services, public services, and imported goods were covered. Findings were that the Rotterdam model had the smallest information inaccuracy for the entire sample period, making it the "best" model. However, Parks mentions that an important difference among the models which may be reflected in the results is that the Indirect Addilog model and Linear Expenditure System both assume additivity and thus allow less flexibility in the adjustment of the coefficients on the price terms than does the Rotterdam model, which is constrained only by homogeneity and symmetry, properties discussed in Chapter I.

Goldman (1971) did a similar comparison with the same models using British and Dutch data from 1922-1939 and 1949-1963 for foods, "vice" (including tobacco and beverages), durables, and a remainder. His conclusions were that the Indirect Addilog had the worst fit of all the models, and that the Rotterdam performed relatively better than the other two.

Using Dutch data for food, beverages and tobacco, durables, and a remainder from 1922-1963 and British data on the same groups of commodities from 1900-1938, Theil (1975a) determined that the best demand model was the Rotterdam. Based on the average information inaccuracy for all commodity groups taken jointly, the Indirect Addilog was second and the Linear Expenditure System third.

Deaton and Muellbauer (1980) have concluded that the Almost Ideal Demand System is comparable to the Rotterdam and translog models, but has advantages over both. Bewley (1982), interested to see if this claim was in fact true, compared the Rotterdam model, the Almost Ideal Demand System, the Indirect Addilog model, and a "naive" model. He used an Australian time-series from 1960-1975 of per capita expenditure and prices for food, alcohol and tobacco, clothing, rent, durables, transportation and communication, and other expenditures. The information inaccuracy difference between the Almost Ideal Demand System and the Rotterdam model is small. Supporting earlier claims, both models performed better than the Indirect Addilog which did, however, do better than the naive model.

Yoshihara (1969) theoretically and empirically compared the Rotterdam, Indirect Addilog, double logarithmic, and Linear Expenditure System models and found that the Linear Expenditure System explained the pattern of Japanese demand best. He determined that both the Rotterdam and double logarithmic models were unsatisfactory from a theoretical viewpoint. Both models violated his empirical conditions that the income elasticities should not all be equal to one. Yoshihara believes that when a consumer is presented with rising income and constant prices, his expenditure pattern is more likely to change than stay constant. This constraint, however, implies that the utility function must be non-homogeneous which is a property not allowed in this study; homogeneity of degree zero in prices and income of the demand equations was imposed on all estimations.

The Indirect Addilog and Linear Expenditure System models satisfied the theoretical properties, and so were fitted to Japanese data on per capita consumption for 1902-1960. Based on sum of squares of residuals, the conclusion was that the Linear Expenditure System provided a better fit.

In this paper, Chapter I will outline the theoretical implications and restrictions implied by utility maximizing behavior. The next five chapters will each be devoted to an individual functional form specification, including explanations of estimation procedures and results. Chapter VII, the final chapter, provides conclusions about the five demand models based on the preceding results.

CHAPTER I

DEMAND FUNCTIONS AND RESTRICTIONS

To obtain a system of demand equations by conventional microeconomic theory, we must assume the existence of an underlying utility function,

$$U = f(q_1, q_2, \ldots, q_n) \tag{1.1}$$

which the representative consumer wishes to maximize subject to a budget constraint. The utility function is assumed to be strictly increasing, quasi-concave, and twice differentiable.

Ordinary demand equations can be obtained from the direct utility function, or from the associated indirect utility function. To arrive at the demand equations via the direct utility function, one must explicitly solve a constrained optimization problem. A Lagrangean function is formed to maximize utility subject to the budget constraint

$$L = f(q_1, q_2, \ldots, q_n) + \lambda\left(M - \sum_{i=1}^{n} p_i q_i\right) \tag{1.2}$$

where λ is the Lagrangean multiplier, M is nominal income, and p_i equals the nominal price of the ith good. First order conditions are found by differentiating L with respect to q_i and λ, resulting in

$$\partial L/\partial q_i = U_i - \lambda p_i \qquad \text{for } i = 1, 2, \ldots, n \tag{1.3}$$

$$\partial L/\partial \lambda = M - \sum_{i=1}^{n} p_i q_i \tag{1.4}$$

where $U_i = \partial U/\partial q_i$. Notice that the right hand side of (1.4) is equal to the budget constraint in implicit form. Setting (1.3) and (1.4) equal to zero, we obtain the (n+1) first order conditions

$$U_i = \lambda p_i \qquad \text{for all } i = 1, 2, \ldots, n \tag{1.5}$$

$$\sum_{i=1}^{n} p_i q_i = M \tag{1.6}$$

Solving for λ in (1.5) and equating the resulting equations establishes that

$$U_1/p_1 = U_2/p_2 = \ldots = U_n/p_n \tag{1.7}$$

which expresses mathematically the economic principle that the marginal utility per dollar of all goods $i=1, \ldots, n$ is equal when utility is maximized, assuming second order conditions are met. This can be shown graphically in a two good case, depicted in Figure 1.

At points q_1^* and q_2^*, we have obtained the highest attainable level of utility which is the highest indifference curve given our budget constraint.

Demand functions for the n commodities are found by substituting U_i/p_i into the differential of L with respect to λ and solving the resulting equation for q_i^*. The q_i^* functions form what is known as a system of "ordinary" demand equations, as opposed to "compensated" demands, denoted q_i^c, where expenditures are minimized subject to a fixed level of utility.

To insure that the first order conditions yield a global maximum, second order conditions must be checked. Assuming the first order conditions are satisfied, it is a sufficient condition for a global maximum if the utility function, U(q), exhibits strict quasi-concavity in all of the positive, or first quadrant of n dimensional space.

Given that utility is a function of quantities consumed, and that the function is strictly quasi-concave, monotonically increasing, and differentiable, a specification for demand functions to be empirically estimated can be derived from the first order conditions of utility maximization as shown above. In some cases, this may be more easily said than done because these conditions frequently cannot be solved explicitly for the demand functions, and when they are, the resulting equation may be difficult or impossible to estimate.

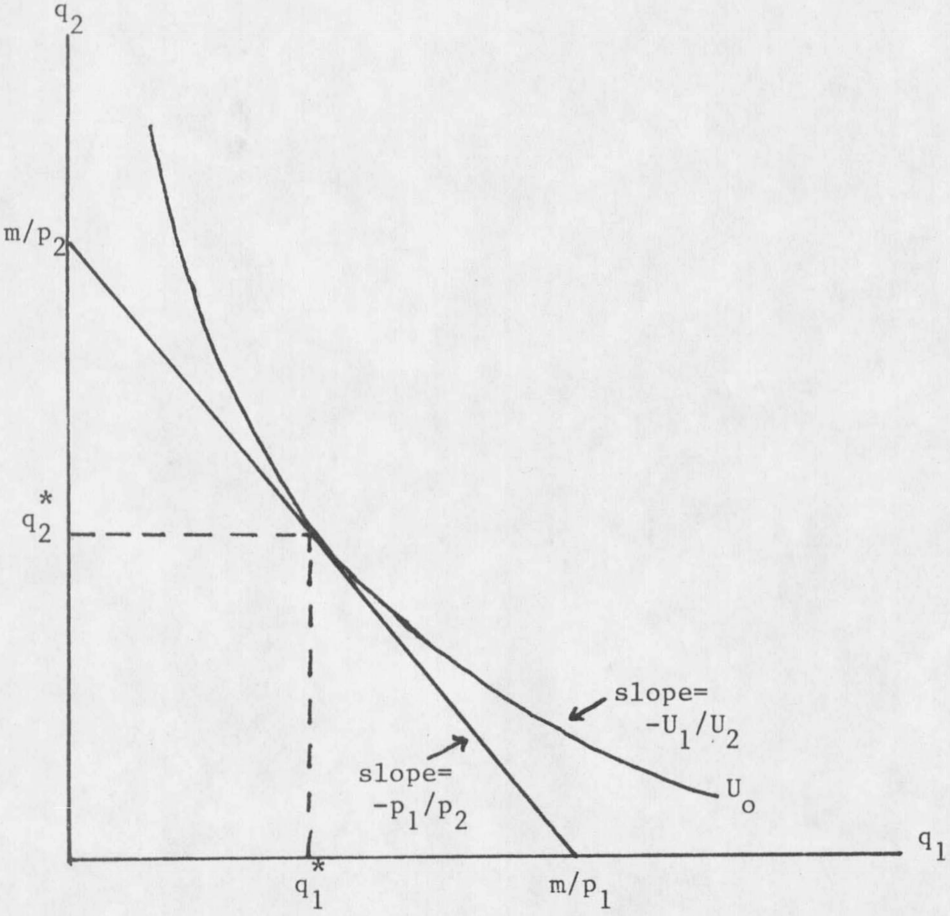

Figure 1. Two good case of utility maximization.

Duality theory provides a convenient way of obtaining a functional specification for demand functions, yet avoiding the problem of explicitly solving an optimization problem. The dual approach makes it possible to go from preferences to behavior and back again in two steps. Therefore, construction of preference consistent demand functions becomes straightforward.

Duality is based on an indirect utility function

$$\widetilde{U} = g(p_1, p_2, \ldots, p_n, M) \qquad (1.8)$$

defined to be the maximum utility obtainable at the given prices and income level. The indirect function is obtained by substituting the ordinary demand equation for each good into the direct utility function; the resulting indirect utility is thus a function of prices and income rather than a function of quantities of goods consumed, $q_1, \ldots, q_n$. A system of ordinary demand equations is obtained from the indirect utility function by applying Roy's Identity,

$$q_i^* = -\partial\widetilde{U}/\partial p_i \Big/ \partial\widetilde{U}/\partial M \qquad (1.9)$$

Another approach to applying duality theory is to substitute U into compensated demand functions obtaining ordinary demand functions; the compensated demand functions are obtained by applying Shepard's lemma to an indirect expenditure function. However, utility, a non-estimatable variable, is an argument in the resulting demand equations, thus explaining why ordinary rather than compensated demands are estimated in empirical studies.

As a practical matter, the dual approach is sometimes easier to apply, likely to be more flexible to measure, and able to more conveniently analyze problems than the primal approach. Whichever approach is chosen to arrive at the system of demand equations, these equations must satisfy three restrictions that do not depend on functional form or the approach to obtaining specifications of a system of demand equations. Adding-up is one of these restrictions where

$$\sum_{i=1}^{n} p_i q_i = M \qquad (1.10)$$

or total expenditures sum to total income. However, this restriction is more relevant to complete systems of equations where estimates of the demand for all commodities is desired, rather than estimation of demand for a subsystem such as the one in this study.

The other two restrictions are: (1) each equation must be homogeneous of degree zero in prices and income, and (2) the demand system must satisfy what are known as Slutsky conditions, which are mathematical restrictions on derivatives of the demand equations.

Homogeneity[2] is based on the assumption that individual consumers make their decision based on relative prices, not absolute prices. The notion of money illusion is dismissed.

The Slutsky equation is written

$$\partial q_i^*/\partial p_j = \partial q_i^c/\partial p_j - q_j^*(\partial q_i^*/\partial M) \qquad i, j = 1, \ldots, n \qquad (1.11)$$

where q_i^* and q_i^c denote the ordinary and compensated demand functions, respectively. This equation shows a separation of the reaction of a consumer to a price change of good j on the quantity demanded of good i into two parts, the income effect, $q_j^*(\partial q_i^*/\partial M)$, and the substitution effect, $\partial q_i^c/\partial p_j$. The income effect takes into account the change in quantity demanded due to a change in real income of the consumer which is due to the nominal

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[2] A function is defined to be homogeneous of degree k if

$$f(tq_1, tq_2, \ldots, tq_n) = t^k f(q_1, \ldots, q_n)$$

where k is a constant and t is any positive number. Therefore, if a function is to be homogeneous of degree zero, then

$$f(tq_1, \ldots, tq_n) = t^0 f(q_1, \ldots, q_n) = f(q_1, \ldots, q_n)$$

In other words, this means that if all prices and income are multiplied by a positive constant t, the quantity demanded of the goods i = 1, . . ., n must remain unchanged. This restriction is easily imposed by employing the necessary and sufficient condition that all prices and income are in ratio form, i.e., divide each by a price deflator.

price change. The substitution effect incorporates the change in quantity demanded of a good because of a change in its relative price.

The conditions which are derived from the Slutsky equation and which all demand functions must satisfy are

$$\partial q_i^*/\partial p_j + q_j^*(\partial q_i^*/\partial M) = \partial q_j^*/\partial p_i + q_i^*(\partial q_j^*/\partial M) \quad (1.12)$$

the cross-substitution effect k_{ij} exactly equals k_{ji}.

$$\sum_{i=1}^{n} p_i(\partial q_i/\partial p_j) + q_j = 0 \quad (1.13)$$

or, the sum of all price and income elasticities exactly equals zero. And

$$\partial q_i^c/\partial p_i < 0 \quad (1.14)$$

and own-substitution effect k_{ii} is negative.

Along with the general restrictions on demand functions discussed above, some particular properties of the utility function are required. These include separability and aggregation. A weakly separable utility function is defined as a function in which the marginal rate of substitution between two variables belonging to the same group be independent of the value of any other group (Leontief 1947). It can be written as

$$U(q) = \phi[U_1(q_1), \ldots, U_s(q_s)] \quad (1.15)$$

where $(U_1, \ldots, U_s)$ is a function of s variables and, for each s, $U_s(q_s)$ is a function of subvector q_s alone (Goldman and Uzawa 1964).

To be able to partition the consumption set into subsets which would include commodities that are closer substitutes or compliments to each other than to goods in other subsets is a desirable property, particularly in cases such as this study. Gorman (1959) proves that if specific utility functions are to be homogeneous of degree one in prices and income, we should never group luxuries, near-luxuries, and necessities together. This justifies looking at the portion of income that individuals will allocate to individual goods, such as meats, a subset of a larger group, food, which are closer substitutes than, for example, beef and furniture. For a more in depth discussion of separability definitions, see Goldman and Uzawa.

Phlips (1974) and Pollack (1971) feel that additivity is defensible only if the arguments of the utility function are taken to be broad aggregates of goods, such as food and clothing, rather than individual commodities. Additivity extends the definition of separability to items from a pair of different groups; that is, the marginal rate of substitution between dress slacks and steak should be independent of the consumption of tents.

We would also like our utility function to be applicable to both the individual and the aggregate. A representative consumer exists if the market behavior of an aggregate of different consumers is the same as the market behavior of a number of identical hypothetical consumers, each with the same level of income (Muellbauer 1976). This presents the aggregation problem. Does the representative consumer exist, and if so, does he reflect the behavior of the average consumer?

Muellbauer (1975) establishes necessary and sufficient conditions for the aggregation relations to be consistent in functional form with the individual relations. The most general condition is called "generalized linearity" (GL), where the relative marginal value shares are independent of income or utility, i.e.,

$$\partial/\partial y(\partial w_i/\partial y \big/ \partial w_j/\partial y) = 0 \qquad \text{for all i, j} \tag{1.16}$$

More restrictively, aggregate market demand equations are consistent with individual demand equations corresponding to some level of income, which does not vary as relative prices vary if and only if "price independent generalized linearity" (PIGL) holds. This is defined by budget shares of the form

$$w_i(M, p) = \log M \, A_i(p) + B_i(p) \tag{1.17}$$

or