An analysis of factors affecting the demand for milk in Montana
by John Elliott Barkell

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Applied Economics
Montana State University

Abstract:
The purpose of this study is to identify the factors that influence consumption of fluid dairy products in Montana and to measure the impact on consumption of a change in these factors.

Eyen though actual consumption of fluid milk at the retail level is not readily observable, a close proxy is the volume of milk utilized for fluid milk products.

A dynamic demand model is formulated based on the theory of consumer choice, factors suggested by prior studies, and on the socioeconomic characteristics of Montana. Maximum Likelihood Estimation of the statistical model is performed using a non linear least squares procedure.

Factors found to significantly influence milk consumption are price, income, the age structure of the population and season of the year. Consumers appear to respond slowly to changes in the economic factors with only about 50 percent of the ultimate response completed in one year. Consumption is found to be price inelastic in the short run with an elasticity of -.33. However, in the long run after complete adjustment occurs, demand is found to be elastic with an estimate of the long run price elasticity of -2.58.

An inverse relationship between consumption and income was estimated when income was entered in a strictly additive form. It is shown that this results from the interaction between income and the age structure of the population.

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Date July 11, 1980

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JOHN ELLIOTT BARKELL

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Approved:

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Head, Major Department

Graduate Dean

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ABSTRACT

The purpose of this study is to identify the factors that influence consumption of fluid dairy products in Montana and to measure the impact on consumption of a change in these factors. Even though actual consumption of fluid milk at the retail level is not readily observable, a close proxy is the volume of milk utilized for fluid milk products.

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Chapter 1

INTRODUCTION

This study examines the demand relationships for fluid milk products in Montana from 1964 through 1976. Specifically, this analysis is directed to:

(1) Identifying the factors significantly influencing fluid milk consumption in Montana.

(2) Measuring the magnitude of consumer response to changes in these factors, and

(3) Analyzing the time path of consumer adjustment to changes in the influencing factors.

The first of the specific objectives is accomplished by analyzing (1) the theory of consumer behavior, (2) existing prior studies, and (3) the economic and socioeconomic structure of Montana's economy and population. Chapters 1 and 2 summarize this analysis.

The second objective is accomplished by utilizing econometric procedures to estimate the parameters underlying the statistical demand relationship. Chapters 3 and 4 outline the procedure and summarize the results of the statistical analysis.

The final objective is accomplished by statistically analyzing the dynamic nature of consumer response to changes in the factors upon which economic decisions are based. This objective is accomplished by defining a theoretical behavioral relationship believed to explain observed changes in consumption over time and statistically estimating the parameters that underly the theoretical relationship.

More specifically, this study statistically examines the aggregate retail demand relationship for Grade A, Class 1 dairy products in Montana. Quarterly time series data is used in the analysis for the years 1964 to 1976.

Retail level demand is estimated for the consumption of the following milk products: whole milk, low fat and skim milk, chocolate milk and drink, buttermilk, half and half, commercial cream and whipping cream.

Three economic agents influence the demand structure of the fluid dairy product market in Montana: Consumers, the dairy industry (producers and processors) and price regulators.

Consumers seek to maximize their total utility as a function of the quantities of commodities consumed subject to an income constraint. Producers and processors seek to maximize their net return subject to the price received for their product and the prices paid for the factors of production. Price regulators must insure that adequate supplies of wholesome milk products are available to consumers at all times at the lowest possible cost in accordance with state law. The characteristics of each of these economic sectors will be examined in the remainder of this chapter.

Consumers

Dairy product purchases represent about 16 percent of the total expenditure for food and 2.3 percent of all expenditures in the United States [13]. Next to meat, dairy products comprise the single most important group of commodities in the diet. Fluid milk products are the most important element within the dairy product group, accounting for well over one-half of all expenditures for dairy products in the United States.

Annual per capita consumption of fluid milk products in Montana has fallen in recent years. In 1967, 246 pounds of milk were consumed per person compared to approximately 236 pounds per person in 1976. Nationally, a decreasing trend in consumption was also observed over the same period. National per capita consumption in 1964 was estimated to be nearly 300 pounds per person. By 1975, national consumption had fallen to 248 pounds. Currently Montanans appear to consume slightly less milk per person than the national average [4]. This may be due to several factors. First, per capita income in Montana is lower than the national average. Second, Montana is basically an agricultural, rural state. This is likely to exert some influence on consumption. These factors are analyzed in greater detail in latter sections of this chapter. Table 1 summarizes the historical trend in per capita milk consumption in Montana.

Rather dramatic decreases in per capita consumption occurred

Table 1

Per Capita Consumption of Fluid
Milk in Montana

(1967-1976)

Pounds per person per year

Year	Per Capita Consumption[1/]
1967	246
1968	246
1969	236
1970	239
1971	244
1972	248
1973	239
1974	234
1975	229
1976	236

Source; A Report of Milk Utilization in Montana; Department of Business Regulation; Milk Control Division.

[1/] Calculated as follows: (Total Class 1 unit sales - fluid milk sales to other states) ÷ Bureau of Census population estimates.

between the years 1972-1975. This was a period of rapid milk price increase. Table 2 summarizes the changes in the "order" price of one-half gallon homogenized milk over this period. This rapid price escalation in both nominal and in real terms probably explains much of the decline in consumption during this period.

There have also been significant changes in the types of fluid milk products consumers have demanded. In 1967 nearly 74 percent of all fluid milk consumed was homogenized. Low fat milk comprised only 16.5 percent of milk sales. However, a dynamic shift from whole milk to low fat milk occurred after 1967. Homogenized milk dipped to a market share of 49.5 percent of sales while low fat milk rose to 39 percent. Table 3 shows this change in demand over time.

This change in consumer tastes is probably attributable to several sources. First, there has been an increased awareness of the possible health hazards associated with cholesterol and fat. Low fat milk possesses the desirable attributes of fluid milk with reduced cholesterol levels. Low fat milk is also priced somewhat lower than whole milk. Generally, one half gallon of low fat milk is 2 cents less than whole milk.

Prior studies have shown that the economic and demographic characteristics of consumers are factors that can influence fluid milk consumption. Several of the characteristics of Montana's populace that may be important determinants of milk consumption patterns are:

Table 2

Changes in the Montana Order Price
Per One-Half Gallon Homogenized Milk
(1973-1975)

Date of Change	New Price (Cents per one-half gallon)	Cumulative Change in Price since January 1, 1973 (cents per one-half gallon)
January 1973	65	0
February 1973	67	2
May 1973	69	4
October 1973	71	6
November 1973	75	10
February 1974	76	11
March 1974	79	14
April 1974	82	11
June 1974	85	20
August 1974	84	19
September 1974	83	18
October 1974	84	19
December 1974	86	21
April 1975	85	20
July 1975	86	21
August 1975	85	20
October 1975	86	21
December 1975	87	22

Source: Office of Business Regulation; Milk Control Division.

Table 3

Consumption of Fluid Milk Products by Type[2/]
(1967-1976)
(percent of total sales)

Year	Raw Milk	Pasturized Milk	Homogenized Milk	Chocolate Milk	Low Fat	Butter Milk	Skim Milk	Chocolate Drink	All Cream
1967	.8	.7	73.7	1.2	16.5	1.3	3.2	.09	2.3
1968	.6	.4	70.2	1.2	20.4	1.4	3.1	.2	2.3
1969	.6	.3	66.5	1.5	24.2	1.4	2.9	.2	2.2
1970	.6	.2	63.2	1.8	27.8	1.4	2.7	.09	2.1
1971	.5	.2	60.1	2.3	30.5	1.4	2.7	.09	2.0
1972	.6	.1	57.8	2.6	32.6	1.4	2.9	.10	1.9
1973	.6	.4	54.5	2.5	34.6	1.5	3.7	.5	1.8
1974	.7	.4	51.7	2.4	37.0	1.6	4.4	.04	1.8
1975	.6	.4	50.4	2.5	38.0	1.7	4.6	.04	1.7
1976	.7	.04	49.5	2.7	39.1	1.7	4.5	.05	1.7

Source: A Report of Milk Utilization in Montana; Department of Business Regulation; Milk Control Division 1967-1976.

[2/] Data unavailable prior to 1967.

(1) Disposable income levels

(2) Changes in the distribution of income

(3) Changes in total population

(4) Rural/Urban composition

(5) Age structure

(6) Education level

(7) Racial composition

(8) Sex composition

Disposable Income Levels

Disposable per capita income in Montana in unadjusted terms has risen steadily since 1964. The trend has also been upward when adjusted by the consumer price index. Table 4 summarizes the trend in disposable income levels since 1964.

Income Distribution

Changes in the distribution of income quite likely affect aggregate demand for many commodities including fluid milk. Three sectors of the population structure are of interest when analyzing income distribution: (1) Rural and Urban, (2) White and Non-white, and (3) Male and Female.

Table 5 shows changes in income distribution of rural and urban residents between 1959 and 1970.

Major distributional changes may be observed in table 5, notably

Table 4

Disposable Income per Person in Montana
1964-1975
(dollars per person per year)

Year	Per Capita Disposable Income (Unadjusted)	Real per Capita Income (1967 = 100)
1964	2030	2185
1965	2181	2308
1966	2352	2420
1967	2449	2449
1968	2543	2440
1969	2705	2464
1970	3035	2610
1971	3125	2576
1972	3508	2730
1973	4038	3034
1974	4193	2839
1975	4667	2895

Source: Survey of Current Business.

Table 5

Percentage Distribution of Money Income of Families and Median Family Income, Urban and Rural Sectors, Montana and United States, 1959 and 1970

	Montana						United States					
	Urban		Rural Non-Farm		Rural Farm		Urban		Rural Non-Farm		Rural Farm	
	1959	1970	1959	1970	1959	1970	1959	1970	1959	1970	1959	1970
Under $3,000	15.2	8.63	22.4	11.81	31.7	14.77	16.4	N.A.*	28.9	N.A.*	47.1	N.A.*
$ 3,000 - 5,999	36.0	16.16	41.9	20.01	38.1	24.63	31.7	N.A.*	36.8	N.A.*	31.3	N.A.*
6,000 - 9,999	35.2	32.85	27.9	34.03	18.1	26.89	34.1	N.A.*	25.2	N.A.*	14.8	N.A.*
10,000 - 14,999	10.1	27.26	5.9	23.36	7.6	18.38	12.3	N.A.*	6.6	N.A.*	4.6	N.A.*
15,000 - 24,999	2.8	11.97	1.5	8.85	3.3	10.53	3.9	N.A.*	1.8	N.A.*	1.6	N.A.*
25,000 and over	.8	3.13	.5	1.94	1.2	4.80	1.5	N.A.*	.7	N.A.*	.6	N.A.*
	100.0	100.00	100.0	100.00	100.0	100.00	100.0	—	100.0	—	100.0	—
Median Income												
1959	5918		5050		4289		6166		4750		3228	
1970	6766		8237		7365		N.A.*		N.A.*		N.A.*	

Source: Montana Economic Study, Part I, Vol. 3, pp 6-7 and 1970 Census of Population: General, Social and Economic Characteristics, Montana, p. 133.

*Not available.

the displacement of urban mean income from the highest level in 1959 ($5918 compared to $5050 for rural non-farm and $4289 for rural farm) to lowest level in 1970 ($6766 compared to $8237 rural non-farm and $7365 rural farm). However, rural non-farm and rural farm sectors continue to maintain large segments in the poor category (under $6,000). Compared to the 1959 national median income for these groups Montana compared favorably, being slightly higher in the rural non-farm and rural farm groups and slightly lower in the urban group.

From the standpoint of income distribution among racial groups, non-whites comprise a disproportionate share of the state's poor individuals. In 1959, 84 percent of non-whites had incomes less than $6,000 compared to 58 percent of the white populace. In 1970, 50 percent of the Blacks, 37 percent of the Spanish, and 71 percent of the Indians in Montana had incomes less than $6,000 compared to 28 percent of the white sector. Table 6 shows income distribution between racial groups in 1959 and 1970.

Some distributional changes have occurred in the incomes of males and females. As table 7 shows, the median income of women is much lower than that of men both in 1959 and 1970. However, there is a significant trend to a greater proportion of women entering the work force and to higher incomes for those employed. In 1959, 77 percent of the men employed had incomes less than $6,000 compared to 98 percent of the women employed. By 1970, 52 percent of the men had

Table 6

Percentage Distribution of Families by Level of Money Income and by Color, Montana 1959 and 1970.

Income Level	White 1959	White 1970	Non-white 1959	Non-white 1970 Negro	Non-white 1970 Spanish	Non-white 1970 Other (Indian)
Under $1,000	4.23	2.17	13.60	4.76	2.70	7.75
$ 1,000 - 1,999	6.62	3.01	19.63	3.26	2.88	13.95
2,000 - 2,999	8.60	4.76	16.60	11.87	5.15	18.06
3,000 - 3,999	11.21	5.37	14.66	8.90	9.38	10.78
4,000 - 4,999	12.88	5.87	11.17	12.46	8.70	11.03
5,000 - 5,999	14.24	6.99	8.74	8.90	8.39	9.06
6,000 - 6,999	11.52	8.13	3.73	6.82	11.34	4.36
7,000 – 7,999	8.42	8.45	4.53	14.24	6.43	8.60
8,000 - 8,999	6.35	8.62	2.25	7.72	10.36	3.41
9,000 - 9,999	4.24	7.48	1.36	2.08	5.02	3.72
10,000 and over	11.68	39.15	3.72	18.99	29.65	9.28
	100.00	100.00	100.00	100.00	100.00	100.00

Source: Bureau of Census: 1970 Census of Population: General, Social and Economic Characteristics - Monana, p. 133.

incomes less than $6,000 while 90 percent of the women employed did. While it is difficult to draw conclusions since the income data is not in real terms it does appear that women in the lower income ranges progressed more rapidly than males. However, men entered the upper income ranges in much greater proportion than did women.

Rural/Urban Composition

Since 1950 there have been significant changes in the rural/urban character of the state. In 1950, there were 75,000 more people living in rural areas than in urban areas. Urban residents comprised only 44 percent of the state's population. By 1960, the rural/urban mix was approximately equal. However, urban residents outnumbered rural residents by 47,000 persons in 1970. Urban residents rose from 50 percent of the population to 53 percent in the ten year period.

Figure 1 shows the population change from the rural countries in Montana to the urban counties between 1960 and 1970.

This shift in rural/urban composition is further intensified by the fact that many of those now classified as rural residents are actually nonfarm also. It is expected *a priori* that a population that is primarily urban and non-farm will consume more fluid milk at the retail level than a population that is rural and agriculturally oriented. This is due, at least in part, to home production and consumption of fluid milk as well as greater distances from retail outlets.

Table 7

Distribution of Income Levels of Persons 14 Years and Over by Sex, Montana, 1960 and 1970.

Income Level	Male 1960	Male 1970	Female 1960	Female 1970
	- Percent -			
No Income	8.2	8.7	46.5	37.8
With Income	91.8	91.3	54.5	62.2
1 - 999	14.8	12.0	48.3	34.3
1,000 - 1,999	13.4	10.6	19.7	20.7
2,000 - 2,999	10.8	8.6	12.5	12.2
3,000 - 3,999	12.0	7.0	10.2	10.5
4,000 - 4,999	13.4	6.2	5.1	7.3
5,000 - 5,999	12.6	7.3	2.1	4.9
6,000 - 6,999	8.0	8.4	.8	3.8
7,000 - 7,999		8.6		2.3
8,000 - 8,999	9.5	7.4	1.6	1.4
9,000 - 9,999		5.0		.8
10,000 and over	5.5	18.8		1.8
Total	100.0	100.0	100.0	100.0
Median Income ($)	3,910	5,751	1,035	1,760

Source: Bureau of Census, 1970 Census of Population: General Social and Economic Characteristics, Montana, p. 114.

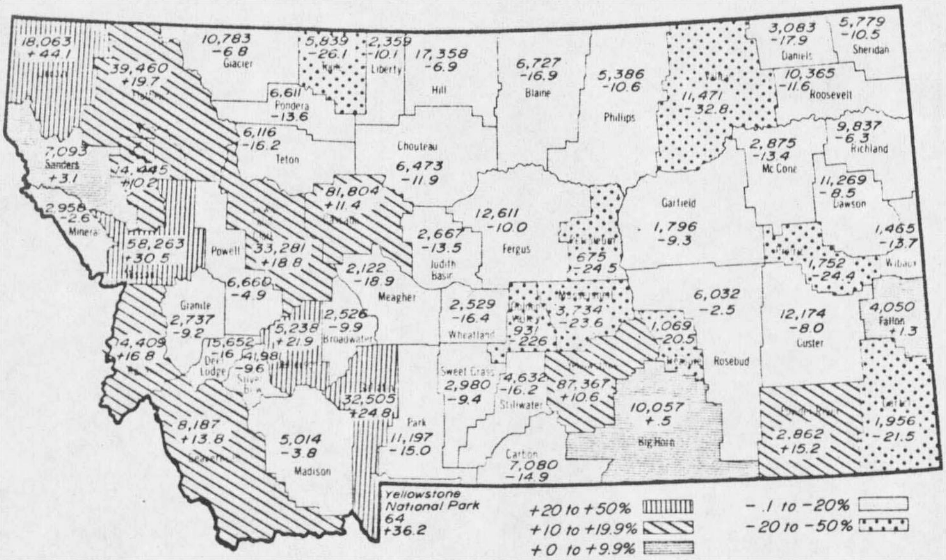

Figure 1. 1970 County Population and Percentage Change in County Population, Montana, 1960 to 1970.

Upper figure represents county population. Lower figure represents percentage change in population.

Source: Montana Data Book, Department of Planning and Economic Development, Helena, Montana, 1970, p. 4.17.

In 1960, there were about 27,000 farms in Montana of which 38 percent reported at least one dairy cow [7]. By 1969, 2000 fewer farms were still in operation and only 25 percent of the remaining farms had at least one dairy cow. It is expected that a shift in the nature of the urban/rural and agricultural character of the state should exert some influence on retail milk consumption. Unfortunately, detailed data useful in the statistical analysis is not available to measure the quantitative effects of this structural change.

Age Structure

The age structure of the population has also changed a great deal since 1950. Between 1950 and 1960 the proportion of young individuals (defined as 18 years of age and less) in the total population rose from 33.0 percent to 38.6 percent. However, after 1960 the population became progressively older. By 1970, only 36.4 percent of the total population of Montana was 18 years of age or less. By 1975, roughly 33 percent of the population was categorized in the "young" group.

Several prior studies [3, 4, 24] have shown that per capita milk consumption decreased as age of the population increased. Thus, a population comprised of older individuals, such as in Montana, would be expected to consume less milk than a younger population when the effects of all other factors are accounted for.

Education Level

The general education level has risen slightly in recent years. In 1960, the average education level was 12.1 years of education (the average number of years of education of an individual over 25 years old). This level rose to 12.3 years in 1970. Some studies have associated greater fluid milk consumption with higher education levels. However, this increase in the average education level was not expected to have a major influence on aggregate milk consumption. For this reason, education was not utilized as an explanatory variable in the statistical model.

Racial Composition

As with education, racial composition changed to a slight extent during the period of this study. In 1950, 96.8 percent of the population was white. By 1960, the proportion dropped slightly to 96.4 percent. An additional decline to 95.5 percent was noted in 1970 [16]. While there is evidence that non-whites consume less milk than whites [4], it is not likely to be important in Montana due to the relatively homogeneous nature of the racial distribution over time.

Sex Composition

Other socio-economic variables have been identified in prior studies. Women have been shown to consume more milk per capita than men. Detailed statistical data on sex composition was not available for this study, however, it is not expected to significantly influence the results.

The Montana Dairy Industry

The dairy industry in Montana is characterized by two distinct sectors: Dairy Producers and Dairy Processors and Distributors. While the two sectors are termed "the dairy industry", it is not uncommon for the goals and objectives of the sectors to be conflicting. Producers, for example, seek to obtain the highest possible farm price for their raw milk since this is the major source of their revenue. Dairy processors, on the other hand, seek the lowest possible raw milk price since this is a major input (and variable cost factor) in the production of processed dairy products. The nature of raw milk along with the different objectives of the two sectors has led to government involvement in dairy marketing in order to insure a constant milk supply for the general public. Much of the remainder of this chapter addresses the role of government in the dairy industry and analyzes the trends and characteristics of the sectors of the dairy industry.

Milk Marketing in Montana

Milk in the Montana market is categorized according to standards of sanitation in production and product utilization. Raw milk produced for utilization as fluid products must meet strict standards of sanitation. Milk produced under these standards is labeled "Grade A." Milk produced under less stringent standards for farm use or manufactured products is labeled "Grade B" and is not eligible for fluid utilization. Grade A milk producers must be licensed by the Milk Control Division of the Office of Business Regulation in accordance with state law. Sanitation standards are enforced by the State Department of Livestock.

Grade A milk is further categorized according to its ultimate utilization. Fluid products such as whole milk and cream are labeled "Class 1." Manufactured products such as ice cream, cottage cheese, and non-fat dry milk for human consumption are "Class II" products. Hard cheese, butter and powdered milk for non-human use are grouped into "Class III" products. Table 8 summarizes the distribution of Grade A production into the various classes of products.

Currently about 93 percent of all milk produced in Montana is eligible for fluid use [21]. While all Grade A milk qualifies for use as fluid products, only about 75 percent is ultimately consumed as fluid milk or cream. The remainder is processed into manufactured dairy products such as ice cream and cheese.

Dairy producers are paid for their raw milk in accordance with its ultimate utilization. This "blend price" is based on the percent of the stock of raw milk that enters the market in each of the three

Table 8

Classes of Milk Products in Montana

Class 1	Class 2	Class 3
Whole Milk (Homogenized and/or pasturized testing at least 3.25%)[3/]	Non-fat dry milk (Human food)	Butter
Chocolate Milk (test at least 3.25%)	Cottage Cheese	Cheddar Cheese
Low fat milk (.5% to 2%)	Ice Cream	Process Cheese
Buttermilk (testing 2% or less)	Ice Milk	Livestock feed
Skim Milk (testing less than .5%)	Sherbert	Powder Milk (non-human use)
Chocolate Drink (testing less than .5%)	Condensed milk	
Half and Half Cream (testing at least 10.5%)	Yogurt	
Commercial Cream (testing at least 18%)		
Whipping Cream (testing at least 30%)		

Source: Office of Business Regulation; Milk Control Division.

[3/] Testing refers to fat content.

milk utilization classes. Generally Class 1 products have the highest valuation and Class 3 the lowest. Thus, at least at the farm level, high Class I utilization is desirable.

Table 9 summarizes the historical trends in Grade A milk production by source of receipt. Two major sources of Grade A milk are available: Local production (including independent producers, producer/distributors and processor owned herds) and imports from neighboring states.

By far most Grade A milk is from local sources. Imports comprised only 4.8 percent of all Grade A milk processed in 1976. While imports of Grade A milk have remained relatively stable over the time period of this study, exports have risen substantially (in fact, since 1975 exports of Grade A milk have actually exceeded imports).

Producer/distributor production of Grade A milk (that is, dairy producers that both produce and process their own milk for on-farm sale) has remained nearly constant since 1964. This source of production accounts for about 1 percent of all Grade A milk utilized. Processor owned herds, once accounting for over 2 percent of the Grade A milk produced, now account for less than 1 percent. Table 10 summarizes the trends in Class I utilization by source of production.

As with Grade A milk, most Class I milk is obtained from local independent producers (about 99 percent). Over 90 percent of

Table 9

Grade A Milk Production in Montana
(1964 - 1976)
Millions of Pounds

Year	Local Production			Import/Export	
	Independent Producers	Producer/ Distributors	Processor Owned Herds	Imports	Exports
1964	192.0	2.0	N/A[a]	7.4	N/A[4]
1965	200.6	1.6	N/A	16.9	N/A
1966	212.0	2.7	N/A	13.6	N/A
1967	207.5	2.5	5.3	10.5	1.8
1968	211.7	2.2	5.6	8.4	2.6
1969	207.2	2.1	5.8	8.5	5.8
1970	215.3	2.1	5.1	11.1	4.8
1971	221.8	1.8	5.2	12.6	6.6
1972	228.8	2.4	5.6	11.2	5.4
1973	225.0	2.3	2.2	13.2	6.7
1974	215.6	2.4	1.7	13.8	8.3
1975	221.4	2.4	0.7	13.1	23.2
1976	226.6	2.1	0.0	12.1	19.5

Source: A Report of Milk Utilization in Montana; Department of Business Regulation; Milk Control Division

[4] Data not available 1964-1966.

Table 10

Class I Utilization in Montana by Milk Source
(1964 - 1976)
Millions of Pounds

	Local Production		Imports/Exports		
Year	Independent Producers (including processor owned Herds)	Producer/ Distributor	Imports	Exports	Total Class I Milk Utilized in Montana
1964	164.6	2.0	7.0	NA	173.6[5/]
1965	158.3	1.6	16.6	NA	176.6[5/]
1966	164.6	2.6	13.3	NA	180.4[5/]
1967	162.1	2.0	9.9	1.8	172.2
1968	164.1	1.6	8.0	2.6	171.1
1969	160.9	1.4	8.1	5.9	164.4
1970	158.7	1.5	10.9	4.0	167.1
1971	165.0	1.5	10.3	3.5	173.8
1972	170.8	1.5	11.1	3.8	180.2
1973	165.9	1.7	12.7	5.7	175.2
1974	162.8	2.0	13.1	6.3	172.2
1975	172.7	2.1	12.3	15.1	172.3
1976	176.8	1.9	11.7	11.9	178.4

Source: A Report of Milk Utilization in Montana; Department of Business Regulation; Milk Control Division.

[5/] Overstates actual utilization in Montana by the amount of Class I Milk exported.

producer/distributor Grade A milk goes to fluid products, yet this source only accounts for about 1 percent of total Class I utilization.

Exports of Class I products have increased in importance since they were first reported in 1967. Exports have risen from an estimated 1.2 million pounds in 1967 to nearly 12 million pounds in 1976. Imports, on the other hand, have remained quite stable over the same period. Since 1975, Class I exports have exceeded imports by a slight amount. Utilization of exported fluid milk from Montana is given in Table 11 for 1967-76.

The Producer Sector

Table 12 presents historical trends in the number of Grade A producers, cows in production and average production per cow from 1964 to 1976. Since 1964, the trend has been to fewer yet larger milk producers in Montana. In 1964, there were 461 licensed Grade A producers (including producer/distributors) while in 1976 there were 271, a decrease of 41 percent. Herd size per producer, on the other hand (Grade A cows per producer), rose from 44 cows per producer in 1966 (the first year of available data on cow numbers) to 77 cows per producer in 1976. Production per cow has increased from 10,780 pounds per cow in 1966 to slightly over 11,000 pounds in 1976.

There are substantially fewer producer/distributors currently than in the past. In 1964, 16 producer/distributors were in operation

Table 11

Utilization of Fluid Milk Exported from Montana
(1967 - 1976)
Millions of pounds

Year	Class I	Class II	Class III
1967	1.8	0.0	0.0
1968	2.6	0.0	0.0
1969	5.9	0.0	0.0
1970	4.0	NA	0.8
1971	3.5	2.5	0.6
1972	3.8	1.4	0.2
1973	5.0	0.6	0.4
1974	6.2	0.5	1.5
1975	15.1	1.2	6.9
1976	11.9	1.4	6.3

Source: A Report of Milk Utilization in Montana; Office of Business Regulation; Milk Control Division.

Table 12

Grade A Milk Producers in Montana[6/]
(1964 - 1976)

Year	Number of Independent Producers	Number of Grade A Milk Cows[6/]	Average Production Per Cow
1964	461	NA	NA
1965	465	NA	NA
1966	443	19,916	10,780
1967	410	20,005	10,761
1968	410	20,267	10,723
1969	383	19,607	10,862
1970	346	20,567	10,819
1971	337	20,657	11,078
1972	310	20,965	11,295
1973	303	20,464	11,362
1974	262	19,479	11,279
1975	261	20,354	11,030
1976	271[7/]	20,921	11,041

Source: A Report of Milk Utilization in Montana; Office of Business Regulation; Milk Control Division.

*NA = Data not available

[6/]Includes producer/distributor hers.

[7/]Includes "out of state" producers licensed in Montana.

compared to 10 in 1976.

Receipts from the dairy production sector of the Montana economy represent about 2 percent of total agriculture receipts received by farmers from marketing all commodities. While total receipts from dairy product sales have risen slowly in unadjusted terms since 1964, the importance of dairy products relative to other commodities has fallen from the standpoint of income earned. Table 13 summarizes receipts from dairy production between 1964 and 1975.

The cash receipts of Montana's dairy producers account for about 5 percent of the total receipts for dairy products in the Mountain States area. Next to Wyoming, Montana's dairy industry is the smallest in the Intermountain area [15]. Nationally, Montana producers rank 44th in terms of cash receipts from marketing milk and cream.

The Distributor Sector

There have been some significant changes in the distributor sector since 1964. In 1964, there were 37 Montana based distributors and 16 out-of-state distributors operating in Montana. By 1976, there were only 14 Montana distributors and 14 out-of-state distributors still in operation. Much of this change in structure is the result of a shift to larger plants capable of achieving economies of scale in milk processing [21].

Table 13

Cash Receipts from Dairy Products

Year	Receipts from Dairy Products (Million $)	As Percent of Total Receipts for All Commodities
1964	13.7	3.6
1965	13.7	3.3
1966	14.4	2.8
1967	15.2	3.2
1968	15.7	3.1
1969	16.5	3.1
1970	15.9	2.6
1971	16.6	2.7
1972	17.8	2.2
1973	19.1	1.7
1974	22.3	1.9
1975	22.6	2.1

Source: Montana Agricultural Statistics.

Milk Price Administration in Montana

Some form of milk price regulation has existed in Montana since the Depression precipitated unstable marketing conditions in the 1930's. In 1935, the Montana legislature enacted Chapter 189, Laws of 1935, which established sanitation standards and set minimum order prices. This act was intended to provide temporary relief to the dairy industry as a response to the economic turmoil of that time.

In 1939, Montana undertook permanent milk price regulation. Currently, the Milk Control Division of the Office of Business Regulation enforces milk control policy in accordance with the Executive Reorganization Act of 1971. Minimum prices of various fluid milk products are established by the Montana Milk Control Board under provisions of Montana Milk Control Division Laws, Chapter 4, Sections 27-401 through 27-429.

At the retail level, a minimum price is set for each fluid milk product and container size, with lower minimum sales prices set for institutions such as schools and hospitals (Federal Government Installations are exempt from price controls). Historically, the minimum order price has been the effective market price.

Prior to 1974, a separate order price was established for three geographic regions in the state.[8/] Within each region, supply and

[8/] While the state was divided into 12 geographic regions, only three order prices were established. Separate prices pertained to the Bozeman-Missoula area, Northeast Montana and a third price for all remaining areas.

demand factors were assumed to be homogeneous. Those regions that supplied surplus quantities of milk were usually allowed to set lower retail prices than areas with deficit production levels. In 1974, regional price differentiation was eliminated and a single set of prices applied statewide.

Prior to 1971, increases in the order price of retail milk products and producer blend price had to be supported by increased costs of production. This procedure had several weaknesses. First, historical costs usually lagged behind actual cost of production. By the time cost data had been analyzed by the milk control board, it was often no longer relevant to the actual conditions in existence.

Another problem with cost of production analysis is the lack of a single cost of production. Each producer has a different cost of production based upon the efficiency of his operation. It is at best difficult to determine whose historical cost data to utilize in setting increments in milk price.

Also, several of the input costs of milk production are price determined. For example, high cow prices will increase the cost of production. Thus, the use of cost of production as the basis for milk pricing policy can lead to cycles of high prices determining high production costs because market prices of dairy cows tend to move directly with milk prices, at least in the short run.

In 1971, flexible pricing formulas were adopted at both the producer and retail levels. The formulas base product price on various economic indicators of production cost, levels of economic activity such as income, and the opportunity costs facing the dairy sector. While these formulas are more responsive to changes in economic conditions, there is still no guarantee the established price will equate the demand for and supply of fluid dairy products in Montana.

Review of Prior Studies

Due to lack of adequate data, no prior studies have undertaken empirical investigation of the demand relationships in Montana for beverage milk products. However, several recent studies of a national or regional scope do exist and may be used to validate the results of the current study within the limitations of the data source and method of analysis.

Comparison with prior studies must be done with caution. The results of older studies of the 1940's and 1950's may be outdated due to changes in consumer attitudes. It is unlikely that consumers view milk products today in the same manner as in the 1940's. Aggressive advertising campaigns by the dairy industry along with improved nutritional and health qualities of fluid milk have certainly changed consumer tastes and preferences over the past 30 years.

We might expect studies based on cross sectional data to yield different estimates than time series studies. Estimated elasticities from cross sectional data usually may be interpreted as "long run" elasticities while those from time series data tend to be more "short run", or possibly an average of short and long run [4]. We should also expect individuals in various regions of the country to respond differently due to differences in the economic and demographic characteristics of the region.

A recent study by Boehm and Babb [4] used cross section/time series analysis based on consumer panel data obtained during the period April 1972 to January 1974 by the Market Research Corporation of America (MRCA). The data base was comprised of over 800,000 purchases of selected dairy products by 5,000 families distributed nationally.

Boehm and Babb estimated the price elasticity for fluid milk to be -0.16 in the short run and -1.63 in the long run. The short run price response was measured using time series analysis with long run response measured using cross section data.

The estimated income elasticity of 0.05 was smaller than prior studies had indicated. The study concluded "household income did not exert a significant influence on consumption rates". Household composition, season of the year and race were other factors found to significantly influence consumption. Also, households in the mountain states (which included Montana) were found to consume less milk than those in the Northeastern and Midwestern states and slightly more than those in the South.

William T. Boehm [3] also analysed household demand for fluid milk products in a 1976 study using a different source of data. Federal Order Market data corresponding to 22 Standard Metropolitan Statistical Areas (SMSA's) were developed into a statistical model using a pooled cross section. Boehm identified three categories of non economic variables believed to exert influence on consumption levels. These were demographic variables (age, sex, education and race), physical environment variables (temperature, rainfall, employment opportunities, institutional constraints), and product environment (advertising, attitudes, merchandising techniques, promotions, health consideration).

Price elasticity for the mountain region was estimated to be -0.159. A positive but numerically small income elasticity was estimated. While income was shown to exert little effect on consumption, distribution of income was found to be very important.

Age structure was also found to be significant. Increases in population in the age groups under 5 years and between 44 and 65 years of age significantly increased per capita consumption. Less substantial increased consumption levels were identified for those between 5 and 17. Per capita consumption fell for those 65 years old and older.

Johnson [11] concluded in a study published in the Proceedings of the Sixth National Symposium on Dairy Market Development (1966) that the price elasticity for milk has been decreasing over time. In 1930, the price elasticity for fluid milk was believed to be about -0.5 for U.S. markets. Price elasticity has decreased to -0.3 in 1950 and -0.2 in the 1960's. Johnson concluded the decreasing trend of price elasticity was due to increasing real income levels which enabled consumers to purchase the same amount of milk regardless of price.

Brinegar [5] studied consumer response to changing milk price and income level in a community in eastern Connecticut from 1947 to 1949. From data obtained in consumer interviews, the short run price elasticity was estimated to be -0.48. No long run price elasticity was estimated. The long run income elasticity (the long run defined as 1 year) was estimated to be 0.24. Family composition was found to be an important explanatory variable in that a significant positive relationship between the number of children under 18 years old and total consumption of milk was observed.

George and King [10] estimated the demand for food commodities including fluid milk in the United States using both time series and cross section data. Both price and income elasticities were estimated at the retail level. Farm level elasticity was then derived from the statistical results at the retail level. The short run retail price elasticity was estimated to be -0.35 while the short run farm price elasticity was -0.32. Long run price elasticities were not estimated.

The long run income elasticity measured based on cross section data was 0.37.

A 1971 study of consumers in the Gainesville, Florida area by Prato [20] estimated the price elasticity of demand to be -5.7. Prato points out that this value may be distorted due to the manner in which a weighted average price was constructed. Consumers in higher income households did purchase greater quantities of milk than lower.

In 1972, Thomas and Waananen [24] conducted a study to exmaine the consumption and use patterns of certain fluid milk products in eight western cities. Approximately 2,000 consumers were interviewed in 1967 concerning their purchase patterns. Although this study was not a statistical analysis and did not quantitatively measure consumer response, several factors influencing milk consumption were identified.

Milk consumption was inversely related to the age of the purchaser. With few exceptions households purchased less milk per capita as the age of the head of household increased. Per capita consumption by both sexes was found to decline sharply with the advent of adulthood with older adults consuming very little fluid milk or its products.

A slight increase in consumption was associated with increased income levels. The study surmized "as incomes increase the consumption of fluid milk by children stabilizes, possibly even increases somewhat, while consumption of fluid milk by adults declines". Other factors identified in this study which affected fluid milk consumption were education levels, race, family size, sex of individual, and prices of milk substitutes such as coffee, tea, fruit juice and others.

In summary, most prior studies estimate the price elasticity of demand to range from -0.16 to -0.4 in the short run and greater than unity in the long run. Income response is much less clear. Some studies do associate greater consumption with higher income yet the magnitude of response measured by the income elasticity is usually small. Other significant factors that explain observed consumption levels include age, education, household composition, region and season of the year. *A priori* these same factors are expected to be important in Montana.

Chapter 2

THEORY OF CONSUMER BEHAVIOR

The foundations of the theory of consumer behavior are well documented in economic literature. The purpose of this chapter is to present an outline of the relevant aspects of the economic theory in order to develop a method of analysis to examine the behavior of consumers in the fluid dairy product market in Montana.

Theory of Demand

The objective of each consumer in the economy is to attain the highest level of utility or satisfaction possible via the consumption of goods and services available to him. However, no individual has unlimited income. Thus, it is assumed the desire of the rational consumer is to maximize utility subject to the limitations of income.

There is an infinite number of commodity combinations available to the consumer. We assume the individual is aware of all the possibilities he faces (he possesses perfect information) and is able to rank each possibility ordinally.

Implicitly, the ability to ordinally rank various commodity bundles is represented by the individual's utility function

$$U\ (q_1,\ \ldots,\ q_n)$$

where q_i is the level of consumption of the i^{th} commodity. The utility function enables the individual to rank commodity bundles such that he either prefers one bundle to another or is indifferent between bundles.

We assume the utility function is a continuous, single valued function of all commodities available. No meaningful numeric (cardinal) value of utility is attached to any commodity bundle. The only valid economic interpretation of utility is in the preference relationship of the possible combinations.

The form of the utility function is influenced by the tastes and preferences of the individual and by various socioeconomic factors such as age, sex and race. The structure of the utility function is assumed constant over the period of the statistical analysis even though changes probably occur over time. In general, no two individuals would have the same utility function since each would place different values on the possible consumption opportunities.

Since the consumer attempts to attain the highest level of utility subject to the income constraint, his actions may be represented by maximing a Lagrangian function

$$L = U(q_1, \ldots, q_n) - \lambda \left(\overline{Y} - \sum_{i=1}^{n} p_i q_i\right)$$

where $\overline{Y} = \sum_{i=1}^{n} p_i q_i$ is the budget constraint of the individual, p_i is the price of the i^{th} commodity, and λ is the Lagrangian multiplier representing the marginal utility of income.

The demand function for each good is derived from the 1st order conditions for a maximum. This is accomplished by setting the partial derivatives of L with respect to each good and λ equal to 0. This yields (n+1) independent equations.

$$\frac{\delta L}{\delta q_1} = \frac{\delta U}{\delta q_1} - \lambda p_1 = 0$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\frac{\delta L}{\delta q_n} = \frac{\delta U}{\delta q_n} - \lambda p_n = 0$$

$$\frac{\delta L}{\delta \lambda} = \overline{Y} - \sum_{i=1}^{n} p_i q_i = 0$$

The first n elements of the (n+1)x1 vector represent the necessary conditions for constrained utility maximization. The (n+1) element insures all income earned by the individual is spent (saving is also considered a good which yields utility).

This system of (n+1) equations in (n+1) unknowns may be solved for each q_i. Generally, this would yield:

$$q_i = f(p_1, \ldots, p_n | \overline{Y})$$

This represents the demand for the i^{th} good as a function of the prices of all goods consumed by the indiviudal and the fixed level of income.

The demand relationships derived in this manner have the following properties:

(1) They are single valued functions of prices and incomes. The socioeconomic factors and tastes do not appear explicitly in the demand function. Yet, the shape of the utility function and, hence, the demand relationship is closely related to these factors.

(2) The demand relationships are homogeneous of degree 0. Therefore, if all prices and income are doubled there will be no change in the quantity demanded of the i^{th} good. This assumes no money illusion exists.

The preceding demand relationship has several implications for practical econometric estimation:

(1) The demand function relates quantity consumed to all prices appearing in the individual's utility function. From a statistical point of view, it is impossible to estimate a demand function which includes all prices. It does seem reasonable to include only those prices closely related to the commodity under investigation. This presents a theoretical justification for including the prices of substitutes and complements and excluding prices of goods that are "want independent".

(2) Since the demand relationships derived above displays the absence of money illusion, relative prices and real income are the relevant variables. This justifies deflating

monetary variables by a measure such as the consumer price index to differentiate between a change in price due to inflation versus a change in the relative price which is economically relevant.

Consumer Response to Changing Relative Price

Consumer response to changes in relative prices of commodities is composed of two effects. First, consumers respond to "pure" changes in relative prices while holding purchasing power constant. As relative prices rise less of the good is purchased [2]. This is the "first fundamental law of demand" and represents the substitution effect. Higher relative prices induce the consumer to search for substitute commodity bundles composed of less of the higher priced good. Second, a change in the price of a good, holding all other prices and nominal income constant, causes the purchasing power of the individual to change. Higher relative prices reduce the opportunity set or real income facing each consumer. Again, the consumer is forced to seek an alternative commodity bundle.

If the typical consumer responds to higher (lower) income levels by purchasing more (less) of the good, the income effect reinforces the substitution effect and an inverse relationship between quantity consumed and price must exist. On the other hand, if a consumer purchases more of a good as income falls, the income effect counters the negative substitution effect. Only if this opposite income effect is of sufficient magnitude to offset the negative substitution effect could we observe a direct relationship between prices and quantity consumed. This would be a rare occurrence.

Price Elasticity of Demand

Consumer response to changing prices is measured by the price elasticity of demand. Price elasticity is defined as the ratio of the percentage change in quantity consumed to the percentage change in relative price.

$$\varepsilon = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

The elasticity coefficient is a number independent of the unit of measurement of either price or quantity. Generally, the value of the elasticity changes along each point on the demand curve. Most empirical studies measure elasticity at the mean value of each variable.

The value of the price elasticity coefficient for a normal good ranges from $-\infty$ to 0. Three ranges have particular economic relevance.

(a) Values of ε between 0 and -1 are termed "inelastic". When elasticity falls in this range a change in price elicits a less than proportional change in quantity consumed.

(b) A value of $\varepsilon = -1$ is called "unitary" elasticity and indicates proportional changes in both price and quantity.

(c) A value of $\varepsilon < -1$ is called "elastic" demand and implies a greater than proportional change in quantity consumed for a given price change.

In general, the availability of close substitutes and the proportion of the budget spent on the particular commodity determines the magnitude of the price elasticity of demand. Usually, a good with a greater number of acceptable substitutes will have a relatively higher elasticity since it is less costly for a consumer to search for substitutes when a great number exist. The greater the proportion of the budget spent on a good, the greater the price elasticity of demand. Generally, the greater the amount spent on a commodity the greater the potential net gains from searching for substitutes as price rises. For those goods comprising a small part of the budget, the cost of searching for a substitute may outweigh the potential benefits and consumers are likely to respond less to a given price change. Also, income effects are greater when the commodity comprises a large part of the total budget.

A second fundamental law of consumer behavior theory is: "The price elasticity of demand for a good is more elastic in the long run than in the short run" [2]. Consumers are generally unable or unwilling to respond immediately or fully to changing market conditions. This is the result of habit formation by consumers and a lack of immediate market information. This implies ultimate consumer response is distributed over several time periods after the initial price disturbance. Chapter 3 will fully develop this "distributed lag" concept.

Price Elasticity and Producer Revenue

The total revenue earned by a producer or processor is closely related to the manner in which consumers react to changing market conditions. When price changes, there are two counteracting forces affecting processor revenue.

(1) As price rises, consumers respond by purchasing less of the good. This represents a reduction in total revenue to the processor.

(2) As price rises, each unit of commodity sold by the processor becomes more valuable or increases total revenue.

Which of these forces dominates determines whether total revenue increases, decreases or remains the same when price changes. Mathematically, we may relate the change in total revenue to given price changes in the following manner:

$$\frac{\delta(\text{Total revenue})}{\delta(\text{Price})} = Q(1 + \varepsilon)$$

where Q is the quantity consumed and ε is the price elasticity of demand. Thus, if the price elasticity is unitary, total revenues remain unchanged since a price rise leads consumers to reduce consumption proportionately. When demand is elastic (i.e., less than -1) the loss from reduced consumption is greater than the increase from the per unit price rise and total revenue falls as price per unit rises. For inelastic demand, consumers respond less than proportionately to changing price, therefore, total revenue increases with per unit price increases.

Demand at the Farm Level

The demand for fluid milk at the farm level is a derived demand. That is, the demand for raw milk from the producer is derived from the demand for fluid milk at the retail level [8]. The same factors that affect price elasticity at the retail level affect elasticity at the farm level. However, at the farm level an additional factor, the size and stability of the marketing margin, is an important determinant of price response.

In markets such as that for fluid milk where the farm to retail marketing margin is constant, farm price elasticity is easily measurable from the statistical analysis of the retail demand relationship. In such a market the primary (or retail) demand curve and the derived (farm level) demand curve will be parallel. At any given quantity of fluid milk consumption the distance between the curves is the marketing margin. This relationship is shown in Figure 2.

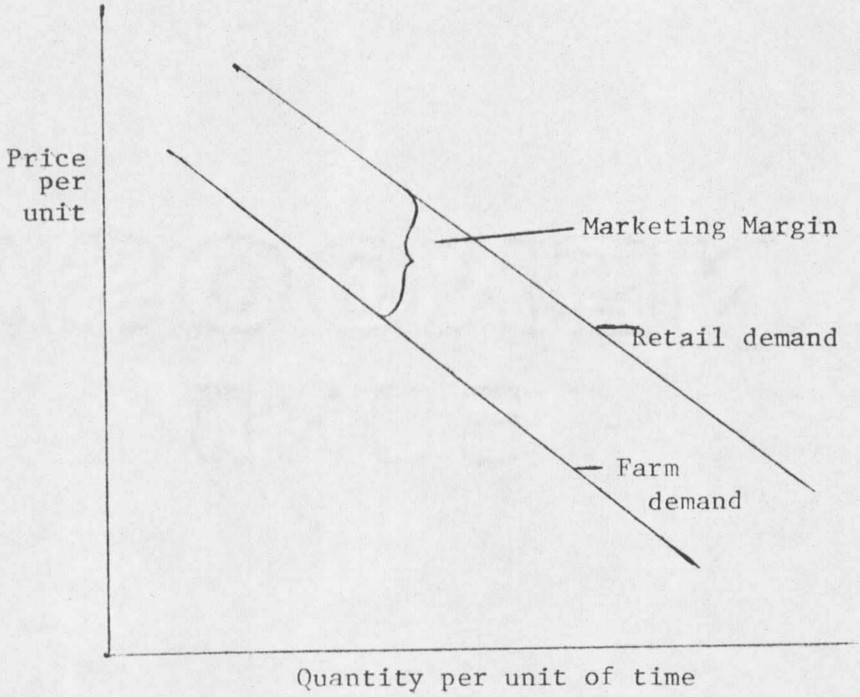

Figure 2. Demand at the Retail and Farm Levels With Constant Absolute Marketing Margins

It may be shown that with a constant absolute marketing margin the following relationship between retail and farm level elasticity holds

$$E_f = E_r (P_f/P_r)$$

Where E_f is the price elasticity at the farm level; E_r is price elasticity at the retail level; and (P_f/P_r) is the ratio of farm to retail price.

Since P_f is less than P_r, elasticity at the farm level will be less than elasticity at the retail level.

In the long run, assuming perfect competition and no excess profits in either the primary or farm level market, the relationship between primary demand and derived demand depends on whether the industry is subject to constant, increasing or decreasing cost.

Response to Changing Income Levels

Consumers may respond in one of three ways to changing income levels. For "normal" goods consumption increases as income rises. If consumption remains unchanged after income rises the good is "income neutral." If consumption decreases as income rises the good is termed "inferior." The magnitude of response is measured by the income elasticity coefficient. Normal goods have an income elasticity greater than 0 while inferior goods have negative income elasticity.

All goods can be normal but it is not possible for all goods to be inferior. This is readily observed from the following relationship derived from the definition of income elasticity:

$$\sum_{i=1}^{n} \alpha_i \cdot \eta_i = 1$$

Where α_i is the proportion of the budget spent on each good and η_i is the income elasticity of each commodity purchased. From this relationship we note, in general, goods comprising a large portion of the consumer's budget such as dairy products are likely to possess income elasticities which are small. It is also possible for a good to be normal for a relatively low income level and inferior at some higher level.

Applications of the Theory

There are several difficulties encountered in applying the theory of consumer demand to empirical studies:

(1) Since it is impossible to statistically estimate demand functions containing all prices found in the budget of an individual it is necessary to abstract from reality. The model which is developed is not able, nor is it intended, to predict consumer response exactly. It is merely an approximation of what we believe to be economic reality.

(2) The theoretical framework which has been developed is for an individual or "typical" consumer. More generally, however, we are interested in the market or the aggregate demand of all consumers for a particular commodity.

In theory, we are able to combine the demand schedule of every individual by adding the total quantity demanded at each price and obtaining the market demand schedule. However, it is impossible to estimate the demand for each individual in the market. We must rely on aggregate data for those variables such as quantity consumed and income which are different for each consumer. Since each individual possesses a different utility function which cannot be aggregated into a "market utility function", the statistical model is not strictly based on economic theory. For example, the market demand for goods may depend not only on income level but also on the manner in which income is distributed. While a change in the distribution of income may shift the market demand curve, the use of aggregate data does not permit the detection of the shift. The results obtained from the use of aggregate data may be biased and this possibility should be recognized.

Summary

The theory of demand developed in this chapter, along with a knowledge of the dairy industry, presents a theoretical justification for those variables expected to be important determinants of the level of milk consumption in Montana. Based upon economic theory, an inverse relationship between the relative price of milk and total consumption is expected. Since few close substitutes for fluid milk exist, it is likely the price elasticity is small in the short run and greater in the long run.

Prior studies suggest that income would be expected to have a positive effect on consumption. The income elasticity may be of small magnitude since milk products do comprise a relatively large share of consumer expenditure. Several socioeconomic factors may also be important such as season of the year, rural-urban composition, population, family composition and age distribution.

Chapter 3

THE STATISTICAL MODEL

The purpose of this chapter is to outline statistical and econometric procedures which appear to be appropriate for obtaining empirical estimates of consumer response in the fluid milk market. The role of demand theory developed in Chapter 2 compliments the statistical method. A given data base may have a multitude of statistical explanations. Yet, only the statistical results based on sound economic principles are relevant.

The Statistical Demand Relationship

The statistical model representing the fluid milk demand structure is assumed to be [17]

$$(1) \quad q_t^* = \beta_o + \sum_{i=1}^{K} \beta_i X_{it} + e_t,$$

where q_t^* is the long run equilibrium level of fluid milk consumed and X_{it} represents the value of the i^{th} explanatory in time period "t". The set of explanatory variables $(X_1, \ldots, X_k)$ are the economic and socioeconomic factors believed to determine the quantity of fluid milk consumed in any time period. The vector of β_i's respresents the parameters underlying the statistical relationship. Each β_i describes the marginal effect of a small change in the i^{th} explanatory variable on q_t^*. The error term, e_t, accounts for the stochastic nature of the empirical demand relationship.

The disturbance term e_t is assumed to possess the following properties:

$$E(e_i) = 0 \quad \text{(all i)}$$

$$E(e_i e_j) = 0 \quad (i \neq j)$$

$$E(e_i^2) = \sigma^2 \quad \text{(all i)}$$

The nature of the stochastic error term implies the long run demand function does not hold exactly each period. The disturbance term accounts for such factors as errors in measurement of the quantity of fluid milk consumed and unobservable factors which may affect consumption.

The variable q_t^* may be interpreted as the quantity of fluid milk that would be consumed if all factors were to remain unchanged for a sufficient time period. However, most market factors do not remain constant over time. Consumers develop habit patterns that change slowly. As a result, consumers are unable or unwilling to adjust immediately as market factors change. Consequently, q_t^*, the long run equilibrium, is not readily observable.

It is assumed that consumers adjust to changing conditions in a systematic manner. The actual change in consumption between periods is assumed to be a proportion of the difference between the long run desired level and the quantity consumed last period. This adjustment mechanism is a behavioral relationship described by the following difference equation:

$$(2) \qquad q_t - q_{t-1} = \alpha[q_t^* - q_{t-1}] + U_t$$

The quantity $(q_t - q_{t-1})$ is the observed change in the quantity consumed between time period t and t-1. The adjustment parameter α is assumed to possess a positive value less than unity. This adjustment mechanism implies consumers adjust rapidly when α is near 1 and slowly for values of α near 0.

The adjustment mechanism is stochastic. The error term U_t is assumed to possess the classical properties of zero mean and constant variance. Thus, the process of adjustment outlined above may not hold exactly each period, but deviations from the theoretical adjustment pattern are assumed to average to 0 over time.

Solving the difference equation (2) for q_t^* yields

$$(3) \qquad q_t^* = q_t/\alpha - [(1-\alpha)/\alpha]q_{t-1} - U_t/\alpha$$

The unobservable quantity q_t^* is eliminated upon substitution of equation (3) into the long run demand function (1). The demand function is transformed into

$$(4) \quad q_t = \alpha\beta_o + \alpha \sum_{i=1}^{K} \beta_i X_{it} + (1 - \alpha)q_{t-1} + (U_t + \alpha e_t),$$

which can be directly estimated. This procedure provides a theoretical justification for including the lagged value of the dependent variable as an additional explanatory variable in order to eliminate the unobservable long run equilibrium quantity and to capture the habit persistence or inertia effects of consumer behavior. The demand relationship is now dynamic as represented by the 1st order difference equation (4).

Some Statistical Considerations

(A) A classical assumption of the standard regression model is that all explanatory variables are nonstochastic. Inclusion of lagged values of the dependent variable introduces random elements into the data matrix in violation of the classical assumptions [23].

(B) The parameter estimate of α must be positive and less than one. Violation of this assumption implies the difference equation (4) is unstable since the variance of q_t increases with the sample size [23].

The disturbance term of equation (4) is uncorrelated with the lagged dependent variable since q_{t-1} depends on $e_{t-1}, \ldots, e_1$ but not on e_t. In the absence of autocorrelation, ordinary least squares estimation provides consistent parameter estimates and we could proceed as usual provided the sample size is large [23].

However, the Durban-Watson statistic is asymptotically biased when the lagged dependent variable is treated as an ordinary explanatory variable. Hence, this statistic may not have the "power" to detect autocorrelation among the successive error terms. Inclusion of the lagged dependent variable with autocorrelation among the disturbances results in inconsistent estimates since q_{t-1} and

e_t are then correlated.

If each (1) and (2) are redefined with $E(q_t^*)$ and $E(q_{t-i})$ replacing q_t^* and q_{t-i}, then the final equation replacing (4) is

$$(4)' \quad E(q_t) = \alpha\beta_o + \alpha \sum_{i=1}^{K} \beta_i X_{it} + (1-\alpha)E(q_{t-1}).$$

This approach implies that the mathematical relationships (1) and (2) are more nearly valid when q_t^* and q_t are defined with the "statistical errors" removed from them. The ultimate consideration is what constitutes these "statistical errors" e_t and U_t which are primarily "ignorance terms" in our empirical model with aggregate data. The operational model is obtained by adding an "ignorance term" to (4)' above where $q_t = E(q_t) + v_t$.

Then one can also directly interpret (4)' without any motivation for its existence except a distributed lagged response with respect to the variable. Substitution of the expectation of the lagged dependent variable for the actual lagged value introduced nonlinearity among the parameters of the statistical model. Using only one independent variable for simplification, the statistical model now becomes

$$(i) \quad q_t = \beta_o + \beta X_t + \lambda[E(q_{t-1})] + v_t$$

Replacing the expectation by

$$E[q_{t-1}] = \beta_o + \beta X_{t-1} + \lambda E(q_{t-2})$$

makes apparent the intrinsically nonlinear relationship among the parameters.

Estimation using ordinary least squares is no longer possible. It is therefore necessary to employ a nonlinear estimation procedure. For this purpose, maximum likelihood estimates may be obtained using nonlinear least squares estimation [12]. It may be shown that under general conditions the maximum likelihood estimator is consistent, asymptotically efficient and asymptotically normal [23].

The formulation of (i) implies a geometric distributed lag has been imposed upon all explanatory variables. Successive substitution of $E(q_{t-i})$ for each prior period results in

$$\text{(ii)}\quad q_t = \alpha(1 + \lambda + \lambda^2 + . . .) + \beta(X_t + \lambda X_{t-1} + \lambda^2 X_{t-2} + \ldots) + v_t$$

which implies observed consumption this period is a weighted linear combination of all explanatory variables over all prior periods.

If a permanent (one time) change in a variable such as price occurred while all other variables remained constant, equation (ii) suggests quantity consumed in the first time period would change by

$$\beta(\Delta P)$$

In period $(t + 1)$ quantity consumed would change by

$$\beta(\Delta P)(1+\lambda).$$

The total or "long run" consumer response would be

$$\beta(\Delta P)(1 + \lambda + \lambda^2 + \ldots).$$

The long run price elasticity suggested by this formulation of the statistical model is:

$$E_{LR} = \frac{\beta}{1-\lambda}(\bar{p}/\bar{q})$$

The long run elasticity, evaluated at the sample means, is found by dividing the parameter estimate of price by one minus the parameter estimate of the expectation of the lagged dependent variable and multiplying this quantity by the ratio of the mean values of price and quantity consumed.

Hypothesis Tests

Tests of significance on the estimated parameters may be performed provided the sample is of sufficient size. Asymptotic variances of the estimators may be obtained from the diagonal elements of the inverse of the information matrix. Standard, single parameter hypotheses tests may be performed using a "t" statistic since the parameter estimates are distributed asymptotically normal. It should be noted that the test statistic is only approximately distributed as a "t" distribution since the linear approximations do not estimate the nonlinear relationships among the parameters exactly and because sample size is not infinite.

Properties of the Estimators with Serially Correlated Disturbances

It may be shown that when the disturbances are serially correlated the estimators are consistent but no longer asymptotically efficient. Confidence intervals and hypothesis tests conducted with inefficient estimators yield biased results when the usual parameter covariance matrix is assumed.

Some desirable large sample properties may be regained by estimating the autocorrelation parameters and transforming the model. It is assumed the disturbances are generated in the following manner

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \ldots + \rho_s u_{t-s} + e_t$$

where $\rho_1, \ldots, \rho_s$ are the coefficients of autocorrelation and e_t possesses the classical properties.

The general procedure is outlined below with only one independent variable and first order autocorrelation

(5) $q_t = \alpha + \beta X_t + U_t$

(6) $q_{t-1} = \alpha + \beta X_{t-1} + U_{t-1}$

Equation (5) represents the model the current period and equation (6) represents the model the prior period. Multiplying (6) by ρ and subtracting from (5) yields

(7) $q_t - \rho q_{t-1} = \alpha(1-\rho) + \beta(X_t - \rho X_{t-1}) + (u_t - \rho u_{t-1})$

which reduces to

$$q_t = \alpha(1-\rho) + \beta(X_t - \rho X_{t-1}) + \rho q_{t-1} + e_t$$

In the more general case of an n^{th} order autocorrelation, the transformation becomes

$$(8)\quad q_t - \sum_{i=i}^{n} (\rho_i q_{t-i}) = \alpha(1-\sum_{i=1}^{n} \rho_i) + [X_t - \sum_{i=1}^{n} (\rho_i X_{t-i})] + e_t$$

The disturbance term of the transformed model (8) possesses the classical properties. Thus, the estimates of the parameters will be consistent and "asymptotically equivalent to the best-linear-unbiased estimators" [12].

Autocorrelation also introduces nonlinearity into the regression model. Maximum likelihood estimates[1/] of the autocorrelation parameters may be obtained using an iterative technique [11]. This procedure introduces several statistical complications:

(1) Two degrees of freedom are lost for each autocorrelation parameter estimated. One degree of freedom is lost in the actual estimation of ρ_i and another due to a lost observation in the transformation process. This presents difficulties if the sample size is small and if a high degree of autocorrelation is encountered.

(2) The small sample properties of maximum likelihood estimators are not generally known.

(3) Equation (8) becomes an n^{th} order difference equation with the order determined by the degree of autocorrelation. For the solution of the difference equation to be stable it is necessary for the n roots of the characteristic equation to be less than one in

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1/ See Appendix A for this maximum likelihood estimation procedure.

absolute value.

Testing the Stability of High Order Difference Equations

It is often difficult to determine the roots of the characteristic equation when the difference equation exceeds third order. This is sometimes the case when a high order of autocorrelation necessitates transforming the original statistical model such as in equation (8). However, it is still quite easy to test for stability (even if the roots are unknown) using the "Schur theorum" [6]. This procedure is outlined below.

"The roots of the n^{th} degree polynomial equation

$$a_o b^n + a_1 b^{n-1} + \ldots + a_{n-1} b + a_n = 0$$

will all be less than unity if and only if the following n determinants are all positive."

$$A_o = \left| \begin{array}{c|c} a_o & a_n \\ \hline a_n & a_o \end{array} \right| > 0$$

$$A_1 = \left| \begin{array}{cc|cc} a_o & 0 & a_n & a_{n-1} \\ a_1 & a_o & 0 & a_n \\ \hline a_n & 0 & a_o & a_1 \\ a_{n-1} & a_n & 0 & a_o \end{array} \right| > 0$$

$\vdots$

$$
\vdots
$$

$$
An = \left|\begin{array}{cccc|cccc}
a_o & 0 & \cdots & 0 & a_n & a_{n-1} & \cdots & a_1 \\
a_1 & a_o & \cdots & 0 & 0 & a_n & \cdots & a_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_{n-1} & a_{n-2} & \cdots & a_o & 0 & 0 & \cdots & a_n \\
\hline
a_n & 0 & \cdots & 0 & a_o & a_1 & \cdots & a_{n-1} \\
a_{n-1} & a_n & \cdots & 0 & 0 & a_o & \cdots & a_{n-2} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_1 & a_2 & \cdots & a_n & 0 & 0 & \cdots & a_o
\end{array}\right| > 0
$$

If each determinant possesses a value greater than 0, both the necessary and sufficient conditions for convergence will be satisfied.

Summary

Using the expectation of the lagged dependent variable in dynamic regressions eliminates the random element from the systematic part of the regression equation. The assumed form of the adjustment mechanism using the expectation is no more arbitrary than using the actual lagged value and has consistently displayed better explanatory power in the empirical results.

However, nonlinearity is introduced into the model but presents no major obstacle to estimating the parameters. Maximum likelihood estimation using nonlinear least squares provides desirable asymptotic properties of the estimators when the stochastic disturbance term meets the classical assumptions.

Autocorrelation parameters may be estimated and tested for significance. When significant autocorrelation exists, the regression equation may be transformed and the classical properties of the disturbance terms regained.

Chapter 4

STATISTICAL RESULTS AND ECONOMIC CONCLUSIONS

This chapter summarizes the results of the statistical analysis. A single equation model estimated by nonlinear least squares was utilized. It was assumed that supply and demand functions for fluid milk in Montana are independent. It was postulated that consumers base their decisions on price the preceding and earlier periods. Since prices of fluid milk products were administered at both the farm and resale levels during the period of the analysis, it was possible to treat this variable as a nonstochastic explanatory variable. These assumptions imply the supply and demand sectors of the fluid milk market are represented by a system of recursive equations.

An aggregate model was chosen which incorporated population explicitly as an exogenous variable as opposed to development of a per capita consumption model. This formulation allowed population to be partioned into two age groups that were permitted to interact with income.

The Statistical Model

The following is the statistical model developed for this study

$$q_t = \beta_0 + \beta_1 X_{1t} + \beta_3(X_{2t} \cdot X_{3t}) + \beta_4(X_{4t}) + \beta_5(X_{2t} \cdot X_{4t}) + \beta_6(X_{5t-1})$$

$$+ \beta_7(Q1) + \beta_8(Q2) + \beta_9(Q3) + \lambda(E(q_{t-1})) + e_t,$$

where

q_t = observed consumption in class 1 milk in period t (in units of 10,000 pounds),

X_{1t} = adjusted order price per half-gallon whole milk (cents),[1/]

X_{2t} = total disposable income in period t (real income in millions of dollars),[1/]

X_{3t} = total population 18 years old and younger,

X_{4t} = total population over 18 years old,

$X_{5(t-1)}$ = adjusted price paid to manufacturers for non-fat dry milk, lagged one time period (in cents per pound),[1/]

Q_1 = binary variable representing the first calendar quarter (1 if 1st quarter; 0 otherwise),

Q_2 = binary variable representing the second calendar quarter (1 if 2nd quarter; 0 otherwise),

Q_3 = binary variable representing the third calendar quarter (i if 3rd quarter; 0 otherwise), and

$E(q_{t-1})$ = expectation of the lagged dependent variable.

[1/] Monetary variables adjusted using consumer price index (1967 = 100).

Total Class 1 Utilization

Total consumption of fluid milk in Montana is not directly observable at the retail level. However, it is possible to construct a relatively close proxy to actual consumption by using the quantity of fluid milk utilized in Class I products.

Monthly Class I utilization, for the period January 1964 to December 1976, obtained from the official records of the Montana Milk Control Division, was aggregated to quarterly quantities in units of ten thousand pounds. This data is available since state law requires dairy processors to account for all milk purchased from producers. However, only an approximation to actual utilization is available for several reasons. First, the quantity of milk exported from Montana as Class I products has only been recorded since 1967. However, up until 1975 net imports had been relatively constant and represented about 4 percent of total consumption.

Data on Class I utilization from processor owned herds and producer/distributors was not available on a monthly basis. This utilization from independent producers, accounting for 94 percent of actual utilization, was available for analysis on a quarterly basis. Since the utilization from net imports, producer/distributors and processor owned herds has fluctuated only slightly over the period of analysis the parameter estimates should not be greatly affected by its exclusion. It can be shown that if this unobservable quantity was exactly constant over the period of the analysis the parameter estimates (with the exception of the intercept) would not be affected at all. Thus, the statistical estimates should not be severly distorted by the unavailable data on consumption. We might expect the coefficient of determination to be lower, however, since the measurement error in the dependent variable will increase the unexplained variation somewhat.

Retail Price

There was no single retail price for Class I products in Montana over the period of this analysis. A vector of prices existed covering each product and container size. The largest single product utilization in the Class I group during this period was whole milk (homogenized and/or pasturized milk testing not less than 3.25 percent butter fat). It was also known that the majority of consumers purchased whole milk in one-half gallon containers. Thus, the retail price of one-half gallon whole milk was believed to accurately reflect the price facing the typical consumer. The order price of whole milk in one-half gallon containers reported by the Montana Milk Control Division was used to represent the entire Class I price vector. While a weighted average price combining each Class I product would have been preferable theoretically, date limitations made its construction impossible.

There has been no retail price competition in the dairy industry in Montana since price regulation was introduced. The Montana Milk Control Board set the minimum order price charged to consumers, and historically, this legal minimum became the effective retail price. The order price corresponds closely to the actual market price during the period of the study. During the time period when regional pricing was in effect, the price believed to have been charged most consumers was used. This presented no particular problem since regional prices generally varied by a small differential.

Retail milk price was deflated by the consumer price index to distinguish between price changes due to increases in the general price level and changes in relative prices. The adjusted retail price of whole milk per half gallon, measured in 1967 dollars, rose from a minimum value of 50¢ per half gallon in 4th quarter 1972 to a high value of 57¢ in 3rd quarter 1974. During this same period, the unadjusted price rose from 63¢ per half gallon to 85¢ per half gallon. The rapid milk price increases of the 1970's provided the price variation necessary to statistically estimate consumer reaction to changing price levels.

Income

Three alternative measures of income, all adjusted by the consumer price index, were utilized to estimate the demand relationship: Total disposable income, total personal income, total non-farm income. Yearly income values in millions of dollars were obtained from the Survey of Current Business [22] and translated to quarterly values by linear interpolation. Total disposable income was believed to be the income measure with the greatest economic relevance since it represented the actual income available for the purchase of goods and services. Total personal income was included as an alternative measure.

Aggregate consumer response to changing non-farm income was also investigated. Since Montana is an agriculturally oriented state with a large rural population, which probably consumes less milk at the retail level, it was possible that the farm income component of total personal income (approximately 11 percent in 1969) unduly biased the statistical estimate of the income elasticity. It was believed use of non-farm income might present a clearer insight to consumer income response.

It was also believed that response of the typical consuming family to changing income levels was intimately related to the composition of the family. This hypothesis is supported by prior studies [3, 4, 24]. It was hypothesized that families composed of younger family members (defined as 18 years old or younger) would on average consume more milk as income increased than would families with an older age distribution. This hypothesis was testing using interactions between disposable income and two variables constructed by partioning total population into groups 18 years of age and less and over 18 years of age.

Adjusted Dry Milk Price

The closest dairy substitute for fluid milk is non fat dry milk. An increase in the relative price of fluid milk is expected to induce some consumers to enter the dry milk market and conversely, an increase in the real price of dry milk should increase the consumption of fluid milk.

The cross price effects between fluid consumption and dry milk price ideally require actual retail price of dry milk. However, this data was unobservable due to lack of adequate historical data and because of price variation between brands and regions. Thus, the price per pound received by manufacturers of non fat dry milk was used as a proxy for retail dry milk price. The manufacturing price differed from the retail price by the marketing margin and transportation cost which were assumed to be constant over the period of the study. Dry milk price at the manufacturing level was also deflated by the CPI and lagged one time period (3 months) to account for the time required for dry milk to reach the retail market.

The cross-price response between fluid milk and non-dairy substitutes such as coffee and carbonated beverages was not investigated due to a lack of adequate data. Though some prior studies have shown a substitution effect to exist [4], it was not believed to be of significant magnitude to cause serious distortions in the statistical results.

Population

The level of population is an important determinant of total quantity of fluid milk consumed. Equally important, however, may be the age structure of population in that we would expect an older population to consume less fluid milk than a younger population. In part this is due to the beneficial dietetic attributes of milk in the growth process of younger children and the health warnings regarding consumption by older adults (as well as changes in tastes and preferences associated with changing age).

Annual population estimates were obtained from the Department of Commerce, Bureau of the Census, Series P-25 [19] and linearly interpolated to quarterly values. To capture changes in the structure of the population, total population was partitioned into two age classes as detailed under the description of the income variable above.

Seasonal Effects on Milk Consumption

Possible seasonal effects on milk consumption were analyzed by incorporating binary variables into the statistical model. Two alternative seasonal formulations were tested:

(1) It was hypothesized the seasonal effects shifted the intercept of the statistical model only while the slope (i.e., price parameter) remained unchanged.

(2) It was hypothesized the seasonal effects changed the slope parameter on price while the intercept remained unchanged.

Procedures outlined in Kmenta [12] were used to test these hypotheses.

Hypothesis 1 was tested by adding three additional explanatory variables in the model in such a manner that each seasonal variable was valued at 1 for the months associated with its respective quarter. The fourth quarter was used as a base period to determine if statistically significant differences in milk consumption existed between quarters.

Hypothesis 2 was tested by interacting retail milk price in each quarter with the binary variables described above. No significant differences between seasonal variables were detected and the numerical results are not reported in this study.

A third hypothesis that both the intercept and slope of the regression change each quarter could have been tested by partitioning the sample into each quarter's observations and performing individual regressions on the four subsamples. However, the limited size of the available data did not permit this approach to be utilized.[2/]

The Statistical Results

The parameter estimates and associated statistical measures for the milk demand equation are given in Table 14.

Stability of Model

Dynamic stability of the autoregressive error structure in the statistical model was tested using "Schur's Theorem" described in Chapter 3. The four determinants were evaluated using a computer routine. In each case the value of the determinant was positive. It can be concluded, based on this test, that the model is dynamically stable. Appendix C summarizes the results of the application of "Schur's Theorem" to this statistical model.

[2/] This formulation assumes the error variance is different for each quarter's observation. Alternatively, a test of the same hypothesis with the added assumption of homogeneous error variance could have been performed by including both the seasonal dummies and the interaction of each dummy with the other independent variables in the statistical model.

Table 14. Statistical Results[3/]

Variable Name	Parameter Estimate	Standard Error	Approximate "t" Value
Intercept	-1245.1	519.96	-2.395
Adjusted retail price	- 24.55	4.169	-5.889*
Interaction: Young population X total disposable income	.00344	.000544	6.324*
Old population	8.143	1.135	7.174*
Interaction: old population X total disposable income	-.00314	.000313	-10.032*
Dry milk price t-1	1.487	.3961	3.754*
Binary Variable 1st Quarter	86.36	106.34	.812
Binary Variable 2nd Quarter	-173.32	82.19	-2.109*
Binary Variable 3rd Quarter	132.53	109.86	1.206
Expectation of the lagged dependent variable	.8723	.0382	22.84*
1st Order serial correlation parameter	-.4695	.1490	-3.151*
2nd Order serial correlation parameter	-.4604	.1424	-3.233*
3rd Order serial correlation parameter	-.5462	.1424	-3.836*
4th Order serial correlation parameter	-.3340	.1490	-2.242*
Adjusted Multiple R-Squared	.5975		
Standard Error of Estimate	122.8986		

*Significant at the 95 percent level.

3/For reader convenience the partial derivatives of consumption with respect to old population and income are reported in text of chapter.

Consumer Response to Changing Price

Consumers appeared to respond strongly to changing price levels in the fluid milk market. The size of the estimated price coefficient and its associated "t" statistic (-5.889) indicated a strong inverse relationship between relative price and quantity of milk consumed. In the short run (a time period less than three months) a 10 percent increase in retail milk price decreased quantity consumed by 3.3 percent, all other variables constant. The short run price elasticity of -.33 was slightly greater than several recent studies indicated [3, 4]. However, it was nearly identical to the elasticity estimated by George and King [10]. This value does not seem unreasonable based on prior studies.

An approximate 95 percent confidence interval (see Appendix B) of -.22 and -.44 was also computed for the short run elasticity.

The point estimate of the long run price elasticity was -2.58. (If price changes by 10 percent while all other factors remain constant, quantity consumed will fall nearly 26 percent in the long run after all adjustments are completed.) This elasticity measure was considerably greater than other studies had reported. However, an approximate 95 percent confidence interval (calculated in Appendix B) around the long run elasticity (-4.29 to -.87) did bracket the point estimate of -1.62 by Babb and Boehm [4]. It appears consumers ultimately respond to changing price with greater than proportional changes in the consumption of fluid milk after total adjustment has been realized.

The wide confidence interval on the long run price elasticity indicates rather imprecise estimation. While it may be concluded with some degree of confidence that demand is price elastic in the long run, the actual magnitude of consumer response may not have been accurately estimated.

It should be emphasized that the interval estimate of the long run elasticity is only an approximation. Since the long run elasticity is nonlinear in the parameter estimates, it is necessary to obtain a linear approximation via a Taylor series expansion (see Appendix B). As a result, the accuracy of the interval estimate is conditional upon the accuracy of this linearization technique. Since the Taylor expansion is only approximately normally distributed (and only in large samples because the maximum likelihood estimates are asymptotically normal), use of the "t" distribution in interval estimation may introduce further distortion of the interval estimates. The magnitude of this potential distortion and its effect on the interval estimate is impossible to determine.

The ultimate consumer response appeared to be distributed over a lengthy time period. Given a permanent one time price change, (ΔP), the sequence of adjustment which would occur, assuming all other factors remain constant, is portrayed in Table 15. Most consumer response would be completed in a three to four year period (12 to 16 time periods of the quarterly model) and two-thirds of the adjustment is completed after two years. Therefore, three to four years might be considered the long run for policy decisions based upon the results of this analysis. One year might be called an intermediate run since about 50 percent of the total adjustment is accounted for in a year. The lengthy period of adjustment suggested by the results of the analysis was unexpected. Consumers would be expected to react quickly in markets for perishable commodities such as fluid milk where purchases are made frequently. The same factor that caused the interval estimate to be so wide probably accounts for this lengthy adjustment (i.e., imprecise estimation of λ). Most factors will change before total adjustment occurs. Therefore, the process of adjustment outlined above is a theoretical chain of actions that is unlikely to actually be observed.[4/]

Dynamic Elasticity

A concept similar to analysis of the time path of quantity

[4/] Many alternative lag structures were tested on the price variable, however, no significant relationships were discovered.

Table 15

Rate of Adjustment Over Time

Number of Quarters since initial price disturbance	Actual adjustment completed since initial price disturbance [5/]	Percent of ultimate adjustment since initial price disturbance [6/]
1 (immediate adjustment)	$(\Delta P)(\beta)$	13%
2	$(\Delta P)(\beta)(1+\lambda)$	24%
3	$(\Delta P)(\beta)(1+\lambda+\lambda^2)$	34%
4	$(\Delta P)(\beta)(1+\lambda+\lambda^2+\lambda^3)$	42%
5	.	50%
6	.	57%
7	.	62%
8	.	67%
(ultimate adjustment)	$(\Delta P)(\beta)(1+\lambda+\lambda^2+\ldots) = \frac{(\Delta P)(\beta)}{(1-\lambda)}$	100%

[5/] λ equals .87 (see Table 14).

[6/] The percentage of adjustment completed is found in the following manner:

$$\%\text{ adjustment} = \frac{\text{actual adjustment completed}}{\text{total ultimate adjustment}}$$

Therefore in the immediate time period (1)

$$\%\text{ adjustment} = \frac{(\Delta P)(\beta)}{\frac{(\Delta P)(\beta)}{(1-\lambda)}} = (1-\lambda) \text{ or } 13\% \text{ (since } \lambda = .87\text{)}.$$

adjustment is that of "dynamic elasticity." As noted in Chapter 2 demand becomes more elastic with increasing increments of time. As a result, the price elasticity of demand also increases in absolute value as time passes until the total potential adjustment is realized. The dynamic nature of the elasticity may be analyzed as follows [see 18].

By definition a dynamic elasticity is given by

$$E_p(\tau) = \frac{\overline{P}_t}{\overline{Q}_t} \frac{Q_{t+\tau} - Q_t}{\Delta P_t}$$

where: $Q_{t+\tau} - Q_t$ is the adjustment in quantity of fluid milk consumed between the intial price disturbance at time (t) and the number of time periods in which adjustment has occurred (τ).

ΔP_t is the initial price disturbance. Price is assumed to change by ΔP and remain at the new level indefinitely.

$\frac{\overline{P}_t}{\overline{Q}_t}$ is the ratio of the mean values of price and quantity consumed over the period of the analysis.

τ is the number of periods of adjustment.

Thus in the 1st time period:

$$E_p\ (1) = \frac{\overline{P}_t}{\overline{Q}_t} \frac{Q_{t+1} - Q_t}{\Delta P_t}$$

However, $Q_{t+1} - Q_t$ is equal to (ΔP) (β) as shown in Table 16.

Therefore,

$$E(1) = \frac{\bar{P}_t}{\bar{Q}_t} \frac{(\Delta P)\ (\hat{\beta})}{\Delta P}$$

$$= \frac{\bar{P}_t}{\bar{Q}_t} \quad \hat{\beta}$$

which is the short run elasticity for period 1.

Taking $\tau = 2$,

$$E_p(2) = \frac{\bar{P}_t}{\bar{Q}_t} \frac{Q_{t+2} - Q_t}{\Delta P}$$

$$= \frac{\bar{P}_t}{\bar{Q}_t} \frac{(\Delta P)(\hat{\beta})\ (1+\lambda)}{\Delta P}$$

$$= \frac{\bar{P}_t}{\bar{Q}_t} \quad (\hat{\beta})\ (1+\lambda)$$

Upon realization of the ultimate adjustment

$$\lim_{\tau \to \infty} E_p(\tau) = \frac{\bar{P}_t}{\bar{Q}_t} \quad \frac{(\Delta P)\ (\hat{\beta})\ (\ 1 + \lambda + \lambda^2 + \ldots)}{\Delta P}$$

$$= \frac{\bar{P}_t}{\bar{Q}_t} \quad \frac{\hat{\beta}}{(1 - \lambda)}$$

which is the "long run" price elasticity of demand.

Table 16 summarizes the time path of adjustment of the dynamic elasticity where:

Table 16

Time Path of Dynamic Elasticity

Number of Quarters Since Initial Price Disturbance	Adjustment in Quantity Consumed	Value of Dynamic Elasticity
1	$(\Delta P)\ (\beta)$	-.33
2	$(\Delta P)\ (\beta)\ (1+\lambda)$	-.61
3	$(\Delta P)(\beta)(1+\lambda + \lambda^2)$	-.87
4	$(\Delta P)(\beta)(1+\lambda + \lambda^2 + \lambda^3)$	-1.08
5	$(\Delta P)(\beta)\ (1+\lambda + \lambda^2 + \lambda^3 + \lambda^4)$	-1.27
6	.	-1.44
7	.	-1.58
8	.	-1.72
∞	$(\Delta P)(\beta)\ \frac{1}{1 - \lambda}$	-2.58

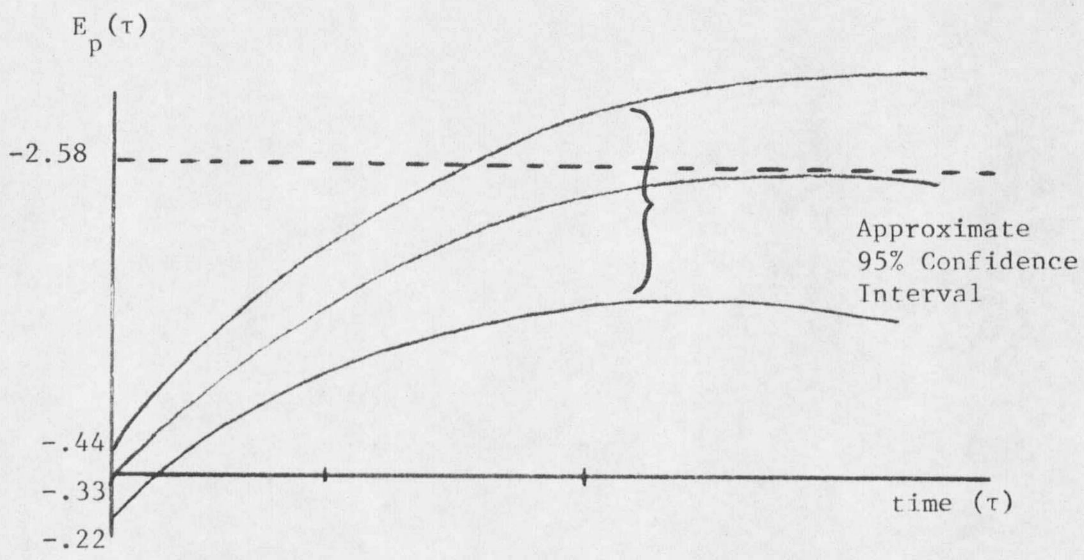

Figure 3. Adjustment of Dynamic Price Elasticity of Demand with Approximate Confidence Interval

$\hat{\beta} = -24.55$

$\lambda = .87$

$\overline{P}_t / \overline{Q}_t = 0.0135$

Thus, based on this adjustment pattern, the price elasticity becomes "elastic" within one year of an initial price disturbance if all other factors remain constant during the same time period.

Estimated "Farm Level" Price Elasticity from "Retail" Data

It was possible to obtain estimates of "farm level" price elasticity from the statistical results at the "retail" level. As was shown in Chapter 2, the retail and farm demand curves will be parallel when the marketing margin is contant.

The marketing margin existing in the 4th quarter 1976 was chosen since this was the most recent data available. Farm level elasticity was based on the following formula:

$$E_f = E_r \left(\frac{P_f}{P_r}\right)$$

where E_f = "Farm" level price elasticity

E_r = Estimated "retail price elasticity (Measured at 4th quarter 1976 prices)

P_f = Farm level price of one-half gallon homogenized milk

P_r = Retail price of one-half gallon homogenized milk

$E_f = -(.29)\ (41/86)$

$E_f = -.14$

Thus at the most recent levels of price and quantity, farm level price elasticity is considerably less elastic than retail price elasticity, viz., one-half as much.

Consumer Response to Changing Levels of Real Income

The response of consumers to changing income levels is difficult to interpret from the results of this study. Table 17 summarizes the values of the parameter estimates, standard errors, t values and the income elasticities measured at the mean values of consumption for the various measures of income. In all cases a signficant, inverse relationship between income and consumption was observed. These findings conflict with most previous studies and prior expectations.

A possible explanation of these results may be found in the interaction of income and the age structure of the population. Intuitively, a family's response to changing levels of income will depend on the age structure of the family. Families composed of younger members are likely to respond differently to changes in income than will families with an older age structure (this hypothesis is supported by [4,24]). Specifically, a family with a number of young children is likely to increase consumption of fluid milk as income increases much more than a family comprised mainly of adults.

It was not possible to observe the age structure of each family

Table 17

Statistical Measures of the Relationship Between Consumption of Fluid Milk and Income

Measure of Income	Parameter Estimates	Standard Error	T Value	F Value	Income Elast.
Total Disposable	-0.6112	.0576	-10.61		-.29
Total Personal	-0.5665	.0659	- 8.60		-.31
Total Non Farm	-1.5829	.2129	- 7.43		-.76
Interactions: Young/Total Disposable Income	0.00344	.000544	+ 6.32		
Old/Total Disposable Income	-0.00314	.000313	-10.13		
Income effect[7/] of interactions (population variable at means)	-1.329			18.42	-.26

[7/] The partial derivative of consumption with respect to income using the statistical model including interactions was computed as follows:

$E(q)$ is the expected level of consumption

P_y is population 18 years old and less

P_o is population over 18 years old

Inc is total disposable income (in real terms)

The partial effect of a change in income measured at the mean values of P_y, P_o is given:

(i) $$\frac{\delta(E(q))}{\delta(Inc)} = \beta_y\ (\overline{P}_y) + \beta_o(\overline{P}_o)$$

Where:

$\overline{P}_y$ is the average population under 18 years old (1964-1976)

Table 17, Continued

$\overline{P}_o$ is the average population over 18 years old (1964-1976)

Substituting statistical estimates for the parameters yields:

$$\frac{\partial(E(q))}{\partial(Inc)} = (.00344)(256.4) + (-.0034)(456.7)$$

$$= -1.329$$

purchasing fluid milk products in Montana. However, a proxy to this relationship using interactions between real disposable income and population gave quite good results. The statistical estimates of the interaction parameters (β_y = .00344 and β_o = -.00314) were highly significant with t values of 6.324 for the young/income interaction and -10.032 for the old/income interaction. The positive estimate for the young/income interaction implies that as income increased, given the average population under 18 years of age, aggregate consumption of fluid milk products also increased. The negative parameter on the old/income interaction implies that as income increased, given the average population over 18 years old, fluid milk consumption decreased. These results suggest that fluid milk products are normal goods for families comprised mainly of younger members and inferior goods for older individuals.

As (i) in Table 17 indicates, the overall change in aggregate consumption for changes in aggregate income depends on the relative magnitudes of the population under 18 years old and over 18 years old. The negative value of the partial income parameter i.e. $\frac{\partial(E(q))}{\partial(Inc)}$ may be explained by the fact that nearly two-thirds of Montana's population exceeds 18 years of age.

Thus, it appears the negative aggregate income response measured by disposable personal and non-farm income may have been the result of two counteracting forces: as income increased (1) the proportion of the population under 18 years of age increased consumption and (2) the proportion of the population over 18 years decreased consumption. This latter effect tended to dominate and, as a result, the negative aggregate response to changing income was observed.

The partial income effect was tested for significance in the following manner. The statistical model was estimated with the interaction terms included and the residual sum of squares computed. The interaction terms were then excluded from the statistical model and the residual sum of squares recomputed. An "F" statistic was constructed as follows:

$$F = \frac{SSE\omega - SSE\Omega}{SSE\Omega} \cdot \frac{n-k}{q}$$

Where SSEω = the residual sum of squares when the interaction terms are omitted

SSEΩ = the residual sum of squares when the interaction terms are included

The hypothesis that income had no effect on consumption (i.e. $\beta_y = \beta_o = 0$), was tested using a "F" test based on 2 and ∞ degrees of freedom in the numerator and denominator respectively. The computed "F" value was 18.43 and implied a significant relationship existed between aggregate income and fluid milk consumption during the time period of the analysis.

While the study suggests that fluid milk is a normal good for younger individuals and inferior for older, this conclusion must be considered carefully due to the nature of the aggregate data utilized. Relatively high intercorrelations between variables were present. It is possible that a trend in the tastes of the older population could easily be confounded with the response to income. Such a downward trend in milk consumption by adults could be associated with the "cholesterol scare" created by the medical profession.

Adjusted Dry Milk Price

The parameter estimate of dry milk price lagged one time period was significant. However, the magnitude of the cross price elasticity of +.1 indicates that while dry milk is a substitute for fluid milk the cross substitution is quite weak. It appears that most consumers in the fluid milk market will react to increasing fluid milk prices by generally reducing fluid milk consumption and/or substitution to non-dairy products rather than substitution to dry milk.[8/]

Serial Correlation

Significant 4th order serial correlation was estimated, however

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8/ This result was expected. The prior study by Thomas and Waananen [24] showed that consumers viewed dry milk as inferior to fluid products. Dry milk was generally evaluated with strong negative feelings with respect to taste and other attributes.

higher orders were not significant. This implies the stochastic disturbances are generated in the following manner.

$$\text{(ii)}\quad e_t = -.4694e_{t-1} - .46043e_{t-2} - .5462e_{t-3} - .3340e_{t-4} + U_t$$
$$(.1490)\qquad (.1424)\qquad (.1424)\qquad (.1490)$$

Equation (ii) indicates that unusually high (low) values of consumption up to one year ago are likely to induce unusually low (high) levels of consumption this period. There is no particular economic explanation for this statistical result. It is possible that specification errors in the model due to unobservable or unmeasureable factors are responsible. Systematic errors in measurement on the dependent variable also might explain this result.

The Lagged Dependent Variable and Its Expectation

Inclusion of the expectation of the lagged dependent variable systematically improved the explanatory power of the model compared to using the actual value of the lagged dependent variable. The multiple coefficient of determination (R^2) rose from .57 using actual lagged values to .78 using the expectation of the lagged values. This represents a signficant reduction in the residual sum of squares.

While there is no precise explanation for the significant increase in explanatory power using the expectation of the lagged dependent variable, it is believed that better theoretical properties of the error term are responsible. It is also likely the nonstochastic difference equation representing the adjustment mechanism models consumer reactions better than the stochastic adjustment mechanism.

Higher order lags on the dependent variable and its expectation introduced severe multicollinearity and did not improve the data fit significantly. This result was anticipated since the dependent variable was expected to be highly correlated between periods.

Season of the Year

Statistically significant seasonality exists in fluid milk consumption. On the average, 1.73 million pounds less milk is consumed during the months of April, May and June (the fourth calendar quarter was used as a period of reference) when the effects of all other variables are accounted for. Based on prior studies [4] lower aggregate consumption was expected for the period May to September.

A full analysis of seasonality was not pursued due to the aggregation of the data on a calendar quarter basis rather than on a season of the year basis.

Population

Increases in population have a direct influence on the total consumption of fluid milk. The partial effect on the young population variable using the young/income interaction term[9/] measured at the

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[9/] Originally young population had been entered as $\beta_1(Y_{ng}) + \beta_2(Y_{ng} \cdot \text{Inc})$. This formulation resulted in high multicollinearity between sample mean value of disposable income is:

$$\frac{\partial E(q)}{\partial (Y_{ng})} = \beta Y_{ng}(\overline{Inc}) = 6.33$$

The partial effect of an increase in the old population measured at the mean income is:

$$\frac{\partial E(q)}{\partial (Old)} = \beta_{Old} + \beta_{Old/Inc} \quad (\overline{Inc})$$

$$= 8.14 - 5.81 = 2.33$$

An increase in the younger population induces a greater increase in total consumption of fluid milk than does an increase in the older population. A 10 percent increase in the younger population increases consumption by 4.1 percent while a 10 percent increase in the older population increases consumption by 2.7 percent.

Summary

Based upon the statistical results of this analysis it may be concluded:

(1) In the short run (a period of three calendar months) demand appears to be inelastic. Thus, changes in the price of fluid milk are likely to elicit less than proportional changes in fluid milk consumption. A 10 percent change in price is expected to change consumption inversely by 3

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9 Continued/ β_1, β_2. Thus, the (Y_{ng}) variable was dropped from the model.

percent.

(2) There is evidence consumer response becomes more elastic with the increasing time lag between price changes and consumption. A time period of one year after a price change is required for price elasticity to become "elastic." In the long run after complete consumer adjustment to price changes has occurred (holding all other influencing variables constant over the same period) a 10 percent change in price will elicit a 26 percent change in consumption.

(3) It appears the ultimate consumer response is distributed over a lengthy time period. Approximately one year is required for 50 percent of the ultimate response to be completed. The lengthy period of adjustment may not seem realistic since most consumers enter the fluid milk market frequently and probably should adapt to changing economic conditions more rapidly than the study indicates. This result suggests the dynamic elements of the statistical model may be subject to some estimation problems.

(4) There is evidence that the interaction between the age distribution of the population and disposable income is an important determinant of consumption. Results of the analysis imply fluid milk is a normal good for young individuals (under 18 years of age) and families and an

inferior good for older individuals (over 18 years of age) or families composed mainly of adults. Since two-thirds of the population of Montana falls in the older group, the inverse response between consumption and income for older individuals tends to dominate. Thus, an increase in aggregate income led to a decrease in aggregate consumption in the statistical model.

(5) Increases in population size significantly increases the total consumption of fluid milk. However, a 10 percent increase in the younger population increases consumption to a greater degree than a 10 percent increase in the older population (4.1 -vs- 2.7 percent).

(6) Seasonality in fluid milk consumption is significant. Holding all other factors constant, less milk is consumed in the months of April, May and June than in other months. This may be the result of increased competition from non-dairy beverages during the late spring and early summer.

(7) While nonfat dry milk is a substitute for fluid milk, the relationship is quite weak. A 10 percent increase in dry milk price led to increased fluid milk consumption of 1 percent. It appears consumers respond to higher fluid milk prices by consuming less fluid milk and/or substitution to non-dairy beverages rather than by consuming greater

quantities of dry milk.

Limitation of This Study

Several statistical and data limitations should be considered in evaluating the validity of the results and interpretations of this analysis:

(1) The properties of maximum likelihood estimators in non-linear estimation are asymptotic and thus are usually not known for small samples. This study utilized 40 observations and it is likely that the sample is not of sufficient size to fully capture the desirable large sample properties. Therefore the statistical estimates and tests of significance may be misleading.

(2) Some bias is introduced by the unavailability of data for exports and other unobservable elements (i.e. producer/distributors; processor owned herds). Since these elements were not constant over the period of the study the estimates may be affected to some extent.

(3) Using the retail price per half gallon of whole milk to represent the price vector rather than constructing a weighted average is likely to affect the results to some degree. This is probably not a major problem since there was not a great deal of variation in the price relationship

between the various forms of fluid milk or container sizes.

(4) Several socioeconomic variables that may have changed over the analysis (such as rural/urban composition, income distribution, etc.) for which data were not available may have introduced specification errors in the statistical model.

(5) A number of measurement errors in the fluid milk utilization of certain years (1964, 1973) undoubtedly affected the estimates somewhat. These errors amounted to about 7 percent of utilization.

APPENDICES

APPENDIX A

The likelihood function is constructed assuming a first order serial correlation scheme and one explanatory variable for simplicity. The regression equation is transformed as shown on page 58 to regain the classical properties of the disturbance

$$Y_t - \rho Y_{t-1} = \alpha\gamma(1-\rho) + \beta\gamma(X_t - \rho X_{t-1}) + (1-\gamma)\,[E(Y_t) - \rho E(Y_{t-1})] + U_t$$

where $U_t = E_t - \rho E_{t-1}$.

Constructing the logarithm of the likelihood function (assuming normality) yields

$$L = -\frac{n-1}{2}\log(2\pi\sigma^2 U_t) - \frac{1}{2\sigma^2}\sum_{t=1}^{n}(Y_t^* - \hat{Y}_t^*)^2$$

where $Y^*_t - \hat{Y}^*_t$ is

$$Y_t^* - \hat{Y}_t^* = (Y_t - \rho Y_{t-1}) - \alpha\gamma(1-\rho) - \beta\gamma(X_t - \rho X_{t-1}) - (1-\gamma)\,[E(Y_t) - \rho E(Y_{t-1})].$$

The maximum likelihood estimates (conditional on Y_1, the presample value) of α, γ, β, and ρ are obtained by maximizing L using a non-linear technique [12].

The derivation of (ii) from (i) in the text of Chapter 3 makes it clear that $E(Y_t)$ is implicitly a function of the independent variable X_t back to the beginning of the sample. The presence of an autoregressive error term does not change this result because $E(Y_t)$ is defined unconditionally with respect to information associated with the autocorrelation in the error term.

APPENDIX B

Placing Confidence Intervals on Short and Long Run Elasticities

Short Run Elasticities

By definition the short run price elasticity of demand is:

$$\varepsilon = \frac{\partial q}{\partial p} \cdot \frac{p}{q}$$

Where: ε is the short run price elasticity of demand and p is price and q is quantity consumed.

The elasticity may be estimated by multiplying the parameter estimate of the price variable by the ratio of the mean values of the price and quantity variables:

$$\hat{\varepsilon} = \hat{\beta}\ (\bar{p}\ /\ \bar{q})$$

The estimated elasticity is a function of the random variable and, thus, is itself a random variable.

The variance of the estimate is given by:

(i) $\quad Var(\hat{\varepsilon}) = (K^2)\ Var\ (\hat{\beta})$

Where K is the ratio of the mean values of price and quantity.

The standard deviation of the elasticity estimate is the square root of the variance:

(ii) $\quad \sigma_{\hat{\varepsilon}} = K\sigma_{\hat{\beta}}$

Where $K = \bar{p}/\bar{q} = 0.0135$.

By definition a 95 percent confidence interval is given by:

$$\varepsilon \pm t(.025,\ n-p)\ (Ks_{\hat{\beta}})$$

$$(-.33) \pm (1.96)(.0135 \cdot 4.17), \quad -.44 < \varepsilon < -.22$$

where n-p is the number of degrees of freedom associated with the t statistic and $s_{\hat{\beta}}$ is the estimated standard deviation of β (standard error). This is a conditional confidence interval given the particular sample value of K.

Long Run Elasticities

The long run elasticity is estimated by:

$$\text{(iii)} \quad \hat{\eta} = \frac{\partial q}{\partial p} \cdot \frac{p}{q} \cdot \frac{1}{1-\hat{\lambda}}$$

$$\text{(iv)} \quad \hat{\eta} = \frac{\hat{\beta} \hat{K}}{1-\hat{\lambda}}$$

where $\hat{\lambda}$ is the parameter estimate of the lagged dependent variable or its expectation. The estimate of the long run elasticity is a random variable that is a nonlinear function of $\hat{\beta}$ and $\hat{\lambda}$.

The variance of the random variable may be approximated by "linearizing" [1] via a Taylor series expansion and dropping terms of second order and higher.

$$\text{(v)} \quad \text{Var}\,(\hat{\eta}) = K^2 \,\text{Var}\, \frac{\hat{\beta}}{1-\hat{\lambda}}$$

$$K^2 \left[\left(\frac{\partial f}{\partial \hat{\beta}}\right)^2 \text{var}\,(\hat{\beta}) + \left(\frac{\partial f}{\partial \hat{\lambda}}\right)^2 \text{var}(\hat{\lambda}) + 2 \left(\frac{\partial f}{\partial \hat{\beta}}\right)\left(\frac{\partial f}{\partial \hat{\lambda}}\right) \text{cov}\,(\hat{\beta},\hat{\lambda}) \right]$$

where $f = \dfrac{\hat{\beta}}{1-\hat{\lambda}}$.

$$\text{Cov}\,(\hat{\beta},\hat{\lambda}) = \hat{\rho}(s_{\hat{\beta}})(s_{\hat{\lambda}})$$

where ρ is the estimated parameter correlation between the random variables $\hat{\beta}$ and $\hat{\lambda}$, and $s_{\hat{\beta}}$ and $s_{\hat{\lambda}}$ are the estimated parameter standard errors.

$$\frac{\partial f}{\partial \hat{\beta}} = \frac{1}{(1-\hat{\lambda})} ; \qquad \frac{\partial f}{\partial \hat{\lambda}} = \frac{\hat{\beta}}{(1-\hat{\lambda})^2}$$

$\hat{\lambda} = 0.8723$ $\qquad$ $\hat{\beta} = -24.5540$

$s_{\hat{\lambda}} = .0382$ $\qquad$ $s_{\hat{\beta}} = 4.1691$

$$\frac{\partial f}{\partial \hat{\beta}} = 7.8309; \qquad \frac{\partial f}{\partial \hat{\lambda}} = -1505.4567$$

$$\mathrm{Cov}(\hat{\beta},\hat{\lambda}) = .0090$$

Substituting these values into equation (v) yields:

$$\mathrm{Var}\ (f) = 4160.9$$

$$s_{\hat{f}} = 64.51$$

$$s_{\hat{\eta}} = s_{\hat{f}} K = .8709$$

A 95 percent confidence interval is then given as:

$$\hat{\eta} \pm t(.025,\ n-p)\ s_{\hat{\eta}}$$

$$-2.58 \pm 1.96\ (0.8709)$$

$$-4.29 < \eta < -.87$$

APPENDIX C

Testing the Dynamic Stability of the Statistical Model

Since the statistical model is a fourth order difference equation upon transformation, four determinants must be evaluated in order to utilize "Schur's Theorem." The characteristic equation, determinants and determinant values are:

$$y_{t+4} + .4695y_{t+3} + .4604y_{t+2} + .5462y_{t+1} + .3340y_t = C$$

$$\Delta_1 = \begin{vmatrix} 1 & 0.3340 \\ 0.3340 & 1 \end{vmatrix} = 0.8884$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 0.3340 & 0.5462 \\ 0.4695 & 1 & 0 & 0.3340 \\ 0.3340 & 0 & 1 & 0.4695 \\ 0.5462 & 0.3340 & 0 & 1 \end{vmatrix} = .6377$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & 0 & 0.3340 & 0.5462 & 0.4604 \\ 0.4695 & 1 & 0 & 0 & 0.3440 & 0.5462 \\ 0.4604 & 0.4695 & 1 & 0 & 0 & 0.3340 \\ 0.3340 & 0 & 0 & 1 & 0.4695 & 0.4604 \\ 0.5462 & 0.3340 & 0 & 0 & 1 & 0.4695 \\ 0.4604 & 0.5462 & 0.3340 & 0 & 0 & 1 \end{vmatrix} = .4273$$

$$\Delta_4 = \begin{vmatrix} 1 & 0 & 0 & 0 & 0.3340 & 0.5462 & 0.4604 & 0.4695 \\ 0.4695 & 1 & 0 & 0 & 0 & 0.3340 & 0.5462 & 0.4604 \\ 0.4604 & 0.4695 & 1 & 0 & 0 & 0 & 0.3340 & 0.5462 \\ 0.5462 & 0.4604 & 0.4695 & 1 & 0 & 0 & 0 & 0.3340 \\ 0.3340 & 0 & 0 & 0 & 1 & 0.4695 & 0.4604 & 0.5462 \\ 0.5462 & 0.3340 & 0 & 0 & 0 & 1 & 0.4695 & 0.4604 \\ 0.4604 & 0.5462 & 0.3340 & 0 & 0 & 0 & 1 & 0.4695 \\ 0.4695 & 0.4695 & 0.5462 & 0.3340 & 0 & 0 & 0 & 1 \end{vmatrix} = .2799$$

Based on this test, it was concluded the statistical demand relationship estimated was dynamically stable.

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