Effects of electron-positron pairs on active galactic nuclei
by Bradley Gene Tritz

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics
Montana State University

Abstract:
Electron-positron pair creation is investigated in and around accretion flows within active galactic nuclei. First, two-temperature accretion disk models from the 1970's are re-examined, and the effects of pairs on disk structure are calculated. It is found that thermal pah- production produces insufficient pairs to greatly affect the disk, but that nonthennal processes produce sufficient pairs to significantly alter flow structure for a wide range of parameters. Then improvements are made upon earlier studies of pair-induced over- cooling of two-temperature spherical accretion flows. It is found that thermal processes produce insufficient pairs to cause over-cooling; however, nonthennal pair production can be an effective cooling agent. Finally, the origin, vertical structure, and radiation spectrum of steady-state pair cascade atmospheres surrounding accretion flows are investigated, by developing computer codes to model radiative transfer, scattering, pair production, and pair annihilation. It is found that substantial pair atmospheres may develop above accretion flows which emit even a small fraction of their luminosity as gamma- radiation. The radiation spectrum emitted by the flow may be significantly reprocessed during transit through the atmosphere, leading to interesting observational consequences. The conclusion is reached that under modest and reasonable assumptions, pairs can significantly alter the accretion flows within, and the radiation spectra emitted from, the central engines of active galactic nuclei.

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ABSTRACT

Electron-positron pair creation is investigated in and around accretion flows within active galactic nuclei. First, two-temperature accretion disk models from the 1970's are re-examined, and the effects of pairs on disk structure are calculated. It is found that thermal pair production produces insufficient pairs to greatly affect the disk, but that nonthermal processes produce sufficient pairs to significantly alter flow structure for a wide range of parameters. Then improvements are made upon earlier studies of pair-induced over-cooling of two-temperature spherical accretion flows. It is found that thermal processes produce insufficient pairs to cause over-cooling; however, nonthermal pair production can be an effective cooling agent. Finally, the origin, vertical structure, and radiation spectrum of steady-state pair cascade atmospheres surrounding accretion flows are investigated, by developing computer codes to model radiative transfer, scattering, pair production, and pair annihilation. It is found that substantial pair atmospheres may develop above accretion flows which emit even a small fraction of their luminosity as gamma-radiation. The radiation spectrum emitted by the flow may be significantly reprocessed during transit through the atmosphere, leading to interesting observational consequences. The conclusion is reached that under modest and reasonable assumptions, pairs can significantly alter the accretion flows within, and the radiation spectra emitted from, the central engines of active galactic nuclei.

CHAPTER 1

INTRODUCTION

Among the most intriguing objects in nature are the quasars, the brightest and most distant objects known. In the years since their discovery in the 1960s, astronomers and astrophysicists have come to the realization that these objects are (exceedingly bright) members of a much larger and tremendously varied group: the active galaxies. These galaxies derive their name from the fact that the galactic central core region, called the galactic nucleus, is unusually energetically active. Some measure of appreciation for the extraordinary nature of active galactic nuclei (AGNs) may be gained from a glance at a few of their many spectacular characteristics. For example, the central engine of AGNs (the region where most of the energy is produced) is thought to be some 10 orders of magnitude smaller than the host galaxy as a whole, but can, in the case of bright quasars, outshine the rest of the galaxy by a factor of 10^3 or more. Some AGNs (in what are known as radio galaxies) are seen to propel powerful and remarkably collimated beams of radiation and matter (cosmic jets) which fuel vast radio-emitting structures (radio lobes) a thousand times larger than the host galaxy. And many AGNs exhibit the intriguing ability to radiate more or less uniformly across some 11 or so decades of frequency, from radio through hard gamma-rays. Many of the prominent spectral features observed in AGNs, along with many theoretical considerations, are consistent with the now almost universally accepted model of an AGN as an accreting supermassive black hole ($M=10^{6-9}M_{sol}$), residing in the center of the galaxy. If any readers are not familiar with basic AGN theory and observation, they are urged to look over the necessarily brief AGN primer found in the Appendix, before reading the remainder of this paper.

This thesis deals with the effects of electron-positron pairs in AGNs. In this introductory chapter we shall first review simple arguments for the possibility of pair creation in AGNs. We then illustrate the need for studying pairs in AGNs by examining the literature, noting what work has been done in this field and what important lines of inquiry have been inadequately treated. Finally we propose the specific goals we wish to achieve in this thesis, and the methods by which we shall achieve them.

The Possibility of Pair Creation in Active Galactic Nuclei

Nonthermal Pair Creation

Of the AGNs close enough and bright enough to be viewed in γ-rays, many are found to be strong γ-ray emitters (see Appendix). Jelley (1966) was the first to suggest that electron-positron pair creation may occur in AGNs, when γ-rays interact with low energy photons through the quantum electrodynamical process:

$$\gamma+\gamma\rightarrow e^{+}+e^{-} \quad . \tag{1.1}$$

The subsequent 15 years or so saw relatively little progress made in investigating the consequences of this possibility. (Notable exceptions include Bonometto and Rees 1971, Herterich 1974, and Cavallo and Rees 1978.) This was due, in large part, to the exceedingly complex physics involved in accelerating particles to energies high enough to produce the γ-radiation necessary for pair creation. (If electrons produced the $h\upsilon>100$MeV photons observed in some sources, they would have to be ultrarelativistic, with Lorentz factors $\gamma>10^2$.) Plausible acceleration mechanisms include: strong shock waves, violent magnetic reconnection, and strong electric fields, among others. Interest in the field was renewed when Guilbert, Fabian, and Rees (1983) sidestepped the stubborn issue of acceleration physics and made the obvious but quite fruitful suggestion that much could be learned about AGN behavior by simply modeling particle acceleration as a continuous injection of some high-energy, nonthermal distribution of particles (usually electrons) throughout the emission region. The reasoning was that: (1) the violent variability seen in AGN spectra is a strong indication of chaotic conditions in which at least some particle acceleration is guaranteed; and (2) preliminary studies have shown that most of the mechanisms listed above may produce quite similar power-law particle spectra, so that only a few free parameters (power-law slope, upper and lower energy limits, and total injected power) would be needed to model a generic acceleration mechanism. The newly-accelerated particles would then cool by

Compton upscattering soft photons, by bremstrahlung emission, or by synchrotron emission, producing γ-radiation in the process, which would then produce pairs.

As Lightman and Zdziarski (1987) point out, acceleration of some electrons to $\gamma >> 1$ is energetically feasible because typical black hole accretion efficiencies (ratio of luminosity to rest mass accretion rate; see the Appendix) of 10% make available an energy up to 0.1 $m_p c^2$ per ionization electron (where m_p is the proton mass and we assume ionized hydrogen is accreting). This scheme is the basis for all "nonthermal" pair creation models considered in this paper.

If a significant fraction of accretion energy is indeed channeled nonthermally to γ-radiation, the criterion for significant pair production is that the source be sufficiently compact and luminous, and the resulting photon densities great enough, so that γ-rays will interact and pair produce instead of escaping from the system. This point may be quantified by a simple, widely used, estimate involving the compactness parameter l (Guilbert, Fabian, and Rees 1983), which is a dimensionless ratio of source luminosity L to source size r:

$$l = \frac{L\sigma_T}{4\pi r m_e c^3} , \qquad (1.2)$$

where σ_T is the Thomson cross section. When $l >> 1$ pair production should be important, with most ≈1MeV γ-rays pair producing instead of escaping. When $l << 1$ most of these photons will escape and little pair production is expected (see Chapter 2).

When applied to AGN central engines, the compactness parameter tells us on both theoretical and observational grounds that pair production could be important in these objects. Theoretically, following Lightman and Zdziarski (1987) and others, we may rewrite l as product of dimensionless ratios

$$l = (1/2)\,(m_p/m_e)\,(L/L_{Edd})\,(r_g/r)\ , \qquad (1.3)$$

where L_{Edd} is the Eddington luminosity (see Appendix), r_g=2GM/c^2 is the Schwartzchild radius of a black hole of mass M, and G is the gravitational constant. For $r \approx 10 r_g$, typical of many models, l can exceed unity even for sub-Eddington luminosities.

Observationally, we can estimate the compactness parameter by calculating L from an observed spectrum (the distance to the object must be known), and estimating source size r by the timescale of variabilty Δt (see the Appendix):

$$R \approx c\Delta t\ , \qquad (1.4)$$

to obtain the observed compactness parameter:

$$l_{obs} = \frac{L_{obs}\sigma_T}{4\pi \Delta t_{obs} m_e c^4}\ . \qquad (1.5)$$

AGN surveys (see Lightman and Zdziarski 1987 for a complete list) indicate values of l spread fairly uniformly over the interval 10^{-3} to 10^{1}, indicating the possible importance of pairs in many AGNs.

If γ-ray production does occur and if the source is sufficiently compact, so that some pairs are produced, there exists the possibility that these pairs will initiate a "nonthermal pair cascade" producing many more pairs. The cascade functions as a positive feedback loop in which the initial relativistic pairs cool by upscattering ambient soft photons to γ-ray energies; these new gamma rays produce a second generation of pairs, which will produce more γ-rays, which will produce more pairs, and so on. Whether these nonthermal processes will be effective or not is a complicated and highly model-dependent question.

Thermal Pair Creation

In contrast to the unequal sharing of energy characterizing nonthermal models, in models of "thermal" pair production, available energy is shared more or less equitably among electrons. Here the importance of pair production depends critically on the electron temperature T_e. If the dimensionless temperature $\theta_e \equiv kT_e/m_e c^2$ is greater than unity, most electrons will be relativistic and readily produce γ-rays, which in turn will produce pairs. If θ_e is not far below unity, the high energy tail of the Maxwell-Boltzmann distribution remains relativistic, and some γ-ray and pair production may result. Many AGN accretion flow models exhibit electron temperatures around $0.1<\theta_e<0.5$, indicating the possibility of significant thermal pair production. Whether pairs will indeed be important is, as was the case for nonthermal models, a difficult and highly model-dependent question.

In addition to photon-photon pair creation, there exist many other pair producing reactions, including photon-electron $\gamma+e \rightarrow e+e^+e^-$, electron-electron $e+e \rightarrow e+e+e^+e^-$, photon-proton $\gamma+p \rightarrow p+e^+e^-$, and so on. The photon-electron cross section is down by roughly a factor of α_f, the fine structure constant, from the photon-photon cross section. The electron-electron cross section is down by an additional factor of α_f, and the corresponding proton rates are even lower. Photon-photon reactions appear always to be the dominant pair creation process in AGNs (see Svensson 1987) because of the large value of its cross section and because particle densities remain too low in most AGN models.

Direct Observation of the 511 KeV Annihilation Line in AGN Spectra

No direct observation of the 511 keV annihilation feature has been reported in extragalactic AGN spectra. This is by no means a death nell to the theory of pairs in AGNs, however. Many theories have been set forth in which the annihilation line is scattered or absorbed on its way out of the source, or is merely intrinsically too weak to be detected. New more powerful detectors slated for the coming few years, and improved modeling (this paper, for example), should shed considerable light on the subject.

We note that an on-again, off-again annihilation line, carrying 10^{37}erg/s (Phinney 1983), has been observed coming from our Galactic center. Our Galactic nucleus has long been suspected to be weakly active, and if these observations are confirmed, we will have the first (but only) direct evidence of pairs in AGNs.

The Investigation of Pair Creation in Specific Accretion Models

Current theory tells us that matter accreting onto a black hole may assume any number of geometrical forms. However, if any angular momentum is originally present as mass is fed into the system from great distances, or if the hole is spinning and the Lense-Thirring effect is in operation, the accretion flow should settle into some sort of accretion disk in the region near the hole (see the Appendix). It is in this central portion of the accretion flow where most of the gravitational potential energy is released. Any atmosphere of matter or radiation present in the space surrounding the flow (above or below the disk) may scatter, absorb, or re-emit portions of the primary spectrum emitted from the accretion flow, constituting so-called spectral reprocessing. What we see through our telescopes and detectors is the reprocessed primary spectrum. Any successful model of AGN central engines must therefore address both accretion flow and atmosphere. We now consider specific important AGN flow and atmosphere models in which the effects of pair creation could be great, but for which this possibility has not been adequately investigated, or investigated at all.

A quick survey of conventional wisdom tells us that accretion flows can surround themselves with at least five fundamentally different kinds of atmospheres : (1) radiation pressure driven winds, in which intense radiation from the inner regions of the flow blows matter (pairs or otherwise) outward (Leighly 1990); (2) evaporative/diffusive atmospheres, in which particles with sufficiently high velocities may overcome any (as yet unknown) containment forces and "evaporate" from the surface of the flow (Shakura and Sunyaev 1973); (3) magneto-coronal atmospheres, in which shear amplified magnetic fields induce an instability by which matter in magnetic flux tubes becomes buoyant and rises up out of the flow, perhaps resulting in magnetic flares, prominences, and a high temperature corona much like those seen on the sun (Galeev, Rosner and Vaiana 1979); (4) cosmic jets, in which a complex of ordered magnetic and electric fields tap into the rotational energy of the (spinning) black hole and propel radiation and/or matter out more or less along the rotational axis (Phinney 1983; Burns and Lovelace 1982); and (5) pair cascade atmospheres, in which γ-rays, emitted from the flow, or produced by, say, high temperature electrons in a mageto-corona, initiate a pair cascade which populates the atmosphere with pairs (see comments in Svensson 1985).

Of these five models, only pair winds and jets have received a great deal of treatment (see the above references). We turn our attention to one of the neglected areas, the pair cascade atmosphere. If the assertion by Guilbert, Fabian, and Rees (1983) is true, and AGN accretion flows are indeed natural γ-ray emitters, pair production should not only occur inside the flow, but outside as well, and some sort of pair atmosphere should form. This has yet to be investigated in any detail at all. The task is a difficult one: the situation is intrinsically three-dimensional; the outer limits and spatial structure of the atmosphere are a priori unknown; the atmosphere may be optically thin, so that radiative transfer will link together all regions of the atmosphere; the atmosphere depends critically on the emission spectrum and geometry of the accretion flow, which in turn may be affected by the atmosphere; and several physical processes (scattering, pair creation, pair annihilation, radiative transfer, particle transfer, ect.) are concurrently at work.

A survey of the literature reveals a large number of accretion disk models that have been proposed over the last twenty years or so. Many of these are variations on one or the other of the two most widely referenced models, the cool, optically thick, geometrically thin disk of Shakura and Sunyaev (1973), and the hot, optically thin, geometrically thin, two-temperature disk of Shapiro, Lightman, and Eardley (1976). These two models were originally formulated as pair-free, without taking into account any possible pair creation. Let us turn our attention to the two-temperature disk. As we note in the Appendix, two-temperature accretion flows are predicted for a large range of flow conditions in AGNs. The presence of pairs could significantly alter the state of matter and radiation within the disk, affecting disk structure and emission spectrum. Before the present work, the only attempt to calculate equilibrium pair densities in the two-temperature disk was by Liang (1979), who noted correctly that the high electron tempertures in the pair free model ($T_e=10^{9-10}$K) indicated pairs might be important. His results, which showed that thermal pair production was very important, with the density of pairs greatly exceeding the density of protons, were seriously questioned however, when Svensson (1985) and others found his methods to suffer many critical flaws. Also, no investigations into nonthermal pair production have been conducted on this or any other disk whatsoever.

Inflated accretion flows (spherical or quasi-spherical geometry) have been investigated somewhat more thoroughly for the effects of pairs than have the disks (see, for example: Svensson 1987; Lightman and Zdziarski 1987; Done and Fabian 1989; Fabian et al. 1986; and Begelman, Sikora, and Rees 1987). As was the case with the disks, there are many models of spherical accretion from which to choose, but we shall again limit discussion in the present paper to two-temperature class of flows. It is of course desirable that two-temperature flows be investigated for the effects of pairs in both their thin disk and inflated limits. About the only work done in the spherical limit was the pioneering paper of Begelman, Sikora, and Rees (1987), who used crude estimates of both thermal and nonthermal pair creation rates to estimate the effects of pairs on the thermal instability to possible collapse of the innermost regions of the ion pressure supported flow due to the overcooling of ions by pairs through Coulomb collisional energy transfer. The collapse, and particularly the size of the collapsed region, influence heavily the geometry and primary spectrum of the flow (see Begelman, Sikora, and Rees 1987). A revisitation of the work of Begelman, Sikora, and Rees (1987) utilizing more detailed calculations of pair creation rates is needed.

The Present Problem: Detailed Calculations of the Effects of Electron-Positron Pairs in Two-Temperature Accretion Flows; and the Origins, Structure, and Spectral Reprocessing of Pair Cascade Atmospheres in Active Galactic Nuclei

In this thesis we propose to remedy several important deficiencies in the development of accurate, self-consistent models of AGN central engines in the following manner. (1) We will investigate the effects of pairs on the classic two-temperature disk model of Shapiro, Lightman, and Eardley (1976). We will do this by improving the original pair free model, generalizing it to accommodate the presence of any pairs, developing accurate thermal and nonthermal models of pair creation within the disk, and numerically solving the model to determine the nature and extent of the effects of pairs. (2) We will improve upon the pioneering work of Begelman, Sikora, and Rees (1987) by using more accurate treatments of both thermal and nonthermal pair production to determine the nature and extent of the effects of pairs on the over-cooling and collapse instabilities in two-temperature quasi-spherical accretion flows. (3) We will develop detailed numerical models of pair cascade atmospheres. Computer codes will be developed which, for any primary spectrum emitted by an accretion flow, will numerically solve for the three-dimensional structure of, and the spectral reprocessing by, the resulting atmosphere. Two distinct models will be developed. The first is a "pair cascade shower atmosphere," which employs many simplifying assumptions. The second, called "the general cascade atmosphere," is a more accurate model with many simplifying assumptions relaxed. We will make primitive comparisons with observation where appropriate and possible.

CHAPTER 2

PAIR PRODUCTION WITHIN TWO-TEMPERATURE ACCRETION FLOWS

Two-temperature accretion flows, in which protons maintain higher temperatures than electrons, are an important class of accretion flows, and are predicted to be present in many AGN central engines (see the Appendix). To investigate the effects of pairs on the structure and properties of two-temperature accretion flows, we adopt two extreme cases, a disk and quasi-spherical approach, which together, hopefully, would give some realistic insight to the actual physical situation. In disk accretion, the two-temperature hot region should be geometrically thick (see Appendix). However, a realistic model of such an inflated torus is non-existent. Therefore, we adopt the hot two-temperature thin disk model of Shapiro, Lightman, and Eardley (1976), the best available at the moment. They estimate that the thin disk model should remain acceptably accurate (to within a factor of about 2) even when the half-thickness h of the disk grown to the order of the radial distance r from the center of the disk: $h \sim r$. Our results should be accurate within the region $h < r$ and at least qualitatively valid up to moderately inflated states, $h \sim r$, at which point spherical or quasi-spherical inflow has become appropriate (White and Lightman 1989). In our quasi-spherical model we adopt the approach of Begelman, Sikora and Rees (1987) where the effect of the angular momentum is taken into account by allowing the infall velocity v to be less than the free fall velocity by a factor μ. This crudely models accretion as matter spirals, rather than directly falls, in toward the black hole. The quasi-spherical results should be valid in geometries ranging from moderately thick tori, h~r, to essentially spherical inflow, h>r.

If the electron temperature is sufficiently high, bremsstrahlung could become an important cooling mechanism (see White and Lightman 1989). However, we adopt the view that the observed broad-band composite spectra of continuum radiation from Seyfert nuclei and typical radio-quiet quasars are more naturally explained by unsaturated Comptonization of soft photons by energetic electrons (Tsuruta 1988), and thus we do not discuss bremsstrahlung models in this paper. (See, however, White and Lightman 1989 for discussion of *one*-temperature bremsstrahlung disks which do not contradict the observational absence of a Wien hump in low luminosity Seyferts, for instance, but do appear to be inconsistent with gamma-ray observations and observed power-law slopes.) The soft photons may refer, e.g., to the UV bump observed in many of these objects or infrared emission through cyclotron higher harmonics (Takahara and Tsuruta 1982). When Comptonization is unsaturated, the outcoming radiation has a power-law spectrum (e.g., Shapiro, Lightman and Eardley 1976). Our thermal models are, therefore, constructed to satisfy the important constraint that the energy slope α of the observed power-law X-ray spectra is ~0.3-1 for AGNs in general, and ~0.7 for Seyfert 1s in particular.

Within our two-temperature accretion flows near a black hole the proton temperature is comparable to its virial value, which is ~10^{11}K to 10^{12}K, while the expected electron temperature of around 10^9K is of the order of its rest mass energy (Shapiro, Lightman and Eardley 1976). Under such high temperatures the thermal production of electron-positron pairs could be important. On the other hand, it has been pointed out that pairs can be produced even more efficiently through a nonthermal process (e.g. see Rees 1984, Guilbert, Fabian and Rees 1983, Fabian et al. 1986). These authors argued that in the environment of the accretion flows near a black hole some fraction of electrons could be accelerated effectively to highly relativistic energies through shocks, magnetic reconnections, etc., and these electrons will produce pairs through a cascade process, described in detail in Chapter 3. Therefore, in our work we consider both thermal and nonthermal pair production, in both our disk and quasi-spherical models.

Two-Temperature Disk

Model with Thermal Pair Production

We refer the reader to the pair-free disk structure equations of Shapiro, Lightman, and Eardley (1976). Taking $n_e=n_i+2n_+$ to be the total electron density (ionization electrons plus pairs), where n_i is the ion (proton) density, and n_+ is the positron (=pair) density, we accommodate the presence of pairs by formally including the pair contributions to the gas pressure and mass density in the equation of state and the condition of hydrostatic equilibrium:

$$P = n_e kT_e + n_i kT_i \ , \qquad (2.1)$$

$$P/h = (n_e m_e + n_i m_p)(GM/r^3)h \ . \qquad (2.2)$$

Here T_e is the electron (and positron) temperature; T_i the ion (proton) temperature; P the pressure; M the mass of the accreting black hole ($10^7 M_{sol}$ for models presented in this chapter); and h the disk half-thickness at radius r where h<<r is the thin disk criterion. We ignore the radiation pressure contribution to the total pressure P in equation (2.1). Radiation pressure will become important at super-Eddington accretion rates (see Appendix) and should be considered. However, at such high accretion rates we find that the flow inflates considerably (see "Results" section of this chapter) and our thin disk analysis no longer holds. Equation (2.2) represents the balance between the vertical component of the gravitational force $F_g=(GM\rho/r^2)(h/r)$ and the pressure force $F_p=P/h$. Here we have invoked the thin disk assumption h<<r to use the small angle approximation $h/(r^2+h^2)^{1/2} \approx h/r$. The electron contributions to the pressure and density will be important in pair-dominated plasmas, but remain small when pair densities are low. The equation of conservation of angular momentum remains unaltered from Shapiro, Lightman, and Eardley (1976), (shear stress)(area)r=(angular momentum transfer):

$$\alpha_{vis} P \quad (2\pi r \cdot 2h) \quad r = (GMr)^{1/2} \quad \dot{M}\Phi \ . \tag{2.3}$$

Here $\dot{M}$ is the mass accretion rate; α_{vis} the so-called "viscosity parameter" satisfying the usual viscosity law: (shear stress)=α_{vis}P, with $0.01<\alpha_{vis}<1$ (see Shakura and Sunyaev 1973 and Shapiro, Lightman and Eardley 1976); and $\Phi(r)=1-(6GM/c^2/r)^{1/2}$, which reflects the absence of viscous stresses at the disk's inner edge where matter leaves its quasi-Keplerian orbit and quickly falls toward the horizon (see the Appendix; Shapiro and Teukolsky 1983).

We use the expression of Stepney and Guilbert (1983) for the collisional energy transfer rate from the hot ions to the cooler electrons, which is considerably more accurate than the treatment in Shapiro, Lightman, and Eardley (1976),

$$\dot{u}_{ep} = \frac{3m_e}{2m_p} n_e n_i \sigma_T c \ln\Lambda \frac{k(T_i - T_e)}{K_2(1/\theta_e) K_2(1/\theta_i)}$$

$$\times \left[\frac{2(\theta_e+\theta_i)^2+1}{\theta_e+\theta_i} K_1\left(\frac{\theta_e+\theta_i}{\theta_e\theta_i}\right) + 2K_0\left(\frac{\theta_e+\theta_i}{\theta_e\theta_i}\right) \right] \tag{2.4}$$

where $\theta_e=kT_e/(m_e c^2)$, $\theta_i=kT_i/(m_p c^2)$, $\ln\Lambda\approx25$, σ_T is the Thomson cross section, and K_i is the modified Bessel function of the second kind of order i. This expression results from integrating the Rutherford differential cross section for coulomb collisions over relativistic Maxwell-Boltzmann distributions using relativistically correct two-body reaction rates formalism (see Weaver 1967). Taking the low-temperature limit $1 \gg \theta_e \gg \theta_i$ (involving asymptotic expansions of the $K_i(z)$; see Abromowitz and Stegun 1965), the approximate form of equation (2.4) used by the above-mentioned studies is recovered: $\dot{u}_{ep} \propto (T_i - T_e) T_e^{-3/2}$.

Ion energy balance is accomplished by matching ion cooling, equation (2.4), to viscous heating of the disk at all r:

$$\dot{u}_{ep} = \dot{u}_{vis} , \tag{2.5}$$

where shear stresses heat the disk at the rate (see Shapiro, Lightman, and Eardley 1976)

$$\dot{u}_{vis} = 3/(8\pi)\dot{M}(GM/r^3)\Phi/h . \tag{2.6}$$

In order to calculate pair production from photon-photon and photon-electron interactions the photon spectrum within the disk must be specified (the electron distribution will be addressed shortly). However, the radiation spectrum depends upon the particular electron cooling mechanisms at work. Facilitating comparison with the original pair-free disk model of Shapiro, Lightman, and Eardley (1976), we follow these authors and assume the existence of a copious source of soft photons (UV or softer) which are Compton upscattered (Comptonized) off the hot relativistic electrons and subsequently escape from the disk. This is the so-called "Comptonized soft-photon" model. This approach has a number of commendable features.

First the form of the Comptonized spectrum depends very weakly, or not at all, upon the spectral shape, mean photon energy, and strength, of the soft source (Lightman and Zdziarski 1987). This eliminates a build-up of free parameters needed to specify the soft source. Secondly extensive monte carlo and analytical work of others (Pozdnyakov, Sobol, and Sunyaev 1977 and 1979; Zdziarski 1985; Sunyaev and Titarchuk 1980) has provided accurate, convenient, analytic fits to Comptonized soft spectra in idealized geometries. Thirdly unsaturated Comptonization easily provides the UV-X-ray power-law characteristics of most Seyfert 1 spectra (see Appendix). Finally a soft UV source would be a natural consequence of any cooler (10^{4-5}K) gas clouds in and around the hot (10^{9-12}K) disk we are considering. Alternative electron cooling mechanisms (bremsstrahlung and Comptonized bremsstrahlung) have been investigated by White and Lightman (1989).

The Comptonization process proceeds as follows. Soft photons of initial (dimensionless) energy x_i are repeatedly upscattered by thermal electron of (dimensionless) temperature θ_e until either they escape from the disk or reach energies of $x \approx \theta_e$, after which the process saturates and further scatterings result in no net energy transfer between photon and electron populations. Saturation is marked by the development of thermal population of photons, the Wien spectral component.

Those photons not reaching thermal energies before escape will in general form a power-law distribution, as we now demonstrate. Photons far below the electron thermal energy ($x_i \ll \theta_e$) will undergo a mean energy amplification given by $A=1+4\theta_e+16\theta_e^2$. This popular formula conveniently combines the nonrelativistic ($\theta_e<<1$) result $A_{nr}=1+4\theta_e$ with that in the extreme relativistic ($\theta_e>>1$) case $A_{er}=16\theta_e^2$ in a smooth manner. We let τ_e be the optical depth to Thomson scattering ($\tau_e \equiv n_e \sigma_T h$) and note $e^{-\tau_e}$ is roughly the mean probability of escape per scattering. The mean probability that a photon anywhere within the system will scatter rather than directly escape is $P_{sct}=1-e^{-\tau_e}$. If the system is optically thin, $\tau_e<$(a few), the probability of exactly k scatterings before escape $P^k_{sct}e^{-\tau_e}$ can be approximated as simply P^k_{sct}. After k scatterings the photon energy has risen to $x=x_iA^k$, producing a photon spectrum (for $x<\theta_e$): $L(x)=L(x_i)P^k_{sct}$. We can rewrite P^k_{sct} as follows:

$$P^k_{sct} = \left(A^{\log P_{sct}/\log A}\right)^k$$

$$= \left(\frac{x}{x_i}\right)^{\log P_{sct}/\log A} \qquad (2.7)$$

and obtain the power law $I(x) \propto x^{-\alpha}$ with energy index

$$\alpha = \frac{-\log P_{sct}}{\log A} . \qquad (2.8)$$

This useful result was first shown by Y.B. Zeldovich and numerically verified by Pozdnyakov, Sobol, and Sunyeav (1977) and many others.

We can therefore safely treat the radiation field within the disk as a superposition of a power law and a Wien component, following Zdziarski (1985):

$$n(x) = \frac{1}{2}N_P(x/\theta_e)^{-\alpha}(e^{-x/\theta_e})/x + \frac{1}{2}N_W(x/\theta_e)^3(e^{-x/\theta_e})/x \qquad (2.9)$$

where $x=h\nu/(m_e c^2)$ is the dimensionless photon energy, α is the energy index of the power-law portion of the spectrum, N_W is the total density of Wien photons, and N_P is related to the power-law photon density (Svensson 1984). Using numerical monte carlo calculations, Zdziarski (1985) has found that equations (2.8) and (2.9) together are quite accurate if the relative strength of the two spectral components is given by the fitting formula:

$$N_W/N_P = [\Gamma(\alpha)/\Gamma(2\alpha+3)]P_{sct} , \qquad (2.10)$$

where Γ is the Euler gamma function.

The power-law index α is fixed by observation in our models (a free parameter, in other words). Many Seyfert 1 spectra exhibit $\alpha \approx 0.7$ in the X-ray region (Mushotzky 1984, and the Appendix) and we set $\alpha = 0.7$ for models presented in this chapter. Equation (2.8) is therefore a constraint between τ_e and θ_e.

We normalize the photon spectrum by equating, at each radius, radiative luminosity to viscous heating:

$$\dot{u}_{vis} = u_{rad}/[\max(1,\tau_e)h/c] , \qquad (2.11)$$

where $u_{rad}=(3/2)kT_eN_W + (1/2)\Gamma(1-\alpha)kT_eN_P$ is the photon energy density within the disk, obtained by integrating equation (2.9).

Pair equilibrium requires the balance of pair creation, annihilation, and inflow:

$$0 = \dot{n}_+ = \dot{n}_+^{cre} + \dot{n}_+^{ann} + \dot{n}_+^{inf} . \qquad (2.12)$$

We take annihilation to proceed at the rate (see Svensson 1982):

$$\dot{n}_+^{ann} = -(3/32)g_a\sigma_Tc(n_e-n_i)(n_e+n_i) , \qquad (2.13)$$

with:

$$g_a = [1+2\theta_e^2/\ln(1.12\theta_e+1.3)]^{-1} . \qquad (2.14)$$

This is a convenient and quite accurate fit to the exact result. (The exact rate is obtained by integrating the annihilation cross section, given in Chapter 4, over a relativistic Maxwell-Boltzmann distribution of electrons and positrons). If pairs are advected inward with the flow of the ions, then the disk geometry casts pair inflow into the form:

$$\dot{n}_+^{inf} = (hr)^{-1} \frac{d}{dr}(hrv_{acc}n_+) , \qquad (2.15)$$

where:

$$v_{acc} = \dot{M}/[4\pi hr(n_i m_p + n_e m_e)] \qquad (2.16)$$

is the net inward velocity of accretion, obtained from examining mass conservation.

Pairs will be produced through photon-photon interactions $\gamma+\gamma \rightarrow e^+e^-$ and photon-electron interactions $\gamma+e \rightarrow e+e^+e^-$. All other pair producing reactions (electron-electron, along with all proton and three-body reactions) are neglected (see Svensson 1984 and Liang 1979). The division of the photon spectrum into two components, equation (2.9), requires five pair production rates to be found : interactions between (1) power-law photons, (2) power-law and Wien photons, (3) Wien photons, (4) power-law photons and electrons and (5) Wien photons and electrons:

$$\dot{n}_+^{cre} = \dot{n}_+^{PP} + \dot{n}_+^{PW} + \dot{n}_+^{WW} + \dot{n}_+^{Pe} + \dot{n}_+^{We} . \qquad (2.17)$$

The utility of the fit in equation (2.9) is presently seen by the fact that we may analytically reduce all three photon-photon rates down to the same single integral over the cross section, and the two photon-electron rates down to a (different) single integral over the cross section.

We shall first derive the $\gamma+\gamma$ pair creation rate $\dot{n}_+^{\gamma\gamma}$ from the interaction between two isotropic populations:

$$n_1(x_1) = \tfrac{1}{2}N_1 \, (x_1/\theta_1)^{-\alpha_1} \quad e^{(-x_1/\theta_1)}/x_1 \qquad (2.18)$$

and:

$$n_2(x_2) = \frac{1}{2}N_2\ (x_2/\theta_2)^{-\alpha_2}\ e^{(-x_2/\theta_2)}/x_2\ . \tag{2.19}$$

The probability per unit time of absorption by population 2 photons for a photon from population 1 with energy x_1 is

$$\frac{d\tau(x_1)}{dt} = \int_0^{\pi}\int_0^{\infty} \sigma_{\gamma\gamma}(s)\ \ c(1-\cos\theta)\ \ \frac{1}{2}\sin\theta n_2(x_2)\ \ dx_2 d\theta\ , \tag{2.20}$$

where θ is the collision angle between x_1 and x_2, $\frac{1}{2}\sin\theta n_2(x_2)dx_2d\theta$ is the differential population 2 photon density, $c(1-\cos\theta)$ is the relative velocity of x_2 along the direction of motion of x_1, and $\sigma_{\gamma\gamma}(s)$ is the total pair production cross section, which is a function of the center-of-momentum (c.m.) frame energy γ of either photon (see Gould and Schréder 1967):

$$\sigma_{\gamma\gamma}(s) = \frac{1}{2}\pi r_0^{\ 2}\ (1-\beta^2)\left((3-\beta^4)\ \ln\frac{1+\beta}{1-\beta}\ -2\beta(2-\beta^2)\right). \tag{2.21}$$

Here $\beta=(1-\gamma^{-2})^{1/2}$ is the c.m. velocity of either electron, $s=\gamma^2$, $2s=x_1x_2(1-\cos\theta)$, and r_0 is the classical electron radius. Switching integration variables from θ to s yields:

$$\frac{d\tau(x_1)}{dt} = \frac{c\pi r_0^2}{x_1^2}\int_0^\infty dx_2 \frac{n_2(x_2)}{x_2^2}\int_0^{x_1x_2} ds \frac{2\sigma_{\gamma\gamma}(s)s}{\pi r_0^2} . \tag{2.22}$$

Defining the inner integral as the dimensionless function $\varphi(x_1x_2)$ (see Gould and Schréder 1967) and noting the total creation rate will be the integral of equation (2.22) over $dn_1(x_1)$, we have the symmetric form

$$\dot{n}_+^{\gamma\gamma} = c\pi r_0^2 \int\int \frac{\varphi(x_1x_2)}{x_1^2x_2^2} n_1(x_1)n_2(x_2)dx_1dx_2 . \tag{2.23}$$

Inserting the spectral functions of equations (2.18) and (2.19) into equation (2.23), defining $s_0=x_1x_2$ and switching variables from x_2 to s_0 we have

$$\dot{n}_+^{\gamma\gamma} = c\pi r_0^2 \frac{N_1N_2}{4}\int_1^\infty ds_0 \;\; \varphi(s_0) \;\; s_0^{(-3-\alpha_2)} \quad \theta_1^{\alpha_1}\theta_2^{\alpha_2}$$

$$\times \int_0^\infty dx_1 \;\; x_1^{(-1-\alpha_1-\alpha_2)} \quad e^{(-x_1/\theta_1-s_0/(x_1\theta_2))} . \tag{2.24}$$

The x_1 integral can be found in Gradshteyn and Ryzhik (1980) as #3.471.9. Inserting a factor $1/(1+\delta_{12})$ to avoid double counting in the case populations 1 and 2 are the same population, we arrive at

$$\dot{n}_+^{\gamma\gamma} = (3/16) \quad \sigma_T c \, \frac{N_1 N_2}{1+\delta_{12}} \, (\theta_1\theta_2)^{(\alpha_1+\alpha_2)/2}$$

$$\times \int_1^\infty ds_0 \quad \varphi(s_0) \quad s_0^{-(6+\alpha_1+\alpha_2)/2} \quad K_{\alpha_2-\alpha_1}\left(\frac{4s_0}{\theta_1\theta_2}\right)^{1/2} . \qquad (2.25)$$

This result was found by Svensson (1984). Here $\dot{n}_+^{PP}$ is obtained when $\alpha_1=\alpha_2=\alpha$, $N_1=N_2=N_P$, $\theta_1=\theta_2=\theta_e$ and $\delta_{12}=1$; $\dot{n}_+^{PW}$ is obtained when $\alpha_1=\alpha$, $\alpha_2=-3$, $N_1=N_P$, $N_2=N_W$, $\theta_1=\theta_2=\theta_e$ and $\delta_{12}=0$; and $\dot{n}_+^{WW}$ is obtained when $\alpha_1=\alpha_2=-3$, $N_1=N_2=N_W$, $\theta_1=\theta_2=\theta_e$ and $\delta_{12}=1$. An accurate analytic fit to $\varphi(s_0)$ is found in Gould and Schréder (1967), after correcting a critical misprint. Equation (2.25) is integrated numerically for various θ_e and the results stored for tabular interpolation.

We next calculate the pair production rate from interactions between an isotropic Maxwell-Boltzmann electron (and positron) distribution at temperature θ_1

$$n_1(p_1) = N_1 p_1^2 \, \frac{e^{-(p_1^2+1)^{1/2}/\theta_1}}{\theta_1 K_2(1/\theta_1)} \qquad (2.26)$$

and the isotropic photon distribution given by equations (18) or (19):

$$n_2(p_2) \;=\; \tfrac{1}{2}N_2\;(p_2/\theta_2)^{-\alpha_2}\quad e^{(-p_2/\theta_2)}/p_2\;. \tag{2.27}$$

Here p_1 and p_2 are the electron and photon momenta respectively. (We suppress factors of c and m_ec^2 throughout, so that the dimensionless energy-momentum relationships read x=p for photons and $\gamma^2=p^2+1$ for electrons.) We will first calculate the differential reaction rate dR (cm^{-3} sec^{-1}) in a beam of electrons with density dn_1 and energy $\gamma_1=(1-\beta_1^{\,2})^{1/2}$ colliding with a photon beam of density dn_2 and momentum p_2, where the collision angle is θ. We temporarily switch to the electron rest frame (signified here by primed quantities) where the physics is much simpler and calculate the (Lorentz-invariant) rate

$$dR \;=\; dR' \;=\; dn_1'dn_2'\quad c\sigma_{\gamma e}(p_2')\;, \tag{2.28}$$

where

$$p_2' \;=\; p_2\gamma_1\quad(1-\beta_1\cos\theta)\;. \tag{2.29}$$

is the rest-frame photon momentum. Here c is the relative velocity and $\sigma_{\gamma e}(p_2')$ is the total photon-electron pair creation cross section. Accurate analytic fits to $\sigma_{\gamma e}(p_2')$ are calculated by Haug (1981) and presented by Stepney and Guilbert (1983). Invariance of the dot product of the four-vector currents $j_1=n_1(1,\beta_1\hat{v}_1)$ and $j_2=n_2(1,\hat{v}_2)$ yields the density transformation:

$$dn_1'dn_2' = dn_1 dn_2 \; (1-\beta_1\cos\theta) = dn_1 dn_2 \; p_2'/(p_2\gamma_1) \; , \tag{2.30}$$

where the second equality follows from equation (2.29). We now substitute equation (2.30) into equation (2.28) and integrate over the (isotropic) distributions to obtain the total rate

$$\dot{n}_+^{\gamma e} = \frac{1}{2}c \int\int\int n_1(p_1)n_2(p_2) \; \frac{p_2'}{p_2\gamma_1} \; \sigma_{\gamma e}(p_2') \; dp_1 dp_2 du \; , \tag{2.31}$$

where $u = \cos\theta$ and the factor of 1/2 removes a spurious factor of 2 introduced by the u integration. We change variables from u in favor of p_2' via equation (2.29)

$$\dot{n}_+^{\gamma e} = \frac{1}{2}c \int_4^\infty dp_2' \; \sigma_{\gamma e}(p_2')p_2'$$

$$\times \int_0^\infty dp_2 \; \frac{n_2(p_2)}{p_2^2} \int_{p_1^{min}}^\infty dp_1 \; \frac{n_1(p_1)}{\gamma_1^2\beta_1} \; , \tag{2.32}$$

where:

$$p_1^{min} = \frac{1}{2} \left| \frac{p_2'}{p_2} - \frac{p_2}{p_2'} \right| . \tag{2.33}$$

The lower limit to the p_2' integration represents the reaction threshold, which defines a minimum energy state in the c.m. frame. Threshold for this reaction occurs when all three electrons (incident plus created pair) are at rest in the c.m. frame after the interaction, giving a total c.m. energy of $E_{cm}=3$. Before the interaction this energy is carried by the incident photon x_{cm} and the incident electron, whose c.m. energy is determined by the requirement that the incident momenta of the photon and electron must be the same. That is,

$$E_{cm} = 3 = x_{cm}+(x_{cm}^2+1)^{1/2} . \tag{2.34}$$

This expression gives the c.m. photon threshold energy $x_{cm}= \frac{4}{3}$. The Lorentz factor γ_{rel} of the relative velocity β_{rel} between the c.m. and elctron rest frames is

$$\gamma_{rel} = (1+x_{cm}^2)^{1/2} , \tag{2.35}$$

which gives $\gamma_{rel}= \frac{5}{3}$ and $\beta_{rel}= \frac{4}{5}$. We then Lorentz transform x_{cm} back to the rest frame

$$x_{rest} = x_{cm}\gamma_{rel} \quad (1+\beta_{rel}) , \tag{2.36}$$

and obtain the threshold value $x_{rest}=\frac{4}{3}\frac{5}{3}(1+\frac{4}{5})= 4$.

The lower limit p_1^{min} to the p_1 integration is found by examining equation (2.29) for the smallest value of $p_1(=\gamma_1\beta_1)$ for any choice of p_2, p_2' and $\cos\theta$. If p_1 is considered a function of the variables $p_2'/p_2 \equiv a$ and $\cos\theta \equiv u$, it is immediately obvious that for any $a<1$ the minimum for p_1 occurs when $u=+1$, yielding $p_1^{min}=(a^{-1}-a)/2$. For any $a>1$ the minimum occurs when $u=-1$, yielding $p_1^{min}=(a-a^{-1})/2$. These two results are equivalent to p_1^{min} given in equation (2.33).

Substituting the spectral shapes of $n_1(p_1)$ and $n_2(p_2)$ from equations (2.26) and (2.27) into equation (2.32) gives

$$\dot{n}_+^{\gamma e} = \frac{c}{2}\int_4^\infty dp_2' \;\sigma_{\gamma e}(p_2')p_2'$$

$$\times \int_0^\infty dp_2 \;\frac{1}{2}N_2(p_2/\theta_2)^{-\alpha_2} \quad e^{(-p_2/\theta_2)}/p_2{}^3$$

$$\times \int_{p_1^{min}}^\infty dp_1 \;N_1p_1{}^2 \;\frac{e^{-(p_1{}^2+1)^{1/2}/\theta_1}}{\theta_1 K_2(1/\theta_1)\gamma_1{}^2\beta_1}\;. \tag{2.37}$$

The p_1 integral is trivial (writing p_1 and β_1 as functions of γ_1) and produces:

$$\dot{n}_+^{\gamma e} = \frac{cN_1N_2\theta_2{}^{\alpha_2}}{4K_2(1/\theta_1)}\int_4^\infty dp_2' \;\sigma_{\gamma e}(p_2')p_2'$$

$$\times \int_0^\infty dp_2\ p_2^{(-\alpha_2-3)}\ e^{-[\theta_2^{-1} + (2\theta_1 p_2')^{-1}]p_2 - p_2'/(2\theta_1 p_2)} . \qquad (2.38)$$

The p_2 integration is done with the help of integral #3.471.9 in Gradshteyn and Ryzhik (1980), and we obtain the final result (changing the dummy variable to x for ease of notation):

$$\dot{n}_+^{\gamma e} = \frac{cN_1N_2}{2K_2(1/\theta_1)}\theta_2^{\alpha_2} \int_4^\infty dx\ \sigma_{\gamma e}(x)x \left(\frac{x}{(2\theta_1/\theta_2)+(1/x)} \right)^{-(2+\alpha_2)/2}$$

$$\times K_{-2-\alpha_2}\left[2\left(\frac{x}{2\theta_1}\left[\frac{1}{\theta_2} + \frac{1}{2x\theta_1} \right] \right)^{1/2} \right] . \qquad (2.39)$$

where $\dot{n}_+^{Pe}$ results when $N_1=n_e$, $\theta_1=\theta_2=\theta_e$, $\alpha_2=\alpha$, and $N_2=N_P$; and $\dot{n}_+^{We}$ results when $N_1=n_e$, $\theta_1=\theta_2=\theta_e$, $\alpha_2=-3$, and $N_2=N_W$. This result was found by Zdziarski (1985). Equation (2.39) is integrated numerically for various θ_e and the results stored for tabular interpolation.

Equations (2.1)-(2.3), (2.5), (2.8), and (2.10)-(2.12) constitute the eight equations necessary for the solution of the eight variables describing the thermal disk at any radius r: n_e, n_i, T_e, T_i, P, h, N_P, and N_W. Model parameters to be specified are: M, $\dot{m} \equiv \dot{M}/\dot{M}_{Edd}$, α_{vis}, and α. Here, $\dot{M}_{Edd}c^2$ is the Eddington luminosity. Results are given in the "Results" section of this chapter.

Model with Nonthermal Pair Production

We adopt the scheme (see, e.g. Lightman and Zdziarski 1987) whereby some fraction ε_e of the power developed through viscous dissipation within the disk goes to the acceleration of some electrons to relativistic energies ($\gamma >> 1$). The γ-rays produced as these relativistic electrons quickly cool will produce pairs, which, in turn, cool and produce additional γ-rays. Depending upon conditions, several generations of pairs may be produced in a nonthermal pair cascade process. Detailed investigations into the efficiency of producing pairs through this process have been carried out for the idealized case of uniform, spherical sources (see Lightman and Zdziarski 1987, Svensson 1987, Done and Fabian 1989). These authors find that for a wide range of parameters, the "pair-yield," PY (that fraction of energy initially directed to nonthermal electron acceleration which ultimately appears as rest mass energy of pairs), is solely a function of the "electron compactness parameter," l_e:

$$PY = \begin{cases} 2\times10^{-3} l_e & 3<l_e<50 \\ 0.1 & l_e>50 \end{cases} \qquad (2.40)$$

where (see, e.g., Begelman, Sikora, and Rees 1987):

$$l_e = (L_e/r)\sigma_T/(m_e c^3) \ , \qquad (2.41)$$

and L_e is the total power given to electron acceleration in a source of size r. Detailed investigation (or any investigation) of pair cascades in the context of a two-temperature (or any) disk has not yet been done. As a starting point then, we apply equations (2.40) and (2.41) to the disk, noting (see, e.g. Lightman, Zdziarski, and Rees 1987) that this treatment would be appropriate in the case of marginally thick disks ($h/r < 1$) with high viscosity ($\alpha_{vis} \geq 0.1$). Then:

$$L_e = \varepsilon_e L = \varepsilon_e (1/12)\dot{M}c^2 \qquad (2.42)$$

is the power going to nonthermal electron acceleration throughout the disk. Here we have integrated equation (2.6) to obtain $L = (1/12) \dot{M} c^2$, the total viscous dissipation of the disk (see Shapiro, Lightman, and Eardley 1976). The pair production rate is now

$$\dot{n}_+^{cre} = PY \varepsilon_e \dot{u}_{vis} / (2m_e c^2) \ . \qquad (2.43)$$

We retain the annihilation and inflow terms from the thermal model, as well as equations (2.1)-(2.3). The passage of energy through a nonthermal channel we reflect in a modified ion energy balance equation, which replaces equation (2.5):

$$\dot{u}_{ep} = (1-\varepsilon_e)\,\dot{u}_{vis} \ . \qquad (2.44)$$

We no longer require detailed specification of the photon spectrum, so we replace equation (2.8) by the condition of unsaturated Comptonization:

$$y \equiv 4\theta_e \; (4\theta_e+1) \;\; \max(\tau_e,\tau_e^2) \;\; = 1 \; . \qquad (2.45)$$

Equations (2.1)-(2.3), (2.12) [with pair creation given by (2.43), annihilation by (2.13), and inflow by (2.15)], (2.44), and (2.45) are solved for the now six variables n_e, n_i, T_e, T_i, P, and h. The list of parameters to be specified is M, $\dot{m}$, α_{vis}, and ε_e. Results are presented in the "Results" section of this chapter.

Two-Temperature Quasi-Spherical Flow

Model with Nonthermal Pair Production

We next investigate pair-induced over-cooling of quasi-spherical two-temperature flows, extending the treatment of Begelman, Sikora, and Rees (1987). Here we approximate the inflow as spherical, but adjust the accretion velocity to some fraction μ of its freefall value to accommodate in a simple manner any angular momentum of the flow. Mass conservation then gives:

$$n_i \; = \dot{M} \; / \; (m_p v_{acc} \, 4\pi r^2) \; , \qquad (2.46)$$

where

$$v_{acc} \; = \mu \; (2GM/r)^{1/2} \; ; \qquad 0<\mu<1 \; . \qquad (2.47)$$

Begelman, Sikora, and Rees (1987) define a critical radius, r_{crit}, interior to which, Coulomb energy loss from ions to electrons overwhelms ion heating:

$$\dot{u}_{ep} = \dot{u}_{gr} \quad \text{at} \quad r_{crit} \ , \tag{2.48}$$

where $\dot{u}_{ep}$ is given by equation (2.4) and $\dot{u}_{gr}=GM\dot{M}/(4\pi r^4)$ is the rate of gravitational energy dissipation at r. We set the ion temperature to remain near its virial value ($T_i>>T_e$) outside the stongly cooled region:

$$(3/2)kT_i = \varepsilon_{th}(GMm_p/r) \ , \ \varepsilon_{th}<1 \ . \tag{2.49}$$

Adopting a nonthermal scheme similar to that used with the disk flow, we divert a fraction ε_e of $\dot{u}_{gr}$ to accelerate a small nonthermal population of electrons, and write pair production as

$$\dot{n}_+^{cre} = PY\, \varepsilon\dot{u}_{gr} \,/\, (2m_ec^2) \ , \tag{2.50}$$

where equation (2.40) gives PY, equation (2.41) gives l_e, and now the total power given to electron acceleration is:

$$L_e = \varepsilon_e L = \varepsilon_e(1/6)\dot{M}c^2 \ . \tag{2.51}$$

Here we have assumed radiation is produced down to at least $r=6GM/c^2$. Integrating gravitational potential energy dissipation down to this radius gives the total luminosity $L=(1/6)\dot{M}\,c^2$. We approximate pair inflow:

$$\dot{n}_+^{inf} = \frac{1}{r^2}\frac{d}{dr}(r^2 n_+ v_{acc}) \tag{2.52}$$

by

$$\dot{n}_+^{inf} = n_+ v_{acc}/r \ . \tag{2.53}$$

Electron cooling will be unsaturated Comptonization: y=1 [equation (2.45)]. We solve equations (2.46), (2.48), (2.49), (2.12) [with pair creation given by (2.50), inflow by (2.53), and annihilation by (2.13)], and (2.45) [with $\tau_e \equiv n_e \sigma_T r$ defining the electron scattering optical depth for spherical geometry] for r_{crit}, $T_e(r_{crit})$, $T_i(r_{crit})$, $n_i(r_{crit})$, and $n_e(r_{crit})$. Model parameters are M, $\dot{m}$, μ, ε_{th}, and ε_e. Results are presented in the "Results" section of this chapter.

Model with Thermal Pair Production

In our final model we examine the effect of thermal pair production on the two-temperature, quasi-spherical flow. Pair creation is accomplished through the same physical processes assumed in the thermal disk model. Again, a Comptonized photon spectrum is given by equation (2.9). The seven flow variables at the critical radius [r_{crit}, $T_e(r_{crit})$, $T_i(r_{crit})$, $n_i(r_{crit})$, $n_e(r_{crit})$, $N_P(r_{crit})$, and $N_W(r_{crit})$] are found by solving equations (2.46), (2.48), (2.49), (2.8) & (2.10) [both with $\tau_e = n_e \sigma_T r$], (2.12) [with pair creation given by (2.17), annihilation by (2.13), and inflow by (2.53)], and (2.54):