



Three dimensional finite element analysis of matrix cracks in multidirectional composite laminates
by Modayur Shrinivas

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Mechanical Engineering
Montana State University
© Copyright by Modayur Shrinivas (1993)

Abstract:

This thesis presents the results of three dimensional finite element analyses of cross-plyed ([0/90/0]) and angle-plyed ([0/±45]s) laminates with cracks in the off-axis plies. The materials are modelled with homogenous orthotropic layers or regions. The effects of actual microstructures are considered in two stages, first by including matrix interlayers between plies, and then by including discrete strands of fibers within the layers. The discrete strands model the structure of the reinforcement in typical wind turbine blades and other common composites. This work does not model interply delamination regions, which will be introduced in subsequent studies.

The [0/90/0] laminates were analyzed for two different sets of material properties representing glass fiber/epoxy and carbon fiber/epoxy composites. These two classes of laminates were analyzed for three different models: a) no epoxy layer between plies, b) uncracked epoxy layer between plies, and c) cracked epoxy layer between plies. Stress concentrations were lower in general in carbon/epoxy laminates than in glass/epoxy laminates. The three-dimensional analysis appears necessary to predict the effects of off-axis ply cracking on axial-ply failure for the glass/epoxy case. The presence of an epoxy interlayer reduces the stress concentrations considerably. While most of the stress components increase moderately if the interlayer is cracked (relative to an uncracked layer), some interlaminar stresses were reduced by cracking the layer. The interlayer results are consistent with experimental observations of O0 ply failure resulting from off-axis ply cracking in fatigue if the layers were stitched tightly together, preventing the formation of an interlayer.

The [0/90/0] laminate was also analyzed with a discrete band of high fiber material surrounded by pure matrix in the O0 ply, instead of homogenous ply properties. The crack then penetrated through the 90° ply, the epoxy interlayer, and the epoxy region in the O0 ply. This geometry corresponds to many composites composed of strands which are woven or stitched together. The shape of the strand cross-section was also found to be important. In general, the axial stresses in the high fiber region were increased relative to homogenous layers, but the other stress components were not strongly affected.

Similar results were obtained for a [0/±45]s laminate with both homogenous plies and a O0 ply with a strand structure. The laminates contained crossing matrix cracks in the (+) and (-) 45° plies. Stress concentrations at the intersection were higher than for the [0/90/0] case due to the off-axis ply orientation and the greater proportion of off-axis plies. Stress concentrations appear high enough to cause O0 ply failure when the ±45 plies crack, even at low applied stresses. The interlaminar stresses are also very high, and should lead to interply delamination. Both of these findings are consistent with experimental observations.

**THREE DIMENSIONAL FINITE ELEMENT ANALYSIS OF MATRIX CRACKS
IN MULTIDIRECTIONAL COMPOSITE LAMINATES**

by

Modayur Shrinivas

a thesis submitted in partial fulfillment
of the requirements for the degree

of

Master of Science

in

Mechanical Engineering

**MONTANA STATE UNIVERSITY
Bozeman, Montana**

May 1993

71378
Sh 866

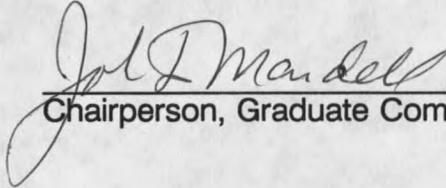
APPROVAL

of a thesis submitted by

Modayur Shrinivas

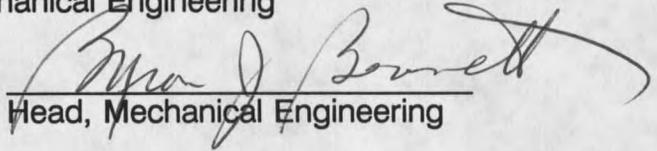
This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

4/22/93
Date


Chairperson, Graduate Committee

4/22/93
Date

Approved for Mechanical Engineering


Head, Mechanical Engineering

4/30/93
Date

Approved for the College of Graduate Studies


Graduate Dean

STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a master's degree at Montana State University, I agree that the library shall make it available to borrowers under rules of the library.

If I have indicated my intention to copyright this thesis by including a copyright notice page, copying is allowable only for scholarly purposes, consistent with "fair use" as prescribed in the U.S. Copyright Law. Requests for permission for extended quotation from or reproduction of this thesis in whole or in parts may be granted only by the copyright holder.

Signature M. Shumway

Date 04/22/93

ACKNOWLEDGEMENTS

There are a few people to whom credit should be extended for making this research and thesis possible. First and foremost, I would like to extend my thanks to Dr. John Mandell, who provided the inspiration and guidance that gave shape and substance to this thesis. I wish to thank Dr. Jerry Dwyer for his proofreading and constructive criticism. I am grateful to my fellow graduate students and friends who helped me during this research in one way or the other. I wish to acknowledge Dr. VanLuchene of the Civil Engineering Department and the staff of the Chemical Engineering Department for providing the computing resources. Special mention must be made of the National Center for Super Computing Applications, for the use of their super computer. Financial support for this research was provided by Sandia National Laboratories, for which I am grateful. I thank my family who stood behind me all through my pursuit of a graduate degree.

TABLE OF CONTENTS

	Page
1. INTRODUCTION	1
Damage	1
Damage in Cross-ply Laminates	1
Fatigue of Wind Turbine Blade Materials	4
Thesis Objective and Approach	6
Thesis Organization	7
2. TEST PROBLEM	8
Introduction	8
Problem Formulation	8
Results and Discussion	12
Summary	18
3. ANALYSIS OF [0/90/0] COMPOSITE LAMINATES WITH A CRACKED 90° ply	20
Introduction	20
Model Geometry	20
Models Analyzed	22
Problem Formulation	22
Results and Discussion	28
Summary	64
4. ANALYSIS OF [0/90/0] LAMINATES WITH A DISCRETE HIGH FIBER STRAND	66
Introduction	66
Model Analyzed	66
Results and Discussion	75
Summary	86
5. ANALYSIS OF [0/±45] _s LAMINATE WITH CRACKED ANGLE PLIES	88
Introduction	88
Model	88
Results and Discussion	94
Summary	106

TABLE OF CONTENTS-Continued

6. ANALYSIS OF A $[0/\pm 45]_s$ LAMINATE WITH A DISCRETE HIGH FIBER STRAND	108
Introduction	108
Model Analyzed	108
Results and Discussion	112
Summary	123
7. CONCLUSIONS AND RECOMMENDATIONS	124
Conclusions	124
Recommendations	126
REFERENCES	128
APPENDICES	132
Appendix A - Static Analysis Using ANSYS 4.4a	133
Appendix B - Three Dimensional Anisotropic Solid Element - STIF64	135
Appendix C - Wavefront Solver in ANSYS	138

LIST OF TABLES

Table	Page
1. Material Properties	28
2. STIF64 Element Description	135

LIST OF FIGURES

Figure		Page
1.	Radiograph showing transverse crack, longitudinal cracks and delaminations in a cross-ply laminate. The schematic shows details around a delamination. From [1].	3
2.	Test Problem - Model Geometry	11
3.	Test Problem - Present Study.	14
4.	Test Problem - Comparison. Stresses in the $[\pm 45]_s$ Laminate . . .	15
5.	Interlaminar Stress in the $[0,90]_s$ Laminate. Normal Stress S_z along $z=h$ for $b/h=8$	16
6.	Interlaminar Stress in the $[0,90]_s$ Laminate (near free edge). Normal Stress S_z along $z=h$ for $b/h=8$	17
7.	$[0/90/0]$ Composite Laminate with a Cracked 90° ply (without epoxy interlayer)	21
8.	$[0/90/0]$ Composite Laminate with a Cracked 90° Ply (with epoxy interlayer)	23
9.	$[0/90/0]$ Composite Laminate with a Cracked 90° Ply (Quadrant modelled)	24
10.	Finite Element Mesh of the $[0/90/0]$ Laminate	26
11.	Finite Element Mesh of the $[0/90/0]$ Laminate (Close-up View) . .	27
12.	Through-thickness Stresses. $[0/90/0]$ Glass/Epoxy Laminate . . .	30
13.	Through-thickness Stresses - Comparison (glass/epoxy laminate)	33
14.	Stress in the Y direction in the 90° Ply (glass/epoxy laminate) . . .	34
15.	Stresses in the Y Direction at the $0/90$ Interface (glass/epoxy laminate)	35
16.	Stress in the Y direction in the 0° Ply	36

LIST OF FIGURES-Continued

Figure	Page
17. Through-thickness Stresses at the crack (carbon/epoxy laminate)	37
18. Through-thickness Stresses - Comparison (carbon/epoxy laminate)	38
19. Stress in the Y direction in the 90 ⁰ Ply (carbon/epoxy laminate) .	40
20. Stresses in the Y Direction at the 0/90 Interface (carbon/epoxy laminate)	41
21. Stress in the Y direction in the 0 ⁰ Ply (carbon/epoxy laminate) ..	42
22. Through-thickness Stresses. [0/90/0] Glass/Epoxy Laminate ...	43
23. Through-thickness Stresses - Comparison (glass/epoxy laminate)	44
24. Stress in the Y direction in the 90 ⁰ Ply (glass/epoxy laminate) ...	45
25. Stresses in the Y Direction at the 0/90 Interface (glass/epoxy laminate)	46
26. Stress in the Y direction in the 0 ⁰ Ply (glass/epoxy laminate)	47
27. Through-thickness Stresses at the crack (carbon/epoxy laminate)	48
28. Through-thickness Stresses - Comparison (carbon/epoxy laminate)	49
29. Stress in the Y direction in the 90 ⁰ Ply (carbon/epoxy laminate) .	50
30. Stresses in the Y Direction at the 0/90 Interface (carbon/epoxy laminate)	51
31. Stress in the Y direction in the 0 ⁰ Ply (carbon/epoxy laminate) ..	52
32. Through-thickness Stresses. [0/90/0] Glass/Epoxy Laminate ...	54
33. Through-thickness Stresses - Comparison (glass/epoxy laminate)	55
34. Stress in the Y direction in the 90 ⁰ Ply (glass/epoxy laminate) ...	56

LIST OF FIGURES-Continued

Figure	Page
35. Stresses in the Y Direction at the 0/90 Interface (glass/epoxy laminate)	57
36. Stress in the Y direction in the 0° Ply (glass/epoxy laminate)	58
37. Through-thickness Stresses at the crack (carbon/epoxy laminate)	59
38. Through-thickness Stresses - Comparison (carbon/epoxy laminate)	60
39. Stress in the Y direction in the 90° Ply (carbon/epoxy laminate) .	61
40. Stresses in the Y Direction at the 0/90 Interface (carbon/epoxy laminate)	62
41. Stress in the Y direction in the 0° Ply (carbon/epoxy laminate) . .	63
42. Cracking in 0° Ply of [0/±45] _s Triax glass/polyester Wind Turbine Blade Material, Showing High Fiber Content Strands with Matrix in between [25]	67
43. Model Geometry of the [0/90/0] Laminate	71
44. Cross-section of the [0/90/0] Laminate at the Crack Rectangular High Fiber Region	72
45. Cross-section of the [0/90/0] Laminate at the Crack Rounded High Fiber Region	73
46. Finite Element Mesh at the Crack - Rounded High Fiber Region .	74
47. Stresses around the High Fiber Region - Effect of Corner Angle .	76
48. Through Thickness Stresses in the High Fiber Region	78
49. Through Thickness Stresses in the Average Fiber Region	79
50. Stresses in the 0° Ply Parallel to the X axis	81
51. Stresses in the Y direction at the High Fiber/Epoxy Interface . . .	83

LIST OF FIGURES-Continued

Figure	Page
52. Stresses in the Y direction at the High Fiber/Epoxy Interface . . .	84
53. Deformation in the Laminate (deformation exaggerated)	85
54. $[0/\pm 45]_s$ Laminate Model Geometry	89
55. Cross-section of the $[0/\pm 45]_s$ Laminate	91
56. Finite Element Mesh of the $[0/\pm 45]_s$ Laminate	92
57. Finite Element Mesh of the $[0/\pm 45]_s$ Laminate (Close-up View). .	93
58. Through Thickness Stresses at the Intersection of +45 and -45 Cracks	95
59. Through Thickness Stresses at the Intersection of +45 and -45 Cracks (Comparison)	98
60. Along -45 Crack/Epoxy Layer Interface from point (1) to point (2)	99
61. Along +45 Crack/Epoxy Layer Interface from point (1) to point (2)	100
62. Through Thickness Stresses near the Crack Intersection passing through +45 Crack Only	101
63. Through Thickness Stresses near the Crack Intersection Passing Through -45 Crack Only	102
64. Through Thickness Stresses Passing Through +45 Crack Only	104
65. Through Thickness Stresses Passing Through -45 Crack Only	105
66. $[0/\pm 45]_s$ Laminate with Discrete High Fiber Strand - Model Geometry	109
67. $[0/\pm 45]_s$ Laminate with Discrete High Fiber Strand - Cross Section	110
68. Through Thickness Stresses at the Intersection of +45 and -45 Cracks	113

LIST OF FIGURES-Continued

Figure		Page
69.	Through Thickness Stresses at the Intersection of +45 and -45 Cracks (Comparison)	115
70.	Along +45 Crack/Epoxy Layer Interface. (along 1-2-3)	116
71.	Along -45 Crack/Epoxy Layer Interface. (along 1-2-3)	117
72.	Stresses Around the High Fiber Region at the +45 Crack Plane .	119
73.	Through Thickness Stresses, passing through +45 Crack Only . .	120
74.	Through Thickness Stresses, passing Through -45 Crack Only . .	121
75.	* STIF 64 3-D Solid Element	137

ABSTRACT

This thesis presents the results of three dimensional finite element analyses of cross-plyed ($[0/90/0]$) and angle-plyed ($[0/\pm 45]_s$) laminates with cracks in the off-axis plies. The materials are modelled with homogenous orthotropic layers or regions. The effects of actual microstructures are considered in two stages, first by including matrix interlayers between plies, and then by including discrete strands of fibers within the layers. The discrete strands model the structure of the reinforcement in typical wind turbine blades and other common composites. This work does not model interply delamination regions, which will be introduced in subsequent studies.

The $[0/90/0]$ laminates were analyzed for two different sets of material properties representing glass fiber/epoxy and carbon fiber/epoxy composites. These two classes of laminates were analyzed for three different models: a) no epoxy layer between plies, b) uncracked epoxy layer between plies, and c) cracked epoxy layer between plies. Stress concentrations were lower in general in carbon/epoxy laminates than in glass/epoxy laminates. The three-dimensional analysis appears necessary to predict the effects of off-axis ply cracking on axial-ply failure for the glass/epoxy case. The presence of an epoxy interlayer reduces the stress concentrations considerably. While most of the stress components increase moderately if the interlayer is cracked (relative to an uncracked layer), some interlaminar stresses were reduced by cracking the layer. The interlayer results are consistent with experimental observations of 0° ply failure resulting from off-axis ply cracking in fatigue if the layers were stitched tightly together, preventing the formation of an interlayer.

The $[0/90/0]$ laminate was also analyzed with a discrete band of high fiber material surrounded by pure matrix in the 0° ply, instead of homogenous ply properties. The crack then penetrated through the 90° ply, the epoxy interlayer, and the epoxy region in the 0° ply. This geometry corresponds to many composites composed of strands which are woven or stitched together. The shape of the strand cross-section was also found to be important. In general, the axial stresses in the high fiber region were increased relative to homogenous layers, but the other stress components were not strongly affected.

Similar results were obtained for a $[0/\pm 45]_s$ laminate with both homogenous plies and a 0° ply with a strand structure. The laminates contained crossing matrix cracks in the (+) and (-) 45° plies. Stress concentrations at the intersection were higher than for the $[0/90/0]$ case due to the off-axis ply orientation and the greater proportion of off-axis plies. Stress concentrations appear high enough to cause 0° ply failure when the ± 45 plies crack, even at low applied stresses. The interlaminar stresses are also very high, and should lead to interply delamination. Both of these findings are consistent with experimental observations.

CHAPTER 1

INTRODUCTION

Damage

One of the first mechanisms of failure to occur in many composite laminates under static or fatigue loading is the development of cracks in the cross plies and angle plies [1]. These cracks are often the cause of the development of other types of damage like delamination which finally lead to catastrophic failure of the whole laminate. The word "damage" is often used loosely to describe some undesirable change which is thought to have occurred to a material or structure [2]. The damage and its effect on the structure are separate entities. A particular way to define damage is to identify it with the changes in the constitution (microstructure or macrostructure, depending on the scale) of the material. The changes in the material (or structural) response (i.e. properties) can then be related to the appropriate measures of the underlying changes of the material constitution. Thus, damage and its consequences are separate entities.

Damage in Cross-ply Laminates

Cross-ply laminates are composed of unidirectional layers (with parallel aligned fibers). The direction of the layers (plies) alternate in mutually perpendicular directions, such as 0/90/0/90..., where the angle given is relative to the load direction. This class of laminates has been studied extensively and the data

concerning damage and its effect have been richly documented. In the following, a brief description of damage in cross-ply laminates of carbon/epoxy is given. Further details are described in Jamison et al. [3].

When a cross-ply laminate of carbon/epoxy is loaded in tension along the longitudinal direction, a number of cracks appear in the transverse plies at a certain value of the applied stress. The cracks span the thickness of the transverse plies and almost immediately grow in the transverse direction (Stage I). As the stress is increased, more cracks appear in the transverse direction between previously formed cracks. The planes of the cracks are roughly normal to the direction of stress and the spacing is fairly uniform. Under fatigue loading, the crack spacing is found to decrease further and a minimum spacing may eventually result irrespective of the loading path. Beyond this saturation spacing, further loading appears to change the nature of the cracking process. Figure 1, taken from [3] shows the crack pattern in Stage II. The horizontal lines are 90° ply cracks (cracks parallel to the fibers of the 90° plies) and the vertical lines are axial cracks parallel to the fibers in the 0° plies, which are symmetrically placed on both sides of the transverse plies. The dark zones in the radiograph seen at intersections of transverse and longitudinal lines were shown by Jamison et al. [3] to be delaminations (cracks in the plane of the sheet between the 0° and 90° plies). The final stage, Stage III of the damage development in cross-ply laminates was shown by Jamison et al. [3] to consist of coalescence of delaminations in regions between two longitudinal cracks, and fiber failures in the axial (0°) plies.

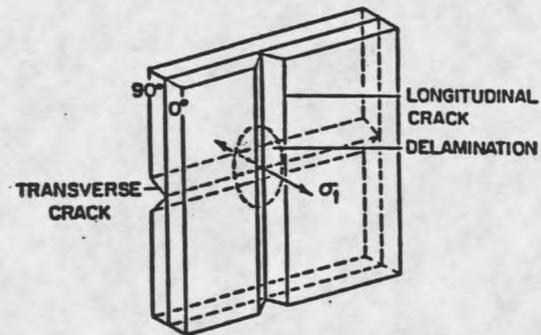
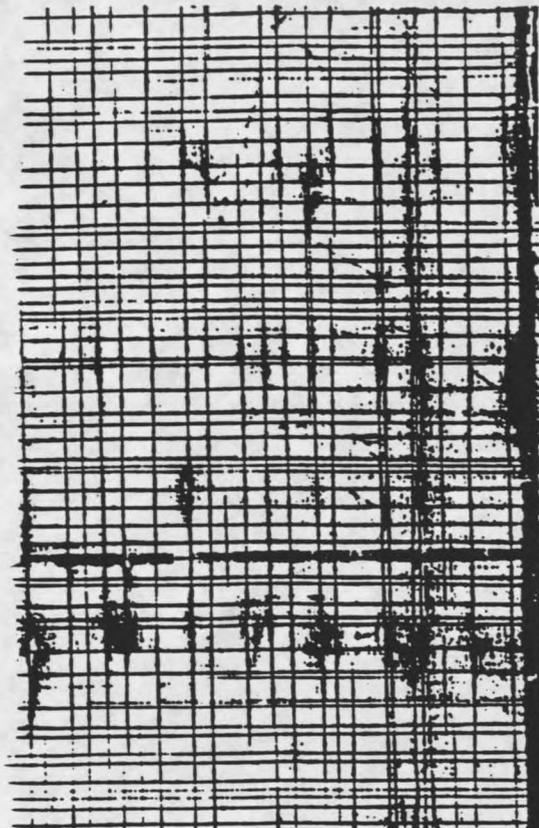


Figure 1. X-ray radiograph showing transverse crack, longitudinal cracks and delaminations in a cross ply laminate. The schematic shows details around a delamination. From [1].

The high mechanical performance of fiber reinforced plastics is generally governed by the fiber properties. However, the matrix properties can be important under certain loading conditions and fiber configurations [4]. Matrix micro-damage occurs in cross-ply fiber reinforced plastics even when the matrix has a higher failure strain than the fibers [5-12]. Generally, the onset of matrix micro-failure in glass-reinforced polyester specimens occurs between 0.2 and 0.5% strain and is associated with the characteristic "knee" found in the stress-strain curves of cross-ply laminates [4].

Fatigue of Wind Turbine Blade Materials

The high cycle fatigue resistance (lifetime in the 10^6 to 10^9 cycle range) of composite materials used in wind turbine rotor blades has been recognized as a major uncertainty in predicting the reliability of wind turbines over their design lifetime [13,14]. These blades typically experience 10^8 to 10^9 significant fatigue cycles over their lifetime of 20 to 30 years. This is well beyond the cycle range of most aerospace structures.

As with other laminates, the fatigue behavior of these materials is distinguished by several important general features [14-17] :

1. Failure is usually progressive, resulting from the gradual accumulation and interaction of dispersed damage, rather than by the nucleation and growth of a dominant crack.

2. As damage accumulates, the constitutive relations of the material may change significantly.
3. A number of distinct damage modes can be identified, including fiber dominated tension and compression, matrix dominated cracking parallel to fibers and interlaminar cracking between plies. While some of these may cause failures directly, certain modes like matrix cracking may have an indirect effect on failure by causing load transfer to fibers.
4. Under tensile loading, the strains to produce matrix cracking are generally well below those to produce fiber failure. As a consequence, in multidirectional composites, cracking tends to initiate first in domains where fibers are at the greatest orientation relative to maximum tensile stress (like 90° plies). Cracking accumulates in these domains and then in domains of lesser orientation (like the 45° plies). Delamination between plies may occur at cut edges, ply terminations or at the intersection of matrix cracks in adjoining plies. Finally, gross failure occurs by fiber breakage in any domains which have plies with the least orientation to maximum stress (like the 0° plies).
5. Large-scale delamination between plies has been a significant failure mode for composite structures, particularly with out-of-plane loads.
6. Theoretical models for damage progression and failure are under development, but no general approach to lifetime prediction for

composites is widely accepted. Only delamination failures have a well developed theoretical context in classical fracture mechanics.

Cracks in composite laminates affect the stiffness and strength of the laminate on the macro scale. The stress distributions around the cracks can be quite complex even under relatively simple static loading. Analytical approaches are often not adequate for completely describing the stress distributions around cracks. A numerical approach is much more feasible under these circumstances. The cracks are sites of high stress gradients and the problem is completely three dimensional. The nature of the problem is amenable to finite element analysis.

Thesis Objective and Approach

Point 3 cited above is the particular focus of this thesis. The stress redistribution due to matrix cracking is especially important as it may lead to laminate failure. The thesis explores the load transfer to the plies which have fibers oriented parallel to maximum applied stress. This knowledge is important as it addresses stress distribution on a micro scale and the possible course of future delamination as well as fiber failures.

The nature of the stress gradients near the crack are examined using three dimensional finite element analysis. Particular attention is paid to interlaminar shear stresses as these stresses are responsible for delamination at a later stage. The type of laminates that have been studied are the $[0/90/0]$ and $[0/\pm 45]_s$; in each case, crack symmetries representative of experimental observation are considered.

Three levels of representation of the material structure are included : uniform orthotropic plies, uniform orthotropic plies with a pure matrix layer between each ply and plies with discrete strands of high fiber content. The latter is the closest representation of the actual materials used in wind turbine blades.

Thesis Organization

Chapter 2 gives the results of a test case for comparison with literature results. This establishes the validity of the finite element model and mesh refinement chosen for the actual case. Chapter 3 presents the results and discussion of the effects of 90^0 ply matrix cracking on the stress field, including the effects of ply elastic constants and a matrix interlayer between plies (both cracked and uncracked layers). Chapter 4 extends these results to discrete fiber strands rather than uniform plies. Chapter 5 analyzes the $[0/\pm 45]_s$ laminates, considering similar structural parameters. Chapter 6 explores the stress distributions in a $[0/\pm 45]_s$ laminate with a discrete fiber strand in the 0^0 ply. Chapter 7 discusses conclusions and recommendations.

CHAPTER 2

A TEST PROBLEM - INTERLAMINAR STRESSES IN COMPOSITE LAMINATES

Introduction

The response of a finite-width composite laminate under uniform axial extension is analyzed using finite element techniques. These techniques are employed to obtain solutions for stresses throughout the region of interest with special emphasis on the laminate free edges. Results for material properties typical of a high modulus graphite epoxy system are presented and are compared with several literature solutions to the same problem. The purpose of this exercise is to establish the accuracy of the method employed in this study and to establish the finite element mesh refinement needed to obtain accurate results for stress gradients of this general type.

Problem Formulation

Symmetric laminates are laminates in which plies are stacked symmetrically about the mid-thickness, such as $[+45,-45,-45,+45]$. The analytical technique for determining the in-plane, elastic response of a laminated composite called the lamination theory (see Ref.[18] for a detailed discussion of this theory), is based upon the assumption that a state of plane stress exists for symmetric laminates under in plane tractions. When the laminate is composed of layers of different orientations, lamination theory implies certain impossible boundary tractions on a

free edge [19-21,28]. This problem is modelled without such assumptions using finite element techniques and the results are compared with published results.

Consider a laminate loaded by tractions applied only on its ends $x = \text{constant}$ (Figure 2), such that the stress components are independent of x . Now, assuming Saint Venant's principle holds for a laminate, this stress distribution will exist in regions sufficiently removed from areas of load introduction.

The strain-displacement relations are as follows

$$\begin{aligned} \epsilon_x &= u_{,x} & \epsilon_y &= v_{,y} & \epsilon_z &= w_{,z} \\ \gamma_{yz} &= w_{,y} + v_{,z} & \gamma_{xz} &= w_{,x} + u_{,z} & \gamma_{xy} &= v_{,x} + u_{,y} \end{aligned}$$

where a comma denotes partial differentiation. The u, v and w are displacements along x, y and z directions respectively and ϵ and γ denote normal and shear strains respectively. By Saint Venant's principle, the equilibrium conditions take the reduced form,

$$\begin{aligned} \tau_{xy,y} + \tau_{xz,z} &= 0 \\ \sigma_{y,y} + \tau_{yz,z} &= 0 \\ \tau_{yz,y} + \sigma_{z,z} &= 0 \end{aligned}$$

Since we are concerned only with symmetric angle-ply laminates under extensional loading, we can enforce symmetric boundary conditions for the displacements with respect to the x - y and x - z planes.

(1) x - y plane

$$u(x,y,z) = u(x,y,-z)$$

$$v(x,y,z) = v(x,y,-z)$$

$$w(x,y,z) = -w(x,y,-z)$$

(2) x-z plane

$$v(x,y,z) = -v(x,-y,z)$$

$$w(x,y,z) = w(x,-y,z)$$

These symmetry conditions are for the whole laminate. When half a quadrant of the model is built to exploit these symmetry conditions, appropriate boundary conditions (B.C.'s) are imposed as will be explained later.

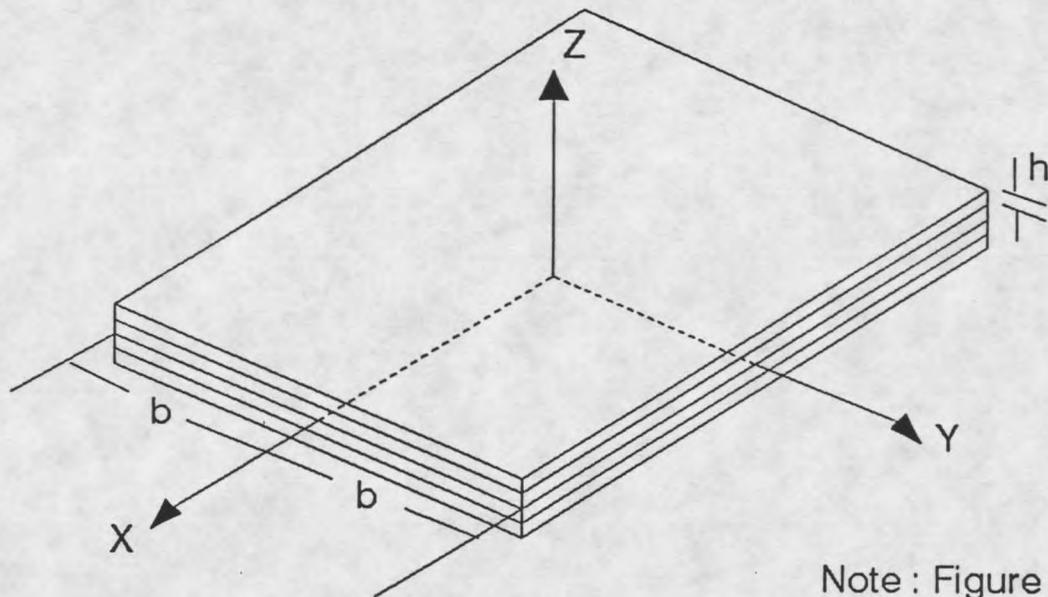
The problem was solved using the finite element method. By this method, the continuous material region was replaced by discrete elements. Discrete values of the dependent variables (like stresses) are determined at discrete points (nodes) on the elements. A large number of elements are required at the material free edges to capture the high stress gradients in these areas. So, in building the model, a fine mesh is used near the free edges to capture the high stress gradients in these regions. The finite element method software ANSYS 4.4a was used and STIF64 anisotropic brick elements (refer Appendix B for details on this element type) were used to build the F.E. model. This element is a complete 3-D anisotropic element which is required by the nature of our problem.

The boundary conditions imposed were

$$u(0,y,z) = 0 \quad v(x,0,z) = 0 \quad w(x,y,0) = 0$$

and the extension condition

$$u(X_0,y,z) = U$$



Note : Figure not to scale.

Figure 2. Test Problem - Model Geometry.

U is a displacement boundary condition value from which the uniform axial strain can be calculated as,

$$U/X_0 = \epsilon_x$$

The F.E. mesh is graded so that smaller elements are used near the laminate free edges. The smallest element size used near the free edge was $h/25$. Different element sizes were used near the laminate free edge and the results compared. For an element size less than $h/25$, stress values near the free edge were not significantly different from the values for $h/25$. Since lowering the element size increases the number of degrees of freedom, the smallest element size was kept at $h/25$ ($0.04 h$). The problem was modelled and solved on an IBM RS/6000 workstation.

Results and Discussion

The laminate under consideration is a $\pm 45^\circ$ laminate under a uniform axial strain, ϵ_x . Laminate theory predicts a uniform, planar state of stress in each layer with the axial stress component σ_x , and a non-zero in-plane shear stress component τ_{xy} , which arises from the shear coupling term of the layer stiffness matrices. These results are accurate for laminates of theoretically infinite width. However, they are inaccurate for a finite width laminate since the in-plane shear stress is required to vanish along the free edge (see Figure 2).

A high modulus carbon/epoxy system was modelled with the geometric relationship $b=8h$, and material properties,

$$E_{11} = 20.0 \times 10^6 \text{ psi}; \quad E_{22} = E_{33} = 2.1 \times 10^6 \text{ psi}; \quad G_{12} = G_{13} = G_{23} = 0.85 \times 10^6 \text{ psi};$$

$$\nu_{12} = \nu_{13} = \nu_{23} = 0.21$$

where the subscript '1' refers to the fiber direction and '2' refers to the direction perpendicular to the fiber direction in the plane of the laminate and '3' refers to the direction normal to the plane of the laminate. This geometry and properties were chosen to follow the literature case for comparison in References [22] and [23] as well as for comparison with a solution for $[0,90]_s$ laminates by Wang and Dickson [24].

Figure 3 shows the stress distribution at the interface of $\pm 45^\circ$ plies. Actually, the results are plotted at $z = .992 h$ instead of $z = h$ to avoid stress averaging that takes place at the interface which is a region of two dissimilar layers. The interlaminar stresses decay rapidly away from the free edge.

The results are compared with the works of Pipes and Pagano [22], Puppo and Evensen [21], Wang and Crossman [23] and Wang and Dickson [24]. It is seen quite clearly that the work of Puppo and Evensen, being an approximate formulation, has failed to capture the high stress gradients near the free surfaces. In particular, the interlaminar shear stress τ_{xz} in the Puppo and Evensen work does not appear singular, but rather takes on finite magnitude at the free edge. Comparing the results with those of Pipes and Pagano, it is seen that the stresses σ_x and τ_{xy} peak at a distance of $.05 b$ away from the free edge, and the stress τ_{xz} rises rapidly near the free edge. The results of Pipes and Pagano show no such sharp peak and there is only a very gradual and barely noticeable rise and dip in

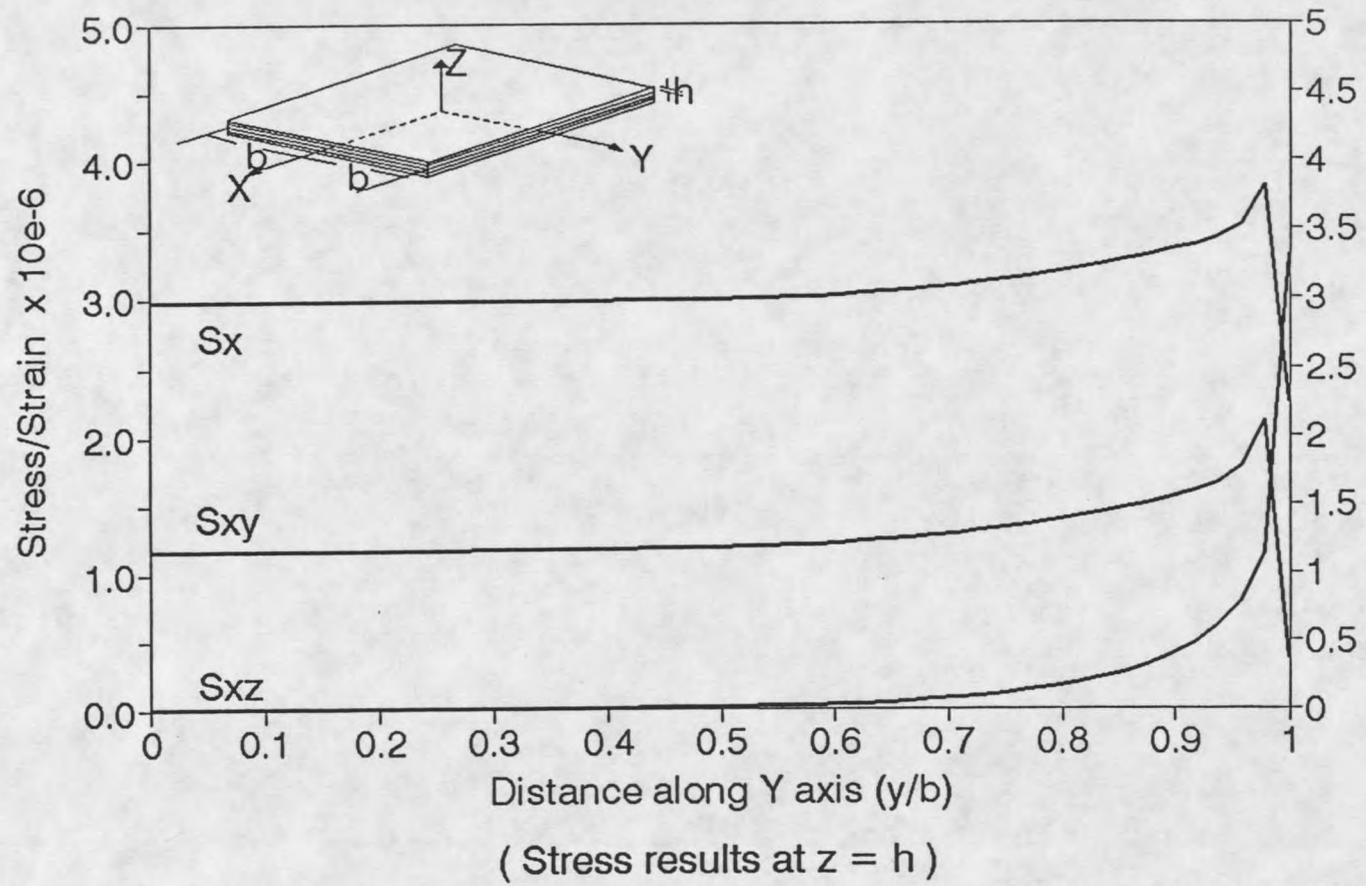


Figure 3. Test Problem - Present Study.

