Abstract:
In this thesis the effects of absorptivity and diffusivity on the displacement of the back surface of a specimen, when its front surface is irradiated with a high energy laser whose temporal pulse shape is represented by a Dirac-delta function, are analyzed for the one-dimensional case. When the specimen is irradiated with laser energy thermoelastic waves are generated inside the specimen. These waves travel from the front to the back surface of the specimen and cause displacement of that surface. Governing differential equations and corresponding boundary conditions approximating this phenomenon are set up. These equations are then solved by Laplace transform method to obtain expressions for temperature distribution inside the specimen and displacement of its back surface.

A sharp spike in displacement-time graph is observed which agrees with the experimentally obtained data. It is also observed that at low absorptivity diffusivity has negligible effect on the displacement-time characteristic. But in the case of high absorptivity, displacement increases significantly with increase in absorptivity. Again for very small diffusivity, peak displacement decreases with increase in absorptivity while for other values of diffusivity peak displacement increases with increase in absorptivity. Also peak displacement does not increase much with increase in diffusivity when the diffusivity is already high.

All of these observations can be explained by considering the heat distribution inside the specimen as a series of discrete point heat sources. The resultant displacement in this case is then the summation of all the displacements due to each individual point heat source.

Using this solution as a Green’s function, the author can obtain displacement of the back surface of the specimen for any arbitrary laser pulse shape. Again this formulation helps to determine the relative effects of absorptivity and diffusivity on the displacement characteristic.
THE EFFECTS OF THERMAL DIFFUSION AND OPTICAL ABSORPTION ON LASER GENERATED ULTRASONIC WAVES IN A SOLID

by

Amitava Roy

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science

in

Mechanical Engineering

MONTANA STATE UNIVERSITY
Bozeman, Montana

July 1989
APPROVAL

of a thesis submitted by

Amitava Roy

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

Date
Chairperson, Graduate Committee

Approved for the Major Department

Date
Head, Major Department

Approved for the College of Graduate Studies

Date
Graduate Dean
STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a master’s degree at Montana State University, I agree that the Library shall make it available to borrowers under rules of the Library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Dean of Libraries when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my written permission.

Signature: [Signature]
Date: 07/18/89
ACKNOWLEDGEMENTS

I would like to thank Dr. R. Jay Conant for his guidance and the many useful discussions concerning the project.

This work was supported by the Department of Interior’s Bureau of Mines under Contract No. J0134035 through Department of Energy Contract No. DE-AC07-76ID01570. I would like to thank Idaho National Engineering Laboratory for this and in particular Dr. K.L. Telschow for his assistance in this project.
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPROVAL</td>
<td>ii</td>
</tr>
<tr>
<td>STATEMENT OF PERMISSION TO USE</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>x</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Mechanism of Wave Generation</td>
<td>2</td>
</tr>
<tr>
<td>Literature Review</td>
<td>4</td>
</tr>
<tr>
<td>Objective</td>
<td>5</td>
</tr>
<tr>
<td>Experimental Set Up</td>
<td>6</td>
</tr>
<tr>
<td>2. FORMULATION OF THE PROBLEM</td>
<td>9</td>
</tr>
<tr>
<td>3. SOLUTION OF THE EQUATIONS</td>
<td>17</td>
</tr>
<tr>
<td>Solution of Heat Conduction Equation</td>
<td>17</td>
</tr>
<tr>
<td>Solution of Wave Equation</td>
<td>19</td>
</tr>
</tbody>
</table>
## 4. RESULTS AND ANALYSIS

<table>
<thead>
<tr>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specialization of the Solution to Heat Conduction Equation for Different Particular Cases</td>
<td>35</td>
</tr>
<tr>
<td>Evaluation of the Temperature Distribution when the Laser Pulse Shape is Represented by a Dirac-Delta Function</td>
<td>39</td>
</tr>
<tr>
<td>Specialization of the Solution to Displacement Equation for Different Particular Cases</td>
<td>44</td>
</tr>
<tr>
<td>Evaluation of Displacement of the Back Surface of the Specimen when the Laser Pulse Shape is Represented by Dirac-Delta Function</td>
<td>47</td>
</tr>
<tr>
<td>Conclusion</td>
<td>56</td>
</tr>
</tbody>
</table>

## REFERENCES CITED

- 58

## APPENDICES

- 60

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Program Name</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix A</td>
<td>DİSPVAX.EXE</td>
<td>61</td>
</tr>
<tr>
<td>Appendix B</td>
<td>DISPMOD.EXE</td>
<td>71</td>
</tr>
<tr>
<td>Appendix C</td>
<td>TEMP.EXE</td>
<td>80</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Combinations of absorptivity and diffusivity used for obtaining temperature distribution</td>
<td>39</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Schematic of the apparatus</td>
<td>7</td>
</tr>
<tr>
<td>2. (a) Temperature distribution and (b) wave generation along the depth of the specimen due to sudden temperature rise at point P</td>
<td>7</td>
</tr>
<tr>
<td>3. Experimentally obtained displacement variation with time</td>
<td>8</td>
</tr>
<tr>
<td>4. Assumed uniform and non-uniform spatial distribution of laser pulse</td>
<td>10</td>
</tr>
<tr>
<td>5. Temperature distribution for high diffusivity and high absorptivity</td>
<td>40</td>
</tr>
<tr>
<td>6. Temperature distribution for low diffusivity and high absorptivity</td>
<td>41</td>
</tr>
<tr>
<td>7. Temperature distribution for high diffusivity and low absorptivity</td>
<td>42</td>
</tr>
<tr>
<td>8. Temperature distribution for low diffusivity and low absorptivity</td>
<td>43</td>
</tr>
<tr>
<td>9. Displacement as a function of time for high absorptivity</td>
<td>48</td>
</tr>
<tr>
<td>10. Displacement as a function of time for low absorptivity</td>
<td>49</td>
</tr>
<tr>
<td>11. Peak displacement as a function of diffusivity for different absorptivities</td>
<td>50</td>
</tr>
<tr>
<td>12. Visualization of the optically penetrating laser source as a collection of point heat sources, along with the waveforms produced from sources P and Q</td>
<td>52</td>
</tr>
<tr>
<td>13. Generation and propagation of wave through the specimen over a period of time for low absorptivity and low diffusivity</td>
<td>54</td>
</tr>
<tr>
<td>14. Generation and propagation of wave through the specimen over a period of time for high absorptivity and high diffusivity</td>
<td>56</td>
</tr>
<tr>
<td>15. Format for data entry files for program DISPVAX.EXE</td>
<td>62</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>16</td>
<td>Program to calculate displacement of the back surface of a specimen</td>
</tr>
<tr>
<td>17</td>
<td>Format for data entry files for program DISPMOD.EXE</td>
</tr>
<tr>
<td>18</td>
<td>Program to calculate maximum displacement of the back surface of a specimen</td>
</tr>
<tr>
<td>19</td>
<td>Format for data entry files for program TEMP.EXE</td>
</tr>
<tr>
<td>20</td>
<td>Program to calculate the temperature along the depth of a specimen</td>
</tr>
</tbody>
</table>
ABSTRACT

In this thesis the effects of absorptivity and diffusivity on the displacement of the back surface of a specimen, when its front surface is irradiated with a high energy laser whose temporal pulse shape is represented by a Dirac-delta function, are analyzed for the one-dimensional case. When the specimen is irradiated with laser energy thermoelastic waves are generated inside the specimen. These waves travel from the front to the back surface of the specimen and cause displacement of that surface. Governing differential equations and corresponding boundary conditions approximating this phenomenon are set up. These equations are then solved by Laplace transform method to obtain expressions for temperature distribution inside the specimen and displacement of its back surface.

A sharp spike in displacement-time graph is observed which agrees with the experimentally obtained data. It is also observed that at low absorptivity diffusivity has negligible effect on the displacement-time characteristic. But in the case of high absorptivity, displacement increases significantly with increase in absorptivity. Again for very small diffusivity, peak displacement decreases with increase in absorptivity while for other values of diffusivity peak displacement increases with increase in absorptivity. Also peak displacement does not increase much with increase in diffusivity when the diffusivity is already high.

All of these observations can be explained by considering the heat distribution inside the specimen as a series of discrete point heat sources. The resultant displacement in this case is then the summation of all the displacements due to each individual point heat source.

Using this solution as a Green's function, the author can obtain displacement of the back surface of the specimen for any arbitrary laser pulse shape. Again this formulation helps to determine the relative effects of absorptivity and diffusivity on the displacement characteristic.
CHAPTER 1

INTRODUCTION

A laser is a special type of high power light source. The main properties of interest that are different in laser radiation as compared to radiation from conventional light sources are the intensity, direction, monochromaticity, coherence, brightness and high power availability. With a simple system it is easily possible to generate short duration pulses of red and infrared laser light with powers of the order of millions of watts. Several billions to trillions of watts have been obtained in a pulse in a more sophisticated system. Moreover, directionality and focusing properties of laser beams make it possible to deliver high irradiation to a small spot.

When a laser beam strikes the surface of a material, some of the beam’s energy is absorbed and some energy is reflected from the surface. If absorption of laser energy is high, intense heating is produced which may lead to melting or vaporization of the target. Whenever the surface of a body is subjected to rapid transient heating and the temperature is not sufficiently high to cause melting or vaporization of the material, thermoelastic waves are produced inside the specimen. These waves propagate through the body and can be detected by a suitable device within the body. Cracks or voids in the material can alter the form and characteristics of the travelling wave if their dimensions are large compared to the wavelength of the travelling wave. By detecting and analyzing the altered wave pattern, the location, size and shape of the irregularity in the material can be determined. Laser pulses of very short duration and low intensity can produce
thermoelastic waves of sufficiently short wavelength to detect fine micro-cracks, previously undetected by ultrasonic testing methods.

**Mechanism of Wave Generation**

Light is absorbed in opaque materials by interaction with electrons. A quantum of light energy is absorbed by an electron which is raised to a high energy state. The excited electron will collide with neighboring electrons and other particles in its vicinity and give up energy. In this process energy transfer occurs from one particle to another in the material. This is the same collision process which governs the conduction heat transfer phenomenon in a material. When a laser beam strikes the surface of an opaque material, it is absorbed in the same fashion as light. In a good conductor the mean free time between collisions for electrons is of the order of $10^{-13}$ to $10^{-14}$ seconds [1]. Thus during the time in which a laser pulse strikes the surface (in this case $10^{-9}$–$10^{-7}$ seconds), an excited electron will make many collisions with other electrons and lattice particles. Therefore it can be said that when a laser beam is absorbed, optical energy is instantaneously converted to heat energy within a volume in which the laser energy is absorbed. The production and distribution of heat energy is so rapid, compared to the laser pulse duration, that the laws of conservation of energy can be applied in the small volume in which energy is absorbed. Therefore it can be said that the concept of temperature and other usual equations of heat flow are all valid in this case and the use of the continuum mechanics approach is justified. When the laser pulse duration is very short (in the order of pico-seconds) then there is no time for an excited electron to distribute its energy amongst its neighboring particles through the collision process. Hence it cannot be said that heat conduction has taken place during that time interval. Thus if the time of interest is very small,
the continuum approach cannot be employed and this case will require a different
treatment. In this case the laser-pulse duration and the time of interest are long
enough to allow application of the usual laws of heat conduction.

There are several ways by which thermoelastic waves can be generated in a
solid with the help of laser, utilizing different laser systems and/or modification
of the specimen surface on which the laser beam is incident. The simplest among
them is the laser irradiation of a clean specimen surface without any chemical
coating on it by a laser pulse which strikes the metal surface normally. The
intensity of the laser pulse must not be high enough to cause melting or damage
of the surface. The amount of energy absorbed depends on the material's optical
absorption coefficient. The reciprocal of the optical absorption coefficient is the
depth at which the laser intensity drops to $1/e$ of its original intensity where
$e$ denotes the exponential term. For metals optical absorption coefficient varies
between $10^5$/cm. to $10^6$/cm., which is very large. Thus for a metal specimen the
absorption of laser energy as it travels through the specimen is extremely high.
Therefore in each consecutive layer of the specimen, less and less energy is available
for absorption and this results in the production of a steep spatial temperature
gradient within the solid. This temperature gradient, in turn, produces a strain
field (thermoelastic effect). Again the rise of temperature is extremely rapid, as
the laser pulse has a steep temporal gradient and it is of very short duration, which
causes the total laser energy to be absorbed in a very short period of time. The
steep temporal gradient of the temperature produces a rapidly changing strain
field. Due to the rapid change in strain with respect to time, an elastic wave is
generated. Thus when there is no melting and/or vaporization of the material
due to absorption of laser energy, the wave generation in the solid is principally
associated with thermoelastic effects.
In this thesis the effect of absorptivity and diffusivity of a material on the temperature distribution in the specimen and displacement of its back surface, caused by the generated thermoelastic waves, are investigated.

Literature Review

The generation of acoustic pulses in a solid by laser irradiation of its surface was first suggested by White [2], in 1963. White showed experimentally that high frequency elastic waves are produced in the solid by pulses of electromagnetic energy and light, upon their absorption at the surfaces of elastic solids and fluids. In another paper [3] White has analyzed the process of elastic wave production and its propagation through a solid which is subjected to transient heating with a laser, with particular emphasis on the case of the input flux varying harmonically with respect to time. He related the elastic wave amplitude to the characteristics of the heat input flux and the thermal and elastic properties of the body. In the process he showed the proportionality of the stress wave amplitude and absorbed power density. In this paper he assumed that all the heat is absorbed at the surface of the body.

Carome, Clark and Moller [4] have described the exposure of a liquid with high optical absorptivity, to a Q-spoiled ruby laser and the resulting development of stress waves. After them Penner and Sharma [5] have investigated theoretically the thermal stress development in partially transparent rods of infinite length for a one-dimensional geometry prior to ablation and thermal equilibrium. For this they assumed a particular simplified temperature profile along the material depth. J.F. Ready [1] described methods by which one can calculate the temperature distribution for any arbitrary laser pulse shape when the laser energy absorbed drops off exponentially along the specimen depth.
Much later, in 1980, Scruby et. al. [6] performed quantitative experimental measurements in the generation of elastic waves by laser radiation. They found that the thermoelastic source generated both longitudinal (L) and shear (S) waves, but the latter predominates at the epicenter. They recorded the displacement of the surface opposite to the surface on which the laser strikes and observed a sharp spike is present in the displacement, signalling the arrival of the first longitudinal wave. Subsequently, the displacement became negative, i.e. in the direction opposite to the laser propagation.

Telschow and Conant [7] have observed that the spike in the displacement can be explained through the use of one-dimensional models that account for optical penetration and thermal diffusion into the material. They considered a single point source buried at the depth “H” below the surface and they showed that a positive precursor signal is produced. Next, optical penetration is taken into account by distributing point sources with an exponentially decaying magnitude with depth into the material. This model also produced a precursor signal whose shape reflects the temperature profile with depth. Finally the effects of thermal diffusion on the precursor signal were considered. In this case they assumed that all the laser energy is absorbed at the surface.

**Objective**

In this thesis an idealization is made about how the laser-pulse is absorbed by a specimen along its thickness when one surface of it is irradiated by a high energy laser-pulse so that the energy intensity absorbed at any particular depth of the specimen is known. It is assumed that laser energy absorbed in the material decays exponentially along the depth of the specimen. Based on this assumption, a closed form solution of temperature distribution along the specimen depth is
obtained. Then, the displacement of one surface of the specimen is calculated for this temperature distribution and for the one-dimensional-case when both diffusivity and absorptivity in the material are present. After that the influence of optical absorption and diffusivity of the material on the temperature distribution in the material and more importantly, on the elastic waveform generated, is determined.

**Experimental Set Up**

A laser beam, properly aligned so that it strikes the material surface normally, is incident on one surface of the specimen (Figure 1). The laser pulse is absorbed all along the material depth as it travels across the specimen and causes the temperature to rise throughout the specimen depth, though temperature falls off rapidly from the front to the back surface of the specimen, as noted previously and as shown in Figure 2a. Due to the temperature rise, elastic waves are generated at each point P along the depth of the specimen (Figure 2b). The energy carried by each of the waves generated throughout the specimen will decrease at a high spatial rate from the front to the back surface due to less temperature rise from one surface to the other. Each of these waves generated at point P travels toward both surfaces of the specimen as shown in Figure 2b and causes displacement of the surfaces when it arrives. After being reflected at the two surfaces the waves travel in the opposite directions. The displacement of one surface at any particular instant would be due to the sum of the effects of all the waves, both original and reflected, reaching that surface at that instant. Also due to diffusion, the temperature profile through the thickness of the material will change with time. This will modify the different characteristics of the waves generated at each point of the specimen as time proceeds. This phenomenon also affects the displacement of any surface with time. A transducer placed at one surface of the
specimen (Figure 1) measures the movement of that surface. The transducer is maintained axial with respect to the laser beam, i.e. at the epicenter with respect to the acoustic source. Instead of the transducer, any other device (e.g. laser beam) may also be used for measuring the displacement of that surface.

Figure 1. Schematic of the apparatus.

Figure 2. (a) Temperature distribution and (b) wave generation along the depth of the specimen due to sudden temperature rise at point P.
Many experiments like the one described above have been reported previously [1,6]. Typically in most of the cases Q-switched laser pulses were used. Peak pulses were of the order of $10^7$–$10^8$ watts and power densities were less than $10^7$ watts/cm$^2$ to avoid melting or damage of the specimen surface. The total energy carried by each pulse was about 60 mj. The duration of laser pulses were several tens of nano-seconds. The resulting characteristic displacement obtained in many of these experiments is shown in Figure 3, which is taken from [6].

Figure 3. Experimentally obtained displacement variation with time.
CHAPTER 2

FORMULATION OF THE PROBLEM

In this chapter several assumptions regarding heat conduction and wave propagation through the specimen, appropriate to the experimental set up, are made and the governing differential equations together with the corresponding boundary conditions, representing an idealization of the real physical phenomenon, are established. In formulating the problem it must be noted that interest focuses on knowing the temperature distribution within the material and the displacement of one surface of the specimen (shown in Figure 1 as surface B) for only a very short period of time, i.e. the time required for the generated elastic wave to travel not more than three times the depth of the specimen. This is because after this time, there would be repetition of result as the original waves would keep bouncing back and forth from both the surfaces and as time progresses, the new waves produced would have negligibly small amplitude to contribute to the net displacement of the specimen surface, due to a slow drop in temperature with respect to time throughout the specimen after some time. For example, longitudinal wave speed through copper is \(4.66 \times 10^5\) cm/sec. In this case, specimen depth is approximately 2.5 cm. so that the author is interested in the time it takes for the wave to travel 7.5 cm., which can be calculated to be \(1.6 \times 10^4\) ns. The assumptions made in formulating the problem are as follows:

i) The laser-pulse strikes over a large area of the specimen compared to the specimen depth through which heat flow occurs in this short time of observation, which is of the order of \(10^4\) ns. The depth of penetration of heat through a
material in time $t$ is given approximately by the equation:

$$D = \sqrt{4kt}$$

where $D$ is the depth through which heat flows, $k$ is thermal diffusivity, and $t$ is time of interest. For example, in copper (which has a diffusivity of 1.1234 cm$^2$/sec.), heat penetrates up to a depth of .0085 cm. in the time of interest which was calculated before to be $1.6 \times 10^4$ ns. for copper. From reference [1] it can be said that for a Q-switched laser pulse a minimum spot diameter of 0.05 cm. is expected, so that the area affected would be much wider than deep. Therefore heat flow through the target can be treated as one-dimensional, from which it is concluded that there is no variation of temperature in the direction perpendicular to laser axis.

![Figure 4. Assumed uniform and non-uniform spatial distribution of laser pulse.](image-url)
ii) The laser is assumed to be of uniform power density in the transverse direction. This assumption is not good because an actual laser shows considerable spatial non-uniformity (Figure 4). One can idealize the variation of laser intensity in the transverse direction as a Gaussian profile. However this assumption will make the problem two-dimensional. For this case the one-dimensional model would give results of the same order.

Using these two assumptions and the fact that the generated elastic wave is plane, one non-vanishing displacement component in the direction of laser propagation is left. Therefore it can be said that the problem is one-dimensional. Hence, all components of stress and strain tensor, apart from the normal component acting on the plane, perpendicular to laser direction, vanish.

iii) The material is isotropic and homogeneous.

iv) In order to remain in the realm of linear-thermoelasticity, it is assumed that the increment of the material temperature as compared to the reference temperature of the material is small [8, page 99].

v) The material properties do not change with temperature. While this is not strictly true, most changes in the properties of metal tend to be small over fairly wide temperature ranges. For a more complete treatment the temperature variation of the thermal properties must be taken into account.

vi) The thermoelastic coupling between the heat-conduction and the displacement equation for thermoelasticity is ignored by suppressing the term of mechanical origin in the heat-conduction equation. This can be done a for short time of observation [8, page 138], as is the case here.

With these assumptions the governing equations of heat-conduction and displacement can be written as follows:

\[
\frac{\partial^2 \theta}{\partial x^2} - \frac{1}{k} \frac{\partial \theta}{\partial t} = -\frac{A}{K}
\]  

(2.2)
and
\[ C_L^2 \frac{\partial^2 u}{\partial z^2} - \frac{E \alpha}{(1 - 2\nu)\rho} \frac{\partial \theta}{\partial z} = \frac{\partial^2 u}{\partial t^2} \]  

(2.3)

where \( \theta \) is temperature difference with the reference temperature, \( z \) is the coordinate representing depth of the specimen (see Figure 2a), \( t \) is time, \( k \) is thermal diffusivity of the material, \( K \) is thermal conductivity of the material, \( A \) is heat generated per unit time per unit volume of the specimen (it is a function of both time and depth of the specimen), \( C_L \) is longitudinal wave speed through the material, \( u \) is non-vanishing displacement component in the \( z \) direction, \( E \) is Young’s modulus of the material, \( \alpha \) is the coefficient of linear thermal expansion of the material, \( \nu \) is Poisson’s ratio, and \( \rho \) is mass density.

In order to specify the form of \( A \) in Equation (2.2) it is assumed [9, page 72] that: i) the amount of laser energy absorbed by the material (i.e. converted into heat) decreases exponentially along the depth of the specimen; ii) the duration of the laser-pulse shape is infinitely small compared to the time of observation i.e. the time taken by the generated thermoelastic wave to traverse three times the depth of the specimen. Thus the temporal pulse shape of the laser can be represented by a Dirac-delta function. Therefore

\[ A(z, t) \approx b \delta(t) \exp(-bz) \]  

(2.4)

where \( b \) is the absorption coefficient of the material and \( \delta(t) \) is the Dirac-delta function. The factor \( b \) comes in the exponential argument because it determines the amount of heat absorption by the specimen along its depth as laser energy passes through it. After integrating the expression for \( A \), with respect to time and depth, between the limits 0 and \( \infty \), the total energy absorbed by the specimen should be obtained. In this case, the total energy absorbed is assumed to be unity. Therefore to get unity after integration of the expression for \( A \), the expression must
be multiplied by the factor $b$. Thus it can be said that in this case $b$ acts as a normalization factor.

Now the initial and boundary conditions are set, after making appropriate assumptions, of Equations (2.2) and (2.3).

Initial condition of heat conduction equation:

There is only one initial condition for Equation (2.2). It is assumed that initially the specimen is at the reference temperature. Therefore the initial condition of the heat-conduction equation is given by

$$\theta(z,0) = 0 \quad (2.5)$$

Boundary conditions of heat conduction equation:

In order to formulate the two boundary conditions of Equation (2.2), the physics of the problem must be considered. For short laser pulses heat is confined to a small area and heat losses from the area are generally small compared to the radiation flux incident on the area. While laser flux densities of interest are of the order of $10^6$ watts/cm$^2$ or greater, even at elevated temperature, thermal radiation amounts to be of the order of $10^3$ watts/cm$^2$ for solid materials. Moreover, the author is interested in a small interval of time. Therefore during that time the total loss of heat from the material surface will be negligibly small. In that case the external surface at which the laser strikes the specimen can be treated as insulated and

$$\frac{\partial \theta}{\partial z(z=0)} = 0 \quad (2.6)$$

For a majority of materials, thermal diffusivity is quite low and the rate at which heat is propagated through the solid is considerably less than the rate at which the elastic wave travels. For example, in the case of copper, as shown before, during the time the elastic wave travels three times the depth of the
specimen (which is calculated to be $1.6 \times 10^{-4}$ ns.), heat penetrates up to a depth of .0085 cm., which is negligibly small compared to the 2.5 cm. depth of the specimen. Thus in the time of observation, the surface which is opposite to the one on which the laser beam is incident will have no information about the heat that propagates through it. Therefore the specimen can be considered a half-space with respect to heat conduction. The limitation of this assumption is the neglect of heat conduction from the region which is very near to the transducer. But in case of metal, absorptivity being very high, almost all the laser energy is absorbed very near to the surface on which the laser beam is incident. As such the assumption will introduce negligible error. Moreover, as the depth in this half-space goes to infinity, the temperature should be bounded, i.e.

$$\lim_{x \to \infty} \theta(z,t) \text{ is bounded} \quad (2.7)$$

In solving the wave equation, the finite depth of the specimen is taken into consideration.

**Initial conditions of wave equation:**

As the wave equation (2.3) is second order in time, there will be two initial conditions. It is assumed that there is no displacement of the specimen initially. This gives

$$u(z,0) = 0 \quad (2.8)$$

The material is initially at rest, that is, the velocity of any material particle is zero at the initial instant. Therefore

$$\frac{\partial u}{\partial t} \big|_{t=0} = 0 \quad (2.9)$$
Boundary conditions of wave equation:

The boundary conditions for the wave equation are derived from the fact that two boundaries are stress free. Hence

\[ \sigma_{zz}(0, t) = 0 \quad (2.10) \]
and

\[ \sigma_{zz}(D, t) = 0 \quad (2.11) \]

where \( D \) is depth of the specimen and \( \sigma_{zz} \) is non-vanishing stress-component.

Also, from the Duhamel-Neumann equation for isotropic material and one-dimensional case,

\[ \sigma_{zz} = (2\mu + \lambda)\epsilon_{zz} - \frac{E\alpha}{(1 - 2\nu)} \theta \quad (2.12) \]

where

\[ \mu = \frac{E}{2(1 + \nu)} \]
\[ \lambda = \frac{E\nu}{(1 + \nu)(1 - 2\nu)} \]

\( \epsilon_{zz} \) is non-vanishing strain component and \( \mu \) and \( \lambda \) are the well known Lame constants. Applying Equation (2.12) to the first boundary condition (Equation (2.10)),

\[ \epsilon_{zz}(z=0) = \frac{E\alpha}{(1 - 2\nu)(2\mu + \lambda)} \theta_{(z=0)} \]
\[ = \alpha(1 + \nu) \quad (2.13) \]

Therefore, since

\[ \epsilon_{zz} = \frac{\partial u}{\partial z} \quad (2.14) \]

the result is

\[ \frac{\partial u}{\partial z}(z=0) = \frac{\alpha(1 + \nu)}{(1 - \nu)} \theta_{(z=0)} \quad (2.15) \]
Similarly, from the second boundary condition (Equation (2.11)),

\[
\frac{\partial u}{\partial z_{(z=D)}} = \frac{\alpha(1 + \nu)}{(1 - \nu)} \theta_{(z=D)}
\]

(2.16)

This completes the formulation of the problem.
CHAPTER 3

SOLUTION OF THE EQUATIONS

In Chapter 2 the problem was formulated. Now both the heat conduction and wave-propagation equations (Equations (2.2) and (2.3) respectively) can be solved. The Laplace transform method is used in solving these equations.

Solution of Heat-Conduction Equation

Taking Laplace transform of Equation (2.2) and applying the initial condition (Equation (2.5)),

\[ \frac{\partial^2 \tilde{\theta}}{\partial z^2} - \frac{s}{k} \tilde{\theta} = -\frac{b}{K} \exp(-bz) \]  

where

\[ \tilde{\theta} = \tilde{\theta}(z, s) = \int_0^\infty \theta(z, t)e^{-st} dt \]

The complete solution of Equation (3.1) is given by

\[ \tilde{\theta}(z, s) = c_1(s) \exp\left(\sqrt{\frac{s}{k}} z\right) + c_2(s) \exp\left(-\sqrt{\frac{s}{k}} z\right) - \frac{b \exp(-bz)}{\rho c(kb^2 - s)} \]  

Applying the boundary condition given by Equation (2.7) to Equation (3.2),

\[ c_1(s) = 0 \]  

Applying the boundary condition given by Equation (2.6) to Equation (3.2),

\[ c_2(s) = \frac{b^2 \sqrt{k}}{\rho c(kb^2 - s)\sqrt{s}} \]  

Therefore the solution of Equation (3.1) is given by

\[ \tilde{\theta}(z, s) = \frac{b}{\rho c(kb^2 - s)} \left\{ \frac{\sqrt{k} \exp\left(-\sqrt{\frac{s}{k}} z\right)}{\sqrt{s}} - \exp(-bz) \right\} \]
and after differentiation,

$$\frac{\partial \theta(z,s)}{\partial z} = -\frac{b^2}{\rho c(kb^2 - s)} \left\{ \exp \left(-\sqrt{\frac{s}{k}} z \right) - \exp(-bz) \right\} \quad (3.6)$$

Inverse Laplace Transform:

In order to take the inverse Laplace transform of Equation (3.5) it is rearranged, using partial fractions as follows:

$$\tilde{\theta}(z,s) = -\frac{b}{2\rho c} \left\{ \frac{\exp \left(-\sqrt{\frac{s}{k}} z \right)}{\sqrt{s}(\sqrt{s} - \sqrt{k} b)} - \frac{\exp \left(-\sqrt{\frac{s}{k}} z \right)}{\sqrt{s}(\sqrt{s} + \sqrt{k} b)} + \frac{b \exp(-bz)}{\rho c(s - kb^2)} \right\} \quad (3.7)$$

The inverses of these transforms can be found in [10]. After taking inverse Laplace transform of Equation (3.7) and after simplifying,

$$\theta(z,t) = \frac{b}{2\rho c} \exp(b^2 kt) \left\{ \exp(bz)erfc \left( b\sqrt{kt} + \frac{z}{2\sqrt{kt}} \right) \right. \\
\left. + \exp(-bz)erfc \left( b\sqrt{kt} - \frac{z}{2\sqrt{kt}} \right) \right\} \quad (3.8)$$

This is the complete solution of the heat-conduction equation and it gives the distribution of temperature along the depth of the specimen for a laser-pulse which has the form of Dirac-delta function with respect to time. This solution is valid for any material with both thermal-conductivity and diffusivity, i.e. this solution is of the most general type as far as material properties are concerned. Now this solution can be used as a Green's function to calculate the temperature distribution due to a laser of any arbitrary temporal pulse shape using the following expression:

$$T(z,t) = \int_0^\infty f(\tau)\theta(z,t - \tau)d\tau$$

where $f(t)$ is the temporal pulse shape of the laser and $\theta(z,t - \tau)$ is obtained from Equation (3.8).
Solution of Wave Equation

Taking the Laplace transform of the differential equation (2.3) and applying both the initial conditions (Equations (2.8) and (2.9)), after simplification using Equation (3.6),

\[
\frac{\partial^2 \bar{u}(z, s)}{\partial z^2} - \frac{s^2}{C_L^2} \bar{u}(z, s) = -E \alpha b^2 \left\{ \exp \left( -\frac{z}{k} \right) - \exp \left( -bz \right) \right\} \frac{(kb^2 - s)}{(kb^2 - s)}
\]

(3.9)

Again

\[
C_L^2 = \frac{\lambda + 2\mu}{\rho} = \frac{E(1 - \nu)}{\rho(1 + \nu)(1 - 2\nu)}
\]

Substituting this value of $C_L$ in Equation (3.9),

\[
\frac{\partial^2 \bar{u}(z, s)}{\partial z^2} - \frac{s^2}{C_L^2} \bar{u}(z, s) = -\frac{\alpha b^2 (1 + \nu)}{\rho c(1 - \nu)} \left\{ \exp \left( -\frac{z}{k} \right) - \exp \left( -bz \right) \right\} \frac{(kb^2 - s)}{(kb^2 - s)}
\]

(3.10)

The complete solution to Equation (3.10) is given by:

\[
\bar{u}(z, s) = A_1(s) \cosh \left( \frac{s}{C_L} z \right) + A_2(s) \sinh \left( \frac{s}{C_L} z \right)
\]

\[-\frac{\alpha b^2 (1 + \nu)}{\rho c(1 - \nu)} \left\{ \exp \left( -\frac{z}{k} \right) - \exp \left( -bz \right) \right\} \frac{(kb^2 - s)}{(kb^2 - s)}
\]

(3.11)

where the particular solution is readily obtained using the method of undetermined coefficient. Now the two boundary conditions are applied to determine the two constants $A_1(s)$ and $A_2(s)$ in Equation (3.11). Applying boundary condition given
by Equation (2.15),

\[ A_2(s) = \frac{\alpha(1 + \nu)C_L}{(1 - \nu)s} \left[ \frac{b^2}{\rho c(kb^2 - s)} \left\{ \frac{b}{\left( b^2 - \frac{s^2}{C_L^2} \right)} \right. \right. \]

\[ - \left. \left. \left. \frac{\sqrt{s}}{\sqrt{k} \left( \frac{s}{k} - \frac{s^2}{C_L^2} \right)} \right\} + \theta(0, s) \right] \]  

(3.12)

Similarly applying boundary condition given by Equation (2.16),

\[ A_1(s) = \frac{\alpha b^2(1 + \nu)C_L}{\rho c(1 - \nu)s(\rho c(kb^2 - s))} \left\{ -\frac{\sqrt{s} \exp \left( -\sqrt{\frac{s}{k}} D \right)}{\sqrt{k} \left( \frac{s}{k} - \frac{s^2}{C_L^2} \right)} \right. \]

\[ + \frac{b \exp(-bD)}{\left( b^2 - \frac{s^2}{C_L^2} \right)} \left\{ \frac{1}{\sinh \left( \frac{s}{C_L} D \right)} - \frac{A_2(s) \cosh \left( \frac{s}{C_L} D \right)}{\sinh \left( \frac{s}{C_L} D \right)} \right. \]

\[ + \frac{C_L \alpha(1 + \nu)\theta(D, s)}{(1 - \nu)s \sinh \left( \frac{s}{C_L} D \right)} \] 

(3.13)

Substituting in the two constants, after simplification and rearranging terms Equation (3.11) can be written as follows:

\[ u(z, s) = W_1(z, s) + W_2(z, s) + W_3(z, s) + W_4(z, s) + W_5(z, s) + W_6(z, s) \]  

(3.14)

where

\[ W_1(z, s) = -C_L \delta T^0 b(U_1(s)) \cosh \left( \frac{z}{C_L} s \right) \frac{\cosh \left( \frac{D - z}{C_L} s \right)}{\sinh \left( \frac{D}{C_L} s \right)} \]  

(3.15)

\[ W_2(z, s) = -C_L \delta (U_2(s)) \cosh \left( \frac{D - z}{C_L} s \right) \frac{\cosh \left( \frac{-z}{C_L} s \right)}{\sinh \left( \frac{D}{C_L} s \right)} \]  

(3.16)

\[ W_3(z, s) = -C_L \delta T^0 b^2 (U_3(s)) \cosh \left( \frac{D - z}{C_L} s \right) \frac{\cosh \left( \frac{-z}{C_L} s \right)}{\sinh \left( \frac{D}{C_L} s \right)} \]  

(3.17)
\[ W_4(z, s) = \frac{C_L \delta T_0 b}{\sqrt{k}} (U_4(s)) \cosh \left( \frac{p - z}{\frac{D}{c_L}} s \right) \frac{\sinh \left( \frac{p}{\frac{D}{c_L}} s \right)}{s}, \] (3.18)

\[ W_5(z, s) = -\delta T_0 b(U_5(z, s)), \] (3.19)

\[ W_6(z, s) = C_L \delta (U_6(s)) \cosh \left( \frac{p}{\frac{D}{c_L}} s \right) \frac{\sinh \left( \frac{p}{\frac{D}{c_L}} s \right)}{s}, \] (3.20)

and

\[ U_1(s) = \frac{\exp \left( -\frac{D \sqrt{z}}{\sqrt{k}} \right)}{\sqrt{k} \Delta (kb^2 - s) \left( \frac{1}{\sqrt{k}} - \frac{s}{\frac{D}{c_L}} \right)} - \frac{b \exp(-bD)}{s(kb^2 - s) \left( b^2 - \frac{s^2}{c_L^2} \right)}, \] (3.21)

\[ U_2(s) = \frac{\theta(0, s)}{s}, \] (3.22)

\[ U_3(s) = \frac{1}{s(kb^2 - s) \left( b^2 - \frac{s^2}{c_L^2} \right)}, \] (3.23)

\[ U_4(s) = \frac{1}{\sqrt{s} \Delta (kb^2 - s) \left( \frac{1}{k} - \frac{s}{c_L^2} \right)}, \] (3.24)

\[ U_5(z, s) = \frac{\exp \left( -\sqrt{\frac{z}{k}} \Delta \right)}{(kb^2 - s) \left( \frac{1}{k} - \frac{s}{c_L^2} \right)} - \frac{\exp(-b\Delta)}{(kb^2 - s) \left( b^2 - \frac{s^2}{c_L^2} \right)}. \] (3.25)
\[ U_6(s) = \frac{\theta(D, s)}{s}, \quad (3.26) \]

\[ \delta = \frac{\alpha(1 + \nu)}{(1 - \nu)} \quad (3.27) \]

and

\[ T_0 = \frac{b}{\rho c} \quad (3.28) \]

Now there is a solution of the wave-equation in the transform space. To get the solution in the z-t space, Equation (3.14) must be inverted.

**Inverse Laplace Transform:**

Inversion of the wave equation requires the inversion of the terms \( U_1(s), U_2(s), U_3(s), U_4(s), U_5(z, s), U_6(s) \) (Equations (3.21)-(3.26)) together with the appropriate hyperbolic coefficients accompanying them in the expressions for the \( W_i(z, s), i = 1 - 6 \) (Equations (3.15)-(3.20)). First consider the effects of the hyperbolic coefficients on the inverse Laplace transform of Equations (3.15) to (3.20). There are two types of hyperbolic terms in the Laplace transforms of the wave equation. From Equations (3.15) and (3.20) the following hyperbolic coefficient is obtained after expansion and using binomial theorem:
\[
\frac{\cosh \left( \frac{s}{C_L} s \right)}{\sinh \left( \frac{D}{C_L} s \right)} = \left\{ \exp \left( \frac{z}{C_L} s \right) + \exp \left( -\frac{z}{C_L} s \right) \right\} \left\{ \exp \left( \frac{D}{C_L} s \right) - \exp \left( -\frac{D}{C_L} s \right) \right\}^{-1}
\]

\[
= \left\{ \exp \left( \frac{z}{C_L} s \right) + \exp \left( -\frac{z}{C_L} s \right) \right\} \left\{ \exp \left( \frac{D}{C_L} s \right) + \exp \left( -\frac{D}{C_L} s \right) \right\}
\]

\[
+ \exp \left( -\frac{3D}{C_L} s \right) + \exp \left( -\frac{5D}{C_L} s \right) + \cdots
\]

\[
= \exp \left( \frac{z - D}{C_L} s \right) \left\{ 1 + \exp \left( -\frac{2D}{C_L} s \right) + \exp \left( -\frac{4D}{C_L} s \right) + \cdots \right\}
\]

Based on the shifting property of \(\exp(-sa)\) the terms inside the first bracket in the above equation represent waves originating at the back surface \((z = D)\) at different times and travelling towards the front surface. For example, the first term represents a wave originating at the back surface at time \(t = 0\), the second term represents a wave originating at the back surface at time \(t = 2D/C_L\) and the third term represents a wave originating at the same surface at time \(t = 4D/C_L\). The terms inside the second bracket in Equation (3.29) represent waves originating at the front surface \((z = 0)\) at different times and travelling towards the back surface. For example, the first term inside the second bracket represents a wave originating at the front surface at time \(t = D/C_L\) and going towards the back surface. Similarly the second and the third term inside the second bracket represent waves originating at the front surface at time \(t = 3D/C_L\) and \(t = 5D/C_L\) respectively and propagating towards the back surface. Actually the
first term inside the second bracket represents the reflection from the front surface of the wave originated at the back surface at time \( t = 0 \). Similarly, the second term inside the first bracket represents the same wave after it reaches the back surface and is reflected toward the front surface at \( t = 2D/C_L \). Therefore it can be said that different waves are reflections of a single original wave at the two surfaces of the specimen. From the hyperbolic term common to Equation (3.16), (3.17) and (3.18),

\[
\frac{\cosh \left( \frac{D-x}{C_L} s \right)}{\sinh \left( \frac{D}{C_L} s \right)} = \exp \left( -\frac{z}{C_L} s \right) \left\{ 1 + \exp \left( -\frac{2D}{C_L} s \right) + \exp \left( -\frac{4D}{C_L} s \right) + \cdots \right\} \\
+ \exp \left( \frac{z-D}{C_L} s \right) \left\{ \exp \left( -\frac{D}{C_L} s \right) + \exp \left( -\frac{3D}{C_L} s \right) + \exp \left( -\frac{5D}{C_L} s \right) + \cdots \right\} .
\]

(3.30)

In this case the first, the second and third terms inside the first bracket represent waves originating at the front surface at times \( t = 0, t = 2D/C_L, t = 4D/C_L \) respectively and going towards the back surface. Similarly the first, second and third terms inside the second bracket represent waves originating at the back surface at time \( t = D/C_L, t = 3D/C_L \) and \( t = 5D/C_L \) respectively and propagating towards the front surface. Here also the different waves are reflections of the wave originated at time \( t = 0 \) at the front surface of the specimen.

Therefore the effect of the hyperbolic terms on the inverse Laplace transform of the \( W_i(z, s), i = 1 - 6 \) (Equations (3.15)–(3.20)) is to shift the time in each term of the inverse Laplace transform of the \( W_i(z, s) \) by subtracting from it the arguments of the corresponding exponential term which multiply it and then multiplying each of the terms by the Heaviside's function having the shifted time as
its argument. Hence the inverse Laplace transform of $U_1(s)$, $U_2(s)$, $U_3(s)$, $U_4(s)$, $U_5(z,s)$ and $U_6(s)$ only must be found.

As the time of observation is the time taken by the elastic wave to travel three times the depth of the specimen (i.e. $t < 3D/C_L$), focus is restricted to only six expansion terms in Equation (3.29) and Equation (3.30). These terms are as follows:

$$\exp\left(-\frac{z}{C_L} s\right), \exp\left(\frac{z-2D}{C_L} s\right), \exp\left(\frac{z+2D}{C_L} s\right),$$

$$\exp\left(\frac{z-D}{C_L} s\right), \exp\left(-\frac{z+D}{C_L} s\right) \text{ and } \exp\left(\frac{z-3D}{C_L} s\right).$$

For computational ease the $W_i(z,s)$ ($i = 1 - 6$) are again rearranged. The following new functions are defined:

$$F_1(s) = \frac{1}{s(s-b^2 k)\left(s-\sqrt{\frac{k}{b}}\right)}, \quad (3.31)$$

$$F_2(s) = \frac{1}{s(s-b^2 k)} \quad (3.32)$$

and

$$F_3(s) = \frac{1}{(s-b^2 k)(s^2-2C_L^2)} = \frac{1}{(s-b^2 k)(s+bC_L)(s-bC_L)} \quad (3.33)$$

Therefore the $U_i(z,s)$ ($i = 1 - 6$) can be rewritten as follows:

$$U_1(s) = C_L^2 \left\{ \frac{\exp\left(-\frac{z}{k}\sqrt{D}\right) F_1(s)}{\sqrt{k}s} - \frac{\exp(-bD)bF_3(s)}{s} \right\}, \quad (3.34)$$

$$U_2(s) = -T_0 \left\{ \frac{b\sqrt{k}}{s^{3/2}(s-kb^2)} - \frac{1}{s(s-kb^2)} \right\}, \quad (3.35)$$

$$U_3(s) = \frac{C_L^2 F_3(s)}{s} \quad (3.36)$$
\[ U_4(s) = \frac{C_L^2 F_1(s)}{\sqrt{s}} \quad , \quad (3.37) \]

\[ U_6(z,s) = C_L^2 \left\{ \exp \left( -\sqrt{\frac{s}{k}} z \right) F_1(s) - \exp(-bz) F_3(s) \right\} \quad (3.38) \]

and

\[ U_6(s) = -T_0 \left\{ \frac{\sqrt{k}}{s^{3/2}} \frac{b \exp \left( -\sqrt{\frac{F}{k}} D \right) \exp(-bD)}{s^{3/2}(s - kb^2)} \right\} \quad (3.39) \]

Now \( F_1, F_2, F_3 \) can be rewritten using partial fractions as follows:

\[ F_1(s) = \frac{1}{b^2 C_L^2} \left\{ \frac{1}{s} + \frac{1}{(\eta^2 - 1)(s - kb^2)} - \frac{\eta^2}{(\eta^2 - 1)\left( s - \frac{C_L^2}{k} \right)} \right\} \quad , \quad (3.40) \]

\[ F_2(s) = \frac{1}{b^2 k} \left\{ -\frac{1}{s} + \frac{1}{s - b^2 k} \right\} \quad , \quad (3.41) \]

\[ F_3(s) = \frac{1}{b^2 C_L^2(\eta^2 - 1)} \left\{ \frac{1}{s - b^2 k} - \frac{\eta b C_L + s}{s^2 - b^2 C_L^2} \right\} \quad (3.42) \]

and

\[ \frac{F_3(s)}{s} = \frac{1}{b^4 k C_L^2} \left\{ \frac{1}{s} + \frac{1}{(\eta^2 - 1)(s - b^2 k)} - \frac{\eta(\eta s + b C_L)}{(\eta^2 - 1)(s^2 - b^2 C_L^2)} \right\} \quad (3.43) \]

where

\[ \eta = \frac{bk}{C_L} \quad . \quad (3.44) \]

From now on:

\[ \frac{z}{\sqrt{4kt}} = X1 \quad , \quad (3.45) \]

\[ \frac{D}{\sqrt{4kt}} = Y1 \quad , \quad (3.46) \]

\[ b^2 kt = X2 \quad (3.47) \]
and

\[ \sqrt{\frac{C_L^2 t}{k}} = X3 \quad (3.48) \]

From [11] and [12], the inverse transforms of the following terms are obtained:

\[
L^{-1}\left\{ \frac{F_1(s)}{\sqrt{s}} \right\} = \frac{1}{b^2 C_L^2} \left\{ 2\sqrt{\frac{t}{\pi}} + \frac{\exp(X2)\text{erf}(\sqrt{X2})}{(\eta^2 - 1)b\sqrt{k}} \right. \\
- \left. \frac{\eta^2 \sqrt{k} \exp(X3^2)\text{erf}(X3)}{(\eta^2 - 1)C_L} \right\}, \tag{3.49}
\]

\[
L^{-1}\left\{ F_1(s) \exp\left( -\sqrt{\frac{s}{k}} z \right) \right\} = \frac{1}{b^2 C_L^2} \left\{ \text{erf}(X1) \\
+ \frac{1}{2(\eta^2 - 1)} \left[ \exp(-bz + X2) \right. \\
\left. \text{erfc}(-\sqrt{X2} + X1) \right] \\
+ \exp(bz + X2)\text{erfc}(\sqrt{X2} + X1) \right. \\
- \left. \frac{\eta^2}{2(\eta^2 - 1)} \left[ \exp \left( -\frac{zC_L}{k} + X3^2 \right) \text{erf}(-X3 + X1) \right. \\
\left. \exp \left( \frac{C_L z}{k} + X3^2 \right) \text{erf}(X3 + X1) \right] \right\}, \tag{3.50}
\]
\[
L^{-1} \left\{ \frac{F_1(s) \exp \left( -\sqrt{\frac{D}{k}} \right)}{\sqrt{s}} \right\} = \frac{1}{b^2 C_L^2} \left\{ \frac{2}{\pi} \exp(-Y^2) \right\}
- \frac{D}{\sqrt{k}} \text{erfc}(Y1)
+ \frac{1}{(\eta^2 - 1)2b\sqrt{k}} \left[ \exp(-bD + X2) \right]
\text{erfc}(-\sqrt{X2} + Y1)
- \exp(bD + X2) \text{erfc}(\sqrt{X2} + Y1)
- \frac{\eta^2 \sqrt{k}}{(\eta^2 - 1)2C_L} \left[ \exp \left( -\frac{C_L D}{k} + X3^2 \right) \right]
\text{erfc}(-X3 + Y1)
- \exp \left( \frac{C_L D}{k} + X3^2 \right) \text{erfc}(X3 + Y1) \right\} \tag{3.51}
\]

\[
L^{-1} \left\{ F_3(s) \right\} = \frac{1}{b^2 C_L^2} \frac{1}{(\eta^2 - 1)} \left\{ \exp(X2) \right\}
- \eta \sinh(bC_L t) - \cosh(bC_L t) \right\} \tag{3.52}
\]

\[
L^{-1} \left\{ \frac{F_3(s)}{s} \right\} = \frac{1}{kb^4 C_L^2} \left\{ 1 + \frac{\exp(X2)}{\eta^2 - 1} - \frac{\eta^2}{\eta^2 - 1} \cosh(bC_L t) \right\}
- \frac{\eta}{\eta^2 - 1} \sinh(bC_L t) \right\} \tag{3.53}
\]
\[ L^{-1}\{U_2(s)\} = T_0 \left\{ -b\sqrt{k} \left[ (kb^2)^{-3/2} \exp(kb^2 t) \text{erf}(b\sqrt{k}t) - \frac{2\sqrt{t}}{kb^2\sqrt{\pi}} \right] \right. \\
\left. - \frac{1 - \exp(kb^2 t)}{kb^2} \right\} \] (3.54)

and

\[ L^{-1}\{U_0(s)\} = -T_0 \left\{ \frac{1}{2} \left[ -4\sqrt{t} \exp\left(-\frac{D^2}{4kt}\right) \right. \right. \right. \notag \\\n\left. \left. \left. + \frac{2D \text{erf}\left(\frac{D}{2\sqrt{kt}}\right)}{kb} \right] \right. \right. \right. \notag \\\n\left. \left. \left. + \frac{1}{kb^2} \left\{ \exp(-Db + kb^2 t) \text{erf}\left(\frac{D}{2\sqrt{kt}} - b\sqrt{kt}\right) \right. \right. \right. \notag \\\n\left. \left. \left. - \exp(Db + kb^2 t) \text{erf}\left(\frac{D}{2\sqrt{kt}} + b\sqrt{kt}\right) \right. \right. \right. \notag \\\n\left. \left. \left. + \exp(-Db)(1 - \exp(kb^2 t)) \right. \right. \right. \notag \\\n\left. \left. \left. \right\} \right. \right. \right. \right. \right. \right. \right. \notag \\\n\left. \left. \left. \right\} \right. \right. \right. \right. \right. \right. \right. \notag \\\n\left. \left. \left. \right\} \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right.
and

\[ A_3(z,t) = -L^{-1}\{U_6(z,s)\} \]  \hspace{1cm} (3.58)  

where all the terms on the right hand side of the expressions are known. After simplification,

\begin{align*}
A_1(t) &= \frac{T_0 k}{2(k^2 b^2 - C_L^2)} \left\{ \exp(-Db + b^2 kt) \text{erfc}\left(b\sqrt{kt} - \frac{D}{2\sqrt{kt}}\right) \\
+ \exp\left(Db + b^2 kt\right) \text{erfc}\left(b\sqrt{kt} + \frac{D}{2\sqrt{kt}}\right) \right\} \\
+ \frac{T_0 b k^2}{2C_L} \left\{ \exp\left(\frac{C_L^2 t}{k} - \frac{D C_L}{k}\right) \text{erfc}\left(\frac{D}{2\sqrt{kt}} + \frac{C_L \sqrt{t}}{k}\right) \\
- \exp\left(\frac{C_L^2 t}{k} + \frac{D C_L}{k}\right) \text{erfc}\left(\frac{D}{2\sqrt{kt}} - \frac{C_L \sqrt{t}}{k}\right) \right\} \\
- \frac{T_0 \exp(-bD)}{b(k^2 b^2 - C_L^2)} \{ C_L \sinh(bC_L t) + kb \cosh(C_L bt) \} ,
\end{align*}

\begin{align*}
A_2(t) &= -\frac{\exp(bk^2 t) \text{erfc}(b\sqrt{kt}) k}{(k^2 b^2 - C_L^2) C_L (k^2 b^2 - C_L^2)} - \frac{bk^2 \exp\left(\frac{C_L^2 t}{k}\right) \text{erfc}\left(\frac{C_L \sqrt{t}}{k}\right)}{C_L (k^2 b^2 - C_L^2)} \\
+ \frac{1}{b(k^2 b^2 - C_L^2)} \{ kb \cosh(bC_L t) + C_L \sinh(bC_L t) \} ,
\end{align*}

\hspace{1cm} (3.60)
\[ A_3(z,t) = -erf\left(\frac{z}{2\sqrt{kt}}\right) \frac{1}{b^2} + \frac{C_L^2}{2b^2(k^2b^2 - C_L^2)} \left\{ \exp(-bz + b^2 kt) \right. \]

\[ \left. \quad \text{erfc}\left(\frac{b\sqrt{kt} - \frac{z}{2\sqrt{kt}}}{2}\right) \right\} \]

\[ - \exp(bz + b^2 kt) \text{erfc}\left(\frac{b\sqrt{kt} + \frac{z}{2\sqrt{kt}}}{2}\right) \}

\[ + \frac{k^2}{2(k^2b^2 - C_L^2)} \left\{ \exp\left(-\frac{C_L z}{k} + \frac{C_L^2 t}{k}\right) \text{erfc}\left(\frac{z}{2\sqrt{kt}} - \frac{C_L \sqrt{t}}{\sqrt{k}}\right) \right. \]

\[ + \exp\left(\frac{C_L z}{k} + \frac{C_L^2 t}{k}\right) \text{erfc}\left(\frac{z}{2\sqrt{kt}} + \frac{C_L \sqrt{t}}{\sqrt{k}}\right) \}

\[ - \frac{\exp(-bz)C_L}{b^2(k^2b^2 - C_L^2)} \left\{ C_L \cosh(C_L bt) + kb \sinh(C_L bt) \right\} . \]

Therefore the complete solution to the wave equation for the time of interest can be written as follows:

\[ u(z,t) = C_L \delta A_1 \left( z, t + \frac{z - D}{C_L} \right) H \left( t + \frac{z - D}{C_L} \right) \]

\[ + A_1 \left( z, t + \frac{z - 3D}{C_L} \right) H \left( t + \frac{z - 3D}{C_L} \right) \]

\[ + A_1 \left( z, t - \frac{z + D}{C_L} \right) H \left( t - \frac{z + D}{C_L} \right) \]

\[ + C_L \delta T_0 \left[ A_2 \left( z, t - \frac{z}{C_L} \right) H \left( t - \frac{z}{C_L} \right) \right] \]

\[ + A_2 \left( z, t + \frac{z - 2D}{C_L} \right) H \left( t + \frac{z - 2D}{C_L} \right) \]

\[ + A_2 \left( z, t - \frac{z + 2D}{C_L} \right) H \left( t - \frac{z + 2D}{C_L} \right) \]

\[ + \delta T_0 b A_3(z,t) \quad \begin{cases} \text{for} & 0 \leq z \leq D \\ \text{for} & 0 \leq t < \frac{3D}{C_L} \end{cases} \]

\[ \text{for} \quad 0 \leq z \leq D \quad 0 \leq t < \frac{3D}{C_L} \]
Specializing this case for the surface $B (z = D)$ in Figure 1,

$$u(D,t) = C_L \delta \left[ A_1(D,t) + 2A_1 \left( D, t - \frac{2D}{C_L} \right) H \left( t - \frac{2D}{C_L} \right) \right]$$

$$+ C_L \delta T_0 \left[ 2A_2 \left( D, t - \frac{D}{C_L} \right) H \left( t - \frac{D}{C_L} \right) \right]$$

$$+ \delta T_0 b A_3 (D, t) \quad \quad (3.63)$$

In Equations (3.62) and (3.63) the $H$ terms are the Heaviside's function.

In order to generate a computer program to calculate the displacement of the back surface, Equation (3.63) is rearranged. The following terms are defined:

$$R_1 = \frac{C_L \delta b k}{pc(k^2 b^2 - C_L^2)} , \quad \quad (3.64)$$

$$R_2 = \frac{\delta b^2 k^2}{pc(k^2 b^2 - C_L^2)} , \quad \quad (3.65)$$

$$R_3 = \frac{C_L \delta}{pc(k^2 b^2 - C_L^2)} , \quad \quad (3.66)$$

$$t_1 = t - \frac{2D}{C_L} , \quad \quad (3.67)$$

$$t_2 = t - \frac{D}{C_L} , \quad \quad (3.68)$$

$$E_1 = \exp \left( -\frac{D^2}{4kt} \right) , \quad \quad (3.69)$$

$$E_{11} = \exp \left( -\frac{D^2}{4kt_1} \right) , \quad \quad (3.70)$$

$$E_{12} = \exp \left( -\frac{D^2}{4kt_2} \right) , \quad \quad (3.71)$$

$$H_1 = H \left( t - \frac{2D}{C_L} \right) , \quad \quad (3.72)$$

$$H_2 = H \left( t - \frac{D}{C_L} \right) , \quad \quad (3.73)$$

$$x_1 = b\sqrt{kt} - \frac{D}{2\sqrt{kt}} , \quad \quad (3.74)$$
\[ x_2 = \frac{D}{2\sqrt{kt}} - \frac{C_L \sqrt{t}}{\sqrt{k}}, \quad (3.75) \]
\[ x_3 = \frac{C_L \sqrt{t}}{\sqrt{k}} - \frac{D}{2\sqrt{kt}}, \quad (3.76) \]
\[ y_1 = b\sqrt{kt} + \frac{D}{2\sqrt{kt}}, \quad (3.77) \]
\[ y_2 = \frac{D}{2\sqrt{kt}} + \frac{C_L \sqrt{t}}{\sqrt{k}}, \quad (3.78) \]
\[ x_{11} = b\sqrt{kt_1} - \frac{D}{2\sqrt{kt_1}}, \quad (3.79) \]
\[ x_{21} = \frac{D}{2\sqrt{kt_1}} - \frac{C_L \sqrt{t_1}}{\sqrt{k}}, \quad (3.80) \]
\[ y_{11} = b\sqrt{kt_1} + \frac{D}{2\sqrt{kt_1}} \quad (3.81) \]

and
\[ y_{21} = \frac{D}{2\sqrt{kt_1}} + \frac{C_L \sqrt{t_1}}{\sqrt{k}}. \quad (3.82) \]

Substituting these terms in Equation (3.63) and after simplification, the expression for the displacement of the back surface of the specimen can be written as follows:

\[ u(z,t) = M_{11} + M_{12} + M_{21} + M_{31} + M_{32} + MH \quad (3.83) \]

where

\[
M_{11} = \frac{R_1}{2} \left( E_1 \right) \left\{ \exp(x_1^2) \text{erfc}(x_1) + \exp(y_1^2) \text{erfc}(y_1) \right\}
+ R_1 \left( E_{11} \right) \left\{ \exp(x_{11}^2) \text{erfc}(x_{11}) + \exp(y_{11}^2) \text{erfc}(y_{11}) \right\} (H_1)
\]

\[ M_{12} = \begin{cases} 
\frac{R_2}{2} \left( E_1 \right) \left\{ \exp(x_2^2) \text{erfc}(x_2) \\
- \exp(y_2^2) \text{erfc}(y_2) \right\} & \text{(for } t < \frac{D}{\zeta_2}) \\
-\frac{R_2}{2} \left( E_1 \right) \left\{ \exp(x_3^2) \text{erfc}(x_3) + \exp(y_3^2) \text{erfc}(y_3) \right\} \\
+ R_2 \left( E_{11} \right) \left\{ \exp(x_{21}^2) \text{erfc}(x_{21}) \\
- \exp(y_{21}^2) \text{erfc}(y_{21}) \right\} (H_1) & \text{(for } t \geq \frac{D}{\zeta_2}) 
\end{cases} \quad (3.85) \]
\[ M_{21} = -2R_1 \exp(ka^2t_2)e\text{rfc}(b\sqrt{kt_2})(H_2) \]
\[ + 2R_2 \exp(kb^2t_2) \text{erfc} \left( b/\sqrt{kt_2} \right)(H_2), \]  
\[ (3.86) \]
\[ M_{31} = -\frac{\delta}{\rho c} \text{erfc} \left( \frac{D}{2\sqrt{kt}} \right) + \frac{R_3}{2} C_L \{ \exp(x_1^2) \text{erfc}(x_1) \]
\[ - \exp(y_1^2) \text{erfc}(y_1) \} , \]
\[ (3.87) \]
\[ M_{32} = \begin{cases} 
\frac{R_2}{2} \{ \exp(x_2^2) \text{erfc}(x_2) \\
+ \exp(y_2^2) \text{erfc}(y_2) \} & \text{(for } t < \frac{D}{\sigma_c} \text{)} \\
-\frac{R_2}{2} \{ \exp(x_2^2) \text{erfc}(x_3) \\
- \exp(y_2^2) \text{erfc}(y_2) \} & \text{(for } t \geq \frac{D}{\sigma_c} \text{)} 
\end{cases} \]
\[ (3.88) \]
and
\[ MH = \begin{cases} 
-R_3(C_L + kb) \exp(C_L bt_2) & \text{(for } t < \frac{D}{\sigma_c} \text{)} \\
-R_3(C_L - kb) \exp(-C_L bt_2) & \text{(for } \frac{D}{\sigma_c} \leq t < \frac{2D}{\sigma_c} \text{)} \\
-R_3(C_L + kb) \exp \left\{ C_L b \left( t - \frac{3D}{\sigma_c} \right) \right\} & \text{(for } \frac{2D}{\sigma_c} \leq t < \frac{3D}{\sigma_c} \text{)} 
\end{cases} \]
\[ (3.89) \]

This completes the solution of the wave equation. This is the complete solution to the wave equation, for the time of interest and for a laser which has a temporal pulse shape of Dirac-delta function. Therefore this solution can be used as a Green's function to calculate the displacement of surface B in Figure 1 for any arbitrary laser-pulse shape as follows:

\[ W(z, t) = \int_0^\infty f(\tau)u(z, t - \tau)d\tau \]

where \( f(t) \) represents the temporal pulse shape of the laser and \( u(z, t - \tau) \) is obtained from Equation (3.83).
CHAPTER 4

RESULTS AND ANALYSIS

Obtained in Chapter 3 were expressions for the temperature distribution in the specimen (Equation (3.8)) and displacement of the back surface of the specimen (Equation (3.83)) when the temporal pulse shape of the laser is represented by the Dirac-delta function. In this chapter the temperature solution (Equation (3.8)) and the displacement solution (Equation (3.83)) are specialized to different particular cases in order to analyze their characteristics in detail. Computer programs were written (presented in Appendix A and B) to calculate temperature distribution along the specimen depth and displacement of the back surface of the specimen for any material properties. A third computer program (presented in Appendix C) calculates the peak displacement for different diffusivities and absorptivities. Since the author's objective is to observe and analyze the effect of diffusivity and absorptivity of a material on the temperature distribution along the specimen depth and displacement of its back surface, when it is irradiated with high energy laser, diffusivity and absorptivity are varied independently in the input to the computer program. Apart from diffusivity and absorptivity, all other input material properties are that of copper.

Specialization of the Solution to Heat-Conduction Equation for Different Particular Cases

The solution to the heat-conduction equation (Equation (3.8)) is now specialized to different particular cases.
Case 1: Temperature at the surface of the specimen for any time.

Taking the limit of Equation (3.8) when \( z \) tends to zero,

\[
\theta_{(z=0)} = \frac{b}{\rho c} \exp(kb^2t)erfc(b\sqrt{kt}) \ .
\]  

(4.1)

Substituting \( t = 0 \) in the above equation, in order to obtain the temperature at the specimen surface at the initial instant and for unit input energy density,

\[
\theta_{(z=0, t=0)} = \frac{b}{\rho c} .
\]  

(4.2)

This is the expression for maximum temperature attained in the specimen during the whole process, as in the very first instant and at the surface the maximum temperature should be obtained. From now on the maximum temperature will be denoted as \( T_0 \).

Case 2: Temperature distribution when the diffusivity of the material is zero.

This means that \( k = 0 \) in Equation (3.8). Since diffusivity is equal to zero, it can be said that conductivity is zero as density and specific heat of the material remain constant. Taking the limit of temperature, as \( k \) tends to zero and applying the relation \( \text{erfc}(-x) = 2 - \text{erfc}(x) \) [10],

\[
\theta = \frac{b}{2\rho c} \left[ \exp(bz)erfc\left(\frac{z}{2\sqrt{kt}}\right) + \exp(-bz) \left\{ 2 - \text{erfc}\left(\frac{z}{2\sqrt{kt}}\right) \right\} \right] .
\]  

(4.3)

Again,

\[
k \to 0 \ , \ \text{erfc}\left(\frac{z}{2\sqrt{kt}}\right) \to 0 \quad \text{(from [10])}
\]  

(4.4)

and

\[
k/K = 1/(\rho c) \ .
\]  

(4.5)

Substituting these values in Equation (4.3),

\[
\theta = \frac{b}{\rho c} \exp(-bz)
\]  

(4.6)

\[
= T_0 \exp(-bz) .
\]
Therefore it can be said that when diffusivity in the material is absent, \((\text{temperature distribution}) = (\text{spatial heat distribution}) \times (\text{a constant factor})\). This specialization agrees with the actual physical phenomenon because in the absence of diffusion, heat cannot propagate through the material. Hence the temperature rise at any particular depth should be proportional to the heat absorbed by the material at that place and if there is no temporal variation of heat absorption, temperature would not also vary with time.

**Case 3: Temperature distribution when time tends to zero.**

As time tends to zero the same result is obtained as in Case 2, because \(k\) and \(t\) always occur together in the solution to heat-conduction equation. This case also agrees with the actual physical situation since in the very initial instant, there is no time for diffusion of heat through the material, even if the diffusion in the material is present.

**Case 4: Temperature distribution when the absorptivity of the material is infinitely large.**

This implies that \(b\) tends to infinity in Equation (3.8). Assume that in Equation (3.8),

\[
\frac{b\sqrt{kt} + \frac{z}{2\sqrt{kt}}}{2\sqrt{kt}} = x
\]  \(\text{(4.7)}\)

and

\[
\frac{b\sqrt{kt} - \frac{z}{2\sqrt{kt}}}{2\sqrt{kt}} = y
\]  \(\text{(4.8)}\)

Therefore as \(b\) tends to infinity both \(x\) and \(y\) go to infinity. Substituting for these values in Equation (3.8), it can be written as
\[ \theta = \left\{ \begin{array}{l} x \exp(x^2) \text{erfc}(x) + y \exp(y^2) \text{erfc}(y) \\ - \frac{z}{2\sqrt{kt}} \exp(x^2) \text{erfc}(x) \\ + \frac{z}{2\sqrt{kt}} \exp(y^2) \text{erfc}(y) \end{array} \right\} \frac{\exp\left(-\frac{z^2}{4kt}\right)}{2\sqrt{kt} \rho c} \] 

(4.9)

Now as \( z \) goes to infinity \( (x \exp(x^2) \text{erfc}(x)) \) tends to one and \( (\exp(x^2) \text{erfc}(x)) \) tends to zero \([10]\). Taking the limiting cases as both \( x \) and \( y \) go to infinity,

\[ \theta = \frac{\exp(-z^2/4kt)}{(\sqrt{\pi kt}) \rho c} \] 

(4.10)

Equation (4.10) is the same equation as obtained in reference \([7]\) for the thermal diffusion model when all the energy is absorbed at the surface of the material. Again if the temperature at the surface of the specimen is calculated for this special case,

\[ \theta = \frac{1}{(\rho c) \sqrt{\pi kt}} \] 

(4.11)

Now if \( k \) or \( t \) go to zero, infinite temperature at the surface of the material is obtained. This is perfectly logical from practical considerations, because with zero diffusivity the total energy which is absorbed at the material surface remains confined to the surface-layer having zero thickness. Therefore the energy density at the surface becomes infinitely large which causes the temperature to rise to infinity. With time going to zero the temperature goes to infinity at the surface because the assumption is that the energy is deposited in the form of a Dirac-delta function, i.e. the amount of energy deposited at the initial moment is infinitely large. With other values of \( z \) (i.e. at different depths other than the specimen surface) and with \( k \) or \( t \) going to zero, temperature is obtained as zero. This is due to the fact that with no diffusion, when all the energy is absorbed at the surface,
no heat can propagate through the material and hence the temperature remains zero along the depth. Similarly when time goes to zero, there is no time for the energy absorbed at the surface to propagate through the material. This causes the temperature to be zero along the material at the initial time.

**Evaluation of the Temperature Distribution when the Laser Pulse Shape is Represented by a Dirac-Delta Function**

The computer program in Appendix A evaluates the temperature distribution when the laser-pulse is represented by a Dirac-delta function. Figure 5–Figure 8 represent the temperature distribution along the depth of the specimen for different time and for different combinations of absorptivity and diffusivity as shown in Table 1, where the units of $b$ are cm.$^{-1}$ and those of $k$ are cm.$^{2}$/sec.

<table>
<thead>
<tr>
<th>Absorptivity</th>
<th>Diffusivity</th>
<th>Absorptivity</th>
<th>Diffusivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$b = 100,000$</td>
<td>Low</td>
<td>$b = 100$</td>
</tr>
<tr>
<td>High</td>
<td>$k = 1.2$</td>
<td>High</td>
<td>$k = 1.2$</td>
</tr>
<tr>
<td>Low</td>
<td>$b = 100,000$</td>
<td>Low</td>
<td>$b = 100$</td>
</tr>
<tr>
<td>Low</td>
<td>$k = .001$</td>
<td>Low</td>
<td>$k = .001$</td>
</tr>
</tbody>
</table>

Table 1. Combinations of absorptivity and diffusivity used for obtaining temperature distribution.
Figure 5. Temperature distribution for high diffusivity and high absorptivity.
Figure 6. Temperature distribution for low diffusivity and high absorptivity.
Figure 7. Temperature distribution for high diffusivity and low absorptivity.
Figure 8. Temperature distribution for low diffusivity and low absorptivity.
The following observations can be made regarding the temperature distribution inside the specimen.

1) In Figure 5 (both diffusivity and absorptivity high) the initial drop in temperature is high. High absorptivity causes high temperature differences between successive layers of the solid, which again, due to high thermal diffusivity produces rapid heat transfer between the layers. This produces rapid drop in temperature initially. After some time, due to diffusion, the temperature difference between different layers decreases and the temperature drop with respect to time at any point of the material becomes slow.

2) In Figure 6 (low diffusivity, high absorptivity) temperature does not decrease with respect to time as rapidly as in case 1. This is because of the fact that due to low diffusivity heat flow across the specimen is very slow.

3) In Figure 7 and in Figure 8, temperature rise is small due to low absorptivity. Also temperature drop with respect to time is negligibly small regardless of the value of diffusivity. This is due to very low temperature difference between consecutive layers of the solid, caused by low absorptivity of the material.

Specialization of the Solution to Displacement Equation for Different Particular Cases

In this section the displacement solution (Equation (3.83)) will be specialized to two different cases.

Case 1: Displacement variation with respect to time when diffusivity of the material is zero.

As $k$ tends to zero,

$$R_1 = 0 \quad \text{(from Equation (3.64))} \quad (4.12)$$

$$R_2 = 0 \quad \text{(from Equation (3.65))} \quad (4.13)$$
\[ R_3 = -\frac{\delta}{\rho C C_L} \quad \text{(from Equation (3.66))} \quad (4.14) \]

and

\[ y_1 \to \infty \quad \text{(from Equation (3.77))} \quad (4.15) \]

Since each term in the right-hand side of Equation (3.83) is bounded for any value of \( k \), substituting the values from Equation (4.12) to Equation (4.15) in Equation (3.83) it can be written, as \( k \) tends to zero, as follows:

\[ u(z, t) = N_{31} + N_H \quad (4.16) \]

where

\[ N_{31} = -\frac{\delta}{\rho c} \text{erfc} \left( \frac{D}{2\sqrt{kt}} \right) - \frac{\delta}{2\rho c} \exp \left( -\frac{D^2}{4kt} \right) \left\{ 2 \exp(x_1^2) - \exp(x_1^2) \text{erfc} \left( \frac{D}{2\sqrt{kt}} \right) \right\} \quad (4.17) \]

and

\[ N_H = \begin{cases} \frac{\delta}{\rho c} \exp(C_L b t_2) & \text{(for } t < \frac{D}{C_L} \text{)} \\ \frac{\delta}{\rho c} \exp(-C_L b t_2) & \text{(for } \frac{D}{C_L} \leq t < \frac{2D}{C_L} \text{)} \\ \frac{\delta}{\rho c} \exp \left\{ C_L b \left( t - \frac{3D}{C_L} \right) \right\} & \text{(for } \frac{2D}{C_L} \leq t < \frac{2D}{C_L} \text{)} \end{cases} \quad (4.18) \]

Taking limit of \( N_{31} \), as \( k \) tends to zero,

\[ N_{32} = -\frac{\delta}{\rho c} \exp(-bD) \quad (4.19) \]

Therefore the solution to the displacement equation as \( k \) tends to zero is given as,

\[ u(z, t) = N_{32} + N_H \quad (4.20) \]

where \( N_{32} \) is given by Equation (4.19) and \( N_H \) is given by Equation (4.18).
Case 2: Displacement variation with respect to time when absorptivity of the material is infinity.

As \( b \) tends to infinity,

\[
R_1 = 0 \quad \text{(from Equation (3.64))} \tag{4.21}
\]

\[
R_2 = \frac{\delta}{\rho c} \quad \text{(from Equation (3.65))} \tag{4.22}
\]

\[
R_3 = 0 \quad \text{(from Equation (3.66))} \tag{4.23}
\]

\[
x_1 \to \infty \quad \text{(from Equation (3.74))} \tag{4.24}
\]

and

\[
y_1 \to \infty \quad \text{(from Equation (3.77))} \tag{4.25}
\]

Similarly \( x_{11} \) and \( y_{11} \) tend to infinity when \( b \) tends to infinity. Putting the above mentioned values in Equation (3.83) the displacement is obtained as,

\[
u(z,t) = N_{12} + N_{21} + N_{33} + N_{34} \tag{4.26}
\]

where

\[
N_{12} = \begin{cases} 
\frac{\delta}{2\rho c} (E_1) \{\exp(x_2^2)erfc(x_2) \\
\exp(y_2^2)erfc(y_2)\} & \text{for } t < \frac{D}{c_L} \\
-\frac{\delta}{2\rho c} (E_1) \{\exp(x_3^2)erfc(x_3) + \exp(y_2^2)erfc(y_2)\} \\
+\frac{\delta}{\rho c} (E_{11}) \{\exp(x_{21}^2)erfc(x_{21}) \\
-\exp(y_{21}^2)erfc(y_{21})\} (H_1) & \text{for } t \geq \frac{D}{c_L}
\end{cases} \tag{4.27}
\]

\[
N_{21} = \frac{2\delta}{\rho c} \exp\left(\frac{C_L^2 t}{k}\right) erfc\left(\frac{C_L \sqrt{t}}{\sqrt{k}}\right) (H_2) \tag{4.28}
\]

\[
N_{33} = -\frac{\delta}{\rho c} erfc\left(\frac{D}{2\sqrt{kt}}\right) \tag{4.29}
\]
and

\[
N_{34} = \begin{cases} 
\frac{\delta}{2\rho c} \left( E_1 \right) \{ \exp(x_2^2) \text{erfc}(x_2) 
+ \exp(y_2^2) \text{erfc}(y_2) \} & \text{for } t < \frac{D}{c} \\
-\frac{\delta}{2\rho c} \left( E_1 \right) \{ \exp(x_3^2) \text{erfc}(x_3) 
- \exp(y_3^2) \text{erfc}(y_3) \} & \text{for } t \geq \frac{D}{c}
\end{cases}
\]  

(4.30)

Evaluation of Displacement of the Back Surface of the Specimen when the Laser Pulse Shape is Represented by Dirac-Delta Function

The computer program in Appendix B evaluates the displacement of the back surface of the specimen. Two plots are generated with two different values of absorptivity, one for high absorptivity (10^5/cm.) and the other for low absorptivity (10^2/cm.). In each case, the displacement is calculated for five different values of diffusivity, from very low to high (.001, .35, .7, 1.1234 and 1.2 cm.^2/sec.). As mentioned earlier all other properties of the material are that of copper. Figure 9 and Figure 10 represent the displacement-time curves for these two cases. Also plotted (Figure 11) are peak values of displacement against diffusivity for different absorptivities, in order to analyze the combined effect of diffusivity and absorptivity on the peak displacement of the metal surface. The following observations can be made from the graphs generated.

1. A positive precursor signal is produced in all the cases.
2. The peak displacement at the back surface occurs when the wave generated at the front surface reaches the back surface.
3. If diffusivity or absorptivity is low then the variation of displacement with respect to time is almost symmetric about the time at which peak displacement occurs.
Figure 9. Displacement as a function of time for high absorptivity.
Figure 10. Displacement as a function of time for low absorptivity.
Figure 11. Peak displacement as a function of diffusivity for different absorptivities.
4. This symmetry disappears gradually with increase in both absorptivity and diffusivity and the spike becomes sharper.

5. For lower values of absorptivity, diffusivity has negligibly small effect on the magnitude of peak displacement. Also the displacement-time graph remains the same for wide variations in diffusivity in case of lower absorptivity.

6. From Figure 11 it can be said that, for very low values of diffusivity, the peak displacement decreases with increase in absorptivity. But for higher values of diffusivity peak displacement increases with increase in absorptivity.

7. It can also be seen from Figure 11 that for high absorptivity, peak displacement increases significantly with increase in diffusivity.

In order to analyze the characteristics of the displacement-time curve it can be said from reference [7] that any point heat source, below the specimen surface, will produce a positive spike and in this case the temperature distribution can be assumed to be consisting of a series of point heat sources having gradually varying intensity (Figure 12). From each point, inside the specimen, due to rapid variation in temperature with time, two waves are generated at the same initial instant and one starts travelling towards the back surface while the other starts travelling towards the front surface (Figure 12). The wave which starts travelling towards the back surface will reach that surface first and produces a displacement of that surface. But due to the different distances the waves have to travel, depending on their points of origin, in order to reach the back surface, they start arriving at the back surface one after the other. The displacement of the back surface due to each individual wave adds up and the displacement of that surface increases gradually. The peak value of displacement occurs when the wave, generated at the front surface of the specimen, arrives at the back surface. After this, the waves which started travelling towards the front surface, start arriving at the back
surface, after getting reflected from the front surface. These waves contribute negative displacement and cause the total displacement to decrease. Thus the displacement, at first, increases up to a certain time and then starts decreasing. This explains the spike in the displacement-time curve as due to a heat source inside the specimen.

![Diagram of laser source and temperature vs depth](image)

**Figure 12.** Visualization of the optically penetrating laser source as a collection of point heat sources, along with the waveforms produced from sources P and Q.

When diffusivity is low, the temperature profile along the material depth remains almost the same with respect to time, as explained earlier. Again waves are produced at any point inside the material only due to rapid change in temperature at that point. Therefore in case of low diffusivity, the spike is due to the waves generated at the initial instant only because after that no wave of significant amplitude is produced as temperature varies very slowly with time after the initial instant. Consider a wave generated at the initial instant at a point buried at a depth $H$ below the front surface of the specimen. The wave going toward the front surface produces a displacement in the negative-$z$-direction. Even after its reflection from the front surface it produces a negative displacement due to
the fact that the front surface is a free surface. The wave going toward the back surface produces a displacement in the positive-$z$-direction. Let these two waves be called F wave and B wave respectively. Referring to Figure 13, it can be said that the time taken by F wave to reach the back surface is

$$t = \frac{D + H}{C_L}.$$

Similarly the time taken by the B wave to reach the back surface is

$$t = \frac{D - H}{C_L}.$$

$D/C_L$ is the time taken by the wave generated at the front surface to reach the back surface and peak displacement occurs at that time as explained earlier. Therefore B wave reaches the back surface $H/C_L$ unit of time before the peak displacement occurs and increases the displacement of the back surface by a fixed amount. In the same way, due to reflection, the F wave reaches the back surface $H/C_L$ units of time after the peak displacement occurs and decreases the displacement of the back surface by the same amount. The net displacement caused by the two waves is the same because they are produced by a single heat source at the same instant and at the same point. As mentioned earlier no new waves are produced at a later time. Therefore the spike, caused by the waves produced at the first instant only in the case of low diffusivity, is symmetric.
Figure 13. Generation and propagation of wave through the specimen over a period of time for low absorptivity and low diffusivity.

When the absorptivity of the material is low, less heat transfer between different layers of the specimen occurs irrespective of the value of diffusivity, as explained earlier. Therefore the spike is again due to the waves generated at the initial instant and the same argument as above explains the symmetry of the spike about the time at which the peak displacement occurs. Also due to low heat transfer between different layers of the specimen, the temperature profile along the depth of the specimen changes very slowly with respect to time, and so the change of temperature with time does not produce any wave. Thus the displacement curve is not only symmetric but it stays the same whatever be the values of diffusivity.

This does not hold true when both absorptivity and diffusivity are large because in that case from many points inside the specimen more than one wave of significant amplitude is generated even after the initial instant due to rapid change in temperature at that point. To explain this, the waves generated at a
differential distance below the front surface (shown in Figure 14a as point Q) at
the initial instant and the waves generated at a point H below (shown in Figure
14a as point P) the front surface at some later time are considered. Let the wave
generated at Q and producing positive displacement be called F1 and the wave
generated at Q but producing negative displacement be denoted by B1. In Figure
14a the amplitude of B1 is shown to be double that of F1 due to the fact that
this diagram represents F1 and B1 after some time of their origin at point Q and
due to reflection of B1 from the front surface, its amplitude is double that of F1.
In the same way as above the wave generated at point P and causing positive
displacement is called F2 and the wave generated at the same point and causing
negative displacement is called B2. As seen from Figure 14b, when F1 passes
the point P its amplitude increases by an amount equal to the amplitude of the
F2 wave. Since both the waves are moving in the same direction F1 retains this
increased amplitude and when it reaches the back surface it causes an increase in
the displacement of the back surface. Therefore the peak displacement increases.
Since B1 and B2 travel in the opposite direction, after B1 crosses the point P its
negative amplitude does not increase. Thus when it reaches the back surface it
causes the displacement to decrease by a smaller amount than it was increased
by the F1 wave when it reached the back surface as shown in Figure 14c. Again
F1 and B1 are equally spaced in time about the time at which peak displacement
occurs, because peak displacement occurs when the F1 wave arrives at the back
surface. Therefore the displacement becomes unsymmetric.

In the case of very low diffusivity, if the absorptivity is high, most of the
energy is absorbed at the surface of the material. Again due to very low diffusivity
heat cannot penetrate inside the specimen. Due to absence of significant amount
of internal heat source the spike becomes shorter. Keeping diffusivity the same
as before (i.e. very low) if absorptivity is now decreased more heat will be able to penetrate inside the specimen as less heat is absorbed at the surface of the specimen. Due to this increase in internal heat source the spike height increases. This explains the increase in the value of peak displacement when absorptivity is decreased, for very low diffusivity. In fact when the peak displacement is calculated for zero diffusivity from Equation (4.21) it is found that the peak displacement decreases with increase in absorptivity, as expected.

Figure 14. Generation and propagation of wave through the specimen over a period of time for high absorptivity and high diffusivity.

Conclusion

The one-dimensional theory and the mathematical model developed in the previous chapters predict the displacement of back surface of a specimen when it is irradiated by a high energy laser pulse whose temporal pulse shape is represented by a Dirac-delta function. Using the solution for this case as a Green's
function, displacement can be obtained for any arbitrary temporal laser pulse shape also. This model is valid for the most general type of material property and in the process it takes into account the effects of both diffusivity and absorbivity of a material. This helps to determine the relative influence of these two properties on the displacement. The results obtained in the previous sections of this chapter agree with those presented in reference [7]. Again as shown before, specialization of the solution to different particular cases yields results obtained before in reference [7] and other papers. Thus the solution obtained in this thesis is the most general solution possible for the one-dimensional case and uncoupled theory of thermoelasticity.
REFERENCES CITED
REFERENCES CITED


APPENDICES
APPENDIX A

PROGRAM NAME: DISPVAX.EXE
Figure 15. Format for data entry files for program DISPVAEX.EXE.

Input files:

1) INPUT.DAT

This file contains the material properties.

Input variables: CLEXP, POISSON, DEN, SPHEAT, CABSORP, WSPEED, QO, DEPTH.

CLEXP: Coefficient of linear expansion.
POISSON: Poisson’s ratio.
DEN: Density.
SPHEAT: Specific heat.
CABSORP: Absorptivity.
WSPEED: Wave speed.
QO: Heat input (in this case it is unity).
DEPTH: Specimen thickness.

2) CONTROL.DAT

This file contains the controlling parameters of the program.

Input variables: RINTTIME, CHANGE1STEP, CHANGE2STEP, CHANGE3STEP, EXTRA.

RINTTIME: Time at which displacement is calculated initially.
CHANGE1STEP: Change in time.
CHANGE2STEP: Change in time.
CHANGE3STEP: Change in time.
EXTRA: Change in time.

3) DIF.DAT

This file contains different diffusivities for which displacement is to be calculated.

Input variables: DIF1, DIF2, DIF3, DIF4, DIF5.

All variables are different values of diffusivity.
Figure 16. Program to calculate displacement of the back surface of a specimen.

************* ** * * * * * ****************************************
*** PROGRAMME TO CALCULATE THE DISPLACEMENT OF THE BACK
*** SURFACE OF A SPECIMEN WHEN ITS FRONT SURFACE IS IRRADIATED
*** BY A HIGH ENERGY LASER PULSE WHOSE TEMPORAL SHAPE IS
*** REPRESENTED BY A DIRAC-DELTA FUNCTION. THE PROGRAM
*** GENERATES DISPLACEMENT VS. TIME DATA FOR DIFFERENT
*** VALUES OF DIFFUSIVITY AND FOR A SINGLE VALUE OF
*** ABSORPTIVITY OF THE MATERIAL.
************* ** * * * * * ****************************************

IMPLICIT REAL*8 (A-H,O-Z)

C PARAMETERS:
C .FT = FINAL TIME OF OBSERVATION
C T = TIME

C DATA INPUT
OPEN(2,FILE='INPUT.DAT',STATUS='OLD').
OPEN(3,FILE='OUTPUT.DAT',STATUS='NEW')
OPEN(4,FILE='CONTROL.DAT',STATUS='OLD')
OPEN(5,FILE='DIF.DAT',STATUS='OLD')

READ(2,*)CLEXP,POISSON,DEN,SPHEAT,
+ CABSORP,WSPEED,Q0,DEPTH
READ(4,*)RINTTIME,CHANGE1STEP,CHANGE2STEP,
+ CHANGE3STEP,EXTRA
READ(5,*)DIF1,DIF2,DIF3,DIF4,DIF5
CLOSE(2)
CLOSE(4)
CLOSE(5)

C STARTING "DO" LOOP FOR CALCULATION OF DISPLACEMENT FOR
C DIFFERENT DIFFUSIVITIES.
DO 20 M = 1,5
  IF(M.EQ.1)DIF=DIF1
  IF(M.EQ.2)DIF=DIF2
  IF(M.EQ.3)DIF=DIF3
  IF(M.EQ.4)DIF=DIF4
  IF(M.EQ.5)DIF=DIFS
  WRITE(*,41)'DIFFUSIVITY=',DIF
  WRITE(3,41)'DIFFUSIVITY=',DIF
  WRITE(4,41)'DIFFUSIVITY=',DIF
  41 FORMAT(' ',A,5X,F16.8)
C CALCULATING VARIOUS CONSTANTS

\[ \text{CONS1} = (\text{DIF} \times \text{CABSORP})^2 \times \text{WSPEED}^2 \]
\[ \text{DELTA} = \text{CLEXP} \times (1.0 + \text{POISSON}) / (1.0 - \text{POISSON}) \]
\[ \text{R1} = \text{WSPEED} \times \text{DELTA} \times \text{CABSORP} \times \text{DIF} / \text{DEN} / \text{SPHEAT} / \text{CONS1} \]
\[ \text{R2} = \text{DELTA} \times (\text{CABSORP} \times \text{DIF})^2 \times \text{DEN} / \text{SPHEAT} / \text{CONS1} \]
\[ \text{R3} = \text{WSPEED} \times \text{DELTA} / \text{DEN} / \text{SPHEAT} / \text{CONS1} \]
\[ \text{DIMEN} = \text{DELTA} / \text{DEN} / \text{SPHEAT} \]

C CALCULATING FINAL TIME OF INTEREST

\[ \text{FT} = 3.0 \times \text{DEPTH} / \text{WSPEED} \]

C CALCULATING THE APPROXIMATE TIME WHEN PEAK DISPLACEMENT C OCCURS

\[ \text{PEAKTIME} = \text{DEPTH} / \text{WSPEED} \]

C WRITING INPUT DATA

WRITE(3,10)'COEFF OF LINEAR EXPANSION = ', CLEXP,
+ 'DENSITY = ', DEN
WRITE(3,10)'POISSON'S RATIO = ', POISSON,
+ 'SPECIFIC HEAT = ', SPHEAT
WRITE(3,10)'ABSORPTION COEFFICIENT = ', CABSORP,
+ 'HEAT INPUT = ', QO
WRITE(3,10)'WAVE SPEED = ', WSPEED,
+ 'DIFFUSIVITY = ', DIF
WRITE(3,10)'SPECIMEN THICKNESS = ', DEPTH,
+ 'FINAL TIME = ', FT
WRITE(*,10)'COEFF OF LINEAR EXPANSION = ', CLEXP,
+ 'DENSITY = ', DEN
WRITE(*,10)'POISSON'S RATIO = ', POISSON,
+ 'SPECIFIC HEAT = ', SPHEAT
WRITE(*,10)'ABSORPTION COEFFICIENT = ', CABSORP,
+ 'HEAT INPUT = ', QO
WRITE(*,10)'DIFFUSIVITY = ', DIF,
+ 'WAVE SPEED = ', WSPEED
WRITE(*,10)'SPECIMEN THICKNESS = ', DEPTH,
+ 'FINAL TIME = ', FT

10 FORMAT( ', A, 2X, E13.5, 5X, A, E13.5)

C STARTING "DO" LOOP FOR CALCULATING DISPLACEMENT AT A FIXED C TIME AT THE SPECIMEN BACK SURFACE (Z=DEPTH).

\[ \text{N} = \text{FT} / \text{CHANGE1STEP} + 1 \]
WRITE(*,*)'MAXIMUM NO. OF TIME STEPS = ', N
T=RINTTIME
DUMMY=0.0
RESULT=0.0
C SETTING COUNTER INITIALLY TO ZERO IN ORDER TO CALCULATE THE C NUMBER OF TIMES DISPLACEMENT IS EVALUATED.
NCOUNT = 0
NFLAG1 = 0
DO 2000 I=1,N
IF(I.EQ.N)THEN
   WRITE(*,*)'MAXIMUM ITERATION IN THE MAIN PROGRAMME EXCEEDED'
   PAUSE
ENDIF

C----------------------------------------------------------------
C FIXING TIME INCREMENT. DIFFERENT TIME REGIONS IN THIS
C SECTION MAY BE ADJUSTED, DEPENDING UPON THE RELATIVE VALUES
C OF ABSORPTIVITY AND DIFFUSIVITY, TO GET A DISPLACEMENT-TIME
C GRAPH OF DESIRED SMOOTHNESS.
IF ( DIF.LT.1) THEN
   IF( (RESULT.LT.DUMMY).AND.(RESULT.LT.5.9D-2).AND.
      (T.LT.0.53665D-5) ) THEN
      T=T+CHANGE3STEP
   ELSE
      IF( (RESULT.LT.DUMMY).AND.(RESULT.LT.1.D-2) )THEN
         T=T+EXTRA
      ELSE
         IF( (T.GE.0.5364805D-5).AND.
             (T.LE.0.5364808D-5) ) THEN
            T = T+CHANGE1STEP
         ELSE
            IF( (T.GE.0.53646500D-5) .AND.
                (T.LE.0.5364999D-5)) THEN
               T = T+CHANGE3STEP
            ELSE
               T = T+EXTRA
            ENDIF
         ENDIF
      ENDIF
   ENDIF
ELSE
   IF( (RESULT.LT.DUMMY).AND.(RESULT.LT.1.D-2) )THEN
      T=T+EXTRA
   ELSE
      IF( (T.GE.0.5364805D-5).AND.
          (T.LE.0.5364808D-5) ) THEN
         T = T+CHANGE1STEP
      ELSE
         IF( (T.GE.0.5364700D-5) .AND.
             (T.LE.0.5364829D-5) ) THEN
            T = T+CHANGE2STEP
         ELSE
            IF( (T.GE.0.5363000D-5) .AND.
                (T.LE.0.5364999D-5)) THEN
               T = T+CHANGE3STEP
            ELSE
               T = T+EXTRA
            ENDIF
         ENDIF
      ENDIF
   ENDIF
ENDIF
ENDIF
ENDIF
ENDIF
ENDIF
Figure 16. (continued)

IF (T.GE.3.D0*DEPTH/WSPEED) GO TO 25
IF (RESULT.LT.DUMMY).AND.(RESULT.LT.3.D-2)) GO TO 25

C CALCULATING THE ARGUMENTS OF HEAVISIDE FUNCTION
T1 = T-DEPTH/WSPEED
T2 = T-2.DO*DEPTH/WSPEED

C CHECKING THE VALUE OF ARGUMENTS OF HEAVISIDE'S FUNCTIONS.
C IF ANY ARGUMENT TURNS OUT TO BE ZERO THEN TIME IS INCREASED
C BY A SMALL AMOUNT SO THAT ARGUMENT IS POSITIVE.
IF (T.EQ.0.DO) THEN
    WRITE(*,*) 'TIME EQUAL TO ZERO'
    PAUSE
ENDIF
IF (T1.EQ.0.DO) THEN
    WRITE(*,*) 'ARGUMENT OF FIRST HEAVISIDE'S FUNCTION IS ZERO'
ENDIF
IF (T2.EQ.0.DO) THEN
    WRITE(*,*) 'ARGUMENT OF SECOND HEAVISIDE'S FUNCTION IS ZERO'
    T1 = T1 + 1.D-14
ENDIF

C SETTING VALUES OF HEAVISIDE FUNCTION ACCORDING TO ITS
C ARGUMENT
NH1 = 0
NH2 = 0
IF (T1.GT.0.DO) NH1 = 1
IF (T2.GT.0.DO) NH2 = 1

C CALCULATING ARGUMENTS OF DIFFERENT ERROR FUNCTIONS IN THE
C DISPLACEMENT SOLUTION. FOR THIS APPROPRIATE FUNCTIONS ARE
C CALLED.
X1 = VALUE1(CABSORP,DIF,T,DEPTH,-1)
Y1 = VALUE1(CABSORP,DIF,T,DEPTH,1)
X2 = VALUE2(WSPEED,DIF,T,DEPTH,-1)
Y2 = VALUE2(WSPEED,DIF,T,DEPTH,1)
X3 = VALUE3(WSPEED,DIF,T,DEPTH,-1)
IF (NH2.EQ.1) THEN
    XH1 = VALUE1(CABSORP,DIF,T2,DEPTH,-1)
    YH1 = VALUE1(CABSORP,DIF,T2,DEPTH,1)
    XH2 = VALUE2(WSPEED,DIF,T2,DEPTH,-1)
Figure 16. (Continued)

\[ Y_{H2} = \text{VALUE2}(\text{WSPEED}, \text{DIF}, T2, \text{DEPTH}, 1) \]

ENDIF

C CALCULATING DIFFERENT CONSTANTS

\[ \text{DUMMY1} = -\text{DEPTH} \times \text{DEPTH}/4.0/D\text{IF}/T \]

IF (\( \text{NH2} = 1 \)) \( \text{DUMMY2} = -\text{DEPTH} \times \text{DEPTH}/4.0/D\text{IF}/T2 \)

C CALCULATING DIFFERENT TERMS IN THE DISPLACEMENT SOLUTION.

C-----------------------------------------------

\[ \text{PRT11} = R1/2.0/D\text{IMEN} \times \text{VALUE}(X1, Y1, 1, \text{DUMMY1}) \]

IF (\( \text{NH2} = 1 \)) THEN

\[ \text{PRTH11} = R1/D\text{IMEN} \times \text{VALUE}(XH1, YH1, 1, \text{DUMMY2}) \]

ELSE

\[ \text{PRTH11} = 0.0 \]

ENDIF

\[ \text{RES11} = \text{PRT11} + \text{PRTH11} \]

C-----------------------------------------------

IF (\( \text{NH1} = 0 \)) THEN

\[ \text{RES12} = R2/2.0/D\text{IMEN} \times \text{VALUE}(X2, Y2, -1, \text{DUMMY1}) \]

ELSE

\[ \text{PRT12} = -R2/2.0/D\text{IMEN} \times \text{VALUE}(X3, Y2, 1, \text{DUMMY1}) \]

IF (\( \text{NH2} = 1 \)) THEN

\[ \text{PRTH12} = R2/D\text{IMEN} \times \text{VALUE}(XH2, YH2, -1, \text{DUMMY2}) \]

ELSE

\[ \text{PRTH12} = 0.0 \]

ENDIF

\[ \text{RES12} = \text{PRT12} + \text{PRTH12} \]

ENDIF

C-----------------------------------------------

\[ \text{RES1} = \text{RES11} + \text{RES12} \]

C-----------------------------------------------

IF (\( \text{NH1} = 1 \)) THEN

\[ \text{CONS21} = \text{CABSOP} \times \text{D SQRT}(\text{DIF} \times T1) \]

\[ \text{CONS22} = \text{WSPEED} \times \text{D SQRT}(T1/\text{DIF}) \]

\[ \text{CONS23} = \text{CABSOP} \times \text{WSPEED} \times T1 \]

IF (DABS(\( \text{CONS21} \)) \( \leq 8.50 \)) THEN

\[ \text{PRT21} = -2.0/D\text{IMEN} \times R1^* + \text{D EXP}(\text{CONS21} \times \text{CONS21}) \times \text{ERFCC}(\text{CONS21}) \]

ELSE

\[ \text{PRT21} = -2.0/D\text{IMEN} \times R1^* \times \text{ASYM}(\text{CONS21}) \]

ENDIF

IF (DABS(\( \text{CONS22} \)) \( \leq 8.50 \)) THEN

\[ \text{PRT22} = 2.0 \times R2/D\text{IMEN}^* + \text{D EXP}(\text{CONS22} \times \text{CONS22}) \times \text{ERFCC}(\text{CONS22}) \]

ELSE

\[ \text{PRT22} = 2.0 \times R2/D\text{IMEN} \times \text{ASYM}(\text{CONS22}) \]

ENDIF

\[ \text{RES2} = \text{PRT21} + \text{PRT22} \]
ELSE
  RES2 = 0.D0
ENDIF

C-----------------------------------------------------------
PRT31 = -DELTA/DEN/SPHEAT/DIMEN*
       ERFCC(DEPTH/2.D0/DSQRT(DIF*T))
PRT32 = R3*WSPEED/2.D0/DIMEN*VALUE(X1,Y1,-1,DUMMY1)
IF (NH1.EQ.0) THEN
  PRT33 = R2/2.D0/DIMEN*VALUE(X2,Y2,1,DUMMY1)
ELSE
  PRT33 = -R2/2.D0/DIMEN*VALUE(X3,Y2,-1,DUMMY1)
ENDIF
RES3 = PRT31 + PRT32 + PRT33

C-----------------------------------------------------------
C CALCULATING THE HYPERBOLIC TERMS
HYCON1 = WSPEED*CABSORP*T1
HYCON2 = WSPEED*CABSORP*(T-3.D0*DEPTH/WSPEED)
HYCON3 = DIF*CABSORP
IF( (NH1.EQ.0).AND.(NH2.EQ.0) ) THEN
  HYPER = -R3/DIMEN*( WSPEED*DEXP(HYCON1)
         + HYCON3*DEXP(HYCON1) )
ENDIF
IF( (NH1.EQ.1).AND.(NH2.EQ.0) ) THEN
  HYPER = -R3/DIMEN*( WSPEED*DEXP(-HYCON1) -
         HYCON3*DEXP(-HYCON1) )
ENDIF
IF( (NH1.EQ.1).AND.(NH2.EQ.1) ) THEN
  HYPER = -R3/DIMEN*( WSPEED*DEXP(HYCON2) +
         HYCON3*DEXP(HYCON2) )
ENDIF

C-----------------------------------------------------------
DUMMY = RESULT
RESULT= RES1 + RES2 + RES3 + HYPER
IF((RESULT.NE.0.D0).AND.(NFLAG1.EQ.0) ) THEN
  WRITE(3,30)T,RESULT
  NFLAG1=I
  NCOUNT = NCOUNT+1
ENDIF
IF(RESULT.GT.1.D-2) THEN
  WRITE(3,30)T,RESULT
  NCOUNT = NCOUNT+1
ENDIF
2000 CONTINUE
25 WRITE(3,*)'NO. OF POINTS = ',NCOUNT
20 CONTINUE
CLOSE(3)
Figure 16. (Continued)

30 FORMAT(' ',E25.16,5X,E25.16)
STOP
END

C***********************************************************************
REAL*8 FUNCTION VALUE1(B,RK,T,D,N)
IMPLICIT REAL*8 (A-H,O-Z)
VALUE1 = B*DSQRT(RK*T) + N*D/2.DO/DSQRT(RK*T)
RETURN
END

C***********************************************************************
REAL*8 FUNCTION VALUE2(CT0,RK,T,D,N)
IMPLICIT REAL*8 (A-H,O-Z)
VALUE2 = N*CT0*DSQRT(T/RK) + D/2.DO/DSQRT(RK*T)
RETURN
END

C***********************************************************************
REAL*8 FUNCTION VALUE3(CT0,RK,T,D,N)
IMPLICIT REAL*8 (A-H,O-Z)
VALUE3 = CT0*DSQRT(T/RK) + N*D/2.DO/DSQRT(RK*T)
RETURN
END

C***********************************************************************
REAL*8 FUNCTION VALUE(X,Y,N,COEF)
IMPLICIT REAL*8 (A-H,O-Z)
IF ( X.GE.0.DO ) THEN
  IF ( (X*X+COEF) .LT. 80.DO ) THEN
    PARTX = DEXP(X*X+COEF) * ERFCC(X)
  ELSE
    PARTX = DEXP(COEF) * ASYM(X)
  ENDIF
ELSE
  PARTX = AERFCC(X,COEF)
ENDIF
ENDIF
IF( (Y*Y+COEF) .LT. 80.DO ) THEN
  PARTY = DEXP(Y*Y+COEF) * ERFCC(Y)
ELSE
  PARTY = DEXP(COEF) * ASYM(Y)
ENDIF
VALUE = PARTX + N*PARTY
RETURN
END

C***********************************************************************
REAL*8 FUNCTION ASYM(A)
IMPLICIT REAL*8 (A-H,O-Z)
PI=DACOS(-1.DO)
SUM=1.DO
DO 10 I=1,100
  IF(I.EQ.100) THEN
    .....
WRITE(*,*) 'MAXIMUM NO. OF ITERATION EXCEEDED IN'
WRITE(*,*) 'FUNCTION "ASYM"'
PAUSE
ENDIF
DEL = (-1.D0)**I*(2.D0*I-1.D0)/(2.D0*A*A)**I
SUM = SUM + DEL
IF (DABS(DEL).LT.1.D-20) GO TO 20
10 CONTINUE
20 ASYM = SUM/DSQRT(PI)/A
RETURN
END
C***********************************************************
REAL*8 FUNCTION AERFCC(X,COEF)
IMPLICIT REAL*8 (A-H,O-Z)
IF ( (X.GT.0.DO) .OR. (COEF.GT.0.DO) ) THEN
  WRITE(*,*)'THIS ARGUMENT IN FUNCTION AERFCC CANNOT'
  WRITE(*,*)'BE POSITIVE'
  IF(X.GT.0.DO) WRITE(*,*)'X'
  IF(COEF.GT.0.DO)WRITE(*,*)'COEF'
  PAUSE
ENDIF
Z=DABS(X)
T=I.DO/(I.DO+O.5D0*Z)
AERFCC = T*DEXP(COEF - 1.26551223D0+T*(1.00002368D0+T*
  +.37409196D0+T*(-.09678418D0+T*(-.18628806D0+T*
  +(.27886807D0+T*(-1.13520398D0+T*(1.48851587D0+T*
  +(-.82215223D0+T*.17087277D0))))))))
IF (X.LT.0.DO) AERFCC=2.D0*DEXP(COEF+Z*Z)-AERFCC
RETURN
END
C***********************************************************
REAL*8 FUNCTION ERFCC(X)
IMPLICIT REAL*8 (A-H,O-Z)
Z=DABS(X)
T=I.DO/(I.DO+O.5D0*Z)
ERFCC=T*DEXP(-Z*Z-1.26551223D0+T*(1.00002368D0+T*
  +.37409196D0+T*(-.09678418D0+T*(-.18628806D0+T*
  +(.27886807D0+T*(-1.13520398D0+T*(1.48851587D0+T*
  +(-.82215223D0+T*.17087277D0))))))))
IF (X.LT.0.DO) ERFCC=2.DO-ERFCC
RETURN
END
C***********************************************************
APPENDIX B

PROGRAM NAME: DISPMOD.EXE
Figure 17. Format for data entry files for program DISPMOD.EXE.

Input files:

1) PKINPT.DAT

This file contains the material properties.

Input variables: CLEXP, POISSON, DEN, SPHEAT, WSPEED, QO, DEPTH.

    Clexp: Coefficient of linear expansion.
    Poisson: Poisson's ratio.
    Den: Density.
    Spheat: Specific heat.
    Wspeed: Wave speed.
    QO: Heat input (in this case it is unity).
    Depth: Specimen thickness.

2) PKCNTRL.DAT

This file contains the controlling parameters of the program.

Input variables: RINTTIME, DELTIME.

    Rinttime: Time at which displacement is calculated initially.
    Deltime: Change in time.

3) ABSORP.DAT

This file contains different absorptivities for which displacement is to be calculated.

Input variables: CABSORP1, CABSORP2, CABSORP3, CABSORP4, CABSORP5, CABSORP6, CABSORP7.

All variables are different values of absorptivity.
**PROGRAMME TO CALCULATE THE PEAK DISPLACEMENT OF THE BACK SURFACE OF A SPECIMEN WHEN IT IS IRRADIATED WITH A HIGH ENERGY LASER WHOSE TEMPORAL PULSE SHAPE IS REPRESENTED BY A DIRAC-DELTA FUNCTION. THE PROGRAMME CALCULATES THE PEAK DISPLACEMENT FOR DIFFERENT DIFFUSIVITIES WHEN ABSORPTIVITY IS HELD CONSTANT.**

IMPLICIT REAL*8 (A-H,O-Z)

C PARAMETERS:
C FT = FINAL TIME OF OBSERVATION
C T = TIME

OPEN(2, FILE='PKINPT.DAT', STATUS='OLD', FORM='FORMATTED')
OPEN(3, FILE='OUTPUT.DAT', STATUS='NEW', FORM='FORMATTED')
OPEN(4, FILE='PKCNTRL.DAT', STATUS='OLD', FORM='FORMATTED')
OPEN(5, FILE='ABSORP.DAT', STATUS='OLD', FORM='FORMATTED')
READ(2,*) CLEXP, POISSON, DEN, SPHEAT, WSPEED, QO, DEPTH
READ(4,*) RINTIME, DELTIME
READ(5,*) CABSORP1, CABSORP2, CABSORP3, CABSORP4
READ(5,*) CABSORP5, CABSORP6, CABSORP7
CLOSE(2)
CLOSE(4)
CLOSE(5)

C STARTING "DO" LOOP FOR CALCULATING DISPLACEMENT FOR VARIOUS VALUES OF DIFFUSIVITY.

DO 20 M = 1, 7
   IF(M.EQ.1) CABSORP=CABSORP1
   IF(M.EQ.2) CABSORP=CABSORP2
   IF(M.EQ.3) CABSORP=CABSORP3
   IF(M.EQ.4) CABSORP=CABSORP4
   IF(M.EQ.5) CABSORP=CABSORP5
   IF(M.EQ.6) CABSORP=CABSORP6
   IF(M.EQ.7) CABSORP=CABSORP7
WRITE(*,*) 'ABSORPTIVITY=', CABSORP
WRITE(3,*) 'ABSORPTIVITY=', CABSORP
DIF=0.0

C SETTING COUNTER TO CALCULATE THE NUMBER OF POINTS.
NCOUNT = 0

C STARTING OF "DO" LOOP TO CHANGE DIFFUSIVITY BY A SMALL AMOUNT.

DO 35 NNN=1, 10000

Figure 18. Program to calculate maximum displacement of the back surface of a specimen
C CALCULATION FOR DIFFUSIVITY .EQ. ZERO.

IF (DIF.EQ.0.D0) THEN
  DIF = DIF + 1.D-14
  DELTA = CLEXP*(1.D0+POISSON)/(1.D0-POISSON)
  R3 = -DELTA/DEN/SPHEAT/WSPEED
  T = RINTTIME
  BIG = 0.D0
  RESULT = 0.D0
  DIMEN = DELTA*DEN/SPHEAT
  DO 29 IJ = 1,2000
       IF (RESULT.LT.BIG) GO TO 25.
       T = T + DELTIME
       IF (T.GE.3.D0*DEPTH/WSPEED) GO TO 25
       IF (IJ.EQ.2000) THEN
         WRITE (*) 'MAXIMUM ITERATION FOR "K=0" EXCEEDED'
         PAUSE
       ENDIF
     T1 = WSPEED*CABSORP*( T-DEPTH/WSPEED )
     T2 = WSPEED*CABSORP*( T-3.D0*DEPTH/WSPEED )
     RES1 = R3*WSPEED*DEXP(-CABSORP*DEPTH)
     IF (T.LT.DEPTH/WSPEED) THEN
       RES2 = -R3*WSPEED*DEXP(T1)
     ELSE
       IF (T.LT.2.D0*DEPTH/WSPEED) THEN
         RES2 = -R3*WSPEED*DEXP(-T1)
       ELSE
         RES2 = -R3*WSPEED*DEXP(T2)
       ENDIF
     ENDIF
     RESULT = (RES1 + RES2)/DIMEN
     IF (RESULT.GT.BIG) BIG=RESULT
     CONTINUE
   ELSE
     IF (DIF.LE..02D0) THEN
       DIF=DIF+.001
     ELSE
       IF (DIF.LE..35D0) THEN
         DIF=DIF+.01
       ELSE
         DIF=DIF+.05
       ENDIF
     ENDIF
     IF (DIF.GT.1.2D0) GO TO 37

C CALCULATING VARIOUS CONSTANTS
CONSl = (DIF*CABSORP)**2.D0-WSPEED**2.D0
DELTA = CLEXP*(1.D0+POISSON)/(1.D0-POISSON)
R1 = WSPEED*DELTA*CABSORP*DIF/DEN/SPHEAT/CONSl
Figure 18. (Continued)

\[
R2 = \text{DELTAV*(CABSORPV*DIF)**2.DO/DEPV/SPHEAT/CONS1}
\]
\[
R3 = \text{WSPEED*DELTAV/DEPV/SPHEAT/CONS1}
\]
\[
\text{DIMEN} = \text{DELTAV/DEPV/SPHEAT}
\]
\[
\text{C CALCULATING FINAL TIME OF INTEREST}
FT = 3.DO*DEPTH/WSPEED
\]
\[
\text{C---------------------------------------------------------------}
\text{C STARTING OF TIME DO LOOP. DISPLACEMENT IS CALCULATED FOR A}
\text{C FIXED TIME AT THE BOUNDARY (Z=DEPTH) OF THE SPECIMEN.}
\text{N = FT/DELTTIME + 1}
\text{T=RINTTIME}
\text{RESULT=0.DO}
\text{BIG=0.DO}
\text{NCOUNT = NCOUNT+1}
\text{DO 2000 I=1,(N+1000)}
\text{IF(I.EQ.(N+1000))THEN}
\text{WRITE(*,*)'MAXIMUM ITERATION IN MAIN PROGRAMME}
\text{EXCEEDED'}
\text{PAUSE}
\text{ENDIF}
\text{IF (RESULT.LT.BIG) GO TO 25}
\text{T = T+DELTTIME}
\text{IF (T.GE.3.DO*DEPTH/WSPEED) GO TO 25}
\]
\[
\text{C CALCULATING THE ARGUMENTS OF HEAVISIDE FUNCTION}
T1 = T-DEPTH/WSPEED
T2 = T-2.DO*DEPTH/WSPEED
\]
\[
\text{C CHECKING THE VALUE OF ARGUMENTS OF HEAVISIDE'S FUNCTIONS.}
\text{C IF ANY ARGUMENT TURNS OUT TO BE ZERO THEN TIME IS INCREASED}
\text{C BY A SMALL AMOUNT SO THAT ARGUMENT IS POSITIVE.}
\text{IF (T.EQ.0.DO) THEN}
\text{WRITE(*,*)'TIME EQUAL TO ZERO'}
\text{PAUSE}
\text{ENDIF}
\text{IF (T1.EQ.0.DO) THEN}
\text{WRITE(*,*)'ARGUMENT OF FIRST HEAVISIDE'S FUNCTION}
\text{IS ZERO'}
\text{T1 = T1 + 1.D-14}
\text{ENDIF}
\text{IF (T2.EQ.0.DO) THEN}
\text{WRITE(*,*)'ARGUMENT OF SECOND HEAVISIDE'S}
\text{FUNCTION IS ZERO'}
\text{T2 = T2 + 1.D-14}
\text{ENDIF}
\]
Figure 18. (Continued)

C SETTING VALUES OF HEAVISIDE FUNCTION ACCORDING TO ITS C ARGUMENT

NH1 = 0
NH2 = 0
IF(T1.GT.0.DO) NH1 = 1
IF(T2.GT.0.DO) NH2 = 1

C CALCULATING ARGUMENTS OF DIFFERENT ERROR FUNCTIONS IN THE C DISPLACEMENT SOLUTION. FOR THIS APPROPRIATE FUNCTIONS ARE C CALLED.

X1 = VALUE1(CABSORP,DIF,T,DEPTH,-1)
Y1 = VALUE1(CABSORP,DIF,T,DEPTH,1)
X2 = VALUE2(WSPEED,DIF,T,DEPTH,-1)
Y2 = VALUE2(WSPEED,DIF,T,DEPTH,1)
X3 = VALUE3(WSPEED,DIF,T,DEPTH,-1)

IF (NH2.EQ.1) THEN
  XH1 = VALUE1(CABSORP,DIF,T2,DEPTH,-1)
  YH1 = VALUE1(CABSORP,DIF,T2,DEPTH,1)
  XH2 = VALUE2(WSPEED,DIF,T2,DEPTH,-1)
  YH2 = VALUE2(WSPEED,DIF,T2,DEPTH,1)
ENDIF

C CALCULATING DIFFERENT CONSTANTS

DUMMY1 = -DEPTH*DEPTH/4.DO/DIF/T

IF (NH2.EQ.1) DUMMY2 = -DEPTH*DEPTH/4.DO/DIF/T2

C CALCULATING DIFFERENT TERMS IN THE DISPLACEMENT SOLUTION.

PRT11 = R1/2.DO/DIMEN*VALUE(X1,Y1,1,DUMMY1)
IF (NH2.EQ.1) THEN
  PRTH11 = R1/DIMEN*VALUE(XH1,YH1,1,DUMMY2)
ELSE
  PRTH11 = 0.DO
ENDIF
RES11 = PRT11 + PRTH11

C--------------------------------------

IF (NH1.EQ.0) THEN
  RES12 = R2/2.DO/DIMEN*VALUE(X2,Y2,-1,DUMMY1)
ELSE
  PRT12 = -R2/2.DO/DIMEN*VALUE(X3,Y2,1,DUMMY1)
  IF (NH2.EQ.1) THEN
    PRTH12 = R2/DIMEN*VALUE(XH2,YH2,-1,DUMMY2)
  ELSE
    PRTH12 = 0.DO
  ENDIF
  RES12 = PRT12 + PRTH12
ENDIF
Figure 18. (Continued)

RES1 = RES11 + RES12

C CALCULATION OF THE TERMS (3+4+5+6+7+8)

IF ( NH1.EQ.1 ) THEN
   CONS21=ABSORP*DSQRT(DIF*T1)
   CONS22=WSPEED*DSQRT(T1/DIF)
   CONS23=ABSORP*WSPEED*T1
   PRT21 = 2.D0/DIMEN*R1*
   + AERFCC(CONS21,CONS21*CONS21)
   PRT22 = 2.D0*R2/DIMEN*
   + AERFCC(CONS22,CONS22*CONS22)
   RES2 = PRT21 + PRT22
ELSE
   RES2 = 0.D0
ENDIF

CALCULATING TERMS (9,10)

PRT31 = -DELTA/DEN/SPHEAT/DIMEN*
   ERFCC(DEPTH/2.D0/DSQRT(DIF*T))
   PRT32 =R3*WSPEED/2.D0/DIMEN*
   + VALUE(X1,Y1,-1,DUMMY1)
   IF (NH1.EQ.0) THEN
      PRT33 = R2/2.D0/DIMEN*VALUE(X2,Y2,1,DUMMY1)
   ELSE
      PRT33 = -R2/2.D0/DIMEN*VALUE(X3,Y2,-1,DUMMY1)
   ENDIF
   RES3 = PRT31 + PRT32 + PRT33

C CALCULATING THE HYPERBOLIC TERMS

HYCON1 = WSPEED*CABSORP*T1
HYCON2 = WSPEED*CABSORP*(T-3.D0*DEPTH/WSPEED)
HYCON3 = DIF*CABSORP
IF ( (NH1.EQ.0).AND.(NH2.EQ.0) ) THEN
   HYPER = -R3/DIMEN*( WSPEED*DEXP(HYCON1)
   + HYCON3*DEXP(HYCON1) )
ENDIF
IF ( (NH1.EQ.1).AND.(NH2.EQ.0) ) THEN
   HYPER = -R3/DIMEN*( WSPEED*DEXP(-HYCON1)
   + HYCON3*DEXP(-HYCON1) )
ENDIF
IF ( (NH1.EQ.1).AND.(NH2.EQ.1) ) THEN
   HYPER = -R3/DIMEN*( WSPEED*DEXP(HYCON2)
   + HYCON3*DEXP(HYCON2) )
ENDIF
RESULT = RES1 + RES2 + RES3 + HYPER 
IF (RESULT .GT. BIG) THEN 
  BIG = RESULT 
ENDIF 
2000 CONTINUE 
ENDIF 
25 T = T - DELTIME 
WRITE (3, 39) DIF, BIG, T 
35 CONTINUE 
37 CONTINUE 
WRITE (3, *) 'NO. OF POINTS=' , NCOUNT 
20 CONTINUE 
39 FORMAT (' ', 3(E20.12, 2X)) 
CLOSE (3) 
STOP 
END 
C*********************************************************** 
REAL*8 FUNCTION VALUE1 (B, RK, T, D, N) 
IMPLICIT REAL*8 (A-H, O-Z) 
VALUE1 = B*DSQRT (RK*T) + N*D/2.DO/DSQRT (RK*T) 
RETURN 
END 
C*********************************************************** 
REAL*8 FUNCTION VALUE2 (CTO, RK, T, D, N) 
IMPLICIT REAL*8 (A-H, O-Z) 
VALUE2 = N*CTO*DSQRT (T/RK) + D/2.DO/DSQRT (RK*T) 
RETURN 
END 
C*********************************************************** 
REAL*8 FUNCTION VALUE3 (CTO, RK, T, D, N) 
IMPLICIT REAL*8 (A-H, O-Z) 
VALUE3 = CTO*DSQRT (T/RK) + N*D/2.DO/DSQRT (RK*T) 
RETURN 
END 
C*********************************************************** 
REAL*8 FUNCTION VALUE (X, Y, N, COEF) 
IMPLICIT REAL*8 (A-H, O-Z) 
PARTX = AERFCC (X, COEF) 
PARTY = AERFCC (Y, COEF) 
VALUE = PARTX + N*PARTY 
RETURN 
END 
C*********************************************************** 
REAL*8 FUNCTION AERFCC (X, COEF) 
IMPLICIT REAL*8 (A-H, O-Z) 
Z = DABS (X) 
T = 1.DO/(1.DO + 0.5D0*Z) 
AERFCC = T*DEXP(COEF - 1.26551223D0 + T*(1.00002368D0 + T*
Figure 18. (Continued)

\[
+ (.37409196D0+T* (.09678418D0+T*(-.18628806D0+T*
+ (.27886807D0+T*(-1.13520398D0+T*(1.48851587D0+T*
+ (-.82215523D0+T*.17087277D0))))))))
\]

IF (X.LT.0.D0) AERFCC=2.D0*DEXP(COEF+Z*Z)-AERFCC
RETURN
END

REAL*8 FUNCTION ERFCC(X)
IMPLICIT REAL*8 (A-H,O-Z)
Z=DABS(X)
T=1.D0/(1.D0+0.5D0*Z)
ERFCC=T*DEXP(-Z*Z-1.2655123D0+T*(1.00002368D0+T*
+ (.37409196D0+T* (.09678418D0+T*(-.18628806D0+T*
+ (.27886807D0+T*(-1.13520398D0+T*(1.48851587D0+T*
+ (-.82215523D0+T*.17087277D0))))))))
IF(X.LT.0.D0)ERFCC=2.D0-ERFCC
RETURN
END

C***********************************************************
APPENDIX C

PROGRAM NAME: TEMP.EXE
Figure 19. Format for data entry files for program TEMP.EXE.

Input files:

1) TEMPIN.DAT

This file contains the material properties and controlling parameters.

Input variables: CABSORP, DEN, SPHEAT, MAXIT, DELZ1, DELZ2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cabsorp</td>
<td>Absorptivity</td>
</tr>
<tr>
<td>Den</td>
<td>Density</td>
</tr>
<tr>
<td>Spheat</td>
<td>Specific heat</td>
</tr>
<tr>
<td>Maxit</td>
<td>Maximum number of iterations allowed</td>
</tr>
<tr>
<td>Delz1</td>
<td>Increment in the value of z</td>
</tr>
<tr>
<td>Delz2</td>
<td>Increment in the value of z</td>
</tr>
</tbody>
</table>

2) TIMEDIF.DAT

This file contains different values of time and diffusivity for which displacement is to be calculated.

Time and diffusivity and read in arrays.
Figure 20. Program to calculate the temperature along the depth of a specimen.

** PROGRAMME TO CALCULATE VARIATION OF TEMPERATURE WITH DEPTH FOR DIFFERENT TIME AND DIFFUSIVITY WHEN LASER-PULSE SHAPE IS REPRESENTED BY A DIRAC-DELTA FUNCTION

```
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION TIME(IO),DIFFUSIVITY(10)
OPEN(2,FILE='TIMEDIF.DAT',STATUS='OLD')
OPEN(3,FILE='TEMPIN.DAT',STATUS='OLD')
OPEN(4,FILE='TEMPOUT.DAT',STATUS='NEW')
C READING DIFFERENT TIMES AND DIFFUSIVITY AT WHICH TEMPERATURE IS TO BE CALCULATED
DO 10 I=1,5
   READ(2,*) TIME(I),DIFFUSIVITY(I)
10 CONTINUE
C READING OTHER INPUT DATA ( MATERIAL PROPERTIES & INCREMENT IN "Z")
   READ(3,*) CABSORP,DEN,SPHEAT,MAXIT,DELZ1,DELZ2
CLOSE(2)
CLOSE(3)
WRITE(*,*)'ABSORPTIVITY= ',CABSORP
WRITE(4,*)'ABSORPTIVITY= ',CABSORP
C STARTING OF "DO" LOOP FOR DIFFERENT DIFFUSIVITIES
DO 20 ND=1,2
   DIF = DIFFUSIVITY(ND)
   WRITE(4,*) '**** DIFFUSIVITY= ',DIF
   WRITE(*,*) '**** DIFFUSIVITY= ',DIF
20 CONTINUE
C STARTING OF "DO" LOOP FOR DIFFERENT TIMES
DO 30 NT=1,5
   T=TIME(NT)
   WRITE(4,*)'TIME=',T
   WRITE(*,*)'TIME=',T
   IF(DIF.LE.1.D-1) THEN
      Z=-DELZ1
   ELSE
      Z=-DELZ2
   ENDIF
   NCOUNT=0
C STARTING OF "DO" LOOP FOR DIFFERENT DEPTH.
DO 40 NZ=1,MAXIT
   IF(DIF.LE.1.D-1) THEN
      Z=Z+DELZ1
   ELSE
      Z=Z+DELZ2
   ENDIF
```
