Multiple embodiment instructional sequence using the computer as the interfacing agent in the instruction of volume of rectangular solids
by Ruth Mary Regling Johnson

A thesis submitted in partial fulfillment of the requirement for the degree of Doctor of Education
Montana State University
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Abstract:
This study measured the effect of two multiple embodiment instructional sequences on the topic of volume of rectangular solids upon student achievement. Instructional sequences investigated were (1) the sequence with computer, (2) the same sequence without computer; and a textbook-based sequence. Independent variables were sequence, ability level, and gender. Dependent variables were scores on post-instruction and post-retention criterion-referenced tests. Test items were divided into these categories: knowledge and comprehension, application and analysis, and total test.

A pretest determined all sequence groups were equal before instruction.

The four-week study was conducted in 21 southwestern Montana classrooms. Classes were randomly assigned to the sequences. Classroom teachers conducted all instruction and testing activities. Post-instruction test was administered after one week of instruction; post-retention test after three weeks of retention activities, one activity each week.

Factorial analysis of scores established the following conclusion. Both multiple embodiment sequences were superior to the textbook sequence except for application and analysis post-instruction scores. The highest achievement was among high ability groups; the lowest achievement was among low ability. Males outperformed females on knowledge and comprehension questions; females outperformed males on the computer sequence; and on both tests, males and females demonstrated equal achievement on application and analysis and total test scores. Two trends were noted: (1) students in the embodiment sequence without computer produced higher scores post-instruction, but students in the computer embodiment sequence scored higher post-retention, and (2) low ability students in the computer sequence scored lower than their counterparts in the other sequences.

The following recommendations were made concerning instruction of volume of rectangular solids. Carefully sequenced multiple embodiment instruction should be used. Use of the computer should be considered in embodiment instruction. Assumption that male achievement is superior to female achievement in this area should not be made. Instruction on this topic should be reinforced over time and should include multiple embodiments.
MULTIPLE EMBODIMENT INSTRUCTIONAL SEQUENCE
USING THE COMPUTER AS THE INTERFACING AGENT
IN THE INSTRUCTION OF VOLUME OF RECTANGULAR SOLIDS

by

Ruth Mary Regling Johnson

A thesis submitted in partial fulfillment
of the requirement for the degree

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APPROVAL

of a thesis submitted by

Ruth Mary Regling Johnson

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

5 June 1989
Date

Chairperson, Graduate Committee

Approved for the Major Department

6/5/89
Date

Head, Major Department

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ABSTRACT

This study measured the effect of two multiple embodiment instructional sequences on the topic of volume of rectangular solids upon student achievement. Instructional sequences investigated were (1) the sequence with computer, (2) the same sequence without computer; and a textbook-based sequence. Independent variables were sequence, ability level, and gender. Dependent variables were scores on post-instruction and post-retention criterion-referenced tests. Test items were divided into these categories: knowledge and comprehension, application and analysis, and total test. A pretest determined all sequence groups were equal before instruction.

The four-week study was conducted in 21 southwestern Montana classrooms. Classes were randomly assigned to the sequences. Classroom teachers conducted all instruction and testing activities. Post-instruction test was administered after one week of instruction; post-retention test after three weeks of retention activities, one activity each week.

Factorial analysis of scores established the following conclusion. Both multiple embodiment sequences were superior to the textbook sequence except for application and analysis post-instruction scores. The highest achievement was among high ability groups; the lowest achievement was among low ability. Males outperformed females on knowledge and comprehension questions; females outperformed males on the computer sequence; and on both tests, males and females demonstrated equal achievement on application and analysis and total test scores. Two trends were noted: (1) students in the embodiment sequence without computer produced higher scores post-instruction, but students in the computer embodiment sequence scored higher post-retention, and (2) low ability students in the computer sequence scored lower than their counterparts in the other sequences.

The following recommendations were made concerning instruction of volume of rectangular solids. Carefully sequenced multiple embodiment instruction should be used. Use of the computer should be considered in embodiment instruction. Assumption that male achievement is superior to female achievement in this area should not be made. Instruction on this topic should be reinforced over time and should include multiple embodiments.
CHAPTER I
INTRODUCTION

Introduction

Mathematics educators, psychologists, and learning theorists have long searched for the processes by which students learn mathematics in order to develop instructional methods that will effectively facilitate such learning. Yet, concern over declining test scores of mathematical skill and concept development has been documented in the past two decades (Flake, 1982; National Assessment of Educational Progress Report (NAEP), 1979a; NAEP, 1979b; NAEP, 1983; National Commission on Excellence in Education, 1983; National Research Council, 1989; Willoughby, 1987). Even though some gain in mathematics achievement was shown in the 1986 NAEP test, it was primarily reflected in lower-order skills (Dossey et al., 1988). Concurrent with the fall of math scores has been a call by several national educational organizations to improve application and analysis skills, to link these to problem solving skills, and to explore ways to incorporate computers into instruction as a means
of enhancing mathematics learning (Committee on Research in Mathematics, Science and Technology, Commission on Behavioral and Social Sciences and Education, National Research Council, 1985; Dolan, 1987; Dossey et al., 1988; Heddens, 1984; National Council of Teachers of Mathematics (NCTM), 1985; National Research Council, 1989; National Science Board Commission on Pre-College Education in Mathematics, Science and Technology, 1983; Suydam, 1984b). Consequently, it is no surprise that a current topic of research is studying how to use the computer as a supplement to mathematics instruction to increase student achievement.

Until the 1960s much of mathematics instruction emphasized teacher show student tell environments, memorization, and drill and practice to promote, reinforce, and retain mathematics knowledge (Byers & Erlwanger, 1985; Heddens, 1984; Shuell, 1986). Traditional elementary school mathematics instruction has consisted of lecture, chalkboard demonstration, and assignment of numerous homework problems from textbooks. Traditional elementary school mathematics learning has consisted of listening, copying, memorizing algorithms, and time-consuming hand calculations to solve the homework assignments. Frequently, this approach has emphasized lower-level computation and comprehension
skills at the expense of the development of higher level application and analytical skills (DeVault, 1981; Dossey, 1989; Dugdale & Kibbey, 1983, 1984; NAEP, 1979a; NAEP, 1979b; NAEP, 1983; NCTM, 1985). This behavioral approach reflected the research of Skinner and Pavlov and most educators viewed mathematics learning and mathematics memory as a "quantitative" measure of learning (Byers & Erlwanger, 1985; Davis, 1983; Gagne, 1985; Shuell, 1986).

However, in the late 1960s and early 1970s a renewed emphasis on "understanding" the concept versus "rote learning" surfaced. This is evidenced by the influence of "meaningful verbal learning" (Ausubel, 1963), developmental stages of learning (Piaget, 1960), "discovery learning" (Bruner, 1975), Rathmell's symbolic-oral-written concept of learning (Payne & Rathmell, 1980), and others. Brownell's (1956) "meaningful arithmetic" was an early harbinger. These process-oriented learning theories produced instructional strategies involving advanced organizers, concrete manipulatives, multiple embodiment instruction, and a shift from teacher-dominated to child-centered, activity-oriented classrooms and laboratories. The "qualitative" instructional approach with regard to meaning and organization of mathematics learning surfaced as the major emphasis in mathematics education during
this time (Wheeler, 1971). It was hoped this approach would develop higher cognitive levels of mathematics learning as well as higher levels of student achievement. However, mathematics instruction in 1983 still consisted primarily of teacher-dominated lecture and demonstration and reliance on textbooks and worksheets (Dossey et al., 1988). Most teachers use a narrow repertoire of instructional practices, though research shows manipulatives used in mathematics instruction increases achievement (McCrary, 1988).

The idea of introducing concepts via concrete materials in elementary mathematics instruction is not new and has been prevalent in the literature since the early 1900s. In 1920, C. L. Hull, for example, noted "a combination of abstract presentations and concrete examples yield a distinctly greater functional efficiency than either method alone" (p. 28). Again, in 1927, J. P. Haynes stated:

It is often necessary for pupils to work with concrete materials; for it is only through actual contact with things, events, qualities, and relations that words, numbers, signs, and symbols acquire meaning. Concrete material is a means of thinking, computing, and verifying results. It provides for learning through several senses (p. 107).

Bruner’s (1971) theory of intellectual development is one theory advocating the use of concrete manipulatives for initial concept building. He believed
knowledge can be represented by a set of actions (enactive), a set of images (iconic), and a set of symbols representing the knowledge concepts and including rules for manipulating the mathematics concept (symbolic). Further, learners proceed through the learning experience in three stages: concrete, semi-concrete and abstract. Piaget (1960, 1967, 1973), Freudenthal (1981), and Van Heile (Hoffer, 1983; Van Heile, 1986) are among the math psychologists and educators who agree with Bruner that students need experiences at the concrete or manipulative level, before they reach the representative or pictorial level. Only then can the abstract level of understanding be reached (Groen & Kieran, 1983; Heddens, 1984).

The Multiple Embodiment Principle proposed by Z. P. Dienes in the 1960s is built upon Bruner’s and Piaget’s theories. The primary focus of Dienes’ theory is that learners acquire mathematical structures through manipulation of physical embodiments, concrete devices that embody the mathematical structure to be learned. The amount of abstraction acquired by the learner is directly proportional to the number of varied experiences embodying the concept (Dienes, 1967).

Retention of knowledge is as important to mathematics instruction as the acquisition of knowledge.
Byers and Erlwanger (1985) contend that "memory plays an essential role in the understanding of mathematics (p. 259)". Gagne and White (1978) acknowledged the importance of memory when they incorporated memory structure into their learning paradigm. Memory involving mathematics learning may be one key to understanding why some students "learn" mathematics by memorizing, others by understanding, and still others learn not at all.

Like mathematics learning, mathematics memory viewed as a quantitative measure (i.e., recall of math facts) was prevalent until the 1960s (Byers & Erlanger, 1985; Norman, 1982). The Ebbinghaus retention curve of verbal memory reflects the rote notion of memory popular in the first part of the 20th century (Underwood, 1982). During this period of behavioral emphasis, drill and practice was a major instructional method. Recently, cognitive psychology has placed a new emphasis on the role of meaning and organization in learning and retention of learning. Byers and Erlanger (1985, p. 261) summarize the current view of memory in mathematics learning:

The crucial question is not whether memory plays a role in understanding mathematics but 'what' it is that is remembered and 'how' it is remembered by those who understand it - as well as by those who do not.

National testing results also indicate the concepts and computation involving the subject of volume are a
matter of concern (Gardner, Rudman, Karlson, & Merwin, 1983a, 1983b; Michigan Department of Education (MEAP), 1983; NAEP, 1983). One reason for this concern may be the minimal attention given to three-dimensional space and geometry in mathematics curriculum (Ben-Haim, Lappan, & Houang, 1985). Shumway (1980) notes the nominal amount of attention given to space and geometry in mathematics research (Shumway, 1980). Tyson-Bernstein (1988) noted the negative effect on the quality of education when curriculum and textbook standards are set by political rather than knowledgeable synthesis.

In addition to the political influence on textbook content, the overuse of textual representation may not be adequate for all learners. Many students learn better with other forms of communication such as graphics, motion, tactile motions, and computer representations (Taylor & Cunniff, 1988). Unfortunately, instructional content is based on textbook content (Dossey, 1988; Nicely, 1985; Taylor & Cunniff, 1988). Two-thirds of all mathematics classes are taught using only one textbook (Committee on Indicators of Pre-College Science and Mathematics, Commission of Behavioral and Social Studies and Education, & National Research Council, 1985). Therefore, supplement of mathematics textbooks to achieve higher level mathematics skills is mandatory (Nicely,
1985). Fortunately, the NCTM is attempting to address this problem by defining appropriate standards for future needs; it is the only educational group to do so (Tyson-Bernstein, 1988).

Another reason for poor achievement on the topic of volume may pertain to poor spatial ability, the ability to visualize and manipulate three-dimensional space (Bishop, 1983; Ben-Haim et al., 1985). One study by Ben Haim et al. (1985) analyzed errors made by students who took the 1983 Michigan Educational Assessment Program test. The test items reviewed measured ability to calculate the volume of rectangular solids. Their findings suggest visualization of three-dimensional objects and mathematical problems dealing with visualization is an area in which students lack skills. The authors suggest initial instruction should include concrete activities. The traditional textbook approach to volume of rectangular solids that uses a two-dimensional semi-concrete format followed by drill and practice type activities is not sufficient (Ben-Haim et al., 1983; Hart, 1981; Nicely, 1985).

Additionally, the traditional mathematics textbook approach follows an instructional progression of length, area, and volume. Some research seems to demonstrate this may not be the best sequence for all students (Hart,
1984). Other research indicates a visual approach to volume is more beneficial to students who have low visualization skills (Gabel & Enoch, 1987). When students do not understand the concept, volume will be interpreted only as a memorized formula (Dugdale & Kibbey, 1983; Gabel & Enoch, 1987; NAEP, 1983).

Just as mathematics learning theories and practices have shifted from quantitative to qualitative emphasis, so has computer-assisted instruction (Chambers & Sprecher, 1983). Until the late 1960s CAI consisted primarily of drill and practice formats. This form of CAI improved memorization and recall of facts, which are lower levels of learning (Bialo & Erickson, 1985; Kulik, Kulik, & Cohen, 1980). Recently, however, development of a new type of software, the simulation, or microworld, offers promise in enhancing concept development in math education (Alesandrini, 1985; Henderson et al., 1983; Chambers & Sprecher, 1983; NCTM, 1985). The ability of the computer to quickly create visual images that represent an abstract mathematical concept holds great promise in helping students acquire those abstract concepts and promoting learning at higher cognitive levels (Bork, 1980; Franz, 1986; Marks, 1985; Reed, 1985; Smith, 1982).
The lack of accessibility of computers for classroom use, lack of funds for software and hardware, and the poor quality of computer software has, in the past, stifled attempts to respond to these national concerns over declining mathematics scores and recommendations for incorporating computer aided instruction into mathematics classrooms (DeVault, 1981; DeVault & Chapin, 1980). Today, however, problems with accessibility, funding, and the quality of software programs have diminished and, in some cases, been completely resolved. A 1984-85 comprehensive national survey revealed that 94% of all public school districts in the United States use computers in instruction; 82% of elementary schools have microcomputers in their classrooms (Hood, 1985). In Montana, 94% of the state's classrooms have access to or contain permanently placed computers (Office of Public Instruction (OPI), 1985b). In Montana's elementary school classrooms the percentage is 84 (OPI, 1987). Public domain software, site licenses, and labs in which all students in a school have access to computers have helped defray costs for both software and hardware purchases. A few studies indicate mathematics instruction incorporating CAI is less expensive than without CAI (Adam, 1989).
A consensus of Montana educational experts concluded top priorities for computer literacy for Montana public schools are: (a) students should be able to use the computer as a tool for inquiry, problem solving, and recreation; and (b) students should participate in a variety of experiences (hands on) in several subject areas (Bruwelheide, 1982). A review of the literature demonstrates that the use of computers as a supplement to instruction can also enhance learning in mathematics (Burns & Bozeman, 1981; Henderson et al., 1983; Kelman et al., 1983).

The impetus for this study came from an internship completed in the Spring of 1987 involving a field test of a multiple embodiment instructional sequence on the subject of volume of rectangular solids. A computer simulation was included as one embodiment. Forty-five students from one town and two rural Montana schools (see Appendix C) participated in the study which consisted of a four-day instructional unit and three weeks of retention activities. Results of the study indicated that use of computer-assisted instruction (CAI) may have been a factor in the increase in student achievement. The positive reaction of both teachers and students indicated to this researcher a need to replicate the pilot study in a more controlled environment. Because
several of the instructional days required more than a 50-minute period, this researcher also decided to increase the length of the instruction time to five 50-minute periods, reconfiguring the research study to a five-day instructional time period.

Statement of the Problem

Concern over the declining test scores in mathematical skill and concept development, including the concept of volume of rectangular solids, has been documented (Gardner et al., 1983; MEAP, 1983; NAEP, 1983). In addition to improving mathematics achievement, educators are concerned about developing mathematics ability at higher cognitive levels. Many national educational organizations urge inclusion of computer-assisted instruction as a means of improving the learning of mathematics.

The problem in this study was to determine the effect of three methods of instruction on student achievement. The three methods included: (a) a multiple-embodiment teaching sequence including the computer as one embodiment, (b) the same sequence with embodiments but without the computer embodiment, and (c) a teaching sequence based on traditional textbook procedures. The dependent variables were mean scores on
criterion-referenced tests. Independent variables were method of instruction, gender, and ability grouping. Instruction consisted of the teaching of volume of rectangular solids. The population were fifth grade students enrolled in school districts in southwestern Montana.

Need for the Study

National testing services have documented the need for an increase in mathematics achievement, including the subject of volume (Gardner et al., 1983; MEAP, 1983; NAEP, 1979a; NAEP, 1979b; NAEP, 1983). Educators are also concerned that much of mathematics instruction does not promote higher cognitive levels of learning (Bialo & Erickson, 1985; MCTM, 1985; Wheeler, 1971). The importance of retention, as well as initial acquisition of mathematics knowledge, is another matter for concern (Byers & Erlwanger, 1985).

The National Science Board of Pre-College Education in Mathematics, Science and Technology (1983), the National Council of Teachers of Mathematics (1985), the National Commission on Excellence in Education (1983), and the National Council of Supervisors of Mathematics (1988) were among the groups who have urged new and continued research to develop improved curricula and
instruction in mathematics, including the use of computers in instruction. Several national mathematics education organizations have urged inclusion of computers into the middle school mathematics curriculum (Johnson, 1982; Commission on Standards for School Mathematics, 1987). The promise of promoting higher cognitive mathematics skill and knowledge is offered by computer simulations (Bork, 1980; Chambers & Sprecher, 1983; Marks, 1985; NCTM, 1985). Yet, computers have not been integrated into the learning of mathematics (Dossey, 1988).

The need for concrete activities as initial experiences in the instruction of volume of rectangular solids has also been documented in studies by Dienes (1967), Reys (1972), Edge and Ashlock (1982), and Ben-Haim et al. (1985), and Bishop (1983) urged research on teaching materials and procedures in the subject areas of space and geometry. However, the use of computers as a supplement to the teaching of volume in order to improve mathematics achievement has not been tested.

Finally, positive results on preliminary field tests of the instructional sequence to be used in this study indicated a need for a carefully controlled experiment. Hart (1981, p. 216) is among several educators who have identified the need for "well-documented evidence that a
particular teaching sequence is preferable to another [in mathematics education]."

**Questions to be Answered**

Questions answered in this study were as follows:

1. Was there a significant difference in student achievement among treatments?

2. Was there a significant difference in student achievement between gender groups?

3. Was there a significant difference in student achievement among ability groupings?

4. Are there interactions between combinations of the independent variables?

These questions are detailed in null hypothesis form in Chapter III.

**General Procedures**

The Campbell-Stanley Nonequivalent Control Group Design (1963) employed an experimental-control model with teaching sequences that included: (a) Treatment I - multiple embodiments including the computer as one embodiment, (b) Treatment II - the same multiple embodiment sequence but without the computer embodiment, and (c) Treatment III - a sequence based on traditional commercial textbook teaching method. A three-week
retention period provided time and activities for all treatments to review material one period per week via:
(a) Treatment I - computer, (b) Treatment II - manipulatives, or (c) Treatment III - pencil and paper problems. A retention test was administered to determine the amount of learning for each treatment.

Two equivalent forms of a criterion-referenced test (CRT) developed by this researcher tested both concept and skill levels of knowledge. Form A served as a post-instruction measure; Form B was used to measure post-retention achievement and a portion of the Form B was used as a pretest. Reliability was determined by the equivalence and stability method; the Pearson R coefficient was .864. Content validity was established with a table of specifications and validation by experts (see Appendix B). Validity of the software was established by two computer education specialists who used criterion established by Alesandrini (1985) and Mezzina (1985) (see Appendix B).

A standardized test of mathematics ability, the Stanford Achievement Test (SAT), was administered to determine ability levels of participating students. A four-task test based on Copeland’s (1979) diagnostic activities with elementary mathematics students was developed by this researcher to determine student
logico-mathematical level, Piaget’s description of mathematics cognitive development (Copeland, 1979, 1984; Groen & Kieran, 1983). Validity was determined with a construct validity method described by Borg and Gall (1983). Reliability was determined with the test-retest method; the Spearman Rho correlation was .8514.

This researcher provided training for 17 classroom teachers from southwestern Montana who conducted the instruction and administered the tests. A student sample size of 419 students was thus generated. Individual structured tests consisting of tasks and questions were conducted by this researcher to determine individual student’s mathematics developmental levels with respect to measurement and conservation of area and volume. The tests consisted of tasks based on Copeland’s (1979) work and were conducted before treatments were administered.

A one-way analysis of variance (ANOVA) on pretest scores was employed to test significant differences of group means in achievement on the pretest to ensure equivalent groups. To test equivalence of ability groups across treatments, two statistics were used: (a) ANOVA of pretest scores; and (b) Kruskal-Wallis non-parametric rank test of scores based on the Piagetian tasks. Factorial analysis of variance was used to determine whether posttests revealed a difference among groups.
Limitations and Delimitations

Limitations of this study were as follows:

1. The study was limited to classrooms of teachers who volunteered to participate in the study.

2. The student sample was from fifth grade mathematics classrooms in southwestern Montana.

3. The time period included one week for instruction and three additional weeks for retention.

4. The subject of this study was volume of rectangular solids.

5. Computer usage was reported by students in Treatment I. Instruction to teachers was to ensure all students had access to CAI during the three weekly retention periods. No other control for time on task was made.

Delimitations of this study were as follows:

1. The population of this study was fifth grade students in southwestern Montana.

2. The experimental group including CAI was limited to classrooms that have at least one Apple IIe or GS available for student use.
Definition of Terms

For the purpose of this study, the following definitions were used:

**computer-assisted instruction (CAI)** - a data processing application using a computer to assist with a student's instruction (Rosenberg, 1984).

**embodiment** - concrete, semi-concrete (pictorial) or symbolic (verbal or written) situation representing a mathematics concept (Indelicato, 1979).

**hardware** - physical equipment used in data processing, as opposed to programs, rules, procedures (Rosenberg, 1984).

**high ability groups** - ability groups in each Treatment determined by a score of 61 or greater on the Stanford Achievement Test.

**high level items** - items on the criterion-referenced test which address application and analysis cognitive skills (see Appendix A; Bloom, 1956).


**low level items** - items on the criterion-referenced test which address knowledge and computation cognitive skills (see Appendix A; Bloom, 1956).
low ability groups - ability groups in each Treatment determined by a score of 49 or greater on the Stanford Achievement Test.

manipulatives - objects operated by learners or teachers (Indelicato, 1979).

medium ability groups - ability groups in each Treatment determined by a score of less than 61 and more than 49 on the Stanford Achievement Test.

microworld - a simulated computer environment (Indelicato, 1979).

multiple embodiment - two or more embodiments of a mathematics concept (Indelicato, 1979).

post-instruction test - criterion-referenced test administered following one week of instruction.

post-retention test - criterion-referenced test administered following three retention activities.

simulation - representation of a system or selected characteristics of that system (Rosenberg, 1984).

software - programs, procedures, rules and documentation pertaining to the operation of a computer system (Rosenberg, 1984).

volume of rectangular solid - volume of a right rectangular prism.
CHAPTER II

REVIEW OF LITERATURE

Introduction

The researcher studied the effect of a multiple embodiment instructional sequence including the computer as one embodiment on student achievement. The dependent variable in the study was student achievement on the subject of volume of rectangular solids. The study also included the effect of the sequence without the computer embodiment or supplement. A control group was taught with a traditional textbook method. Achievement was measured after instruction and after a period of retention activities. The review of literature will include pertinent research on the use of computers in education and the potential to address the need for instructional methods that develop higher cognitive levels of mathematics learning, the importance of memory for mathematics learning, instruction using multiple embodiments, and instruction on volume of rectangular solids, gender, and ability groups in mathematics.
Computers in Public School Education

A review of literature on computer applications in education demonstrates the effectiveness of computer assisted instruction (CAI). Yet, after years of development, from 1955 to the early 1980s, the impact of CAI on education is only beginning to be felt (DeVault, 1981; Henderson, 1983; Henderson, Landesman, & Kachuck, 1985).

CAI programs require active participation on the part of the learner who inputs instructions to be manipulated by the program. The computer acts as a tireless and impartial instructor giving immediate feedback for reinforcement and/or remediation. Verbal or graphic reinforcement for correct responses can increase student motivation and add to the enjoyment of learning (Chambers & Sprecher, 1980; Marks, 1985; White, 1983). Feedback for correction can redirect a learner to remediation and correct errors before the same error is repeated and reinforced (Chambers & Sprecher, 1983; Henderson et al., 1983; Menis, Snyder, & Ben-Kohav, 1980; White, 1983). In this manner a student can predict results and answers to problems; the computer responds with calculated answers and the student can compare his results with that of the computer.
CAI is used in instructional settings as a supplement to or substitute for direct instruction. Supplemental CAI is used as adjunct to traditional classroom instruction and activity; CAI used as substitution replaces other modes of instruction (Henderson et al., 1983; Kulik, Kulik, & Cohen, 1980).

Henderson et al. (1983) discussed the three types of computer assisted instruction programs: tutorial, drill and practice, and simulation. Tutorial programs present new concepts or skills in carefully sequenced steps. Drill and practice software offers learners practice in the development of skills through drill format. Simulations create a model of a real situation, either a life situation or a representation of an experimental situation (Suydan, 1984).

Most pre-1970 studies dealt with tutorial or drill and practice computer assisted instructional programs and were conducted in elementary kindergarten through eighth grade classrooms (Kulik et al., 1980; Willis, 1987). More recent studies indicate an increasing use in all levels of educational climates and CAI use other than tutorial and drill and practice. Kulik, Bangert, and Williams (1983) demonstrated that the positive effects of tutorial and drill and practice CAI appear to weaken as grade level increases through high school. The authors reasoned these types of programs, with their continual
guidance, feedback, and constant call for student input appeal to primary and elementary age learners. Older students, who are more self-directed, prefer self-directed CAI.

Significant positive effects of CAI on student achievement have been reported in the literature. However, DeVault (1981) conducted a review of 32 doctoral dissertations conducted between 1969 and 1979 comparing effective use of CAI and traditional instruction. He found the majority were drill and practice; only five were tutorial, seven concerned programming, and two dealt with problem-solving skills. Of the 30 that compared achievement results, only 12 reported a significant difference.

In a meta-analysis of 59 studies of college teaching conducted during the period from 1967 to 1978, Kulik et al. (1980) found CAI significantly raised achievement scores and decreased the need for direct instructional time. Another meta-analysis of 51 studies of computer-assisted instruction in grades 6 through 12 found a significant increase in achievement and retention scores (Kulik et al., 1983). Criteria for inclusion in both meta-analyses included the following components:
1. Only quantitative based studies with control and experimental groups were used.

2. Studies were carried out in classroom climates, not in computer laboratory climates.

3. No evidence of crippling methodology existed. If identical studies were carried out at the same institution only the most recent was used; if evidence of teaching to criterion tests or clearly different aptitude levels of control and experimental groups were apparent, the study was not included.

4. Analysis included tutorial, drill and practice, and simulation CAI.

5. CAI was used as supplement to instruction, not substitution.

Computer simulation is a new form of computer-assisted instruction (Chambers & Sprecher, 1980; Dugdale & Kibbey, 1983; Henderson et al., 1983). Simulations model a real situation, often one that cannot be experienced by the learner. Computer simulations are often called microworlds (Papert, 1980). For example, a simulated space activity can allow the student to input actions and test results, for instance, while moving, eating, or walking in zero gravity. Thorson (in Chambers & Sprecher, 1983) proposed simulation CAI can bridge the gap between abstract learning and application of that
learning in problem-solving contexts. Computer simulation gives the student experiences in application and analysis levels of cognitive learning (Bridges, 1987; Shaw, Okey, & Waugh, 1984). Students are not simply "doing" math but are "using" mathematics to solve real life problems (Dugdale & Kibbey, 1983, 1984). Magdalene Lampert, Coordinator of the Dilemma Management in Mathematics Project, stated that "doing" math usually means following rules and algorithms; "knowing" math means remembering and correctly applying algorithms. She contends "knowing" is a student's own belief system about math established from his/her own experiences (Instruction for Research on Teaching, 1988). Research in mathematics education has demonstrated that students learn more by using and applying mathematics than simply "learning about" math (Driscoll, 1988).

Microworlds were the main focus of Papert's work with Logo, a simulation in which students manipulate a "turtle" to create and explore geometric forms (Papert, 1980). They were also the basis for Dugdale and Kibbey's (1983, 1984) Green Globs, Tracker, and Graphing Equations combined software package. A new geometry exploration computer package, Geometric Supposer, is another example of simulations used in the instruction of mathematics (Driscoll, 1988).
With the development of simulations, a new body of research dealing with visual computer applications has appeared. Studies on the use of visual and pictorial representation show positive effects for facilitating learning for both adults and children (Alesandrini, 1985; Morris, 1982). Franz (1986) and Smith (1982), for example, demonstrated the potential for visual representation capabilities of the computer in an instructional setting. Shaw and Okey (1985) conducted a study of 173 sixth and seventh grade students on observing, classifying, and recording data. One group used computer simulation, one group used laboratory activities, and a third used a combination of simulations and activities. A fourth control group was instructed via a conventional lecture format. Results showed significantly higher achievement in all three experimental groups. Reed’s 1981 study explored the condition under which computer graphics could improve student estimation of solutions of algebra word problems. The 180 college undergraduate students increased achievement scores. The factors effecting the positive results were: (a) good visual feedback, (b) CAI as supplement, not substitution to instruction, and (c) active participation in the CAI by students. In spite of the positive effects of computer simulations on student
achievement, only 4 to 5% of available software falls into this category (Bialo & Erickson, 1985). Willoughby (1986) estimated that 90% of all CAI addresses lower level computation skills.

Computers in Mathematics Education

The subject area that has made the most use of computer assisted instruction is secondary mathematics (Bialo & Erickson, 1985; Copple, 1981; Keuper, 1985). The Educational Software Selector (TESS), which lists all educational software available through commercial vendors, includes over 4,500 educational programs in 24 subject areas. Twenty six percent was in the area of mathematics, followed by 15% for science (Bialo & Erickson, 1985).

In 1985, Keuper reviewed 25 studies measuring the effect of computer usage in mathematics instruction. The studies were conducted between 1970 and 1983 in grades 7 through 12. Twenty studies reported positive effects on student achievement. Four of the studies reported no effect and one study reported a negative effect. The limitations of Keuper's review were as follows:

1. Not all studies used control and experimental groups.

2. Experimental settings, experimental designs, and types of application varied.
Suydam's 1984 review of microcomputers in mathematics instruction concluded that instruction using CAI as a supplement is at least as effective or more effective in drill and practice type instruction and tutorial CAI use over traditional methods of instruction. Additionally, she reported that problem-solving tasks using a computer increases mathematics understanding.

In a meta-analysis of 40 research findings concerned with computer use in science and mathematics classrooms, Burns and Bozeman (1981) reported CAI used as a supplement to instruction was more effective than traditional instruction alone and increased student achievement in both elementary and secondary mathematics classrooms. Criteria for inclusion into the study included the following requirements:

1. CAI was used as a supplement to, not a substitute for, instruction.

2. CAI was used for elementary and secondary instruction.

3. Studies were conducted in classroom settings only.

4. Only studies that employed control and treatment groups, or where control group data was available relative to predicted performance, were used.

5. The dependent variable was student achievement.
Similar findings were reported in secondary mathematics classrooms by Kulik et al. (1983) and Henderson et al. (1983) when computer-assisted instruction was used as a supplement to traditional instruction. The 1983 Henderson study consisted of two field tests. The first used 36 control and 45 experimental students enrolled in a high school algebra class; the second test used 11 volunteers from an alternative high school. Three computer-video instructional modules designed to teach or re-teach prime numbers, factors, and fractions were employed. Results of the pre-post criterion-referenced tests and the School Learning Questionnaires for both groups demonstrated the modules were positively effective on cognitive outcomes.

Jhin (1971) studied the effect of non-tutorial CAI in the instruction of 11th grade algebra. Independent variables were ability groups and treatments; dependent variables were knowledge of computer principles, abstract reasoning, and algebra. The three month study involved two control and two treatment groups. The researcher concluded CAI was more effective among high ability groups and in increasing knowledge of computers.

The 1982 NCTM Yearbook focused on critical mathematics issues in middle school grades five through nine. Among the issues chosen for printing was the use
of computers in mathematics instruction, including the topics of measurement and geometry. A suggestion for improving instruction in these areas was the use of computer simulations (Thompson, 1982).

Other successful studies and pilots for incorporating CAI into mathematics instruction include graphing equations (Dugdale & Kibbey, 1983, 1984), graphs of calculus equations (Hsiao, 1985; Smith, 1987), algebra (Menis et al, 1980; Morris, 1982; Reed, 1985), problem solving (Steen, 1984), trigonometry (Morris, 1982), and simulated laboratory work (Shaw & Okay, 1985).

Computer-assisted instruction is especially effective with low ability and low achieving as well as remedial math students (Crawford, 1970; Feurzeig, Horwitz, & Niderson, 1981; Harman, 1980; Henderson et al., 1983; Keuper, 1985; Kulik et al., 1983; Menis et al., 1980). The type of CAI used to produce significant achievement among these student populations was drill and practice. Chambers and Sprecher (1980) and Feurzeig et al. (1981) are a few of the researchers who credit this increase in achievement to the immediate feedback, repetitiveness, and patient prodding of the computer program. Kulik et al. (1983) found a decrease in effectiveness of CAI in later school years when self-paced computer packages are used. The authors cite
earlier studies that reported self-paced computer programs provide too little guidance and support for effective use by younger and less able students. Reed (1985) tested a computer simulation with 180 college undergraduates. The program simulated the motion of filling a tank with water and the motion involved in average-speed types of algebra word problems. The simulation presented a concrete example of the events in problems, but it also depended upon subjects’ own abilities to perceive and interpret correctly relevant facts and information. Two results of this study were (a) impairment by previous misconception of the concept resulting in an individual’s inability to "see" changes, and (b) poorer students needed more information and assistance to be able to use the computer in their learning.

Graphic Capability of Computers

Computer usage has demonstrated a positive effect on the facilitation of computation skills of students in mathematics classrooms using drill and practice programs (Burns & Bozeman, 1981; Hartley, 1977; Kulik, Bangert, & Williams, 1983). Comprehension skills have been similarly positively effected with CAI tutorial programs that test learner knowledge of concepts and terminology and ability to reason (Bridges, 1985; Burns & Bozeman,
1981; Dugdale & Kibbey, 1981; Henderson et al., 1983; Kulik et al., 1983). In 1979 the National Assessment of Educational Progress (NAEP) Report warned of the overdevelopment of lower level mathematical computation and comprehension skills at the expense of higher level application and analytical skills in public school mathematics instruction. Too often students developed only the ability to recall and apply a memorized algorithm without understanding the algorithm or its relationship to the problem. When students were forced into simply providing answers without understanding the concept involved, they did not build the foundation for future mathematics study. Dugdale and Kibbey (1983) suggested that as more and more algorithms accumulated in student memory, it became difficult to keep them separated. The National Assessment of Educational Progress Report has recommended "an expanded definition of what is 'basic' to mathematics is crucial to foster student ability to cope with different types of mathematical problems. Students must be introduced to higher level as well as lower level cognitive processes (NAEP, 1979, p.27)."

Computer simulations offer potential instruction that can promote application and analysis of mathematics situations not previously available in the classroom
(Bridges, 1985; Dugdale & Kibbey, 1983, 1984; Franz, 1986; Menis et al., 1980; Reif, 1985). The National Council of Teachers of Education Report (1985) recommended a public school kindergarten through 12th grade curriculum that included use of computer technology to shift instruction away from traditional focus on computation and manipulative skills toward an emphasis on developing mathematical concepts, relationships, and problem-solving skills. For intermediate and secondary students, who are developing or have developed computation and comprehension skills, the computer can alleviate the number crunching aspects of mathematics problem-solving and allow students to concentrate on the application, analysis, and synthesis aspects of the problem so they can develop an understanding of the math process which involves "...the ability to grasp the principle underlying the knowledge and skill (NAEP, 1979, p. 9)." Software programs and computer-based instruction that "lead learners away...[from drill and practice]...and toward exploration" are needed (DeVault, 1981).

NCTM (1985) further recommends the use of computers in mathematics instruction at all levels (K-12) to enhance the concrete to abstract transition necessary for mathematics study. The National Science Board Commission
of Pre-College Education (1983) recommended computer graphics be included in school curriculum "...at the earliest practical grade (p. 1)." "The power of graphic packages makes it easier for students to get a visual sense of geometric concepts (p. 9)." In writing for the 1982 MCTM yearbook, Johnson (1982) urges inclusion of computer simulations to compute and solve problems involving geometry and other mathematics problems. Still, in spite of the apparent need for CAI in mathematics, computers are not widely used in mathematics instruction (LaPointe & Martinez, 1988).

Mathematics is an abstract subject; computer simulation and graphics can illustrate the abstract concepts with visual representations of the concept (Bork, 1980; Marks, 1985; National Science Board Commission on Pre-College Education, 1983; Smith, 1982). It is this ability to present abstract concepts in semi-concrete format that makes simulations valuable for mathematics education ( Flake, 1982; Marks, 1985; Mezzina, 1985; Morris, 1982; Reed, 1985).

The use of special effects of motion and graphics in computer programs offers potential for learning not previously available to mathematics instruction (Alesandrini, 1985; Franz, 1986; Henderson et al., 1983). The primary advantage of graphics is similar to pictorial
and diagramatic aides in textbooks but CAI also has the added facility of motion and variability. With the use of computer simulation, a learner can manipulate a visual representation of mathematical relationships involved in problems. Dugdale and Kibbey (1983) hypothesize the difficulty students have with mathematics was due, at least in part, to a lack of understanding of the relationship between variables and to the mathematical concept as a whole.

Effective CAI graphics are relevant to the topic and content of the lesson (Alesandrini, 1985). Too often graphics add little to the content and are used only to attract a user's attention and keep his/her motivation stimulated. Research demonstrates irrelevant graphics not only detract from learning but may have an adverse effect on memory facilitation (Alesandrini, 1985). Effective graphics also include corrective and reinforcement feedback to facilitate learning (Alesandrini, 1985; Dugdale & Kibbey, 1983, 1984; Franz, 1986; Reed, 1985).

Software packages used for instruction in calculus and trigonometry (Morris, 1982) classrooms have demonstrated the graphics capability of the computer to draw representations of three-dimensional spaces. Similarly, the potential exists for computer simulation
microworlds to represent volume (Ben-Haim et al., 1985). Dugdale and Kibbey (1983, 1984) demonstrated that microworlds can provide an environment that enables learners to apply mathematics in an interesting and challenging way. Microworlds allow learners to focus attention on their own abilities to make use of the mathematics they were learning in the classroom.

**Memory and Mathematics Learning**

In order to understand how to enhance mathematics learning, one must understand how learning takes place. Acknowledgement of the importance of the acquisition, storage and retrieval from memory of mathematics knowledge is noted by the many appearances of the topic in recent studies, books, and papers (Behr & Bright, 1983; Byers & Erlwanger, 1985; Davis, 1983; Marks, 1985; Nicely, 1985; Shuell, 1986).

Various types of mathematics knowledge include (a) rote knowledge (i.e., number facts, procedures, etc); (b) understanding (i.e., relationships of variables to equations); and (c) intellectual skills (applications to problems) (Byers & Erlwanger, 1985). Few will argue the difference between rote replication of memorized, automatic acts and application of mathematical knowledge to solve problems (Dugdale & Kibbey, 1984; Commission on
Standards for School Mathematics, 1987). Mathematics "anxiety" may be due to lack of understanding and ability to apply mathematics to new situations and a continued use of memorized, rote learning (Byers & Erlwanger, 1985; Dugdale & Kibbey, 1984). Memory involving mathematics learning may be one of the factors that distinguish capable students who do well in math from those who do not (Byers & Erlwanger, 1985).

All three types of knowledge are necessary for success in mathematics (Dugdale & Kibbey, 1984; NAEP, 1983; NCTM, 1985). To use mathematics knowledge, one must recall from memory the facts, algorithms, and applications skills learned and stored in memory.

Until 1960 most educators viewed mathematics memory as a "quantitative" measure of learning (Byers & Erlwanger, 1985; Gagne, 1985). One result was an emphasis on repetition; much of the CAI involved drill and practice software (Kulik, Kulik, & Cohen, 1980).

More recently the qualitative role of meaning and organization in mathematics learning has again surfaced as the major opinion. As early as 1932, F. C. Bartlett's experiments resulted in the conclusion that memory was a process, not simply factual recall (Cofer, 1976). In 1957, Helgard stressed "...understanding of principles....for such learning is...likely to remain
permanently available [in memory]" (Hilgard in Byers & Erlwanger, 1985, 262). Bruner (1971) stated learning involves "...internalizing events into a storage system that corresponds to the environment" (p. 5) and refers to retention of knowledge as a main problem. Gagne and White acknowledged the importance of memory in learning when they (1978) adapted memory as a component in their instruction --> memory structure --> learning outcomes paradigm. Piaget and Inhelder (1973) assert knowledge originates from an action, leads to a schemata, or organization of thought, and is stored as a memory image in the brain. Contemporary theory also recognized various modes of memory, including visual, verbal, and tactile (Byers & Erlwanger, 1985). The tactile memory may contribute to remembering concrete experiences in mathematics education (Lesh, 1983). Visual memory may also be an important component, especially as it relates to spatial visualization ability (Lowery & Knirk, 1983).

Two theories emerged during the 1970s that revolutionized the study of memory. First was the distinction between long-term memory (LTM) characterized as permanent, with unlimited capacity and complex retrieval, and short-term memory (STM) characterized as temporary, limited capacity, and easy retrieval (Byers & Erlwanger, 1985; Gagne, 1985; Greeno, 1973; Klatzky,
1980; Weaver, 1980) and a "workplace" for problem-solving
where information from LTM is brought to solve the task
in STM (Greeno, 1973). Recent studies on human brain
anatomy by Dr. Stuart Zola-Morgan and others at the
University of California at San Diego have produced
physical evidence to support the working memory concept
(Beglay et al., 1986). This process approximates, albeit
in a very primitive manner, an information-processing
analogy to computer processing where encoding (input)
enters information (data) into the memory (computer).
Inability to retain or recall information may be due to:
(a) problems with accessing (data) information, or (b)
not knowing what to do with the retrieved data, or (c)
incorrectly or inadequately stored information (Klatzky,
1980; Norman, 1982; Norman & Bodrow, 1976). Therefore,
initial input (knowledge) meaningfully encoded (stored)
aids retrieval and use of knowledge (Gagne, 1985;

Horwitz (1981) hypothesized poor mathematics
students who do not possess formal problem-solving
strategies find visualization useful in reducing the
strain on the limited capacity of STM. His sample
consisted of 36 subjects who had difficulty with college
algebra and a replicated study with 48 highly competent
technical college subjects. The technical students used
visualization skills to solve mathematics problems. The poor-math students used little or no visualization and had difficulty solving the problems.

The second theory on memory is that human memory is organized in a systematic, schematic manner. Gagne (1985) and Byers & Erlwanger (1985) propose various forms of memory storage exist, including ideas, skills, images, and procedures.

The way mathematics knowledge is organized and stored in memory plays a major role in the individual's ability to understand and apply mathematics. Piaget also agrees that memory is supported by existing schemata and that memory for operations and general knowledge differs from memory for specific facts or events (Piaget & Inhelder, 1973). He further claims that what a child remembers depends upon his cognitive developmental level. The role of experience with math concepts at several levels of cognitive development as well as representational systems (written, pictorial, etc.) cannot be underestimated. Many experiences via several sensory inputs are needed before a mathematics concept is understood (Lesh, 1983, p. 77).

Piaget states there are several forms of mathematics knowledge: (a) sensorimotor, instinctual; (b) physical, gained through one's senses; and (c) logico-mathematical,
highest form of mathematics knowledge, gained through one's own experiences, actions, and conclusions (Copeland, 1979). The logico-mathematical knowledge enables the learner to develop a network of mathematics structures and principles (Groen & Kieran, 1983). Some researchers conclude the study of Piagetian mathematics will lead to a "synthesis" of Piagetian structure and information processing in the learning of and memory for mathematics (Copeland, 1984; Gagne, 1985; Groen & Kieran, 1983).

The main point from the cognitive research is that information is stored in an organized manner; retrieval is systematic and depends upon the organization of storage. Consequently, the better a concept is learned, and the more experiences connected with the knowledge, the better the concept is retained and recalled (Byers & Erlwanger, 1982; Gagne, 1985; Klatzky, 1980; Norman, 1982; Underwood, 1982).

The learning of mathematics can be viewed as a process by which learners derive meaning from their memories (Wittock, 1977). But the stored information must be accurate. Also, retrieval and good use of working space processes in the solving of mathematics problems must result in correct application. Continued errors only reinforce the incorrect information stored in
one's memory. A good math student organizes mathematics knowledge so as to minimize the strain on STM; he/she knows which formulas must be memorized and which can be easily recreated. He/she has memorized facts for quick recall. A poor math student relies on a forced attempt to memorize everything (Byers & Erlwanger, 1985; Horwitz, 1981; Gagne, 1985). The emphasis should be on understanding instead of rote performance of a task with no regard to comprehension; not the acquisition of behavior but of knowledge (Shuell, 1986). The interest in CAI simulation as an impetus to promote such learning is understandable.

The Multiple Embodiment Principle

The Multiple Embodiment Principle proposed by Z. P. Dienes in the 1960s states that in order for a concept to be understood it must be presented to the learner in a variety of experiences. Each experience must embody the concept to be learned by:

providing tasks which look quite different but have essentially the same conceptual structure....we can vary the perceptual representation, keeping the conceptual structure constant. For example, parallelograms can be drawn on paper, they can be made out of two congruent triangles, they can be traced out with pegs on a pegboard (Dienes, 1967, p. 13).
Dienes' theory is based on the belief that children learn by passing through stages, including concrete to abstract activity and concepts. Indeed, much of current mathematics educational theory accepts a progression from concrete to semi-concrete to abstract as the sequence through which a learner passes. Advocates maintain mathematics education should consist of initial concrete experiences with physical manipulatives, followed by semi-concrete, pictorial representatives of pictures or drawings and images and, lastly, experiences with abstract representation of the concept such as symbols, numbers, and letters. Several cognitive educators support this premise; for example, Bruner (1971), Piaget (Reys, 1971) and Van Hiele (1986). In the 1960s and 1970s the progression through the three stages was thought to be linear. Recent concepts of cognitive psychology support the thesis that experiences in several representational systems (oral, written, pictorial, manipulative, and real life experiences) facilitate learning and the continual process of learning moves back and forth from one system to another (Behr & Bright, 1983). "Virtually no significant mathematics idea can be adequately understood from within a single representational system" (Lesh, 1983, 76-77).
The repetition of experiences in varying representational systems is different from repeated, identical experiences which may reinforce one component of the concept or associate the concept exclusively with the material used (Behr & Bright, 1983; Dienes, 1967; Wheeler, 1971). Dienes believed children acquire math concepts by individual intuition from their own experiences. "Memories based on the...perception of a model are less complete and less accurate than memorizing based on actions..." (Piaget & Inhelder, 1973, p. 391). Van Hiele voiced a similar belief (Hoffer, 1983; Van Hiele, 1986).

In his review of 11 doctoral dissertations from 1964 to 1977 on mathematics instruction involving multiple embodiments, Indelicato (1979, p. 34-5) found a "...clear mandate to use physical embodiments...in upper elementary grades and junior high school (grades four through eight)." This reinforces earlier, more extensive reviews that conclude "...instruction [using] physical embodiments are an integral part of the concept development and is reflected in positive student achievement" (Indelicato, 1979, p. 35).

Reys (1972) reviewed the literature for research conducted on instruction including multiple embodiments from 1967 to 1970. He found no definite support or
rejection of multiple embodiments as an instructional method, rather that use of multiple embodiments was teacher dependent. Reys states one problem with Dienes' theory is dependency upon the teacher to gather and prepare appropriate lessons. Another problem is the requirement that physical or concrete representation (embodiment) accurately represents the mathematics concept to be represented with a variety of perceptually different modes.

Behr (1976) used five treatments in a study of the effect of multiple embodiments in second grade instruction of two- and three-digit addition and subtraction with and without regrouping. Each treatment involved manipulative, pictorial, and symbolic phases. The manipulative materials used were:

- Treatment 1) counting sticks
- Treatment 2) Dienes' Blocks
- Treatment 3) abacus
- Treatment 4) all three above
- Treatment 5) counting sticks, unifix cubes, Dienes' Blocks, graph paper, abacus and colored chips.

Behr found that multiple embodiments produce an increase in achievement for two-digit but not for three-digit numeration. Although the abacus did not aid learning, all other multiple embodiment treatments proved
more effective than a non-manipulation control group. The study was, however, conducted only in one classroom.

In 1972, a study was conducted to measure effect of manipulative materials used in multiple embodiment instruction of number systems. The subjects were 145 pre-service elementary teachers. Subjects were tested on understanding, performance, and transference of number concepts. A lecture treatment was the control. Neither method was superior (Skipper, 1972).

Jhin (1971) studied the effect of non-tutorial CAI in 11th grade algebra II classes. Independent variables were ability grouping and treatment; dependent variables were knowledge of computer principles, abstract reasoning, and algebra. The three-month study involved two control and two treatment groups. The researcher concluded CAI was more effective among high ability groups and increased achievement of knowledge of computers.

Wheeler (1971) measured the amount of exposure to concrete materials and the effect on regrouping concept in multi-digit addition and subtraction problems. A random sample of 144 second grade students was selected from nine classrooms in three schools. The schools reflected varying utilization of different amounts and types of concrete materials such as bundling sticks,
place value charts, multi-base arithmetic blocks, and the
abacus. One school taught primarily using the symbolic
mode. The dependent variable was achievement scores on
two-digit addition and subtraction problems. The
independent variable was level of abstraction, IQ, and
treatment. Results demonstrated a significant
relationship between number of embodiments and ability to
regroup for two-digit addition and subtraction examples
as shown by performance on multi-digit problems.
Achievement scores increased with the number of
embodiments used. Some children were able to grasp the
abstract concept of multi-digit with no exposure to
concrete materials. Finally, all levels of IQ ability
benefited from exposure to concrete materials. The
experimenter concluded that his study supported Dienes’
theory and recommended exploring use of other types of
embodiments.

More recent studies of multiple embodiment
instruction in mathematics include Edge and Ashlock’s
(1982) study of place value concepts. Forty-two second
grade students from a middle class residential school
were randomly assigned to two groups to compare effect of
single and multiple embodiments in the teaching of place
value of three-digit numbers. The single embodiment was
coffee stirrers which were collected into bundles.
Multiple embodiments included coffee stirrers, Dienes Blocks, and chips. Each treatment included initial instruction with concrete activities. Tests on understanding and transfer of the concept revealed no difference between the use of one or three embodiments. The small N in this study and the lack of a control group, however, reduce the value of the results.

Berlin and White (1986) tested the use of computer simulation as an interactive link from concrete to abstract thought. A random sample of 113 subjects included both second and fourth grade levels from a rural, all-white school and a suburban, all-black school. The study assessed performance on:

1. recognition and duplication of designs;
2. recognition and extension of patterns through seriation; and
3. spatial orientation and discrimination tasks.

Three treatments were:

1. concrete activities only;
2. concrete activities and a computer simulation; and
3. computer simulation only.

Ten task cards required learners to duplicate a color, shape, and orientation pattern on a 10 in. by 10 in. pegboard and extend a pattern around the perimeter of a 6
in. by 6 in. grid with colored cubes. Concrete activities made use of the physical objects; the computer simulation required students to replicate the movement via keyboard input. Following the three-week treatment, paper and pencil instruments tested design recognition and pattern extension. Significant differences resulted in better student performance in grade four over grade two and white over black students. There was a mixed interaction of treatment, gender, and site variables. The problem of all-black urban and all-white suburban sites does not seem to allow a basis of equal comparison. The author concluded there was a mixed effect of computer simulation on achievement and recommended more studies on use of computer as a multiple embodiment.

Only one study concerned the subject of volume. R. L. Johnson (1970) studied effects of three treatments on achievement on the subject matter of perimeter, area, and volume. The three treatments were: (a) a text including diagrams and illustrations and physical manipulation of concrete objects, (b) the text alone, and (c) the written part of the text, excluding diagrams and illustrations. Subjects were 96 students from grades four, five, and six. The individualized instructional lessons were completed in periods of 30 minutes each day. Most students completed the lessons in three weeks, though
four weeks had been allowed. Tests were administered upon completion of the units and a retention test was administered two weeks following the first test. Significant differences resulted between treatment one over treatment two and over treatment three. No difference between treatment two and three was found. As expected, older students responded better than younger students; better readers responded better than poorer readers. There were no gender differences. The study reinforces use of concrete manipulatives. The use of semi-concrete diagrams and illustrations was not student generated but was within the text.

Indelicato's 1979 study attempted to determine whether an increase in achievement could be produced using a variety of physical manipulatives embodying four mathematics concepts: (a) area, (b) ratio and proportion, (c) addition and subtraction of integers, and (d) probability. The treatment took place in four one-week consecutive sessions. There were no retention instructions or activities. One hundred eighty students were randomly assigned to junior high mathematics classes. A 3 X 2 X 3 factorial design represented treatment (zero, one, and three physical embodiments), teacher (2 teachers), and ability (high, middle, and low). Results showed a significant difference in main
effects for treatment in favor of one and three embodiments, but ANOVA analysis on individual mathematics concept instruction resulted in no differences. Therefore, the researcher concluded that all four math concepts contributed to the factorial difference. Retention tests given four weeks later showed no significant difference. There was a significant difference favoring higher ability in all four areas on both post-instruction and retention tests. The effect of the teacher variable was varied according to topic on both post-instruction and retention tests. Transfer items showed no significant difference for any of the independent variables. The researcher and participating teachers felt that, due to amount of time spent on physical activity with embodiments and, consequently, less time with paper and pencil problems, the fact that the one and three embodiment groups did as well demonstrated the worthiness of this approach. Time was not controlled except for length of instructional time. Consequently, the author concluded his study reinforces earlier findings that instruction using "...physical embodiments are an integral part of the concept (development and is reflected in) positive student achievement" (p. 35).
Volume of Rectangular Solids

Mathematics is the science of number (quantity) and space (Bishop, 1983; Sharma, 1985). The emphasis in education, however, is on the former (Bishop, 1983). Usefulness of mathematics in "...describing and quantifying the real world is one of the primary reasons it is a basic school subject" (Ben-Haim et al., 1985, p. 389). The world is three-dimensional but the main tools for teaching mathematics are two-dimensional, e.g., textbooks, pictures, etc. When studying space and/or geometry, the emphasis can be on learning theorems and formulae or on building a system of relationships dealing with the mathematics of space (Bishop, 1983).

One problem in geometry is that it is not possible to draw a generalized diagram. A single, specific triangle is not representative of all triangles; once drawn, it is specific. Many varied diagrams of triangles are needed to illustrate the concept of triangles (Bishop, 1983). One cannot construct or draw a box and say it is all boxes representing all volumes of rectangular solids; rather, many boxes of varying shapes and sizes are needed.

In Bishop's (1983) study of 250 college freshmen, research included testing of three-dimensional situations with two-dimensional format. Actual mathematics
knowledge needed for the test was very limited. Test items consisted of questions similar to: "Shown is a diagram of a cuboid made of small cubes. If the outside of the cuboid is painted red, how many cubes will only have one face painted?" (p. 187). Only 14% of the students successfully answered. Figure 1 shows the illustration described above.

Figure 1. Cuboid figure

Concepts "...are the substance of mathematics knowledge....(they are) more than recall of (facts) and definitions....(they) encompass knowledge of (concept) properties and how (the concept) relates to other math concepts" (Commission on Standards for School Mathematics, 1987, p. 146). Words are a critical dimension when dealing with mathematics concepts. Words identify, categorize, clarify, and represent concepts. In the subject of measurement, "...the number word describes the numerousness of the units into which some continuous dimension of an entity has been divided" (Fuson & Hall, 1983, p. 79); that is, "how many" units are needed in the problem. A student first identifies
the entity and then isolates it from the dimensions which measure the entity. A box, for example, can represent volume as measured by length, width, and height.

Piaget and Inhelder (1973) maintained children younger than 11 or 12 years of age acquire concepts not by analytical thoughts, but by organizing and adapting them from their own perception acquired by interacting with physical objects via their own experiences. Fuson and Hall (1983) identify experiences the learner must pass through to understand a concept. The learner must:

1. Identify the entity and recognize attributes or dimensions pertaining to it.

2. Create (identify) the units into which the dimension is divided.

3. Recognize that all of the entity to be measured must be filled up or used.

4. Count the units and "...make a 'count measure transition', i.e., shift from the counting meaning to the measure meaning of the number word" (e.g., from six to six cm (p. 80). The authors note that in this phase it is more constructive for young learners to have a multiple source of units to avoid having to keep track of reusing the measuring unit and keeping count of the number of times used.
5. In the case of multi-dimensional entities, such a volume, measurement words for the dimensions (e.g., volume given length, width, height) can be obtained by a derived formula (e.g., \( V = L \times W \times H \)).

More skills are needed for comprehension of a measurement word than are needed for memorized formulas. The generalization of a measurement procedure and concept seems to be facilitated by replicating an initial measuring experience across many differing examples and experiences (Fuson & Hall, 1983).

Psychologists have found that school-age children frequently experience visual memory images. This phenomenon diminishes in teen years as abstract thought increases (Sharma, 1985). In Western cultures, this ability is not encouraged, rather the emphasis is on ability to label and categorize quantitatively (Bishop, 1983). The study of space and geometry includes the concept of three-dimensions. One necessary ability in studying three-dimensional mathematics is spatial perception. While there is extensive literature on spatial ability, there is no accepted, clear definition (Clements, 1981). Lowery and Knirk (1983, p. 156), for example, define spatial ability as the ability to "...perceive and mentally retain two and three dimensional objects and their relationship to their
environment," Melancon (1985) as the "ability to rotate and manipulate figures", and McGee (1979, p. 893) as the "...ability to mentally manipulate, rotate, twist, or invert a pictorially presented...object." Clements (1981) found, however, most authors agree that spatial visualization involves an ability to form and manipulate visual images. STM is involved in spatial visualization (Horwitz, 1981; Lowery & Knirk, 1983).

Melancon (1985, p. 3) summarized problems associated with the lack of spatial skills necessary for mathematics learning as follows:

students are suffering from visual atrophy....in mathematics it is important that students be able to think abstractly. The ability to think abstractly may be facilitated by increasing the ability to think visually....visualization abilities include understanding and therefore give the student greater insight into the interrelationships of math concepts.

There is general agreement that a positive correlation between mathematics achievement and spatial perception exists (Bishop, 1983; Clements, 1981; Elmore & Vasu, 1986; Lowery & Knirk, 1982; Young & Ragan, 1980). However, Fennema and Tartre (1985) caution that while acknowledgement of some relationship between spatial visualization and mathematics achievement is evident, the emphasis on spatial visualization as a major component of mathematics achievement is not conclusive. Ben-Haim et al. (1985), too, concluded that, while the conclusion of
a relationship between spatial visualizations and mathematics achievement is generally accepted, studies from 1981 through 1985 covering several subject areas and age groups' results are inconclusive.

There does appear to be a positive relationship between high spatial ability and mathematics ability (Bishop, 1983; Clements, 1981; Lowery & Knirk, 1983). However, Guay and McDaniel (1977) assert that it is important mainly at the elementary level and for both high and low mathematics ability students.

Finally, the literature does demonstrate a definite relationship between spatial visualization and gender (Bishop, 1983; Clements, 1981; Elmore & Vasu, 1986; Fennema & Sherman, 1977; Fennema & Tartre, 1985). Males outperform females on tasks which require three-dimensional thinking and mental manipulation of images (Ben-Haim et al., 1985; Clements, 1981). In middle school mathematics, low spatial visualization ability may be more inhibiting to problem-solving skill in females than in males (Fennema & Tartre, 1985).

Perhaps the first area of mathematics which requires students to read visual data concerning solid objects from pictures is in studying volume. Fourth grade mathematics textbooks include a few "boxes" composed of drawn cubal units (Duncan, et al., 1981; Eicholz, et al.,
1985; Haenisch & Hill, 1985; Nichols et al., 1974; Payne, et al., 1982). National and state test items use similar figures when addressing the concept of volumes of rectangular solids (Ben-Haim et al., 1985; Gardner, et al., 1983a; MEAP, 1983; NAEP, 1979a; NAEP, 1979b). The very poor results are demonstrated in the items shown below in Figures 2 and 3. The percentage of correct answers reported by gender is shown below Figure 3.

Figure 2. Released item from the 1977-78 NEAP Test
What is the volume of each rectangular solid below?

a. 

Answer __________

b. 

Answer __________

Figure 3. Item from the 1977-78 NEAP Test
A rectangular solid is cut into cubes as shown. How many cubes are there?

Answer __________
The percentages of questions answered correctly were:

<table>
<thead>
<tr>
<th>Age</th>
<th>Male</th>
<th>Female</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>77.4</td>
<td>73.7</td>
<td>39.8</td>
<td>29.9</td>
</tr>
<tr>
<td>13</td>
<td>79.2</td>
<td>76.3</td>
<td>62.2</td>
<td>53.4</td>
</tr>
<tr>
<td>17</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Disappointing results on questions pertaining to volume were also reported on the 1982-83 Stanford Achievement Tests (SAT) (Gardner et al., 1983a, 1983b). Two forms of the SAT were administered. Intermediate Level tests were given in grade five in Spring of 1983 and grades six and seven in both Fall, 1982, and Spring, 1982. Illustrated in Figure 4 are items similar to SAT test items along with the percent of correct answers for each group.

**Figure 4. Items from the 1982 SAT**

Each small cube is one cubic unit. How many cubic units are in the volume of these blocks?

- a) 18
- b) 12
- c) 9
- d) 8
- e) NH

**Note:** NH means answer is (N)ot (H)ere
Advanced Level tests are administered to grades seven, eight and nine in the fall and in the spring. A items similar to the volume item on the tests along with the results is shown in Figure 5. The percentages of correct responses are shown below Figure 5.

Figure 5. Item from the 1982 SAT.

Find the volume of the box shown.

\[
\begin{array}{c}
\text{Form E} \\
\text{Grade 7} & 27\% & 33\% \\
\text{Grade 8} & 44\% & 54\% \\
\text{Grade 9} & 58\% & 61\%
\end{array}
\]

A review of the literature by Ben-Haim, Lappan, and Houang (1985) pertaining to instruction to develop spatial perception did not report agreement as to
effectiveness of such instruction between males and females and between age groups. The researchers conducted a study with approximately 1,000 fifth through eighth-grade public school students to determine the difficulties learners have relating to two-dimensional representations of three-dimensional objects. Their findings indicate a need to teach learners to read the two-dimensional representations of solids and provide instruction that initially includes concrete experiences with solid objects as early as middle school years. While developing test items for the study, the authors analyzed errors students made when working with problems concerning volume of rectangular solids. They discovered four categories of errors:

1. counting the number of faces visible on the drawing;
2. counting, then doubling the number of visible faces;
3. counting the number of cubes shown in the drawing; and
4. counting, then doubling the number of visible cubes.

This suggests the students were not aware of the three-dimensional aspect of the solids or did not know how to use the idea of three dimensions to obtain the
answer. Categories one and three indicated to Ben-Haim, Lappan, and Houang that the student was addressing the problem from a two-dimensional format. Categories two and four indicated an awareness of three-dimensional space and some attempt to address the hidden part of the drawing. A three-week training period of spatial visualization activities followed the initial testing as the authors felt that lack of or underdevelopment of that ability was a major factor in dealing with volumes of rectangular solids. A posttest demonstrated the positive effects of such instruction as well as the importance of initial instruction using concrete materials. Ben-Haim et al. concluded initial instruction of volume should include concrete activities.

Proposals to the National Council of Teachers of Mathematics prepared by the Commission on Standards for School Mathematics (1987) agree with Ben-Haim, Lappan, and Houang. In the working draft recommendation, the Commission called for the fifth to eighth grade math curriculum to include "...development of concepts of perimeter, area, and volume through concrete experiences with covering, filling, and counting" (p. 70) and "...visualize and represent...three-dimensional geometric shapes" (p. 73). In their thesis on a child's concept of space, Piaget and Inhelder (1967, p. 77), wrote:
the abstraction of shape actually involves a complete reconstruction of physical shape made on the basis for the subject’s own actions...based originally upon a sensi-motor, and ultimately on a mental, representational space determined by the co-ordination of the actions.

**Gender**

The issue of gender in mathematics education is not limited to spatial visualization ability. The largest gap between male and female achievement, as measured by national tests, is in the mathematics section of the Scholastic Aptitude Test (Sadker, Sadker, & Steindam, 1989), perhaps because women are underrepresented in the high school advanced mathematics tracks (Henderson, 1983). Males also outperformed females on volume questions on the NAEP tests (Gardner, et al., 1983a, 1983b).

Armstrong (1981) found high school males outperform females on higher cognitive level mathematics tasks. On the 1986 NAEP tests, males demonstrated higher achievement than females on higher-level applications, while females demonstrated higher achievement on basic skills and knowledge (Dossey et al., 1988). The difference in achievement by males and females is less obvious in earlier grades. Many researchers maintain gender differences in mathematics at the elementary level do not exist (Pallas & Alexander, 1983).
The issue of gender and CAI in mathematics education has had little attention (Bruwelheide, 1982). Burns and Bozeman's (1981) meta-analysis of 40 studies involving CAI in mathematics reported CAI drill and practice format is more effective for elementary and secondary males, and the effects of drill and practice CAI more effective for males than females in intermediate grades.

The study of gender and multiple embodiments also has had little attention (Indelicate, 1979). When gender is included as a variable, it does not appear to be significant (Indelicate, 1979; Johnson, 1970.)

**Ability Groups**

Effects of CAI in mathematics education varies for ability groups. A meta-analysis by Kulik et al. (1983) of 52 secondary (6-12) mathematics studies using drill and practice and tutorial CAI indicated the effects of CAI were stronger for lower aptitude students than for talented students. Burns and Bozeman's (1981) analysis of 40 studies of drill and practice CAI in elementary and secondary mathematics concluded CAI is more effective with higher and with lower achieving students than instruction without computers. Menis et al. (1980) reported increased achievement in low ability students who used drill and practice CAI to reinforce algebra
skills. Significant increases in achievement among high ability students was reported by Jhin (1971); among low and middle ability groups, CAI was no more effective than traditional instruction. Chambers and Sprecher (1980) found low ability students gained more from CAI than higher ability students. Student self-reports on the 1986 NAEP assessment indicated computers were available primarily to students of higher ability when computers were used in mathematics instruction (Dossey, 1988). The type of CAI was not indicated.

Effects of multiple embodiment instruction on high, medium, and low ability student groups were reported by Indelicato (1979) on four units in seventh and eighth grade mathematics instruction: (a) ratio and proportion, (b) addition and subtraction of integers, (c) area, and (d) probability. High ability learners scored higher on all four topics than did the average and low learners. Wheeler (1971) concluded all IQ levels benefited from multiple embodiment instruction of multi-digit addition and subtraction.

Johnson (1974) conducted a study of the effect of CAI on student achievement in ninth grade mathematics. The author concluded CAI was more effective than Programmed Instruction in speed of work completed among high and average students than among low ability
students. Kieren's (1969) study of 11th grade math students reported more effectiveness with average ability students than with high ability students when computers were incorporated into the instruction. This two year study consisted of a programming phase in which students wrote and used programs to solve problems involving quadratic equations and trigonometry. Jhin's (1971) research with algebra demonstrated CAI was more effective with high ability groups than with medium or low ability groups.

Harman (1980) reported mixed effects on achievement when CAI was used in remedial eighth grade math classes. Crawford (1970) reported an increase in achievement by seventh grade remedial math students with the use of drill and practice format CAI.

One study of multiple embodiments in the instruction of perimeter, area, and volume demonstrated that students with high ability scored significantly better than did students with low ability (Johnson, 1970). Ability was determined by reading ability.

A study by Gordon (1977) determined a profile of characteristics, abilities and factors identifying low mathematics achievement and high mathematics achievement sixth grade students. Gordon's population was 294 students, placed by national test scores into high,
medium and low achievement groups. Students were tested on: (a) four Piagetians test of conservation of number, liquid quantity, weight, and volume; (b) attitude toward mathematics; (c) self-confidence; (d) gender; (e) race; and (f) parent vocation. Findings of the study resulted in the following conclusions: (a) IQ is related to mathematics achievement, (b) ability to conserve increases the chance of success in math, (c) positive attitudes toward math and self-concept contribute to achievement, and (d) reading ability, gender, race, and parent vocation all contribute to success in mathematics.

Conclusion

The review of literature has demonstrated the value of CAI when used as supplement to mathematics instruction. The potential to promote learning at higher cognitive levels has also been documented.

Cognitive psychology embraces a theory of learning that includes organized, stored information and a "workplace" in which knowledge is recalled from memory and processed to solve problems. The better the concept is understood, the better it is organized, stored, and recalled. Memory for mathematics seems to consist of several specific capabilities.
The literature demonstrates gender may be a factor in mathematics learning, particularly on topics requiring spatial visualization, and in mathematics achievement. The issue of gender in computer-assisted instruction and multiple embodiment instruction has less attention than in mathematics education.

Effects of mathematics instruction on ability groups may be a factor in achievement. There is evidence that it may also be a factor in CAI and multiple embodiment instruction.

A multiple presentation of a mathematics concept in several representations, and carried through a concrete to semi-concrete to abstract sequence, holds promise for increasing student understanding of that concept. The use of the computer as one presentation has not been tested; however, the potential to increase student achievement in mathematics is well-documented. Finally, there is evidence that the teaching of volume of rectangular solids is a topic in need of curriculum research.
CHAPTER III

PROCEDURES

Introduction

The problem in this study was to determine the effect of three methods of instruction on student achievement and retention of knowledge of the volume of rectangular solids. One method was an instructional multiple embodiment sequence incorporating computers as a supplement to instruction which includes manipulatives; the second was the sequence without computer supplement but with manipulatives; the third method was a traditional textbook approach.

This chapter includes the following topics:

1. population description;
2. description of treatments;
3. research design;
4. methods of data collection;
5. statistical hypotheses; and
6. analysis of data.
Population Description

The population in this study was fifth grade students enrolled in school districts in southwestern Montana (see Appendix C). Fifth grade was chosen because the topic of interest, volume of rectangular solids, is first addressed in many fifth grade mathematics books (Duncan, et al., 1981; Eicholz, et al., 1985; Haenisch & Hill, 1985; Nichols, et al., 1974; Payne, et al., 1982). Also, many national and state standardized tests do not include volume as an objective until fifth grade or later (Gardner et al., 1983; MEAP, 1983; NAEP, 1983).

Superintendents and principals of public schools and districts in the southwestern Montana area were contacted by this researcher in Spring 1988, and asked if they would recommend fifth grade teachers for participation in the study. Recommended teachers were then contacted by letter. Those who expressed an interest in participation were again contacted by letter and phone with additional information and details.

Workshops detailing study objectives, implementation of the experimental and textbook sequences, use of computer software, if applicable, and activities to be performed by participating teachers were given by this researcher in October 1988. A complete set of instructions, daily attendance and student computer use
logs, computer software (where necessary), tests, and all required student materials were supplied by the researcher. Upon completion, tests were returned to the researcher for evaluation.

Seventeen teachers agreed to participate; one teacher who taught in a departmentalized school was responsible for five classes. Twenty-one classrooms were represented in the study. Teachers and their classes were randomly assigned to one of three treatment groups (see Appendix C). The total number of students completing the study was 419. Completion was determined by (a) less than three days absence during the initial week of instruction, (b) completion of all activities, (c) completion of pretest and SAT, and (d) by completion of at least one of the posttests.

The Montana Office of Public Instruction (1988) reported a K-8 student population of 31,084 for the four counties involved in the study. This researcher used all the teachers who volunteered to participate in the study. Though the N of 419 is sufficient for the population (Krejcie & Morgan, 1970), the population thus generated was not selected by a stratified random process. Therefore, no generalization was attempted. A minimum of 15 subjects per cell is recommended for a factorial design (Gay, 1981; SPSSX, 1986). The population size was
sufficient for a 3 X 3 X 2 factorial design but small
enough group for this researcher to personally inservice
teachers and administer the DLI tests to students in the
field. Additionally, the large sample size gives the
principle of randomness a chance to work (Gay, 1981;
Kerlinger, 1973) and "...increases the power of the
statistic" (Gay, 1981, p. 325).

Description of Treatments

Instruction consisted of the teaching of the concept of
volume of rectangular solids. The time period for
instruction was one week and three weeks for retention
activities. The study consisted of three treatments:

1. Treatment I - a multiple embodiment instructional
sequence including manipulatives, pencil and paper
activities, and the computer as one embodiment.

2. Treatment II - the multiple embodiment
instructional sequence without using the computer as an
embodiment.

3. Treatment III - a sequence based on the
traditional textbook approach.

The length of this study was determined by a time
period sufficient to address all instructional objectives
and allow student exploration, yet fit into time allowed
for the topic in most curriculums, if the subject is
taught at all. A review of selected mathematics textbooks revealed that the subject of volume of rectangular solids is addressed on only two to four pages (Duncan, et al., 1981; Eicholz, et al., 1985; Haenisch & Hill, 1985; Nichols, et al., 1974; Payne, et al., 1982). Further, most public school mathematics curriculums are determined by commercial mathematics textbooks (Committee on Indicators of Pre-College Science and Math, Commission on Behavioral and Social Studies and Education, National Research Council, 1985; Freudenthal, 1981; Nicely, 1985; Willoughby, 1984).

**Multiple Embodiment Instructional Sequence**

An instructional sequence originally developed by Dr. Glenn Allinger, Montana State University Department of Mathematical Sciences, addressed the concept of volume of rectangular solids. The sequence progressed from concrete to semi-concrete to abstract levels and included multiple embodiments, one of which is use of the computer as an interfacing agent. In the winter of 1986 this researcher developed student materials and field tested the sequence. As a result of the field test, this researcher expanded the time from four days of instruction to five days in order to allow complete instruction to take place in five 50-minute periods. Several modifications in materials were made: (a) one
duplicated activity was removed, (b) student materials were consolidated, (c) addition of several discussion questions, and (d) addition of activities to allow the students to "discover" a measuring format for the formula Volume = Length X Width X Height (V = L X W X H).

The lessons begin with students creating boxes. This is done by cutting a pre-determined square or rectangle from centimeter square grid sheets, making a corner cut from each corner, folding and taping the cut grid to make a box. Students proceed by filling the boxes with 1 cm cubes, removing and counting the cubes to determine how many cubes each box will hold. Cubes are then lined vertically on a piece of paper and segments drawn around the cubes to form a bar graph. After several varied boxes are constructed and their volumes are determined, the class, via discussion, arrives at a formula for determining volume without filling and counting. This concrete activity is the first embodiment.

The procedure is repeated in a semi-concrete, or pencil and paper, format with students drawing boxes instead of constructing them by cutting and taping, calculating volumes with the formula, and drawing bar graphs to represent the lined-up cubes. The paper and pencil activity is the second embodiment.
The computer program, which simulates the box making, box drawing, and graphing activities in the sequence, comprises the third embodiment. Student input allows a variety of box sizes and shapes to be created. As with the "hands on" construction, a grid is pictured and a square corner cut is made. The operator inputs the size of grid and corner cut. A record of the volume of each box is also kept and displayed on the monitor.

Abstract activities include determination of volume using symbols (numbers). These activities will consist of "...if given dimensions, find the volume" type of problems.

The objectives of the sequence are:

1. Develop an ability to calculate volume given dimensions in picture form.
2. Develop an ability to calculate volume given dimensions in word problems.
3. Develop an understanding of volume as a measure of three dimensional objects.
4. Develop an understanding that volume is independent of shape.
5. Develop an ability to infer three-dimensional information from two-dimensional representation.
Pedagogical support for the sequence includes:


4. The abstraction of a space shape involves reconstruction of the shape, "...made on the basis of a subject's own actions....based originally upon a sensori-motor, and ultimately on a mental, representational space determined by the coordination of the [subject's] action" (Piaget & Inhelder, 1967).


6. The CAI graphics used in the third embodiment are "...relevant to the topic and content of the lesson"
(Alesandrini, 1985, p. 5) and are presented in a clear, concise manner (Alesandrini, 1985; Bialo & Erickson, 1985; Flake, 1982; two software experts, Appendix B).

7. In order to ensure a correct concept in mathematics, some monitoring of learning must take place (Lesh, Landau, & Hamilton, 1983; Shepard & Ragan, 1982).

8. Mathematics formulas developed and established by students during the introductory lessons result in a better understanding and retention of the formula as well as utilization of the formula in problem solving (Lerch, 1981; Stephens & Olson, 1985).


10. Varied practice of arithmetic can contribute to mathematics understanding (Brownell, 1956) and calculation of the formula is important to the concept of volume because it aids the transition from numbers as boundary lines to the concept of internal space and space occupied (Piaget, Inhelder & Szeminski, 1960).

11. Students often learn more when working together than when working alone as they share views, opinions, and knowledge (Burns, 1988; Dickenson & Vereen, 1983; Wilson, 1987).
Traditional Textbook Approach

A teaching sequence based on traditional textbook instruction was developed by this researcher (see Appendix D). It was based on activities used in 1974-85 mathematics textbooks and was of equal length as the other treatments. Field testing of this instructional approach was made in September 1988, in fifth grade classes in the Bozeman, Montana area that were not involved in the final study. Revisions made were (a) removing errors and/or typing errors in student materials and teacher instructions, (b) incorporation of teacher-led questions into the lesson plans, and (c) addition of one activity on the third day to expand time on that lesson to correspond with the instructional sequence.

The instruction and activities were teacher-directed, beginning with introduction of the "cube" as a three-dimensional object and as a unit of measure. Demonstrations of how to measure and find the volume of rectangular solids (boxes) preceded introduction of the \( V = L \times W \times H \) formula. Reinforcement problems on worksheets concluded the five-day sequence. Sources for the sequence were Duncan, et al. (1981); Eicholz, et al. (1985); Haenisch & Hill (1985); Nichols, et al. (1974); and Payne, et al. (1982). The objectives
of the textbook sequence were the same as those for the multiple embodiment sequence.

Retention Activities

Three sets of retention activities, one each to be given in one-week intervals, were developed by this researcher and field tested in September 1988 in fifth grade classes in the Bozeman area. One class had been involved in the original 1986 field test and those students were assigned activities for Treatments I and II. Another class was involved in treatment field tests and was assigned activities for Treatment III. Each set of activities including problem solving activities, took approximately 30 minutes to complete. Treatment I participated in use activities using the computer. Treatment II used manipulatives and paper and pencil activities similar to drawing boxes and computing volume. Treatment III solved problems found in textbooks similar to "how many cubes will it take to fill a box 1 cm by 2 cm by 2 cm?". All groups were assigned identical calculation problems in addition to the activities (see Appendix D). None of the students in the field test participated in the final study.

As a result of the field test, questions were rewritten so students could understand them with little or no help from their teachers. During the field test,
students kept a log of time on task for all retention activities to ensure retention time on task was equal for all three treatments. This was the only control for student time on task during retention in the final study.

The Software

A Logowriter microworld simulation called Prism4 created by Dr. Lyle Andersen and Dr. David Thomas, Montana State University Department of Mathematical Sciences, and written with Logowriter replicates the semi-concrete paper and pencil portion of the teaching sequence. Student input determines the size of the square corner cut and, therefore, the size and shape of the box constructed from the 2-D pattern. A graph displays resulting volume via a single bar graph. The volume in whole numbers is also displayed.

Review of the literature indicated the following criterion list of educational support for use of this type of software used as supplement to instruction:

1. The content of the software is accurate, clear, and relevant to the instructional topic (Alesandrini, 1985; Mezzina, 1985).

2. Only needed information is displayed on the screen, thus focusing student attention on pertinent information (Alesandrini, 1985; Flake, 1982).
3. The visual display is graphically and textually pleasant (Mezzina, 1985).

4. Instructional computer software should be formatted in a manner familiar to the student (Alesandrini, 1985; Bialo & Erickson, 1985). The drawing, cutting, and folding via computer occurs in the same order as the student activity in the teaching sequence (two software experts, Appendix B).

5. The student interacts with the simulation so he/she is a participator, not just an observer (Chambers & Sprecher, 1980, 1983; Dugdale & Kibbey, 1984; Henderson et al., 1983; Keuper, 1985; Mezzina, 1985; Papert, 1980).

6. The computer simulation replicates the concrete (box making) and semi-concrete (box drawing and marking) activities quickly so the students can compare different sized boxes, allowing them to concentrate on the results and not get lost in the manipulation and calculation actions of the actual activities (Dugdale & Kibbey, 1984; Thomas, Anderson & Allinger, 1987).

7. Students are USING mathematics to solve problems, not DOING mathematics as a rote task (Dugdale & Kibbey, 1984).

8. Many ideas are expressed and understood better through pictures, graphs, and diagrams (Bork, 1980; Marks, 1985; National Science Board Commission of
Pre-College Education in Mathematics, Science and Technology, 1983; Nicely, 1985; Taylor & Cunniff, 1988). The chart and drawings produced by students and computer simulation in the treatment groups represent the concept of volume of rectangular solids (two software instruction experts and two mathematics instruction experts).

Advantages of using Logowriter as the programming language are: (a) the language can be used in several brands of microcomputers; (b) graphics are available; (c) the program is friendly, that is, easy to write and debug; and (d) it has the capability to mix graphics and text (Mezzina, 1985).

Two disadvantages of the Apple version of Logowriter versus the IBM version must be noted. One is the lack of available memory after loading the program. This may result in termination of the program if incorrect input occurs. The user then has to restart the simulation. Another problem is the poorer quality of screen resolution (Thomas et al., 1987). However, the majority of computers used in Montana schools are Apple computers (OPI, 1985). Therefore, the Apple version was used for this study.
Research Design

The pre-post design based on Campbell and Stanley’s (1963) Nonequivalent Control Group Design employed an experimental-control model. Students were randomly placed in one of three treatment groups: (a) the multiple embodiment instructional sequence using the computer as one embodiment, (b) the multiple embodiment sequence without the computer embodiment and, (c) a traditional commercial textbook teaching method. A three-week follow-up period provided time for all groups to review material via (a) computer (Treatment I), (b) manipulatives as used in the sequence (Treatment II), or (c) pencil and paper activities (Treatment III). A retention test was administered to determine the amount of retention for each group. Pretests were administered two to four weeks before instruction began. The post-instruction test was administered on the day following the last day of instruction. The post-retention tests were given the day following the third retention activity. There was a period of three weeks between post-instruction and post-retention tests. All tests were sent to this researcher for scoring.
Nonequivalent Control Group Design

The Campbell-Stanley (1963) Nonequivalent Control Group Design uses a pretest and posttest to measure effect of treatment(s). This design is applicable to assembled groups such as classrooms, where groups are "...as similar as availability permits but not so similar that one can dispense with the pretest" (Campbell & Stanley, 1963, p. 47). Gay (1981) supports use of this design when working with intact groups. He states "...an advantage of this design is that since classes are used 'as is'...." possible effects from reactive interference from variables is minimized (p. 232). Though the groups may differ on the pretest, the design "...may approach true experimentation....the experimenter has natural groups available....," random assignment of groups to treatment or control groups is made and there is no reason to suspect subjects deliberately seek membership in the experimental or design group (Campbell & Stanley, 1963, p. 50). This design controls internal sources of six contaminating variables: history, maturation, selection, testing, instrumentation, and mortality. It does not control interaction of selection and maturation (Campbell & Stanley, 1963; Gay, 1981; Kerlinger, 1973).

Some question of control of regression exists. Stanley and Campbell (1963), however, indicate this is
not a concern if treatment groups have not been selected based on extreme scores on the pretest. Since classrooms were assigned prior to the pretest, this researcher assumes this variable is minimally controlled.

The design does not control the external variable of interaction of testing and treatment. However, since testing is a normal activity in classrooms, Campbell and Stanley (1963) indicate minimum interference from this threat to validity.

It is uncertain whether the variables of interaction of selection and treatment and reactive rearrangement are controlled by this design. No way exists for controlling the fact that experimental and/or control class sections may differ in some important way. But Campbell and Stanley (1963) do conclude the use of an entire intact classroom helps minimize reactive rearrangement and selection and treatment. This researcher recognizes the possibility that factors identified in the G. E. Hawthorne experiments (Gay, 1981) may contaminate this study. However, after a review of research design literature, this researcher has been unable to identify any means to control these factors in this study.
Methods of Data Collection

Standardized Test

Ability groups were determined by the Stanford Achievement Test (SAT), Form E (Gardner et al., 1987a). Of a possible 74 points, the highest scores were 73; the lowest was 15. Students were placed into three groups: high, medium, and low in approximately equal proportions. High ability was determined by a score of 61 or greater. Medium ability was determined by scores between 50 and 60; low by scores of 49 or less. Student numbers in high, medium, and low ability groups were: 147, 140, and 132, respectively.

The mathematics portion of the test included (a) Concepts of Number, (b) Mathematics Computation, and (c) Mathematics Applications. Only Concepts of Number (34 items) and Mathematics Applications (40 items) sections of the battery were used. This researcher did not include the computation section in the test scoring; students were directed to show their work and wrong answers derived from arithmetic errors were not marked wrong. The study was a measure of ability to understand volume of rectangular solids, not ability to do arithmetic. Thus, Computation was rejected as part of the test battery. Results of the test scores designated
assignment to high, middle, or low ability groups, each group being approximately one-third of the population.

Validation of the SAT was established with content validity (Gardner et al., 1987b). Following a review of state and local curriculum guides and textbooks, a table of specifications was prepared. Items were developed and analyzed with a National Item Analysis Program conducted in April 1980, with approximately 10,000 students from across the nation. Additionally, the Otis-Lennon Mental Ability Test was administered to participating students. The final form of the SAT was standardized in September and October 1981, with approximately 100,000 students randomly drawn nationally to represent school population criterion of enrollment, socio-economic status, geography, and public versus private schools (Gardner et al., 1987b).

Reliability coefficients for the SAT Form E were reported as follows:

<table>
<thead>
<tr>
<th></th>
<th>Concept</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuder-Richardson Formula #20:</td>
<td>.88</td>
<td>.91</td>
</tr>
<tr>
<td>Alternate Forms:</td>
<td>.82</td>
<td>.87</td>
</tr>
<tr>
<td>Intercorrelations with Otis Scholastic Ability Test:</td>
<td>.55-.78</td>
<td>.59-.76</td>
</tr>
</tbody>
</table>

Criterion-Referenced Test

Since treatment took place mid-school year, other tests of ability such as the Scholastic Achievement Test,
given by some school districts at the beginning or end of a school year, would not be as reliable as a criterion-referenced test given just before treatment because extraneous variables such as maturation may interfere (Popham, 1981; Stanley & Campbell, 1963). Also, this study is a test of volume of rectangular solids, not general math ability or achievement.

Two parallel, equivalent forms of a criterion-referenced test (CRT) developed by this researcher tested both concept and skill levels of knowledge. The skill level test items measured knowledge and comprehension levels of Bloom’s taxonomy; concept level items measured application and analysis levels of the taxonomy. The CRTs were used as the post-instruction test and as the post-retention test. A portion of the post-retention test was used as a pretest to determine equivalent groups. Reliability was determined by the equivalence and stability method. Validity was established by the content validity method.

Popham (1981) discusses several measures of reliability of criterion-referenced tests. Stability measures "...consistency of a test’s measure over time" (p. 28) by correlating scores of one test administered twice to the same examinees with a "...reasonable interval" between the tests (Popham, 1981, p. 128).
Equivalence measures the degree of equalness between two forms of a test. Because criterion-referenced tests are based on very specific objectives, as opposed to more general objective bases in norm-referenced tests, it is easier to create "...truly equivalent forms (Popham, 1981, p. 29)".

The combination of the above methods of reliability is the equivalence and stability method. To establish this form of test reliability, Form A is administered to a group of examinees; after a time delay, Form B is administered to the same group of test takers. Downie and Heath (1974) note the delay time is an important factor; the longer the delay, the lower the correlation due to intervening variables. The shorter the delay, however, the more the chance that examinees will remember items and react from memory, not knowledge of subject matter. Gronlund (1976) and Popham (1981) warn this method produces the lowest reliability coefficient. Popham (1981) recommends a time interval from two to six weeks. A Pearson-Rho correlation coefficient is computed between the two forms of the test.

Seventy-five fifth grade students in Bozeman area schools were the examinees. These students were not included as subjects in the experimental study. The tests were administered in April 1988, with a four-week
The interval between tests. The Pearson-Rho correlation was .864 and the tests were judged to be reliable.

The content validity strategy was employed to establish test validity. In a criterion-referenced test, the major criterion "...is to provide a clear picture of what it is that an examinee can or can't do...[and therefore], content validity is of major importance" (Popham, 1981, p. 105). Gronlund (1976) defines content validity as "...the extent to which a test measures a representative sample of subject matter content and the behavioral changes under consideration" (pp. 81-82).

One method of obtaining content validity is to define the behavioral objectives in a table of specifications (Guskey, 1985) and submit both table and test to experts in the field who will determine whether or not the test does indeed measure desired knowledge as defined by the table of specifications (Gronlund, 1976; Popham, 1981).

Guskey (1985) describes the process of developing a table of specifications. The first step is to determine what content is to be learned. Second, student behaviors or objectives should be written to reflect the content to be learned. The behavior should represent a range of categories of difficulty as described by Bloom (1956) in his Taxonomy of Educational Objectives. The test
developer then creates a matrix with the levels of the taxonomy on the horizontal axis and the objectives on the vertical axis. Items, identified by question number, are entered into the proper matrix slots. Test items may apply to more than one level. No statistical coefficient is provided with this method (Downie & Heath, 1974; Gronlund, 1976; Kerlinger, 1973; Popham, 1981).

This researcher developed a table of specifications according to Guskey's (1985) description and submitted the test and table to three mathematics education specialists. Two specialists were mathematics educators, one at Montana State University, Bozeman, Montana; and one at Western College, Dillon, Montana. The third was employed by the Bozeman, Montana Public School System. They were asked to determine which items were at computation and comprehension levels and which were at analysis and application levels and to judge whether the test measured the objectives. Eleven items were judged to be at the lower levels; 12 at higher levels. All three specialists judged the test to measure the objectives (see Appendix B).

Validity of The Software

Validity of the software was established by two computer specialists at Montana State University who used
the criterion established by Alesandrini (1985) and Mezzina (1985) (see Appendix B).

The Developmental Level Instrument

A test consisting of four tasks was developed by this researcher with the help of Dr. William Hall, Montana State University Department of Education; and Dr. Richard Copeland, formerly of Florida Atlantic University. They were based on the evaluation and diagnostic work of Dr. Copeland and measured student logico-mathematical knowledge, defined by Piaget as cognitive knowledge and awareness of mathematics (Copeland, 1979, 1984; Groen & Kieran, 1983). A sample of 113 students from the population was tested before the study began. Results were used to establish equivalence among ability groups.

The instrument was field tested in August 1988 with 12 fifth grade Bozeman area students. Modifications entailed rewording of questions to be asked by the interviewer so questions were neutral and ensured clarity to the student being interviewed. The final instrument was given to 18 other fifth grade students from Bozeman Middle School with a six-week interval between first and second administrations.

Reliability for the instrument was determined by the test-retest method. The Spearman-Rho coefficient was
.8514 and the test was judged to be reliable. Validity was established by the construct method. Construct validation is used when the test to be validated measures a psychological quality or idea such as aptitude or readiness (Popham, 1981; Gronlund, 1976). Borg and Gall (1983) state one method to obtain construct validity is to (a) define the observable characteristic evident in the construct, (b) define the behaviors that would be evident with a high and low measure of the construct, and (c) compare results with another assessment of the construct in the individuals measured.

To validate the Developmental Level Instrument (DLI), this researcher defined the Piagetian Stages as described in Copeland (1979, 1984). The definition was given to a teacher who rated her 14 students based on her knowledge of their behavior during the past seven months. Scores were correlated with the researcher’s interview scores. The Spearman Rho correlation was .6208 (p = .0178).

**Statistical Hypotheses**

Questions addressed in this study are stated in the form of null hypotheses.

1. There is no statistically significant difference in mean achievement among treatments as measured by
knowledge and comprehension items on the post-instruction test.

2. There is no statistically significant difference in mean achievement among three ability groups as measured by knowledge and comprehension items on the post-instruction test.

3. There is no statistically significant difference in mean achievement between gender groups as measured by knowledge and comprehension items on the post-instruction test.

4. There is no interaction between treatment and ability as measured by knowledge and comprehension items on the post-instruction test.

5. There is no interaction between treatment and gender as measured by knowledge and comprehension items on the post-instruction test.

6. There is no interaction between gender and ability as measured by knowledge and comprehension items on the post-instruction test.

7. There is no interaction among treatment and ability and gender and grouping as measured by application and analysis items on the post-instruction test.

8. There is no statistically significant difference in mean achievement among treatments as measured by
application and analysis items on the post-instruction test.

9. There is no statistically significant difference in mean achievement among ability groups as measured by application and analysis items on the post-instruction test.

10. There is no statistically significant difference in mean achievement between gender groups as measured by application and analysis items on the post-instruction test.

11. There is no interaction between treatment and ability as measured by application and analysis items on the post-instruction test.

12. There is no interaction between treatment and gender as measured by application and analysis items on the post-instruction test.

13. There is no interaction between gender and ability as measured by application and analysis items on the post-instruction test.

14. There is no interaction among treatment and ability and gender and grouping as measured by application and analysis items on the post-instruction test.

15. There is no statistically significant difference in mean achievement among treatments as
measured by total item scores on the post-instruction test.

16. There is no statistically significant difference in mean achievement among ability groups as measured by total item scores on the post-instruction test.

17. There is no statistically significant difference in mean achievement between gender groups as measured by total item scores on the post-instruction test.

18. There is no interaction between treatment and ability as measured by total item scores on the post-instruction test.

19. There is no interaction between treatment and gender as measured by total item scores on the post-instruction test.

20. There is no interaction between gender and ability as measured by total item scores on the post-instruction test.

21. There is no interaction among treatment and ability and gender and grouping as measured by total item scores on the post-instruction test.

22. There is no statistically significant difference in mean achievement among treatments as
measured by knowledge and comprehension items on the post-retention test.

23. There is no statistically significant difference in mean achievement among ability groups as measured by knowledge and comprehension items on the post-retention test.

24. There is no statistically significant difference in mean achievement between gender groups as measured by knowledge and comprehension items on the post-retention test.

25. There is no interaction between treatment and ability as measured by knowledge and comprehension items on the post-retention test.

26. There is no interaction between treatment and gender as measured by knowledge and comprehension items on the post-retention test.

27. There is no interaction between gender and ability as measured by knowledge and comprehension items on the post-retention test.

28. There is no interaction among treatment and ability and gender and grouping as measured by application and analysis items on the post-retention test.

29. There is no statistically significant difference in mean achievement among treatments as
measured by application and analysis items on the post-retention test.

30. There is no statistically significant difference in mean achievement among ability groups as measured by application and analysis items on the post-retention test.

31. There is no statistically significant difference in mean achievement between gender groups as measured by application and analysis items on the post-retention test.

32. There is no interaction between treatment and ability as measured by application and analysis items on the post-retention test.

33. There is no interaction between treatment and gender as measured by application and analysis items on the post-retention test.

34. There is no interaction between gender and ability as measured by application and analysis items on the post-retention test.

35. There is no interaction among treatment and ability and gender and grouping as measured by application and analysis items on the post-retention test.
36. There is no statistically significant difference in mean achievement among treatments as measured by total item scores on the post-retention test.

37. There is no statistically significant difference in mean achievement among ability groups as measured by total item scores on the post-retention test.

38. There is no statistically significant difference in mean achievement between gender groups as measured by total item scores on the post-retention test.

39. There is no interaction between treatment and ability as measured by total item scores on the post-retention test.

40. There is no interaction between treatment and gender as measured by total item scores on the post-retention test.

41. There is no interaction between gender and ability as measured by total item scores on the post-retention test.

42. There is no interaction among treatment and ability and gender and grouping as measured by total item scores on the post-retention test.

Analysis of Data

Independent variables to be used in the analysis of data were:
1. treatment - three levels: experimental with computer, experimental without computer, and textbook control groups;
2. ability grouping - three levels: high, medium, and low; and
3. gender - male and female.

Dependent variables were:
1. post-instruction achievement total scores;
2. post-instruction achievement scores of knowledge and comprehension level test items;
3. post-instruction achievement scores of application and analysis level test items;
4. post-retention achievement total scores;
5. post-retention achievement scores of knowledge and comprehension level test items; and
6. post-retention achievement scores of application and analysis level test items.

MSUSTAT Statistical Analysis Package, version 4.12, developed by Dr. Richard E. Lund, Montana State University, was used to analyze the data.

Equivalent Groups

Analysis of variance (ANOVA) is used to determine whether samples come from populations with the same mean, i.e., whether populations are equivalent (Spatz & Johnson, 1981). A one-way ANOVA of pretest scores was
employed to test significant initial differences among treatment group means to ensure equivalent groups. The p-value was .9854. The treatment groups were judged to be equivalent.

An interaction between treatment and ability (p = .0581) appeared to demonstrate a trend among low ability students. Two further statistical analyses were conducted to determine whether pre-instruction differences may have existed but were not detected by the pretest ANOVA. Three one-way ANOVAs of pretest scores were used to analyze whether ability groups of the three treatments were equivalent before instruction began. The p-values for each ability group were: (a) high (p = .2306); (b) medium (p = .8578); and (c) low (p = .2270). The Kurskal-Wallis non-parametric rank test (Lund, 1988) was used to determine if ability groups were equivalent as measured by the Developmental Level Instrument (DLI). The p-values for each ability group were: (a) high (p = .3775); (b) medium (p = .5065); and (c) low (p = .6003). Ability groups in Treatments I, II, and III were equivalent before instruction began.

Factorial Analysis of Variance

Kerlinger (1973) states "...Factorial Analysis of Variance is the statistical method that analyzes the independent and interactive effects of two or more
variables on a dependent variable" (p. 245). He continues, "...(interaction between a multiple of independent variables) occurs frequently enough to warrant serious attention....Some behavioral researchers, especially in education...," are aware of the increasing importance of interaction of independent variables and "...interaction [should] be a central concern" (p. 257). This researcher's central question dealt with main effects across treatments. However, as Kerlinger (1981) and Ferguson (1981) point out, interaction is an important question since independent variables affect the dependent variable in varying degrees.

Factorial analysis has several advantages:

1. The researcher is able to manipulate two or more variables simultaneously.

2. Variables not manipulated can be controlled by building them into the design.

3. It is more precise than a one-way analysis.

4. It allows interactive effects of independent variables to be studied (Kerlinger, 1973, pp. 255-6).

**Alpha Level**

Commonly accepted levels of consequence are either .05 or .01 (Ferguson, 1981; Spatz & Johnson, 1981). The chance of making a Type I or Type II error often dictates the choice of alpha. A Type I error occurs when a true
null hypothesis is rejected; a Type II error occurs when a false null hypothesis is retained. A Type I error might result in change in method of instruction for one topic in mathematics when no increase will result in student achievement due to the change in method. A Type II error might result in no change in curriculum when a change could increase student achievement.

As a result of this study, consequences of a Type I error may result in an investment of time for teacher retraining and planning and money for computer hardware and software, with no positive academic results. Since most schools have computers in place (OPI, 1985b; OPI, 1987), the problem concerns time to train or retrain teachers as well as the class time required for the sequence.

Educators are concerned with increasing student achievement (Fullan, 1985). Indeed, most teachers judge their effectiveness in the classroom in relationship to student achievement (Guskey, 1985). Consequences of a Type II error in this study might result in students not being exposed to a program that could increase achievement. One way to control a Type II error is increase the sample size (Ferguson, 1981; Spatz & Johnson, 1981).
This researcher's choice of .05 level is a compromise. Though this researcher is more concerned with a Type II error and might have chosen .10, the large subject size of 419 subjects minimizes this error type.
CHAPTER IV

ANALYSIS OF THE DATA

Introduction

Data reported in this chapter are organized as follows: (a) pretest results, (b) post-instruction results, (c) post-retention results, and (d) comparison of ability level means across treatment. Each posttest includes (a) a score on items that measure lower levels of Bloom’s (1956) cognitive domain (knowledge and comprehension), (b) a score on items that measure higher levels of Bloom’s (1956) cognitive domain (application and analysis), and (c) a total score. Summary tables for each dependent variable, cell count, and the corresponding hypotheses are listed in this chapter.

Analysis of Variance was used to analyze pretest and posttest data. Unequal cell sizes are due to absence of some students on the day the tests were administered. Factorial ANOVA was used to analyze means of ability level measured by pretest scores. Kruskal-Wallis non-parametric rank test was used to compare means of ability level groups as measured by the Developmental
Level Instrument (DLI). Alpha level for each analysis was set at .05. Multiple comparison tests were performed to identify differences when significant differences among three groups was reported.

Pretest Analysis

Subjects in the study were fifth grade students. They were pretested to ensure initial equivalent groups. Table 1 shows summary statistics for the one-way ANOVA.

Test for Equivalence

Table 1. ANOVA of Pretest Scores (N = 419)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of squares</th>
<th>Mean squares</th>
<th>F value</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>.1670</td>
<td>.0836</td>
<td>.01</td>
<td>.9859</td>
</tr>
<tr>
<td>Within</td>
<td>416</td>
<td>2446.1</td>
<td>5.8799</td>
<td></td>
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</tr>
<tr>
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<td>418</td>
<td>2446.2</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Means for Treatments I, II, and III were 6.838 (N = 154), 6.832 (N = 149) and 6.6879 (N = 116). The Treatment Groups were considered equivalent.

Equivalence of Ability Groups as Measured by Pretest Scores

Three one-way ANOVAs were employed to determine whether ability groups were equivalent across treatments. Tables 2, 3, and 4 show summary statistics for the ANOVAs.
Table 2. ANOVA of Pretest Scores of High Ability Groups (N = 147)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of squares</th>
<th>Mean squares</th>
<th>F value</th>
<th>p</th>
</tr>
</thead>
<tbody>
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<tr>
<td>Within</td>
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<tr>
<td>Residual</td>
<td>146</td>
<td>859.97</td>
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</tr>
</tbody>
</table>

Means for ability groups in Treatments I, II, and III were: 7.621 (N = 58), 8.120 (N = 50), and 8.462 (N = 39). The High Ability groups in Treatments I, II, and III were equivalent.

Table 3. ANOVA of Pretest Scores of Medium Ability Groups (N = 140)

<table>
<thead>
<tr>
<th>Source</th>
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<th>Sum of squares</th>
<th>Mean squares</th>
<th>F value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
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<td>Residual</td>
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</tbody>
</table>

Means for ability groups in Treatments I, II, and III were: 7.429 (N = 49), 7.157 (N = 51), and 6.875 (N = 40). The Medium Ability groups in Treatments I, II, and III were equivalent.
Table 4. ANOVA of Pretest Scores of Low Ability Groups (N = 132)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of squares</th>
<th>Mean squares</th>
<th>F value</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>2</td>
<td>10.916</td>
<td>5.4581</td>
<td>1.45</td>
<td>.2388</td>
</tr>
<tr>
<td>Within</td>
<td>129</td>
<td>486.14</td>
<td>3.8684</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>131</td>
<td>497.06</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Means for ability groups in Treatments I, II, and III were: 5.830 (N = 47), 5.771 (N = 48), and 5.162 (N = 37). The Low Ability groups in Treatments I, II, and III were equivalent.

Equivalence of Ability Groups as Measured by Developmental Level Instrument Scores

Three Kruskal-Wallis rank tests were employed to determine whether ability groups were equivalent across treatments as measured by the Developmental Level Instrument (DLI). Tables 5, 6, and 7 show summary statistics for the Kruskal-Wallis statistic.

Table 5. Kruskal-Wallis Rank Test of DLI Scores of High Ability Groups (N = 46)

<table>
<thead>
<tr>
<th>CHISQ (DF = 2)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.941</td>
<td>.3775</td>
</tr>
</tbody>
</table>

The groups were considered equivalent.
Table 6. Kruskal-Wallis Rank Test of DLI Scores of Medium Ability Groups (N = 40)

<table>
<thead>
<tr>
<th>CHISQ (DF = 2)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.356</td>
<td>.5065</td>
</tr>
</tbody>
</table>

The groups were considered equivalent.

Table 7. Kruskal-Wallis Rank Test of DLI Scores of Low Ability Groups (N = 27)

<table>
<thead>
<tr>
<th>CHISQ (DF = 2)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.018</td>
<td>.6003</td>
</tr>
</tbody>
</table>

The groups were considered equivalent.

Post-Instruction Analysis

Analysis of Knowledge and Comprehension Items

The data relating to hypotheses 1, 2, 3, 4, 5, 6, and 7 stated in null form in Chapter III are reported in Table 8.

Hypothesis 1: There is no statistically significant difference in mean achievement among treatments as measured by knowledge and comprehension items on the post-instruction test.

Decision: Reject the null hypothesis at (p = .0001). There was a significant difference in mean scores of treatment groups on knowledge and comprehension
items on the post-instruction test. Both sequence-based instructional treatments (Treatments I and II) scored higher than the textbook treatment (Treatment III). Means for Treatments I, II and III were 8.605, 8.779, and 7.759.

Table 8. ANOVA of Knowledge and Comprehension Items (N = 403)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of squares</th>
<th>Mean squares</th>
<th>F value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMT</td>
<td>2</td>
<td>76.649</td>
<td>38.325</td>
<td>9.93</td>
<td>.0001</td>
</tr>
<tr>
<td>ABL</td>
<td>2</td>
<td>361.04</td>
<td>180.520</td>
<td>46.75</td>
<td>.0000</td>
</tr>
<tr>
<td>GEN</td>
<td>1</td>
<td>17.999</td>
<td>17.999</td>
<td>4.66</td>
<td>.0308</td>
</tr>
<tr>
<td>TMT/ABL</td>
<td>4</td>
<td>21.109</td>
<td>5.277</td>
<td>1.37</td>
<td>.2448</td>
</tr>
<tr>
<td>TMT/GEN</td>
<td>2</td>
<td>13.113</td>
<td>6.556</td>
<td>1.70</td>
<td>.1844</td>
</tr>
<tr>
<td>ABL/GEN</td>
<td>2</td>
<td>4.827</td>
<td>2.414</td>
<td>.63</td>
<td>.5357</td>
</tr>
<tr>
<td>TMT/ABL/GEN</td>
<td>4</td>
<td>26.307</td>
<td>6.577</td>
<td>1.70</td>
<td>.1485</td>
</tr>
<tr>
<td>Residual</td>
<td>385</td>
<td>1486.5</td>
<td>3.861</td>
<td>.</td>
<td></td>
</tr>
</tbody>
</table>

There were significant statistical differences among treatments (p = .0001), among ability levels (p = .0000), and between gender groups (p = .0308).

Hypothesis 2: There is no statistically significant difference in mean achievement among ability levels as measured by knowledge and comprehension items on the post-instruction test.

Decision: Reject the null hypothesis at (p = .0000). There was a significant difference in mean total scores of ability groups on knowledge and comprehension
items on the post-instruction test. Means for ability groups High, Medium, and Low were 9.504, 8.494, and 7.144.

Hypothesis 3: There is no statistically significant difference in mean achievement between gender groups as measured by knowledge and comprehension items on the post-instruction test.

Decision: Reject the null hypothesis at (p = .0308). There was a significant difference in mean total scores of gender groups on knowledge and comprehension items on the post-instruction test. Means for male and female were 8.597 and 8.165, respectively.

Hypothesis 4: There is no interaction between treatment and ability as measured by knowledge and comprehension items on the post-instruction test.

Decision: Retain the null hypothesis at (p = .2448).

Hypothesis 5: There is no interaction between treatment and gender as measured by knowledge and comprehension items on the post-instruction test.

Decision: Retain the null hypothesis at (p = .1844).

Hypothesis 6: There is no interaction between ability and gender as measured by knowledge and comprehension items on the post-instruction test.
Decision: Retain the null hypothesis at ($p = .5357$).

Hypothesis 7: There is no interaction among treatment and ability and gender as measured by knowledge and comprehension items on the post-instruction test.

Decision: Retain the null hypothesis at ($p = .1485$).

Analysis of Application and Analysis Items

The data relating to hypotheses 8, 9, 10, 11, 12, 13, and 14 stated in null form in Chapter III are reported in Table 9.

Table 9. ANOVA of Application and Analysis Scores ($N = 403$)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Squares</th>
<th>F value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMT</td>
<td>2</td>
<td>7.1373</td>
<td>3.5686</td>
<td>.88</td>
<td>.4163</td>
</tr>
<tr>
<td>ABL</td>
<td>2</td>
<td>465.29</td>
<td>232.65</td>
<td>57.26</td>
<td>.0000</td>
</tr>
<tr>
<td>GEN</td>
<td>1</td>
<td>3.6016</td>
<td>3.6016</td>
<td>.89</td>
<td>.3464</td>
</tr>
<tr>
<td>TMT/ABL</td>
<td>4</td>
<td>13.837</td>
<td>34.4593</td>
<td>.85</td>
<td>.4934</td>
</tr>
<tr>
<td>TMT/GEN</td>
<td>2</td>
<td>4.2193</td>
<td>2.1096</td>
<td>.52</td>
<td>.5954</td>
</tr>
<tr>
<td>ABL/GEN</td>
<td>2</td>
<td>4.4105</td>
<td>2.2962</td>
<td>.54</td>
<td>.5816</td>
</tr>
<tr>
<td>TMT/ABL/GEN</td>
<td>4</td>
<td>9.1810</td>
<td>2.2952</td>
<td>.56</td>
<td>.6883</td>
</tr>
<tr>
<td>Residual</td>
<td>385</td>
<td>1564.3</td>
<td>4.0631</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There was a significant statistical difference among ability levels ($p = .0000$).

Hypothesis 8: There is no statistically significant difference in mean achievement among treatments as
measured by application and analysis items on the post-instruction test.

Decision: Retain the null hypothesis at (p = .4163).

Hypothesis 9: There is no statistically significant difference in mean achievement among ability levels as measured by application and analysis items on the post-instruction test (p = .4163).

Decision: Reject the null hypothesis at (p = .0000). There was a significant difference in mean total scores of ability groups on application and analysis items on the post-instruction test. Means for ability groups High, Medium, and Low were 8.595, 7.491, and 5.921.

Hypothesis 10: There is no statistically significant difference in mean achievement between gender groups as measured by application and analysis items on the post-instruction test.

Decision: Retain the null hypothesis at (p = .4934).

Hypothesis 11: There is no interaction between treatment and ability as measured by application and analysis items on the post-instruction test.

Decision: Retain the null hypothesis at (p = .3464).
Hypothesis 12: There is no interaction between treatment and gender as measured by application and analysis items on the post-instruction test.

Decision: Retain the null hypothesis at ($p = .5954$).

Hypothesis 13: There is no interaction between ability and gender as measured by application and analysis items on the post-instruction test.

Decision: Retain the null hypothesis at ($p = .5816$).

Hypothesis 14: There is no interaction among treatment and ability and gender as measured by application and analysis items on the post-instruction test.

Decision: Retain the null hypothesis at ($p = .6883$).

**Analysis of Total Scores Items**

The data relating to hypotheses 15, 16, 17, 18, 19, 20, and 21 stated in null form in Chapter III are reported in Table 10.
Table 10. ANOVA of Total Scores (N = 403)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Squares</th>
<th>F value</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMT</td>
<td>2</td>
<td>75.129</td>
<td>37.564</td>
<td>3.58</td>
<td>.0287</td>
</tr>
<tr>
<td>ABL</td>
<td>2</td>
<td>1645.0</td>
<td>822.48</td>
<td>78.44</td>
<td>.0000</td>
</tr>
<tr>
<td>GEN</td>
<td>1</td>
<td>36.571</td>
<td>36.571</td>
<td>3.49</td>
<td>.0618</td>
</tr>
<tr>
<td>TMT/ABL</td>
<td>4</td>
<td>66.340</td>
<td>16.585</td>
<td>1.58</td>
<td>.1784</td>
</tr>
<tr>
<td>TMT/GEN</td>
<td>2</td>
<td>10.957</td>
<td>5.4785</td>
<td>.52</td>
<td>.5935</td>
</tr>
<tr>
<td>ABL/GEN</td>
<td>2</td>
<td>14.592</td>
<td>7.2962</td>
<td>.70</td>
<td>.4993</td>
</tr>
<tr>
<td>TMT/ABL/GEN</td>
<td>4</td>
<td>27.487</td>
<td>6.8717</td>
<td>.66</td>
<td>.6234</td>
</tr>
<tr>
<td>Residual</td>
<td>385</td>
<td>4036.9</td>
<td>10.486</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There was a significant statistical difference between treatments (\( p = .0287 \)) and ability levels (\( p = .0000 \)).

Hypothesis 15: There is no statistically significant difference in mean achievement among treatments as measured by total scores on the post-instruction test.

Decision: Reject the null hypothesis at (\( p = .0287 \)). There was a statistical difference among groups on total scores on the post-instruction test. Means for treatments I, II, and III were 15.75, 16.23, and 15.16. There was a difference between Treatments II and III.

Hypothesis 16: There is no statistically significant difference in mean achievement among ability levels as measured by total scores on the post-instruction test.

Decision: Reject the null hypothesis at (\( p = .0000 \)). There was a significant difference in mean total
scores of ability groups on total scores on the post-instruction test. Means for ability groups High, Medium, and Low were 18.10, 15.97, and 13.07.

Hypothesis 17: There is no statistically significant difference in mean achievement between gender groups as measured by total scores on the post-instruction test.

Decision: Retain the null hypothesis at (p = .0618).

Hypothesis 18: There is no interaction between treatment and ability as measured by total scores on the post-instruction test.

Decision: Retain the null hypothesis at (p = .1784).

Hypothesis 19: There is no interaction between treatment and gender as measured by total scores on the post-instruction test.

Decision: Retain the null hypothesis at (p = .5935).

Hypothesis 20: There is no interaction between ability and gender as measured by total scores on the post-instruction test.

Decision: Retain the null hypothesis at (p = .4993).
Hypothesis 21: There is no interaction among treatment and ability and gender as measured by total scores on the post-instruction test.

Decision: Retain the null hypothesis at \( p = .6234 \).

Post-Retention Analysis

Three activities were administered during the retention period. Each activity was given in one-week intervals. The post-retention test was administered the week following the last activity.

Analysis of Knowledge and Comprehension Items

The data relating to hypotheses 22, 23, 24, 25, 26, 27, and 28 stated in null form in Chapter III are reported in Table 11.

Table 11. ANOVA of Knowledge and Comprehension Item Scores (N = 401)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of squares</th>
<th>Mean squares</th>
<th>F value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMT</td>
<td>2</td>
<td>20.019</td>
<td>10.010</td>
<td>3.09</td>
<td>.0465</td>
</tr>
<tr>
<td>ABL</td>
<td>2</td>
<td>291.33</td>
<td>145.66</td>
<td>45.00</td>
<td>.0000</td>
</tr>
<tr>
<td>GEN</td>
<td>1</td>
<td>25.611</td>
<td>25.611</td>
<td>7.91</td>
<td>.0049</td>
</tr>
<tr>
<td>TMT/ABL</td>
<td>4</td>
<td>3.935</td>
<td>.98384</td>
<td>.30</td>
<td>.8753</td>
</tr>
<tr>
<td>TMT/GEN</td>
<td>2</td>
<td>32.524</td>
<td>16.262</td>
<td>5.02</td>
<td>.0070</td>
</tr>
<tr>
<td>ABL/GEN</td>
<td>2</td>
<td>9.308</td>
<td>4.654</td>
<td>1.44</td>
<td>.2387</td>
</tr>
<tr>
<td>TMT/ABL/GEN</td>
<td>4</td>
<td>9.563</td>
<td>2.3908</td>
<td>.74</td>
<td>.5661</td>
</tr>
<tr>
<td>Residual</td>
<td>383</td>
<td>1239.7</td>
<td>3.2369</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
There was a significant statistical difference among treatments ($p = .0465$), among ability levels ($p = .0000$), and between gender groups ($p = .0070$). There was a significant interaction between treatment and gender ($p = .0070$).

Hypothesis 22: There is no statistically significant difference in mean achievement among treatments as measured by application and analysis items on the post-retention test.

Decision: Reject the null hypothesis at ($p = .0465$). There was a significant difference in mean scores of treatment groups on application and analysis items on the post-retention test. The mean scores of the sequence, Treatment I and II, were higher than the textbook treatment, Treatment III. Means for Treatments I, II and III were 9.077, 8.968, and 8.549.

Hypothesis 23: There is no statistically significant difference in mean achievement among ability levels as measured by application and analysis items on the post-retention test.

Decision: Reject the null hypothesis at ($p = .0000$). There was a significant difference in mean total scores of ability groups on application and analysis items on the post-retention test. Means for ability groups High, Medium, and Low were 9.007, 8.968, and
8.549. High and low groups differed significantly; the medium group did not differ from high or low groups.

Hypothesis 24: There is no statistically significant difference in mean achievement between gender groups as measured by application and analysis items on the post-retention test.

Decision: Reject the null hypothesis at \( p = .0049 \). There was a significant difference in mean total scores of gender groups on application and analysis items on the post-retention test. Means for males and females were 9.122 and 8.608.

Hypothesis 25: There is no interaction between treatment and ability as measured by application and analysis items on the post-retention test.

Decision: Retain the null hypothesis at \( p = .8753 \).

Hypothesis 26: There is no interaction between treatment and gender as measured by application and analysis items on the post-retention test.

Decision: Reject the null hypothesis at \( p = .0070 \).

There was interaction between treatment and gender. Scores for the groups are reported in Table 12.
Table 12. Interaction of Treatment and Gender Knowledge and Comprehension Score Means

<table>
<thead>
<tr>
<th>Treatment</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>9.09</td>
<td>9.50</td>
<td>8.86</td>
</tr>
<tr>
<td>Females</td>
<td>9.23</td>
<td>8.42</td>
<td>8.33</td>
</tr>
</tbody>
</table>

The interaction is disordinal with females scoring higher in Treatment I and males higher in Treatments II and III.

Hypothesis 27: There is no interaction between ability and gender as measured by application and analysis items on the post-retention test.

Decision: Retain the null hypothesis at ($p = .2387$).

Hypothesis 28: There is no interaction among treatment and ability and gender as measured by application and analysis items on the post-retention test.

Decision: Retain the null hypothesis at ($p = .5661$).

Analysis of Application and Analysis Items

The data relating to hypotheses 29, 30, 31, 32, 33, 34, and 35 stated in null form in Chapter III are addressed in Table 13.
Table 13. ANOVA of Application and Analysis Scores
(N=401)

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Squares</th>
<th>F value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMT</td>
<td>2</td>
<td>46.806</td>
<td>23.403</td>
<td>5.71</td>
<td>.0036</td>
</tr>
<tr>
<td>ABL</td>
<td>2</td>
<td>487.00</td>
<td>243.50</td>
<td>59.46</td>
<td>.0000</td>
</tr>
<tr>
<td>GEN</td>
<td>1</td>
<td>2.867</td>
<td>2.867</td>
<td>.70</td>
<td>.4028</td>
</tr>
<tr>
<td>TMT/ABL</td>
<td>4</td>
<td>37.730</td>
<td>9.430</td>
<td>2.30</td>
<td>.0581</td>
</tr>
<tr>
<td>TMT/GEN</td>
<td>2</td>
<td>10.622</td>
<td>5.311</td>
<td>1.30</td>
<td>.2746</td>
</tr>
<tr>
<td>ABL/GEN</td>
<td>2</td>
<td>14.230</td>
<td>7.115</td>
<td>1.74</td>
<td>.1774</td>
</tr>
<tr>
<td>TMT/ABL/GEN</td>
<td>4</td>
<td>37.194</td>
<td>9.298</td>
<td>2.27</td>
<td>.0611</td>
</tr>
<tr>
<td>Residual</td>
<td>385</td>
<td>1568.6</td>
<td>4.905</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There was a significant statistical difference among treatments (p = .0036) and ability levels (p = .0000).

Hypothesis 29: There is no statistically significant difference in mean achievement among treatments as measured by application and analysis items on the post-retention test.

Decision: Reject the null hypothesis at (p = .0036). There was a significant difference between treatments. Means for Treatments I, II, and III were 8.082, 7.859, and 7.258. Means for Treatments I and II were higher than the mean for Treatment III.

Hypothesis 30: There is no statistically significant difference in mean achievement among ability levels as measured by application and analysis items on the post-retention test.
Decision: Reject the null hypothesis at \( p = .0000 \). There was a significant difference in mean total scores of ability groups on application and analysis items on the post-retention test. Means for ability groups High, Medium, and Low were 9.127, 7.693, and 6.379.

Hypothesis 31: There is no statistically significant difference in mean achievement between gender groups as measured by application and analysis items on the post-retention test.

Decision: Retain the null hypothesis at \( p = .4028 \).

Hypothesis 32: There is no interaction between treatment and ability as measured by application and analysis items on the post-retention test.

Decision: Retain the null hypothesis at \( p = .0581 \).

Hypothesis 33: There is no interaction between treatment and gender as measured by application and analysis items on the post-retention test.

Decision: Retain the null hypothesis at \( p = .2746 \).

Hypothesis 34: There is no interaction between ability and gender as measured by application and analysis items on the post-retention test.
Decision: Retain the null hypothesis at \((p = 0.1774)\).

Hypothesis 35: There is no interaction among treatment and ability and gender as measured by application and analysis items on the post-retention test.

Decision: Retain the null hypothesis at \((p = 0.0611)\).

**Analysis of Total Scores**

The data relating to hypotheses 36, 37, 38, 39, 40, 41, and 42 stated in null form in Chapter III are reported in Table 14.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of squares</th>
<th>Mean squares</th>
<th>F value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMT</td>
<td>2</td>
<td>99.837</td>
<td>49.919</td>
<td>5.27</td>
<td>.0055</td>
</tr>
<tr>
<td>ABL</td>
<td>2</td>
<td>1580.2</td>
<td>790.11</td>
<td>83.34</td>
<td>.0000</td>
</tr>
<tr>
<td>GEN</td>
<td>1</td>
<td>28.534</td>
<td>28.534</td>
<td>3.01</td>
<td>.0828</td>
</tr>
<tr>
<td>TMT/ABL</td>
<td>4</td>
<td>56.531</td>
<td>14.133</td>
<td>1.49</td>
<td>.2042</td>
</tr>
<tr>
<td>TMT/GEN</td>
<td>2</td>
<td>70.856</td>
<td>35.428</td>
<td>3.74</td>
<td>.0247</td>
</tr>
<tr>
<td>ABL/GEN</td>
<td>2</td>
<td>31.695</td>
<td>15.848</td>
<td>1.67</td>
<td>.1893</td>
</tr>
<tr>
<td>TMT/ABL/GEN</td>
<td>4</td>
<td>47.760</td>
<td>11.940</td>
<td>1.26</td>
<td>.2854</td>
</tr>
<tr>
<td>Residual</td>
<td>385</td>
<td>3630.8</td>
<td>9.480</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There was a significant statistical difference between treatments \((p = 0.0287)\) and ability levels \((p = 0.0000)\) and interaction between treatment and gender \((p = 0.0247)\).

Hypothesis 36: There is no statistically significant difference in mean achievement among
treatments as measured by total scores on the post-instruction test.

Decision: Reject the null hypothesis at ($p = .0055$). There was a statistical difference among treatment groups on total scores on the post-retention test. Means for treatments I, II, and III were 17.14, 16.80, and 15.94. Treatment means for Groups I and II were both greater than Treatment III.

Hypothesis 37: There is no statistically significant difference in mean achievement among ability levels as measured by total scores on the post-retention test.

Decision: Reject the null hypothesis at ($p = .0000$). There was a significant difference in mean total scores of ability groups on total scores on the post-retention test. Means for ability groups High, Medium, and Low were 19.15, 16.53, and 14.20.

Hypothesis 38: There is no statistically significant difference in mean achievement between gender groups as measured by total scores on the post-retention test.

Decision: Retain the null hypothesis at ($p = .0828$).
Hypothesis 39: There is no interaction between treatment and ability as measured by total scores on the post-retention test.

Decision: Retain the null hypothesis at (p = .2042).

Hypothesis 40: There is no interaction between treatment and gender as measured by total scores on the post-retention test.

Decision: Reject the null hypothesis at (p = .0247). There was interaction between treatment and gender. Scores for the groups are reported in Table 15 below.

Table 15. Interaction of Treatment and Gender
Application and Analysis Score Means

<table>
<thead>
<tr>
<th></th>
<th>Treatment</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>17.27</td>
<td>17.70</td>
<td>16.17</td>
<td></td>
</tr>
<tr>
<td>Females</td>
<td>17.47</td>
<td>16.46</td>
<td>15.95</td>
<td></td>
</tr>
</tbody>
</table>

The interaction was disordinal with males scoring higher in Treatments II and III.

Hypothesis 41: There is no interaction between ability and gender as measured by total scores on the post-retention test.

Decision: Retain the null hypothesis at (p = .1893).
Hypothesis 42: There is no interaction among treatment and ability and gender as measured by total scores on the post-retention test.

Decision: Retain the null hypothesis at ($p = .2854$).

General Summary Table

Table 16 shows the general summary of $p$-values for all post measures.

Table 16. General Summary Table

<table>
<thead>
<tr>
<th>Variables</th>
<th>PI-lo</th>
<th>PR-lo</th>
<th>PI-hi</th>
<th>PR-hi</th>
<th>PI-tot</th>
<th>PR-tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMT</td>
<td>.0001</td>
<td>.0465</td>
<td>.4163</td>
<td>.0036</td>
<td>.0287</td>
<td>.0055</td>
</tr>
<tr>
<td>LVL</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
<td>.0000</td>
</tr>
<tr>
<td>GEN</td>
<td>.0308</td>
<td>.0049</td>
<td>.3464</td>
<td>.4028</td>
<td>.0618</td>
<td>.0828</td>
</tr>
<tr>
<td>TMT/LVL</td>
<td>.2400</td>
<td>.8753</td>
<td>.4934</td>
<td>.0581</td>
<td>.1784</td>
<td>.2042</td>
</tr>
<tr>
<td>TMT/GEN</td>
<td>.1844</td>
<td>.0070</td>
<td>.5954</td>
<td>.2746</td>
<td>.5935</td>
<td>.0247</td>
</tr>
<tr>
<td>LVL/GEN</td>
<td>.5357</td>
<td>.2387</td>
<td>.5816</td>
<td>.1774</td>
<td>.4993</td>
<td>.1893</td>
</tr>
<tr>
<td>TMT/LVL/GEN</td>
<td>.1485</td>
<td>.5661</td>
<td>.6883</td>
<td>.0611</td>
<td>.6234</td>
<td>.2854</td>
</tr>
</tbody>
</table>

KEY:

Tests

- PI-lo - Post-instruction low level items
- PI-hi - Post-instruction high level items
- PI-tot - Post-instruction total items
- PR-lo - Post-retention low level items
- PR-hi - Post-retention high level items
- PR-tot - Post-retention total items

Variables

- TMT - Treatment
- LVL - Ability Level
- GEN - Gender
CHAPTER V

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

Summary

This study investigated the effect of a multiple embodiment instructional sequence on student achievement in relation to the topic of volume of rectangular solids. The population was 419 fifth grade students from 21 classrooms in southwestern Montana. The study was conducted between November 1988 and February 1989. In October 1988, participating teachers were given inservice and received all student materials, teacher instructions and tests. In November, this researcher randomly chose classrooms from which 113 students were selected and individually tested with the Developmental Level Instrument (DLI) to determine developmental level as described by Piagetian theory of logico-mathematical knowledge. The diagnostic tests were administered before instruction began.

Two to four weeks before instruction began, a criterion-referenced pretest and the Stanford Achievement Test were administered to all 419 participating students
by classroom teachers. The SAT scores were used to assign students to one of three achievement or ability groups; each group included approximately one-third of the total population. Of a total of 74 points on the SAT, 61 or more resulted in placement in the high ability group; a score of 60 or less and 50 or more resulted in placement in the medium group. A score of 49 or less resulted in placement in the low group. The pretest was used to ensure initial equivalence between the three treatments. An ANOVA of prescores resulted in a p-value of .9854; thus the groups were equivalent before instruction. To test the equivalence of ability groups across treatments, three one-way ANOVAs and three Krushal-Wallis non-parametric tests were performed. The groups were found to be equivalent among treatments.

Length of time involved for the actual instruction and retention activities in the study consisted of four weeks with one week for instruction and three weeks for retention activities, one activity per week. The three instructional treatments included: (a) Treatment I - a multiple embodiment instructional sequence including a computer simulation as one embodiment, (b) Treatment II - a multiple embodiment instructional sequence without the computer simulation, and (c) Treatment III - a sequence based on textbook lessons of volume of rectangular
solids. The three retention activities differed as follows: in addition to computing problems, students solved volume problems (a) with the computer (Treatment I), (b) by drawing and labeling boxes (Treatment II), and (c) applying the volume formula (Treatment III). A post-instruction test was administered following the last day of instruction. A post-retention test was administered following three weeks of retention activities.

Dependent variables included three sets of scores on each of two tests: (a) post-instruction test and (b) post-retention test. The three sets of scores measured (a) low level items, composed of knowledge and comprehension cognitive skill levels; (b) high level items, composed of application and analysis cognitive skill levels; and (c) total battery of items. Independent variables were (a) treatment, (b) ability level, and (c) gender.

Conclusions

Conclusions: Post-Instruction

1. The two multiple embodiment instructional sequences did not produce statistically greater achievement on application and analysis items or on total test scores than did the textbook sequence. No
differences were observed among treatments on application and analysis items. On total scores only, Treatments II and III differed significantly.

2. The multiple embodiment sequence produced greater scores on knowledge and comprehension scores than did the textbook sequence. Treatments I and II scored higher than Treatment III on knowledge and comprehension items. The finding supports the research by Indelicato (1979), Johnson (1970), Johnson (1974), and Wheeler (1971).

3. Incorporating the computer as an interfacing agent was as effective as multiple embodiment instruction without computers. No statistically significant differences existed between Treatments I and II on all three post-instruction dependent variables. The question raised in Berlin and White (1986) addressing effective use of the computer to serve as an interactive link between concrete and abstract levels of thought has been addressed in this researcher’s study. However, results of effectiveness of CAI after one week of instruction were tempered by the duplication of instruction in Treatments I and II until the introduction of the computer on the last day of instruction in Treatment I.

4. In each treatment, high ability students demonstrated the greatest achievement following
instruction; low ability students demonstrated the lowest achievement. As expected, means for high ability groups were significantly greater than for medium ability groups. This study supported Gordon (1978) who determined quantitative measures of intelligence and achievement are directly related to achievement in mathematics in middle school students. This study supported the literature on use of multiple embodiment instruction as reported by Indelicato (1979), Johnson (1970), Johnson (1974), and Wheeler (1971).

Comparison of CAI and learner ability is less obvious. Most studies to date involved drill and practice or tutorial CAI and reported greater effectiveness with low ability learners (Chambers & Sprecher, 1980; Crawford, 1970; Feurzeig et al., 1981; Harman, 1980; Henderson, et al., 1983; Keuper, 1985; Kulik et al., 1983, Menis et al., 1980). Burns and Bozeman (1981) also analyzed drill and practice and tutorial CAI; their finding of higher achievement among both low ability and high ability learners coincides, in part, with this study. However, when CAI is used in a simulation format or a programming format, studies have noted increases in effectiveness are found with high ability students but not in other ability groupings (Jhin, 1971; Kieran, 1969; Reed, 1985).
5. Following instruction, males demonstrated greater achievement on low level items than did females in all treatments. This was in agreement with the NAEP 1977-78 released item results, which reported males scored higher than females on all items concerning volume of rectangular solids (Ben Haim, et al., 1985; NAEP, 1979a). Lower level items used in the criterion-referenced test in this study (see Appendix A) were similar to volume items on the NAEP and SAT tests shown in Figures 1, 2, 3, 4, and 5.

6. Males and females demonstrated equal achievement on high level items and on total test scores in all treatments. This conclusion indicated activities and instruction of all treatments were equally effective in producing higher cognitive learning for both males and females. The finding on high level items supported the research of Pallas and Alexander (1983). Total score results supported the research of Indelicato (1979) and Johnson (1970).

7. Treatment did not affect achievement of gender groups. No difference was reported on any of the three dependent post-instruction variables. Indelicato (1979) and Johnson (1970) reported similar results.
Conclusions: Post-Retention

8. The multiple embodiment instructional sequences produced significantly greater achievement on knowledge, comprehension, application, and analysis items than did the textbook sequence. Upon completion of the study students in both Treatment I and II scored significantly higher on all post-retention measures than students in Treatment III. The effectiveness of multiple embodiment instruction is in agreement with the results of research by Edge and Ashlock (1982), Johnson (1970), and Wheeler (1971). Disagreement was found with Reys (1972), and Skipper (1972). Behr’s (1976) findings on two-digit arithmetic agree with the results of this study; his three-digit results disagree. Support was also found with Indelicato’s (1979) results after four weeks of instruction using four one-week units on mathematics topics. However, a retention test administered four weeks later eliminated the significant difference. No retention activities were provided in Indelicato’s study.

Results of his study appear to agree with theories of cognitive psychology of mathematics learning which emphasize the better a concept is learned, the better it is retained and recalled (Byers & Erlwanger, 1985; Gagne, 1985). Cognitive theories of memory and learning state retrieval of knowledge is affected by the quality,
accuracy, and number of stored memories (experiences) of the knowledge (Gagne, 1985). Learners who have difficulty retaining and recalling mathematics concepts have not adapted the concept into memory in an organized manner, but rather rely on memorizing without understanding. Learning based on a student's own actions produces better, more accurate knowledge and recall of that knowledge than does passive, memorized learning (Dugdale & Kibbey, 1983, 1984; Lerch, 1981; Piaget & Inhelder, 1973).

9. Incorporating the computer as an interfacing agent is as effective as multiple embodiment instruction without computers. The post-retention analysis in which Treatment I utilized CAI for each of the three retention activities clearly demonstrated the effectiveness of the computer as one embodiment. The finding of equal effectiveness with incorporation of CAI and no CAI after instruction was reinforced by this post-measure taken upon completion of the four-week study. Though instruction in Treatments I and II differed little during the week of instruction, the effect of a computer embodiment was fully measured by the post-retention tests.

10. Regardless of treatment, the most effective impact on achievement following retention activities was
among high ability learners; the least effective was among low ability learners. The influence of retention activities did change effect on achievement among ability groups. Significance was reported on all dependent measures, post-retention as well as post-instruction. The finding in this study is supported by Piaget's theory that what a child remembers depends upon his cognitive developmental level (Piaget & Inhelder, 1973). Results of this study also support Suydam's (1984) review of mathematics literature which reported an increase in achievement at every elementary grade level, at every achievement level, and every ability level when student-activated or teacher-demonstrated manipulatives are included in instruction. Gordon's (1977) profile of high and low 6th grade mathematics achievers coincides with this study. Piagetian-defined ability to conserve volume as measured by the DLI, appeared to be a factor in student achievement.

11. Males demonstrated greater achievement on low level items than did females following retention activities in all treatments. Retention activities retained performance comparison by males and females. This is consistent with national test scores on mathematics achievement (Gardiner et al., 1983b; MEAP, 1983; NAEP, 1983).
12. Males and females demonstrated equal achievement on application and analysis items and on total test scores. As previously reported, achievement by males and females following instruction was also equivalent. Again, this finding is somewhat surprising due to the alleged superiority of males in mathematics achievement (Sandker et al., 1989), at least at the high school level. As with the post-instruction analysis, final results of this study support Armstrong (1981), maintaining equivalent achievement by both gender groups utilizing higher cognitive skills.

13. Treatment affected achievement on knowledge and comprehension items by gender groups. A disordinal interaction on low level scores was reported. Males in Treatments II and III scored higher than females. This is consistent with national test results which reported male achievement in mathematics was greater than mathematics achievement by females (Sandker et al., 1989). In Treatment I females outperformed males. Both these findings disagree with Pallas and Alexander (1983) who reported no gender differences exist in mathematics at the elementary level. Disagreement is also found with Burns and Bozeman (1981) who reported male superiority in CAI in the intermediate grades. However, the CAI in
Burns and Bozeman (1981) was in drill and practice and tutorial format.

Initial instruction based on the computer embodiment sequence was as effective for males as for females. Apparently retention nullified this effect on achievement that was reported after instruction. Studies have demonstrated females can develop spatial visualization skills and improve achievement on mathematics tests requiring that skill (Ben-Haim et al., 1985; Bishop, 1983). Though no spatial visualization instruction was administered, females in Treatment I appeared to develop that skill over the four-week study. Perhaps the computer simulation reinforced and enhanced the spatial or related skills in females while having little effect on males. Evidently the computer embodiment in Treatment I retention activities was not as effective for males as for females. Berlin and White's (1986) conclusion that students are not influenced in the same manner when CAI is used supported by this study. For fifth grade students in Southwest Montana the effect of CAI on achievement was greater for females than for males.

Concerning the literature on multiple embodiments and gender, Indelicato (1979) and Johnson (1970) found no gender differences. Both treatments I and II are
multiple embodiment sequences. The reported male superiority in Treatment II and female superiority in Treatment I is contradictory and no conclusion on multiple embodiment instruction and gender can be made.

14. Treatment had an effect on gender achievement on total scores. A disordinal interaction on total post-retention scores was detected. On the total score measure, males in Treatment I scored lower than females in Treatment I; males scored higher than females in Treatments II and III.

**Implications**

Though the differences were not significant at .05 alpha level, the following trends were observed:

1. This researcher's hypothesis that computer simulations, such as the one used in this study, operate at a higher level of cognition than traditional paper and pencil activities, such as drawing and labeling, was not supported by the analysis of data \( p = .0581 \). Low ability students in Treatment I scored lower than their counterparts in Treatments II and III on the post-retention test. This researcher compared the mean pretest and DLI scores of ability groups in each treatment to ascertain whether low ability students in Treatment I may have had an initial achievement level or
lower logico-mathematical knowledge level compared to low ability peers in Treatments II and III. Means of low ability level students as measured by the pretest and by the DLI confirm ability level was equivalent among the three treatments prior to instruction. Therefore, the difference in mean scores on the high level items was not attributable to pre-instruction differences. The trend did not occur immediately after instruction as measured by the post-instruction test. Therefore, differences cannot be attributed to one week of instruction. Since the computer was not introduced until the last day of instruction, the effect of the computer was measured during the retention period. Consequently, the interaction is assumed to be due to the effect of the computer on the low level learners in Treatment I.

This trend is supported in the literature. Kutetskii (in Bishop, 1983, p. 182) found students with high ability were able to integrate individual elements of a mathematics problem into a conceptual structure. Reid (1985) and Shaw and Okey (1985) reported similar findings. Students with very low ability saw only disconnected facts and were unable to form a conceptual structure. The trend in this study supports the research of Reed (1985), Kutetskii (in Bishop, 1983), and Shaw and Okey (1985).
For high ability students, entry-level knowledge and ability to accurately perceive math concepts may allow them to obtain maximum learning with CAI (Reed, 1985). Students who have not reached a higher cognitive level of reasoning and thinking may be unable to cope with high level items. The computer embodiment in this researcher's study may have been too abstract for such students to grasp, thus results in lower ability students doing less well than their counterparts who do not use a computer simulation.

2. A trend appears to exist indicating computers as one embodiment of math concepts may produce higher scores on achievement in student learning than the multiple embodiment sequence without computers. In no case was Treatment I statistically greater than II, or vice versa. However, in each significant measure of post-instruction, the mean for Treatment II was higher than the mean for Treatment I; in each measure of post-retention, the mean for Treatment I was greater than II. When mean scores are plotted, the trend can be observed (Appendix E). The observed trend may indicate a direction in the search for ways to improve mathematics application and analysis skills as voiced by the National Assessment of Educational Progress Report (1979); the National Science Board on Pre-College Education in Science, Mathematics
and Technology (1983), National Council of Teachers of Mathematics (1985), and others.

Recommendations for the Profession

Recommendations for Curriculum

1. The use of multiple embodiments in the instruction of volume of rectangular solids should be included in the mathematics curriculum. This study supported the majority of research on multiple embodiments in mathematics education which indicates the effectiveness on learner achievement when multiple embodiment instruction is used. Also supported by this study is current research in cognitive psychology of mathematics learning, indicating a variety of concept representations promotes retention.

2. The inclusion of a computer simulation as one embodiment in a multiple embodiment sequence should be considered. The use of CAI embodiment in this study was as effective as instruction without CAI. Though no affective measures were addressed in this study, one factor that influences achievement in mathematics is attitude toward the subject (Gordon, 1977). Research clearly indicates an increase in attitude toward mathematics when computers are incorporated into instruction (Burns & Bozeman, 1981; Kulik et al., 1983).
Student achievement in mathematics also increases when CAI is included in learning (Burns & Bozeman, 1981; Kulik et al., 1983). Studies also indicate an increase in student motivation to learn (Burns & Bozeman, 1986; White, 1983) when computers are incorporated into instruction. Lastly, a positive correlation exists between attitude toward mathematics and achievement in mathematics (Fennema & Sherman, 1977, 1978; Suydam, 1984c). Measures of motivation, attitude toward mathematics, effect of computer supplement to instruction and the correlation to mathematics achievement may have influenced the results of this study.

 Additionally, there may be a tendency for computer simulation to improve cognitive skills, at least for medium and high ability learners. This recommendation agrees with mathematics education policy makers who urge incorporation of computers into the mathematics curriculum and with researchers who indicate CAI has the potential to increase student achievement, especially at higher cognitive skill levels (Bridges, 1987; Chambers & Sprecher, 1983; Dugdale & Kibbey, 1983, 1984; Hutchinson, 1980; Reif, 1985).

3. Multiple embodiment lessons should incorporate the instruction in a structured sequence that accurately represents the math concept. Additionally, when
developing or implementing multiple embodiments, instructors are cautioned to be aware that each embodiment may have different effects on individual students.

4. Educators should not make the assumption that males outperform females at all cognitive levels of knowledge and in all educational settings. The evidence in this study of achievement by both males and females indicates an instructional sequence that is effective for both gender groups when higher cognitive skills of application and analysis are addressed. The conclusion of equal achievement by gender does not apply, however, to knowledge and computation cognitive skills.

5. Instructional mathematics units should be reinforced over time with retention activities. This may be especially important for instructional units of short duration. The subject of volume of rectangular solids is given little attention in textbooks (Duncan et al., 1981; Eicholz et al., 1985; Haenisch & Hill, 1985; Nichols et al., 1974; Payne et al., 1982; Willoughby et al., 1981). Norman and Bobrow (1976, p. 115) caution that a "period of incubation" may be necessary to reach a solution to a problem. Learning takes time and practice (Norman, 1982). This study demonstrates the need for reinforcement of a topic, particularly a complex topic
such as volume of rectangular solids. Obviously, traditional instruction is not sufficient as is indicated by national tests (Gardner et al., 1983a, 1973b; MEAP, 1983; NAEP, 1983).

6. When using retention activities in multiple embodiment instruction it is important that the retention activities should include multiple embodiments. The effectiveness of the multiple embodiment instruction in this study implies learners benefit from presentation of mathematics concepts in a variety of representational systems. Research in mathematics education supports the teaching of mathematics concepts through a range of conceptual presentations (Copeland, 1984; Bruner, 1971; Dienes, 1967; Lesh, 1983; Lesh et al., 1983; Piaget & Inhelder, 1967, 1973). It follows that retention activities based on those same assumptions will be more effective than non-representational activities and non-multiple embodiment activities.

7. Educational research professionals, including those at university and curriculum development centers, should develop lessons incorporating multiple embodiments in the mathematics curriculum. The sequences should be tested and disseminated to curriculum supervisors and teachers directly responsible for students' education.
Recommendations for Further Study

1. The study of multiple embodiments in mathematics instruction should continue. Incorporation of a variety of embodiments, including computer-assisted instruction, should be investigated.

2. The effect of CAI simulations on students at various cognitive skill levels, in particular those students at a low level of cognitive skills, should be studied. Computer simulations may not produce the highest achievement for every student. The observed trend in this study and other studies, for example, Reed (1985) and Shaw and Okey (1985), should receive further study.

3. A replication of this design to include a second four-week period of retention activities followed by another retention test is recommended. The extended study would allow more student learning time to determine whether the trend toward an increase in achievement when computers are incorporated as one embodiment continues, and if it continues at a significant level.

4. Further research is recommended to explore the potential for CAI to enhance higher cognitive levels of learning. Two questions resulting from this study should receive further study: (a) Does the implication demonstrated in this study toward increasing student
achievement after the use of CAI coincide with the literature alluding to increased student achievement in higher cognitive skills demonstrated by learners with the use of CAI? and (b) Is computer-assisted instruction, when used in a simulation format, at a higher level of instruction than the traditional semi-concrete level of instruction?
References Cited


Stephens, J., & Olson, M. (1985). We should be teaching them more than just symbol manipulation. School Science and Mathematics, 85, 1-10.


APPENDIX A

TESTS
PRETEST
PRE-TEST FOR VOLUME

General Instructions

This is a test of your knowledge of volume of rectangular solids. The test contains 15 questions. There are three parts: 1) computations, 2) multiple choice, and 3) true-false.

Read the directions for each part. If you cannot answer a question, go to the next one. You may use any empty room on the test for your work.

Before you start, write your name, the date, your teacher’s name, your school’s name, and the name of your town in the blanks below.

Name ____________________________

Date ____________________________

Your teacher’s name ____________________________

School ____________________________

Town ____________________________
Part I: Computations: Directions: Compute the volume of the figures described or shown and write the answer in the blank space provided. (1 point each).

This is a cubic unit: 🎯. How many cubic units would be needed to build each of the four solid figures shown below:

1. ___________ cubic units needed for figure 1
   ![Figure 1](image1.png)

2. ___________ cubic units needed for figure 2
   ![Figure 2](image2.png)

3. ___________ cubic units needed for figure 3
   ![Figure 3](image3.png)

4. ___________ cubic units needed for figure 4
   ![Figure 4](image4.png)

5. What is the volume of the box shown at the right?
   ANS ___________ cubic units
   ![Box](image5.png)

Compute the volume of the solid figures described below.

6. If a box is 3 units by 1 unit by 9 units, what is the volume of the box?
   ANS ___________ cubic units

7. A box is filled to the top with 1 inch by 1 inch by 1 inch cubes. If the dimensions of the box are 2 inches by 4 inches by 8 inches, how many cubes would fit into the box?
   ANS ___________ cubes
Part 2: Multiple Choice: Directions: Circle the answer you think is correct. (1 point each)

8. How many unit cubes, □, are needed to build box A?
   a) 14  
   b) 16  
   c) 20  
   d) 24

9. How many unit cubes, □, are needed to build box B?
   a) 22  
   b) 26  
   c) 36  
   d) 48

10. Two boxes are shown below. Circle the statement that is true.
    a) Box 1 has the larger volume.  
    b) Box 2 has the larger volume.  
    c) They have the same volume.

11. Which answer tells the volume of a large box?
    a) 1 yard  
    b) 1 square yard  
    c) 1 cubic yard

Part 3: True-False: Directions: Circling 'T' if the statement is true and 'F' if it is false (1 point each).

12. T  F  The tallest box always has the most volume.
13. T  F  A ball has volume.
14. T  F  A light bulb has volume.
15. T  F  An eraser has volume.
POST-INSTRUCTION TEST
VOLUME TEST (A)

General Instructions

This is a test of your knowledge of volume of rectangular solids. The test contains 21 questions. There are five parts: 1) computations, 2) multiple choice, 3) true-false 4) matching, and 5) short answer.

Read the directions for each part. If you cannot answer a question, go to the next one. You may use any empty room on the test for your work.

Before you start, write your name, the date, your teacher's name, your school's name, and the name of your town in the blanks below.

Name ________________________________

Date ________________________________

Your teacher's name ____________________

School ______________________________

Town ________________________________
Part I: Computations: Directions: Compute the volume of the figures described or shown and write the answer in the blank space provided. (1 point each).

This is a cubic unit: 🎳. How many cubic units would be needed to build each of the four solid figures shown below:

1. _____ cubic units needed for figure 1

2. _____ cubic units needed for figure 2

3. _____ cubic units needed for figure 3

4. _____ cubic units needed for figure 4

5. What is the volume of the box shown at the right?

ANS _____ cubic units

Compute the volume of the solid figures described below.

6. If a box is 3 units by 1 unit by 9 units, what is the volume of the box?

ANS _____ cubic units

7. A box is filled to the top with 1 inch by 1 inch by 1 inch cubes. If the dimensions of the box are 2 inches by 4 inches by 8 inches, how many cubes would fit into the box?

ANS _____ cubes
Part 2: Multiple Choice: Directions: Circle the answer you think is correct. (1 point each)

8. How many unit cubes, \( \square \), are needed to build box A?
   a) 14
   b) 16
   c) 20
   d) 24

9. How many unit cubes, \( \square \), are needed to build box B?
   a) 22
   b) 26
   c) 36
   d) 42

10. Two boxes are shown below. Circle the statement that is true.

   a) Box 1 has the larger volume.
   b) Box 2 has the larger volume
   c) They have the same volume.

11. Which answer tells the volume of a large box?

   a) 1 yard
   b) 1 square yard
   c) 1 cubic yard

Part 3: True-False: Directions: Circling "T" if the statement is true and "F" if it is false (1 point each).

12. T F The tallest box always has the most volume.
13. T F A ball has volume.
14. T F A light bulb has volume.
15. T F An eraser has volume.
Part 4: Matching: Directions: Match the volume shown on the left with the box shown on the right by placing the letter of the box in the blank space (1 point each).

16. 4 cubic units  Box __________
17. 27 cubic units  Box __________
18. 36 cubic units  Box __________
19. 50 cubic units  Box __________

Part 5: Short Answer: Directions: The next two questions go together. Compute the answer to question 20. You can use any blank space on this paper for doing your work. For question 21, write down how you got your answer to question 20. (2 points each)

20. This is a 2 cm cube:

How many of these cubes will it take to fill the 2 cm by 6 cm by 2 cm box shown at the right?

ANS ________________ Cubes

21. Explain how you got your answer to #20.
POST-RETENTION TEST
VOLUME TEST (B)

General Instructions

This is a test of your knowledge of volume of rectangular solids. The test contains 21 questions. There are five parts: 1) computations, 2) multiple choice, 3) true-false, 4) matching, and 5) short answer.

Read the directions for each part. If you cannot answer a question, go to the next one. You may use any empty room on the test for your work.

Before you start, write your name, the date, your teacher's name, your school's name, and the name of your town in the blanks below.

Name ________________________________

Date ________________________________

Your teacher's name ________________________________

School ________________________________

Town ________________________________
Part I: Computations: Directions: Compute the volume of the figures described or shown and write the answer in the blank space provided. (1 point each).

This is a cubic unit: \(\square\). How many cubic units would be needed to build each of the four solid figures shown below:

1. \(\square\) cubic units needed for figure 1

2. \(\square\) cubic units needed for figure 2

3. \(\square\) cubic units needed for figure 3

4. \(\square\) cubic units needed for figure 4

5. What is the volume of the box shown at the right?

   ANS \(\square\) cubic units

Compute the volume of the solid figures described below.

6. If a box is 2 units by 1 units by 7 units, what is the volume of the box?

   ANS \(\square\) cubic units

7. A box is filled to the top with 1 inch by 1 inch by 1 inch cubes. If the dimensions of the box are 2 inches by 5 inches by 6 inches, how many cubes would fit into the box?

   ANS \(\square\) cubes
Part 2: Multiple Choice: Directions: Circle the answer you think is correct. (1 point each)

8. How many unit cubes, , are needed to build box A?
   a) 16
   b) 18
   c) 20
   d) 24

9. How many unit cubes, , are needed to build box B?
   a) 24
   b) 48
   c) 32
   d) 64

10. Two boxes are shown below. Circle the statement that is true.

   a) Box 1 has the larger volume.
   b) Box 2 has the larger volume.
   c) They have the same volume.

11. Which answer tells the volume of a small box?

   a) 1 foot
   b) 1 square foot
   c) 1 cubic foot

Part 3: True-False: Directions: Circling "T" if the statement is true and "F" if it is false (1 point each).

12. T  F  The shortest box always has the least volume.

13. T  F  A can has volume.

14. T  F  A balloon has volume.

15. T  F  A pencil has volume.
**Part 4: Matching**

Directions: Match the volume shown on the left with the box shown on the right by placing the letter of the box in the blank space (1 point each).

16. 12 cubic units Box __________
17. 36 cubic units Box __________
18. 49 cubic units Box __________
19. 64 cubic units Box __________

**Part 5: Short Answer**

Directions: The next two questions go together. Compute the answer to question 20. You can use any blank space on this paper for doing your work. For question 21, write down how you got your answer to question 20. (2 points each)

20. This is a 2 cm cube:

   How many of these cubes will it take to fill the 2 cm by 8 cm by 2 cm box shown at the right?

ANS __________ Cubes

21. Explain how you got your answer to #20.
DEVELOPMENTAL LEVEL INSTRUMENT
Definitions from Copeland and Good

Measurement (C):
- by superposition: placing a unit measure over figure to be measured
- by unit measure: repeated superposition or "iteration"
- by linear measurement or metrical solution: obtain answer by use of a formula

Conservation (G): (or invariance): ability to hold constant a quantity regardless of shape and/or other spatial relations

Reversibility (G): ability to mentally reverse a process

Transitive property of equals (G): if A = B and B = C, then A = C

Characteristics of stages of mathematical and science concepts (Good and Copeland)

Stage 1: pre-operational
primarily sensory; "sight"; lack of understanding

Stage 2: transitional
trial and error; rote measuring; little understanding

Stage 3: concrete operational
does not rely on trial and error; reasoning for measuring, answering; quick response; may rely on physical aides
stage 3.5: uses counting method to build
stage 3: uses metrical method to build (determines side dimensions and builds)

Stage 4: formal operational
can reason without physical aids; conservation of area, internal and occupied volume; able to think at abstract level
Purpose: to determine the ability to measure area and subdivide the area of a figure into units

Materials:
- three cards cut into shapes A, B, and C
- two measuring units: one square, one triangle (half the square)
- magic marker, wet cloth to erase marker strokes

Procedure:
1. Show A and B
2. Ask student to A and B take up the same room?
   - yes or no, How do you know? [they look the same/different]
   - not sure, ask how can you find out? [measure]
3. Ask student to find how much room Shape A contains; may ask if using the unit measure and marker would help
4. Follow procedure for Shape B
5. Ask if A and B then take up the same, more or less amount of room
6. Show C; ask Do B and C have the same room?
7. If unable to measure, ask would the small shapes help?
8. Ask student to find how much room Shape C contains
9. Ask if B and C then take up the same, more or less room

Scoring: (check one)
- (Stage 1): thinks area of A and B is same/different because A and B look same/different
- (Stage 2): can find correct area of A and B with some difficulty but not of C
- (Stage 3): can find A, B easily with unit iteration but not C; counts both units as same amount
- (Stage 3.5): can find correct area of A, B and C with unit iteration
- (Stage 4): can find correct area of A and B with a metrical solution; also finds correct C

Rationale for Scoring
Stage 1: wrongly uses perceptual information; has no idea of solution
Stage 2: may use perceptual information or logical data; does not have concept of conservation of area; may get right answer but has not generalized a logical approach (i.e. $l \times w$) or may count for each shape
Stage 3: uses count and measure method, a logical approach
Stage 3.5: immediate solution by superposition and iteration; correctly applies both unit measurers
Stage 4: uses a metrical solution: can calculate area with length and width measure
TASK 2

MEASUREMENT OF AREA BY MULTIPLICATION

Purpose: to determine the ability to understand multiplication of linear measurements to find area

Materials:
- two laminated drawings of 1) a line segment 3 inches long
  and 2) a 2 X 2 inch square
- marker, wet cloth, ruler

Procedure:
1. Show the 3 inch line segment
2. Ask student to draw a line segment two times as long
3. Show the square
4. Ask student to draw a square 2 times as large; ask How did you know what size to draw?

Scoring:
--- (Stage 2): cannot draw double line segment or square
--- (Stage 3): tries to double one side and produces a rectangle or doubles two sides and produces a square four times original size
--- (Stage 3.5): realizes doubling size of sides of square is too large but unable to draw
--- (Stage 4): realizes area must be 8; and approximates a side between 2.5 and 3 (3 too large; 2.5 too small); may calculate square root of 8

Rationale for Scoring
Stage 2: does not understand linear measurement
Stage 3: understands linear measurement but not to apply to two-dimensional measurement; doubles one or both sides of square
Stage 3.5: thinks the sides of a square must be doubled but then realizes new area will be too large; unable to draw the proper square; does not understand the relationship to area of a square to side measurement
Stage 4: understands the relationship between side dimensions and area of a square
**TASK 3  CONSERVATION AND MEASUREMENT OF VOLUME**

**Purpose:** to determine the ability to understand conservation and measurement of volume

**Materials:**
- about 80 one inch cubes
- one (2 x 3 x 4) rectangular solid constructed from one inch cubes
- cardboard shapes (foundations): 1) 1 x 2, 2) 2 x 3, 3) 3 x 4

**Procedure:**
1. Place the rectangular solid on the 3 x 4 foundation; give student about 30 cubes
2. Ask student to build a building on the 2 x 3 foundation that has the same amount of room as the preconstructed building
3. (Q-1) Does your building have the same or more or less room as my building? How do you know?
   - ANS: more/less, don’t know why
   - same, used same number of blocks as mine
   - same, shape invariant; indicates V = l x w x h
4. Ask student to make a house with those cubes on the 1 x 2 foundation
5. (Q-2) Now, does your building have the same, more or less room as my building? How do you know?
   - ANS: more/less, don’t know
   - same, used same or same number of blocks
   - same, shape invariant
6. (Q-3) Would the buildings take up the same, more or less space in a huge dome? Why
   - ANS: more/less; different shapes

**Scoring:**

--- (Stage 1): cannot build
--- (Stage 2): Q-1, more/less space due to shape; has trouble making correct buildings
--- (Stage 3): Q-1, same [all/same blocks used]; relies on counting the number of blocks to build correctly
   - Q-2, same [same as above]
   - Q-3, more/less space
--- (Stage 3.5): Q-1, same, counts l, w, h to determine total blocks
   - Q-2, same
   - Q-3, more/less space
--- (Stage 4): Q-2, yes [indicates cons. of volume]; doesn’t count to build; calculates/uses dimensions to build
   - Q-3, same space used; understands cons. of interior space

**Rationale for Scoring:**

**Stage 1:** cannot determine a way to build
**Stage 2:** can build one or two buildings with trial and error; cannot conserve volume
**Stage 3:** relies on counting number of blocks to build properly; begins to understand conservation of volume
**Stage 3.5:** counts l, w, h to determine number of blocks in original building and to rebuild; conservation of interior space established
**Stage 4:** counts l, w, h to determine volume to rebuild; conservation of interior volume is established
TASK 4  CONSERVATION AND MEASUREMENT OF VOLUME

Purpose: to determine the ability to understand conservation of volume

Materials:
- three sets of 40 unifix cubes
- drawing of three rectangular solids with the following dimensions:
  A: 12 X 3 X 1
  B: 8 X 3 X 2
  C: 3 X 4 X 3
- pre-constructed buildings (boxes) of A and C made with unifix cubes

Procedure:
1. Ask student to make Box B with the cubes as shown in the drawings
2. (Q-1) Do the three boxes take up the same or less or more amount of space? How do you know?
   Ans: no/yes, different shapes, no reason
       no, measured or counted cubes
       no, dimensions multiplied
3. (Q-2) If I put each one in its own pail full of water, would the same amount of water spill out? How do you know?
   Ans: no, don't know
       no, different number blocks
       no, volume different

Scoring:
___ (Stage 2): has trouble building shape and/or determining space
   Q-1, no/yes [different shape]
___ (Stage 3): uses counting method to build shape and determine space
   Q-2, no/yes but no correct reason for answer [guessed; don't know]
___ (Stage 3.5): uses metrical method to build shape and determine space
   Q-1 and Q-2 same as above
___ (Stage 4): uses metrical method to build
   Q-2, no with answer indicating understanding [same space, same volume, etc]

Rationale for Scoring:
Stage 2: has trouble building; relies on visual perception to matching the drawings
Stage 3: builds by counting; begins to understand conservation of internal volume
Stage 3.5: builds with metrical solution; does not rely on counting; begins to understand conservation of internal space but not space occupied
Stage 4: builds by calculating dimensions; immersion process does not change invariance of volume; understands conservation of volume in terms of space occupied and interior volume
## SCORE SHEET

**Student Name**

**School**

**Teacher**

**Class**

**Gender** M F

**Birthday** _____________ **Age** Years ___ **Months** ___

**Date**

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**Total** _____
APPENDIX B

VALIDATION OF TESTS AND SOFTWARE
CRITERION-REFERENCED TEST
April 20, 1988

Dear:

Please note the attached Table of Specifications and criterion-referenced test for the subject of volume of rectangular solids. The test will be the dependent variable in my research and will be used as a pre and a post test for a week-long teaching sequence in fifth grade mathematics, and again as a post retention instrument.

To obtain validity of this instrument, I am asking three specialists in the area of mathematics to validate the content as an adequate measure of the subject-matter as defined by the objectives. I have listed the objectives on the left of the Table of Specifications; cognitive levels are at the top of the Table. I am also including my description of the objectives.

For your convenience, I have listed Bloom's description of the levels below:

Knowledge: ability to recall specifics as well as universals, methods and processes, structure and patterns.

Comprehension: ability to use materials and ideas without necessarily relating them to other materials or seeing their fullest implementation.

Application: ability to use learned information in particular situations and to apply an appropriate abstraction to a new problem.
**Analysis:** ability to break down information into its component parts thereby making clear the relative hierarchy of ideas and relationships between the ideas.

**Synthesis:** ability to put together elements or parts to form a new whole.

**Evaluation:** ability to make quantitative and qualitative judgements about the extent to which materials, information, methods, or ideas satify a given set of criteria.

My descriptions of things to consider with the stated test objectives follow:

1) Ability to calculate given dimensions with figures: can a student calculate volume when shown a figure with the height, width and length given; can apply to multi-cubed or cubes without the cube divisions.

2) Ability to calculate given dimensions in word problems: can a student calculate volume when height, width and length are given in verbal form.

3) Develop understanding of volume as a measure of three dimensional objects: does the student understand that three dimensions are involved in the volume of an object; that the answer should reflect unit cubed or unit $3$.

4) Develop understanding of volume as independent of shape: objects can be rectangular and non-rectangular as long as they have three dimensions, an awareness that the tallest/smallest object does not necessarily have the most/least volume.
5) Ability to infer 3-D information from 2-D representation: can students use a paper drawing (2-D) to infer the three dimensional information represented in the drawing; does he/she realize "hidden" cubes of a solid object are part of the whole object.

Will you please read the test, mark the cognitive level for each test item by writing the item number in the appropriate box? Of course, one item may apply to two or more objectives. Lastly, please respond to the validation page and return it to me at 213 Reid Hall.

Thanks for taking time to complete this for me.

Sincerely,
VALIDATION OF A CRITERION-REFERENCED TEST
ON THE SUBJECT OF
VOLUME OF RECTANGULAR SOLIDS
FOR
RUTH JOHNSON

I have read the criterion-referenced test to be used in Ruth Johnson's research. I find the contents of the test as a measure of the subject-matter as described by the objectives to be:

✓ Valid

Not Valid, for the reasons listed below.

Signed

Title

Date

Reasons:
VALIDATION OF A CRITERION-REFERENCED TEST ON THE SUBJECT OF VOLUME OF RECTANGULAR SOLIDS FOR RUTH JOHNSON

I have read the criterion-referenced test to be used in Ruth Johnson's research. I find the contents of the test as a measure of the subject-matter as described by the objectives to be:

√ Valid

☐ Not Valid, for the reasons listed below.

Signed

Title

Date

Reasons:

Susan Phillips
Math Specialist, Fort Worth School
7/8/88
I have read the criterion-referenced test to be used in Ruth Johnson's research. I find the contents of the test as a measure of the subject-matter as described by the objectives to be:

- [ ] Valid
- [x] Not Valid, for the reasons listed below.

Signed: [Signature]
Title: Assistant Professor of Mathematics
Date: 25 May 1988

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<td>4) develop understanding of volume as independent of shape</td>
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<td>5) ability to infer 3-D information from 2-D representation</td>
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Placement of Items into cognitive levels:

Lower cognitive levels: 1, 2, 3, 5, 6, 8, 9, 11, 17, 18, 19
Higher cognitive levels: 4, 7, 10, 12, 13, 14, 15, 16, 20, 21
May 5, 1988

Dear

Enclosed you will find a copy of the software I intend to use in my doctoral research. Also enclosed is a criterion list taken from two experts who have studied evaluation of educational software, an explanation of their criterion, and my pedagogical support the use of the software.

The topic of my study is volume of rectangular solids which is studied in fifth grade mathematics. I have included the teaching sequence for your reference.

Briefly, the instructional sequence starts with students cutting a rectangle from a sheet of grid paper. They clip out the corners (square corner nibble), fold and tape sides to make a box. They fill the box with centimeter cubes, remove and count to determine volume. Then they draw the box, record the width, length and height and volume, and draw the volume amount on a bar graph. This completes the concrete phase. The semi-concrete phase consists of drawing and recording information without actually constructing the box. The abstract phase is solving problems when given box dimensions, either in verbal or in pictorial form.

The software is used at the end of the sequence to reinforce the learning. Advantages include quick response to student input of dimensions and corner nibble, allowing the student to predict volume without cutting, etc. and check his answer quickly. He/she can also see the relationship between shape (tallest, flattest), size and volume of the boxes formed. I am enclosing the pedagogical support for the software as stated in my proposal.

Will you please read the criterion information, run the software and check the appropriate information on the check list. Then please respond to the validation page and return everything to me at 213 Reid Hall within the next two weeks.

Thanks for taking time to complete this for me.

Sincerely,

P.S. To run the program, boot in an Apple IIe computer, move cursor to "P4" and type "start" when the cursor blinks in the command center below the little turtle.
From my Proposal:

This researcher's opinion that the P4 software simulation is an effective instructional tool will be supported or rejected with the opinions of two software experts.

Following is a criterion list of educational support for use of the software:

1) the content of the software is relevant to the instructional topic, accurate and clear (Alesandrini, 1985; Mezzina, 1985).

2) only needed information is displayed on the screen, thus focusing student attention in pertinent information (Alesandrini, 1985; Flake, 1982).

3) the visual display is graphically and textually pleasant (Mezzina, 1985).

4) Instructional computer software should be formatted in a manner familiar to the student (Alesandrini, 1985; Blaol and Erickson, 1985). The drawing, cutting and folding via computer occurs in the same order as the student activity in the teaching sequence (two experts).

5) the student interacts with the simulation so he/she is a participator, not an observer (Chambers and Sprecher, 1980, 1983; Dugdale & Kibbey, 1984; Henderson et al., 1983; Keuper, 1985; Mezzina, 1985; Papert, 1980).

6) the computer simulation replicates the concrete (box making) and semi-concrete (paper and pencil) activities quickly so the students can immediately compare results, allowing them to concentrate on those results and not get lost in the manipulation and calculation action of the actual activities (Dugdale & Kibbey, 1984; Thomas, Anderson & Allinger, 1987).

7) students are USING mathematics to solve problems, not DOING mathematics as a rote task (Dugdale & Kibbey, 1984).

8) Many ideas are expressed and understood better through pictures, charts and diagrams (Bork, 1980; Marks, 1985; National Science Board Commission of Pre-College Education in Mathematics, Science and Technology, 1983). The chart and drawings produced by students and computer simulation in the treatment groups represent the concept of volume of rectangular solids (two educational computer experts and mathematics experts).
VALIDATION OF A COMPUTER SIMULATION
ON THE SUBJECT OF
VOLUME OF RECTANGULAR SOLIDS

FOR
RUTH JOHNSON

I have tested the software to be used in Ruth Johnson's research. As defined by criterion from the work of Mezzini and Alesandrini, I find software to be:

☑ Valid.

☐ Not Valid, for the reasons listed below.

Signed

Reasons:

Randy Knuth

Title Director, Institute Computing Lab MSU

Date 5 March 88
Criterion Check List

P4: Program used in Ruth Johnson’s study: Volume of Rectangular Solids

Please check each characteristic below that applies to the software:

Part one: use of graphics (check as many as apply)

1. no graphics

2. graphics used at least 1/4 of the time

3. graphics used at least 1/2 of the time

4. graphics used more than 1/2 of the time

Part two: relevance of graphics

5. Graphics used to decorate

6. Graphics used to motivate or interest

7. Graphics used to direct attention to the content

8. Graphics used to present the content

9. Graphics used to reward correct answers

10. Graphics used to reward wrong answers

Part three: categories of graphics *(see below)*

11. Representational graphics used to present content

12. Representational graphics used to give feedback

13. Analogical graphics used to present content

14. Analogical graphics used to give feedback

15. Abstract graphics used to present content

16. Abstract graphics used to give feedback

Add one point for each item checked; deduct one if you checked 1, 5 or 10. Circle one:

Poor (4 or less)  Good (5-9)  Excellent (10-14)

* -- Representational graphics portray information directly or indirectly. Tangible objects of concepts may be represented; abstract concepts may be represented by showing their effects, results. They may include line drawings and symbols.

* -- Analogical graphics convey a concept or topic by showing something else, implying a similarity. This graphic is based on the assumption that new information will be better learned and remembered if it is related to prior information. Pictures are an example.

* -- Abstract graphics may not actually look like the things they represent but are related conceptually or abstractly. They include graphs, flowcharts, diagrams.
204

The software supports the instruction as follows: (check one)

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<th>Somewhat</th>
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<td>18. The display is pleasing and effective.</td>
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<tr>
<td>19. The content is accurate.</td>
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Sources:
VALIDATION OF A COMPUTER SIMULATION
ON THE SUBJECT OF
VOLUME OF RECTANGULAR SOLIDS

FOR
RUTH JOHNSON

I have tested the software to be used in Ruth Johnson's research. As defined by criterion from the work of Mezzini and Alesandrini, I find software to be:

✓ Valid.

☐ Not Valid, for the reasons listed below.

Signed ________________________________  Title ________________________________

Date ____________

Reasons:  

[Blank space for reasons]
Criterion Check List

P4: Program used in Ruth Johnson's study: Volume of Rectangular Solids

Please check each characteristic below that applies to the software:

Part one: use of graphics (check as many as apply)

1. no graphics
2. graphics used at least 1/4 of the time
3. graphics used at least 1/2 of the time
4. graphics used more than 1/2 of the time

Part two: relevance of graphics

5. Graphics used to decorate
6. Graphics used to motivate or interest
7. Graphics used to direct attention to the content
8. Graphics used to present the content
9. Graphics used to reward correct answers
10. Graphics used to reward wrong answers

Part three: categories of graphics *(see below)

11. Representational graphics used to present content
12. Representational graphics used to give feedback
13. Analogical graphics used to present content
14. Analogical graphics used to give feedback
15. Abstract graphics used to present content
16. Abstract graphics used to give feedback

Add one point for each item checked; deduct one if you checked 1, 5 or 10. Circle one:

Poor (4 or less)  Good (5-9)  Excellent (10-14)

* -- Representational graphics portray information directly or indirectly. Tangible objects of concepts may be represented; abstract concepts may be represented by showing their effects, results. They may include line drawings and symbols.

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</table>

Sources:
DEVELOPMENTAL LEVEL TEST
August 8, 1988

TO: Doctoral Committee for Mrs. Ruth Johnson
FROM: Richard V. Copeland

Mrs. Johnson asked me to edit a diagnostic test she has designed which is based on the research of Jean Piaget. I was glad to do this and edited a second draft as well. I think this test has face validity, and the study can be a valuable addition to the research literature in the field of cognitive development of children.

Richard V. Copeland
APPENDIX C

DESCRIPTION OF SCHOOLS
### DESCRIPTION OF CLASSROOMS

<table>
<thead>
<tr>
<th>School (Site)/District/Class</th>
<th>Size Class and Grades</th>
<th>Class Size</th>
<th>Exp</th>
<th>Tmt</th>
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<tbody>
<tr>
<td>Central (Helena)/I</td>
<td>312 (K-6)</td>
<td>17</td>
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<tr>
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<td>16</td>
<td>21</td>
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<tr>
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<tr>
<td>(Gallatin County)/III</td>
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<td>1</td>
<td>II</td>
</tr>
</tbody>
</table>

Parkview is a departmentalized school. One teacher is responsible for mathematics instruction in all five 5th grade classes.

Size of school district defined by School Law (School Laws of Montana, 1987)

Exp = Years of teaching experience by individual classroom teacher

Tmt = Treatment placement
APPENDIX D

TREATMENTS: TEACHER INSTRUCTIONS AND STUDENT MATERIALS
TREATMENT I
TEACHER INSTRUCTIONS TO STUDENTS

DAY #1

1. Before starting, discuss with the class the following concepts and terminology: square, rectangle, area (as two dimensions; how to compute), length (as one dimension). Have students draw a line; then have them draw a line twice as long. Discuss length as one dimension (length). Have students draw a 1 inch by 2 inch rectangle; then draw a rectangle with twice the area. Discuss area as two dimensions (length and width).

Materials Needed: pencil, paper scissors, masking or scotch tape, 15 one-centimeter cubes per student (they can share), 1 cm by 1 cm grid paper and Student Work Sheet A

II. Construction of box with Rectangular Corner Cut

Objective: 1) to introduce box making and 2) show students why they must cut out a square corner rather than a rectangular corner

A. Distribute grid paper to each student along with pen or pencil, scissors, tape, 1 cm cubes, grid paper and Student Work Sheet A.

B. Have each student prepare a rectangle from grid paper by cutting a 5 cm by 7 cm rectangle. If necessary, show students how to count, mark and cut for the given dimensions.

Then have students shade a 1 cm X 2 cm rectangle at each corner of the cut rectangle (see figure 1). Make certain the students understand that the small squares are 1 cm by 1 cm or 1 cm^2.

C. Instruct students to:
1. cut out the 1 cm X 2 cm shaded corners (see figure 2).
2. fold to form a box and tape the corners.

D. Discuss with the class the following questions:
1. What is wrong with the box? (Ans. two sides are too short/long)
2. If the box were plastic, could you fill it to the top with water? (Ans. no, since two of the sides are shorter than the other two sides, water would spill out)
3. How could the corners have been cut so all the sides were the same height? (Ans. by making square corner cuts)

III. Boxes are constructed from 5 cm X 7 cm rectangles and the volumes of each of those boxes are counted by filling them with 1 cm cubes.

Objective: 1) to show students that boxes of different shapes and sizes will vary in volume, 2) that too large a corner cut will not allow a box to be constructed and 3) introduce record keeping

Box #1: Ask students to:
1. cut a 5 cm X 7 cm rectangle from the grid paper.
2. cut out a 1 cm X 1 cm square from each corner of the rectangle.
3. fold and tape the paper (as done previously) to form a box.
4. draw a sketch of the box on Student Work Sheet A and write the box dimensions along the sides of the box (see figure 3). NOTE: students may need a review of how to draw a box.
5. fill up the box with cubes so it is full but the cubes do not stick out above the top.
6. remove cubes from the box. Line the cubes in a column laid flat on the Cube Count Sheet A. Count the cubes and write the number at the top of the column. Trace around the cube column and remove the cubes. (see figure 4).
7. write the number of cubes that filled the box on the chart at the bottom of Student Work Sheet A.

Box #2: Instruct students to:
1. cut a 5 cm X 7 cm rectangle from the grid paper.
2. cut out a 2 cm X 2 cm square from each corner of the rectangle.
3. continue as above for Box 1, starting with 4.

Box #3:
1. Instruct students to cut a 5 cm X 7 cm rectangle from the grid paper.
2. Ask students to cut a 3 cm X 3 cm square at each corner.
3. Ask if there is a problem? What is wrong? Have students explain what is wrong in one sentence.
Boxes are constructed from 7 cm X 9 cm rectangles, the volumes of each of those boxes are counted by filling them with 1 cm cubes and the results are recorded.

Objective: to 1) transfer information to a chart, 2) create a method of measuring and 3) compare volume of boxes of different shapes and sizes.

Materials Needed: 1 cm X 1 cm grid paper, pencil, scissors, 35 centimeter cubes per student (may share), tape, Student Work Sheet B, and Cube Count Sheet A

Distribute grid paper, 1 cm cubes, tape, Student Work Sheet B, and Cube Count Sheet A

A. Box #1
1. write 7 and 9 at the top of the Cube Count Sheet A where it shows grid size.
2. cut a 7 cm X 9 cm rectangle from the grid paper.
3. cut out a 1 cm X 1 cm square at each corner of the rectangle.
4. fold and tape the paper (as done previously) to form a box.
5. draw a sketch of the box on Student Work Sheet B, and write dimensions along the edges as was done in fig. 3.
6. fill up the box with cubes so it is full but the cubes do not stick out above the top.
7. remove cubes from the box and count them. Instruct students to line cubes in a column on Cube Sheets A. There will be too many cubes (35) for a single column. Ask students for a solution to lining up the cubes and drawing a line around the column (possible solutions: doubling up and making a double column; adding paper for more height; stacking 2 cubes deep, etc). The creativity of developing their own bar graph solution will provide a sense of ownership for the activity.
8. Write the number of cubes that filled the box on the chart at the bottom of Student Work Sheet B.

B. Box #2
2 - 6. Continue as for Box #1, but with 2 cm X 2 cm corner cuts.
7. have students create a column consistent with their solution to #7 in Box #1
8. continue as above

C. Box #3
2 - 8. Continue as for Box #2, but with 3 cm X 3 cm corner cuts.

D. Box #4:
1. Cut a 4 cm X 4 cm square at each corner.
2. Ask if there is a problem? What is wrong? Have students explain what is wrong in one sentence.

NOTE: as students work, walk around the room to make sure dimensions are correct; help as needed with drawings.
DAY #3

WORK IN GROUPS OF TWO OR THREE STUDENTS. HAVE STUDENTS SHARE TASKS OF MAKING, SKETCHING AND RECORDING RESULTS ON STUDENT WORK SHEET C.

Objectives: to 1) transfer information to the chart, 2) compare shapes to find largest volume and 3) introduce the word and the formula for "volume".

Materials Needed: 1 cm X 1 cm grid paper, Cube Count Sheet A, pen or pencil, scissors, 40 centimeter cubes per group, masking tape, Student Work Sheet C

I. Boxes are constructed from 8 X 8 grid and the results are recorded.

Distribute grid paper, cubes, tape Cube Count Sheet A and Student Work Sheet C

A. Box #1: Ask students to:
1. write 8 and 8 at the top of Cube Count Sheet B.
2. cut a 8 cm X 8 cm square from the grid paper.
3. cut out a 1 cm X 1 cm square at each corner of the rectangle.
4. fold and tape to form a box (as previously done).
5. draw a sketch of the box on Student Work Sheet C, and write dimensions along the edges (as previously shown in fig. 3).
6. fill up the box with cubes so the box is full but the cubes do not stick out of the top.
7. remove cubes from the box and count them. Continue as in Day #2.
8. write the number of cubes that filled the box on the chart at the bottom of Student Answer Sheet C for Groups.

Tell students we will now call the total number of cubes the VOLUME of the box.

B. BOX #2
Continue instructions 2 though 8 as above for Box #1, but with 2 cm X 2 cm corner cuts

C. Box #3
Continue instructions 2 through 8 as above for Box #1 but with 3 cm X 3 cm corner cuts

II. Have students place the three boxes side by side to compare sizes and shapes.

Ask them the following questions:
1. Which box has the most volume? (Ans: box 1)
2. Which box is the shortest? (Ans: box 1)
3. Which box is the tallest? (Ans: box 3)
4. Does the shape of the box tell you anything about the volume? (Ans: the squarish (cuboid) box has the largest volume)

III. Ask how the VOLUME of the boxes could be determined without counting the cubes that filled the box. [allow discussion of possible answers. Ans: count the number of cubes along the length, width and height and multiply]
Ask if we could write an equation to tell volume \( V = L \times W \times H \).

Then write the equation on the board.
DAY #4

WORKING ALONE, HAVE STUDENTS CONTINUE AS ABOVE BUT JUST SHADE CORNERS (DO NOT CUT) AND PRETEND TO FOLD (DO NOT FOLD) AS DESCRIBED IN THE FOLLOWING SEQUENCE:

Objective: to 1) move from concrete to semi-concrete work with boxes; determine box of largest volume and 2) introduce the word "nibble".

Materials Needed: various sized cubes (sugar cubes, blocks, etc), pencil, 1 cm x 1 cm grid papers, Student Work Sheet D for Drawn Boxes and Cube Count Sheet B

I. Create drawings of boxes and determine volume

Distribute grid papers, Student Work Sheet D for Drawn Boxes and Cube Count Sheet B

A. "Create" 4 boxes from a 9 cm x 9 cm square having the students only shade in the corners and imagine the folded boxes (1-4 below).

Inform students that we will call the corner cuts "nibbles".

1. Box #1: 1 cm x 1 cm nibble: Instruct students to:
   a. cut a 9 cm x 9 cm square from the grid paper.
   b. SHADE a 1 cm x 1 cm square in each corner (do not cut out).
   c. remember we are calling the corner cut a "nibble".
   d. PRETEND to fold the square with the nibbled corners to make a box.
   e. sketch the box, on Student Work Sheet D as it would look and label the dimensions on the sketch.
   f. complete the Table on the bottom of Student Work Sheet D.

Inform students that we will now just use a sheet to represent that number or volume. One square on Cube Count Sheet B will take the place of one cube.

   g. using the number for volume they have shown in their chart on the Cube Count Sheet B, make a graph to show the volume of the box.

2. Box #2: 2 cm x 2 cm nibble: Ask student to:
   a. use the SAME 9 cm x 9 cm square.
   b. SHADE a 2 cm x 2 cm square in each corner (shade 2X2 over the old 1X1 shading).
   c. continue with d, e, f and g as above for Box 1.

3. Box #3: 3 cm x 3 cm nibble. Ask students to:
   a. use the SAME 9 cm x 9 cm square.
   b. SHADE a 3 cm x 3 cm square in each corner (shade 3X3 over the old 2X2 shading).
   c. continue d through g as described in Box #2.

4. Box #4: 4 cm x 4 cm nibble. Ask students to:
   a. Use the SAME 9 cm x 9 cm square:
b. SHADE a 4cm X 4cm square in each corner (shade 4X4 over the old 3X3 shading).

c. continue d through g as described in Box #2.

B. Again, have students compare shapes, sizes of boxes and volumes.

C. Ask the following questions:
1. What is the relationship between the size of the corner cut and the height of the box? (Ans: the height is the same as 1 side of the square corner cut)
2. In comparing two boxes, will the taller box always have the greater volume> (Ans: no......ask why)

D. Discuss how many possible boxes can be made from various grid sizes. For example:
1. from figures with a 3 cm or a 4 cm side only 1 cm square nibbles are possible. You may make as many boxes as desired by expanding the other side: 3 X 3 (1 box), 3 X 4 (1 box), 3 X 5 (1 box), etc.
2. from figures with a 5 cm or 6 cm side, 1 cm square and 2 cm square nibbles are possible and, therefore, a minimum of two boxes is possible. You may make as many boxes as desired by increasing the length of the other side: 5 X 5 (2 boxes), 5 X 6 (2 boxes), 5 X 7 (3 boxes), 5 X 8 (3 boxes), 5 X 9 (4 boxes), etc.
3. from figures with a 7 cm or 8 cm side, 1 and 2 and 3 cm square nibbles are possible and a minimum of three boxes is possible. And, by increasing the length of the box, a large number of boxes is possible.
The main idea for this extension is that by increasing dimensions of the original grid, you can increase the possible number of boxes. Students should be encouraged to note the pattern of increases (1 cm square nibble: 1 box minimum; 2 cm square nibble: 2 boxes, etc)

E. Ask students to define volume; write their definitions on the back of Worksheet D. Allow discussion of definitions. (emphasize the definition as the amount of space inside an object).

F. Ask students how they can determine the volume of a box (Ans: length X width X height). Original answers might include: counting the cubes, multiplying sides, etc.

G. Ask if a can of coke, a pyramid, a baseball (and other 3-D objects) have volume. Discuss their answers and how they arrived at their conclusions. Emphasize the three-dimensional aspect of these objects; the dimensions do not have to be straight lines but must not be flat (2 dimensions).

H. Show different cubes (1", sugar, any square object). Ask: could these units be used to measure the volume of a box? (Ans: yes, if box is big enough). Emphasize: a cubic unit is used to measure volume, regardless of the size of a cube.
IV. Oral Word Problems: ask the students to get a piece of scratch paper.

A. Draw a chart as shown in Figure 5 on the board and ask students to copy it.

<table>
<thead>
<tr>
<th>Object</th>
<th>L</th>
<th>W</th>
<th>H</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>6</td>
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</tbody>
</table>

B. Tell them you will read some story problems and they must fill in the chart. Fill in the first and second problems as you read them.

C. Problems:
1. I have a box that is 1 centimeter in length, 2 centimeters in width and 3 centimeters in height. What is the volume of my box? Instruct class to fill in the object name (box), length (1 cm), width (2 cm) and height (3 cm). Then figure the volume of the box \( V = L \times W \times H; 1 \times 2 \times 3 = 6 \text{ cubic centimeters, or cm}^3 \).

2. My jewelry box is 3 centimeters long, 3 centimeters wide, and 3 centimeters high. What is the volume of my jewelry box? (fill in object (box), length (3 cm), height (3 cm), width (3 cm) and volume (27 cubic centimeters or 27 cm³)).

3. I have a basket that is 20 inches long, 10 inches wide and 25 inches high. What is the volume of my basket? (5,000 cubic inches, or in³).

4. A rectangular solid is 15 inches long, 12 inches wide and 10 inches high. What is its volume (1,800 cubic inches or 1,800 in³).

5. My sandbox measures 10 feet long, 5 feet wide and 2 feet high. What is the volume of the sandbox? (100 cubic feet or 100 feet³).
DAY #5

Objective: to introduce the computer program P4.

Materials Needed: pencil, scissors, 1 cm X 1 cm grid paper, Student Work Sheet E for Computer Work

I. Review homework.

II. Students work alone. Distribute grid paper and Student Work Sheet E for Computer Work

A. For an imagined box ask students to:
   1. cut out a 11 cm X 12 cm rectangle from the grid paper.
   2. SHADE a 1 cm X 1 cm square in each corner of the rectangle (do not cut).
   3. PRETEND they have folded the paper to make a box.
   4. sketch the box as it would look on Student Work Sheet E, and label the dimensions.
   5. complete the chart and graph on Student Work Sheet E.

B. Tell students we will be working with the computer but they will have to refer to their Student Work Sheet E.

III. Using the computer

A. Boot the computer disk marked P4.

B. Proceed through the program as follows:
   1. Move the cursor down the table of contents until you find P4 and press [return] key.
   2. Wait until the blinking cursor reappears (see figure 6).
   3. Make sure the CAPS LOCK key is depressed.
   5. Directions will appear as shown in figure 7.

   ![Figure 7]

   7. Ask students if the square grid on the computer screen looks like their sketches.
   8. The computer will respond with
      ENTER SIZE OF SQUARE CORNER NIBBLE
   9. Remind students that the nibble is the corner cut out of the boxes they have been making. (NOTE: the program accepts a single digit input which represents a side of the square corner cut.)
I indicates a 1 cm X 1 cm square corner cut. Students will have to be told this is the size of the corner cut.

11. The screen will appear as in figure 8.

C. Ask students the following questions:
1. Does the computer shaded square look like your square?
2. Does the box look like yours (sort of, anyway)?
3. Is the volume the same?
4. Is the graph similar to your graph?

IV. Construct other boxes from the same grid size using the computer:

A. The computer will ask:
ADD ANOTHER FIGURE'S VOLUME? (Y/N/STOP)
The program will continue if you type:
- Y - will use same original rectangular dimensions and allow different nibbles
- N - will let you set new rectangular dimensions
The program will end if you type STOP. However, the last picture will remain on the screen.
Type: Y

B. The screen will display a new grid with the original dimensions and volume(s) and volume graph(s) of previous boxes.

C. The computer will ask:
ENTER SIDE OF SQUARE CORNER NIBBLE
Type: 2 and press [return] key.

D. Note new nibbled grid and box, added dimensions, volume and graph.
E. Continue steps A through D to construct boxes for nibbles of 3, 4 and 5.

F. Ask which dimension box has the largest volume?
F. Try entering 6 for a nibble. Ask students to explain what happens and why it happens. (Ans: the computer will not accept the number because the nibble is too large for the size of the grid)

NOTE: If you hit the return key without a number input at ENTER SIDE OF CORNER NIBBLE, the program will quit and "first doesn't like......" will appear. You will have to restart the program by typing START.

Note: The maximum grid size is 14 BY 14; minimum is 3 BY 3.
V. Make one more sequence of boxes for the students (unless you think they are ready to make their own).

A. Initiate a new sequence by typing M (or START).
B. Type 10 By 13 (for grid size).
C. BEFORE doing any computing, ask students what size nibble they think will produce the largest volume.
D. Type 1 (for nibble).
E. Continue for nibbles 2, 3, etc.
F. Ask if they correctly predicted the largest box and volume. Ask if they can guess what shape will produce the largest volume (flat, skinny or cube-like).
G. Ask the following questions:
1. For a square starting grid with fixed dimensions, what happens to the volume of the boxes as the size of the square corner cuts increases? (Ans: it increases)
2. Does the answer to the previous question depend on the size of the original grid? (Ans: yes, for larger grids the volume of boxes increases, then decreases as you nibble 1 sq. cm, 2 sq cm, 3 sq cm, etc.)
3. Does the answer to the previous question depend on the shape of the starting grid? (Ans: yes)

VI. Continue with students working on their own if time, availability of computers and ability level allow. An activity sheet is included for their use. Have students work in groups of two or three (RH: see Lappan research) on the activity sheet problems. Then check their answers on the computer, each group taking about 5 or 6 minutes time on the computer. If there is not enough time, perhaps they could use the computer during free times that rest of the day.

VII. Handout homework "Volume Problems", which is due the following school day.
Day #6

Objective: Review homework and administer quiz.

Materials: pencil, eraser, test, Activity Sheet #1

I. Have students check their homework answers on the computer in groups of 2 or 3 (about 20 minutes). Review the answers in a whole class situation and answer questions.

II. Administer quiz.
   1. Read general instructions aloud. Make sure each student has completed the cover page information.
   2. Administer test (about 20 minutes).

III. When all students have taken the test, review answers. If a student is absent, have him/her take the test upon return and review individually.

IV. Hand out Activity Sheet #1, which will be due in 1 week.
Week Two: One Week Later

I. Review Activity sheet #1; Watch for:
   1) three-dimensional notation (i.e. cm³, in³, etc)
   2) make sure everyone/group has a chance to work on the
computer for about 20 minutes per group per week
   3) are wrong answers just multiplication or counting
errors, or a lack of understanding the formula

II. Hand out Activity Sheet #2, which will be due in 1 week.

Week Three: One Week Later

I. Review Activity sheet #2; Watch for:
   1) three-dimension notation
   2) equal access to computers

II. Hand out Activity Sheet #3, which will be due in 1 week.

Week Four: One Week Later

I. Review Activity sheet #3

II. Administer final test (the same version as the pre-test).
STUDENT WORK SHEET A

5 cm BY 7 cm Rectangle

Number of Cubes

Box #1

Box #2

Box #3

1. Can we make a box?

2. Why? Explain your answer in one sentence.
STUDENT WORK SHEET B
7 cm BY 9 cm Rectangle

Your Sketches

Box #1

Number of cubes needed to fill Box #1 _______

Box #2

Volume of Box #2 _______cm²
(Number of cubes needed to fill Box #2)

Box #3

Volume of Box #2 _______cm²
(Number of cubes needed to fill Box #3)

Box #4

1. Is there something wrong with this box?

2. Explain your answer in the space below.
Name________________

CUBE COUNT SHEET A
(Cm size)

Grid Size: _____ cm BY _____ cm

Number of Cubes In My Boxes

Box #   Box#   Box #   Box #
STUDENT WORK SHEET C
FOR GROUPS
8 cm BY 8 cm Square

Your Sketches

Box #1

Box #2

Box #3

Table for 8 cm BY 8 cm Square

<table>
<thead>
<tr>
<th>Box</th>
<th>size of corner cut</th>
<th>box dimensions</th>
<th>Volume (number of cubes to fill the box)</th>
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<td></td>
</tr>
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STUDENT WORK SHEET D
FOR PRETENDED BOXES
9 cm BY 9 cm Square

Your Sketches

<table>
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<th>size of corner cut (nibble)</th>
<th>box dimensions</th>
<th>Volume</th>
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<tbody>
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<td>2 cm X 2 cm</td>
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<tr>
<td>3</td>
<td>3 cm X 3 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4 cm X 4 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
STUDENT WORK SHEET E
FOR COMPUTER WORK

11 cm BY 11 cm Rectangle

Cut out an 11 cm BY 11 cm rectangle. In the space below, make a sketch of the rectangle before any corner nibbles are shaded.

Shade a 1 cm BY 1 cm rectangle. Corner nibble on each corner. In the space below, sketch the rectangle with the shaded corners.

Now pretend you have folded and taped the paper to form a box. Draw a sketch of your "Pretend box". Label the dimensions of the box. Draw a column that would be made with the cubes you could have used to fill the box.

<table>
<thead>
<tr>
<th>My Sketch</th>
<th>dimensions of my sketch</th>
<th>Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Your teacher will run a computer program which will draw an 11 cm BY 11 cm rectangle.
1. Does your rectangle look like the computer rectangle?

Your teacher take out a 1 cm BY 1 cm nibble on the computer rectangle.
2. Does the computer shaded grid look like your rectangle with the shaded nibbles?

3. Is your sketch of the box similar to the computer's sketch?

4. Is the volume of your box the same as the computer's volume?

5. Is the computer graph similar to your graph?
4. Fill in the table below as you think the L, W, H and Vol would be for each box indicated by the nibbled corners. You can use your rectangle to shade the corners if you want to.

<table>
<thead>
<tr>
<th>size of nibble</th>
<th>L</th>
<th>W</th>
<th>H</th>
<th>Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm X 1 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 cm X 2 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 cm X 3 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 cm X 4 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 cm X 5 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Did your answers match the computer’s answers?

6. What would happen if you tried to nibble a 6 cm by 6 cm corner?

7. Why would that happen?

8. Sketch the box with the largest volume. Write the dimensions of the box on the sketch.
Name ____________________________

CUBE COUNT SHEET B

Grid Size: _____ cm BY _____ cm

Box #_ Box #_ Box #_ Box #_
BULLETIN BOARD IDEA

Materials Needed:
- tag board of other firm paper
- 1 cm square sugar cubes (Cocktail cubes)
- student made boxes
- computer printout of screen display

Note: to obtain a printout of the screen, connect the computer to a printer. Run the program with all nibbles from a grid size. Type STOP; when the cursor returns to the next line, type PRINTSCREEN

Glue student boxes from one grid size to the tag board and write the box dimensions along the side

Glue a column of sugar cubes to the tag board above the boxes, using one cube to represent a volume of one

Continue for all boxes constructed from an identical sized rectangle or square (i.e. from a 9 X 9 square, boxes with 1, 2, 3, and 4 cm square corner cuts may be added)

Attach a copy of the computer printout

Use original grid size in title (example: Boxes and computer program made from a 9 X 9 Square)
Homework:
Volume Problems

Volume = length \times width \times height
V = L \times W \times H
V = 5 \text{ units} \times 3 \text{ units} \times 2 \text{ units}
V = 30 \text{ cubic units (units}^3\text{)}

1 - 12. Find the volume of each box shown below.

1) \hspace{1cm} 2) \hspace{1cm} 3)
V = \hspace{1cm} V = \hspace{1cm} V =

4) \hspace{1cm} 5) \hspace{1cm} 6)
V = \hspace{1cm} V = \hspace{1cm} V =

7) \hspace{1cm} 8) \hspace{1cm} 9)
V = \hspace{1cm} V = \hspace{1cm} V =

10) \hspace{1cm} 11) \hspace{1cm} 12)
V = \hspace{1cm} V = \hspace{1cm} V =
13 - 24. Find the volume of each box described below.

<table>
<thead>
<tr>
<th></th>
<th>13)</th>
<th>14)</th>
<th>15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1 cm</td>
<td>3 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>W</td>
<td>4 cm</td>
<td>6 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>H</td>
<td>5 cm</td>
<td>2 cm</td>
<td>1 cm</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>16)</th>
<th>17)</th>
<th>18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>4 cm</td>
<td>13 cm</td>
<td>25 cm</td>
</tr>
<tr>
<td>W</td>
<td>5 cm</td>
<td>7 cm</td>
<td>4 cm</td>
</tr>
<tr>
<td>H</td>
<td>3 cm</td>
<td>10 cm</td>
<td>9 cm</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>19)</th>
<th>20)</th>
<th>21)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1 cm</td>
<td>3 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>W</td>
<td>4 cm</td>
<td>6 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>H</td>
<td>5 cm</td>
<td>2 cm</td>
<td>1 cm</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>22)</th>
<th>23)</th>
<th>24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>1 cm</td>
<td>3 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>W</td>
<td>4 cm</td>
<td>6 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>H</td>
<td>5 cm</td>
<td>2 cm</td>
<td>1 cm</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

25 - 28. Find the answers to the story problems.

25. A box is 3 inches long, 2 inches wide and 4 inches high. Find the volume?

26. I have a suitcase that is 1 foot long, 3 feet wide and 8 feet high. How many cubic feet of clothes can I put in the suitcase?

27. A safe is 4 feet wide, 6 feet high and 9 feet long. What is the volume of the safe?

28. A swimming pool measures 30 meters long, 15 meters wide and 6 meters deep. How much water will it take to fill the pool?
ACTIVITY SHEET 1

Name ____________

To find the volume of a rectangular solid, multiply the length by the width by the height.

Volume = L \times W \times H

\[ V = 4 \text{ units} \times 2 \text{ units} \times 3 \text{ units} \]

\[ V = 24 \text{ cubic units (or 24 units}^3) \]

1 - 6. Find the volume of the rectangular solids shown below. Write the volume in cubic centimeters (cm$^3$).

1. \[
\begin{array}{c}
6 \text{ cm} \\
3 \text{ cm} \\
2 \text{ cm}
\end{array}
\] \[ (6 \times 3) \times 2 = \] __________  
volume = __________ cm$^3$

2. \[
\begin{array}{c}
4 \text{ cm} \\
3 \text{ cm} \\
3 \text{ cm}
\end{array}
\] \[ (4 \times 3) \times 3 = \] __________  
volume = __________ cm$^3$

3. \[
\begin{array}{c}
5 \text{ cm} \\
3 \text{ cm} \\
3 \text{ cm}
\end{array}
\] \[ (5 \times 3) \times 3 = \] __________  
volume = __________ cm$^3$

4. \[
\begin{array}{c}
2 \text{ cm} \\
3 \text{ cm} \\
2 \text{ cm}
\end{array}
\] \[ (2 \times 2) \times 3 = \] __________  
volume = __________ cm$^3$

5. \[
\begin{array}{c}
4 \text{ cm} \\
6 \text{ cm} \\
3 \text{ cm}
\end{array}
\] \[ (6 \times 3) \times 4 = \] __________  
volume = __________ cm$^3$

6. \[
\begin{array}{c}
7 \text{ cm} \\
4 \text{ cm} \\
3 \text{ cm}
\end{array}
\] \[ (4 \times 3) \times 7 = \] __________  
volume = __________ cm$^3$

7 - 16. Find the volume of the rectangular solids whose dimensions are listed below. Write the answer in cubic inches (in$^3$).

7. \[ L = 5 \text{ inches} \quad W = 2 \text{ inches} \quad H = 5 \text{ inches} \]
Vol = __________

8. \[ L = 6 \text{ inches} \quad W = 4 \text{ inches} \quad H = 10 \text{ inches} \]
Vol = __________

9. \[ L = 10 \text{ inches} \quad W = 3 \text{ inches} \quad H = 10 \text{ inches} \]
Vol = __________

10. \[ L = 8 \text{ inches} \quad W = 3 \text{ inches} \quad H = 7 \text{ inches} \]
Vol = __________

11. \[ L = 9 \text{ inches} \quad W = 3 \text{ inches} \quad H = 2 \text{ inches} \]
Vol = __________

12. \[ L = 3 \text{ inches} \quad W = 3 \text{ inches} \quad H = 4 \text{ inches} \]
Vol = __________
13. A dog house with a flat roof measures 4 feet long, 4 feet wide and 5 feet high. What is the volume of the dog house?

____________ cubic feet

14. A rectangular solid is three inches in length, five inches in width and 10 inches in height. What is the volume of the solid?

____________ cubic inches

15. A stack of gold bars is 2 meters long, 3 meters wide and 1 meter high. What is the volume of the stack?

____________ cubic meters

16. How much volume is in a box that measures 10 centimeters long, 10 centimeters wide and 20 centimeters high?

____________ cubic centimeters

17 - 19). Complete the charts below. First shade the corner nibble in the grid. Next draw the box that would be made and label the sides. Then find the volume. See the example in 18. When you are finished, run the program, P4, and type in the grid sizes and nibbles. Check the computer screen to see if your answers are correct.

18. Grid Size: 7 BY 5

<table>
<thead>
<tr>
<th>nibble</th>
<th>L</th>
<th>W</th>
<th>H</th>
<th>Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 x 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw your boxes in the space below

19. Grid Size: 10 BY 7

<table>
<thead>
<tr>
<th>nibble</th>
<th>L</th>
<th>W</th>
<th>H</th>
<th>Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 x 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 x 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw your boxes in the space below
21. Run the computer program P4, to see how many different boxes you can make from a 13 BY 13 grid size? In the space below, make a chart and write down the nibbles, length, width, height and volume of each box.
ACTIVITY SHEET 2

Name ________________________________

1) Use the pictures of the cubes to complete the chart below.

<table>
<thead>
<tr>
<th>Cube</th>
<th>length</th>
<th>width</th>
<th>height</th>
<th>number of cubes</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2 - 7. Volume Puzzle. What gets wetter the more its dries?
Break the code to find the answer. Match the answers to the letter in the code strip.

2) \[ L = 2 \text{ cm} \] \[ W = 2 \text{ cm} \] \[ H = 3 \text{ cm} \] \[ \text{Vol} = \]  
3) \[ L = 12 \text{ cm} \] \[ W = 5 \text{ cm} \] \[ H = 7 \text{ cm} \] \[ \text{Vol} = \]  
4) \[ L = 6 \text{ cm} \] \[ W = 6 \text{ cm} \] \[ H = 11 \text{ cm} \] \[ \text{Vol} = \]  

Code Strip

\[ \text{A} = 12 \text{ cm}^3 \] \[ \text{E} = 27 \text{ cm}^3 \] \[ \text{H} = 36 \text{ cm}^3 \] \[ \text{L} = 240 \text{ cm}^3 \] \[ \text{O} = 396 \text{ cm}^3 \] \[ \text{T} = 420 \text{ cm}^3 \] \[ \text{W} = 640 \text{ cm}^3 \]

5) \[ L = 6 \text{ cm} \] \[ W = 3 \text{ cm} \] \[ H = 10 \text{ cm} \] \[ \text{Vol} = \]  
6) \[ L = 3 \text{ cm} \] \[ W = 3 \text{ cm} \] \[ H = 3 \text{ cm} \] \[ \text{Vol} = \]  
7) \[ L = 4 \text{ cm} \] \[ W = 5 \text{ cm} \] \[ H = 8 \text{ cm} \] \[ \text{Vol} = \]  

Answer: \[ 2 \] \[ 3 \] \[ 4 \] \[ 5 \] \[ 6 \] \[ 7 \]
Work in a group of 2 or 3 students for the next questions. Get a piece of grid paper from your teacher. You will also need a pencil and scissors. Cut out the grid sizes and shade the corner nibbles. Sketch the boxes you would make with the grid paper and label the dimensions. Then answer the questions. You do not have to make all the sketches yourself. Have different people in your group do different boxes.

8) What is the volume of the tiniest box you can make?

<table>
<thead>
<tr>
<th>Volume</th>
<th>Grid Size</th>
<th>Nibble</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9) How many different boxes can you make with a 13 X 13 grid? List the nibbles, lengths, widths, heights and volumes in the blanks below. You may not need all of the rows below.

<table>
<thead>
<tr>
<th>Grid Size: 13 BY 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nibble</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Total number of boxes we made

10 - 12. For each grid listed below, which nibble makes the box with the most volume (circle one):

<table>
<thead>
<tr>
<th>Nibbles</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. with a 6 by 10 grid</td>
</tr>
<tr>
<td>11. with an 10 by 14 grid</td>
</tr>
<tr>
<td>12. with a 12 by 14 grid</td>
</tr>
</tbody>
</table>
ACTIVITY SHEET 3

1 - 6. Find the volume of the boxes shown below.

1) 

Volume _______

2) 

Volume _______

3) 

Volume _______

4) 

Volume _______

5) 

Volume _______

6) 

Volume _______

7 - 15. Find the volume of the rectangular solids described below:

<table>
<thead>
<tr>
<th>length</th>
<th>width</th>
<th>height</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>7)</td>
<td>3 meters</td>
<td>6 meters</td>
<td>2 meters</td>
</tr>
<tr>
<td>8)</td>
<td>4 cm</td>
<td>3 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>9)</td>
<td>2 meters</td>
<td>3 meters</td>
<td>7 meters</td>
</tr>
<tr>
<td>10)</td>
<td>5 cm</td>
<td>8 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>11)</td>
<td>6 inches</td>
<td>2 inches</td>
<td>2 inches</td>
</tr>
<tr>
<td>12)</td>
<td>1 feet</td>
<td>2 feet</td>
<td>3 feet</td>
</tr>
<tr>
<td>13)</td>
<td>6 feet</td>
<td>7 feet</td>
<td>10 feet</td>
</tr>
<tr>
<td>14)</td>
<td>8 inches</td>
<td>6 inches</td>
<td>4 inches</td>
</tr>
<tr>
<td>15)</td>
<td>4 meters</td>
<td>7 meters</td>
<td>3 meters</td>
</tr>
</tbody>
</table>
16 - 19. Solve the word problems.

16) What is the volume of a box that is 5 inches long, 10 inches wide and 15 inches high? __________

17) A fish tank is 45 centimeters long, 30 centimeters wide, and 35 centimeters high. What is the volume of the fish tank? __________

18) What is the volume of a cube that is 6 inches by 6 inches by 6 inches? __________

19) A brick wall was 12 feet long, 2 feet wide and 9 feet high. What was the volume of the wall? __________

20 - 23. For the next problems, work with a group of 2 or 3 students.
Use the computer program, P4, to answer the questions. Try different grid sizes and different corner nibbles to find the answers.

20) What is the box with the most volume? volume __________
grid size __________
nibble __________

21) How many different boxes can you make with a 10 BY 12 grid? Fill in the chart with your findings.

<table>
<thead>
<tr>
<th>nibble</th>
<th>length</th>
<th>width</th>
<th>height</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>nibble</td>
<td>length</td>
<td>width</td>
<td>height</td>
<td>volume</td>
</tr>
<tr>
<td>nibble</td>
<td>length</td>
<td>width</td>
<td>height</td>
<td>volume</td>
</tr>
<tr>
<td>nibble</td>
<td>length</td>
<td>width</td>
<td>height</td>
<td>volume</td>
</tr>
</tbody>
</table>

Total boxes: __________

22) Which box has the most volume? circle A or B or C
A. a box made from a 10 BY 7 grid with a 3 corner nibble
B. a box made from a 5 BY 8 grid with a 2 corner nibble
C. a box made from a 3 BY 10 grid with a 1 corner nibble

23) Which box has the least volume? circle A or B or C
A. a box made from a 7 BY 7 grid with a 3 corner nibble
B. a box made from a 8 BY 8 grid with a 2 corner nibble
C. a box made from a 3 BY 3 grid with a 1 corner nibble
TEACHER INSTRUCTIONS TO STUDENTS

DAY #1

I. Before starting, discuss with the class the following concepts and terminology: square, rectangle, area (as two dimensions; how to compute), length (as one dimension). Have students draw a line; then have them draw a line twice as long. Discuss length as one dimension (length). Have students draw a 1" line; then draw a rectangle with twice the area. Discuss area as two dimensions (length and width).

Materials Needed: pencil, paper scissors, masking or scotch tape, 15 one centimeter cubes per student (they can share), 1 cm by 1 cm grid paper and Student Work Sheet A

II. Construction of box with Rectangular Corner Cut

Objective: to introduce box making and show students why they must cut out a square corner rather than a rectangular corner

A. Distribute grid paper to each student along with pen or pencil, scissors, tape, cubes, grid paper and Student Work Sheet A.

B. Have each student prepare a rectangle from grid paper by cutting a 5 cm by 7 cm rectangle. If necessary, show students how to count, mark and cut for the given dimensions.

Then have students shade a 1 cm X 2 cm rectangle at each corner of the cut rectangle (see figure 1).

Make certain the students understand that the small squares are 1 cm by 1 cm or 1 cm^2

C. Instruct students to:
   1. cut out the 1 cm X 2 cm shaded corners (see figure 2).
   2. fold to form a box and tape the corners.

D. Discuss with the class the following questions:
   1. What is wrong with the box? (Ans. two sides are to short/long)
   2. If the box were plastic, could you fill it to the top with water? (Ans. no, since two of the sides are shorter than the other two sides, water would spill out)
3. How could the corners have been cut so all the sides were the same height? (Ans. by making square corner cuts)

III. Boxes are constructed from 5 cm X 7 cm rectangles and the volumes of each of those boxes are counted by filling them with 1 cm cubes.

Objective: 1) to show students that boxes of different shapes and sizes will vary in volume, 2) that too large a corner cut will not allow a box to be constructed and 3) introduce record keeping.

Box #1: Ask students to:
1. cut a 5 cm X 7 cm rectangle from the grid paper.
2. cut out a 1 cm X 1 cm square from each corner of the rectangle.
3. fold and tape the paper (as done previously) to form a box.
4. draw a sketch of the box on Student Work Sheet A and write the box dimensions along the sides of the box (see figure 3). NOTE: students may need a review of how to draw a box.
5. fill up the box with cubes so it is full but the cubes do not stick out above the top.
6. remove cubes from the box. Line the cubes in a column laid flat on the Cube Count Sheet A. Count the cubes and write the number at the top of the column. Trace around the cube column and remove the cubes. (see figure 4).
7. write the number of cubes that filled the box on the chart at the bottom of Student Work Sheet A.

Box #2: Instruct students to:
1. cut a 5 cm X 7 cm rectangle from the grid paper.
2. cut out a 2 cm X 2 cm square from each corner of the rectangle.
3. continue as above for Box 1, starting with 4.

Box #3:
1. Instruct students to cut a 5 cm X 7 cm rectangle from the grid paper.
2. Ask students to cut a 3 cm X 3 cm square at each corner.
3. Ask if there is a problem? What is wrong? Have students explain what is wrong in one sentence.
DAY #2

Boxes are constructed from 7 cm X 9 cm rectangles, the volumes of each of those boxes are counted by filling them with 1 cm cubes and the results are recorded.

Objective: 1) to transfer information to a chart, create a method of measuring and 2) compare volume of boxes of different shapes and sizes.

Materials Needed: 1 cm X 1 cm grid paper, pencil, scissors, 35 centimeter cubes per student (may share), tape, Student Work Sheet B, and Cube Count Sheet A

Distribute grid paper, cubes, tape, Student Work Sheet B, and Cube Count Sheet A

A. Box #1
   1. write 7 and 9 at the top of the Cube Count Sheet A where it shows grid size.
   2. cut a 7 cm X 9 cm rectangle from the grid paper.
   3. cut out a 1 cm X 1 cm square at each corner of the rectangle.
   4. fold and tape the paper (as done previously) to form a box.
   5. draw a sketch of the box on Student Work Sheet B, and write dimensions along the edges as was done in Fig. 3.
   6. fill up the box with cubes so it is full but the cubes do not stick out above the top.
   7. remove cubes from the box and count them. Instruct students to line cubes in a column on Cube Sheets A. There will be too many cubes (35) for a single column. Ask students for a solution to lining up the cubes and drawing a line around the column (possible solutions: doubling up and making a double column; adding paper for more height; stacking 2 cubes deep, etc). The creativity of developing their own bar graph solution will provide a sense of ownership for the activity.
   8. Write the number of cubes that filled the box on the chart at the bottom of Student Work Sheet B.

B. Box #2
   2 - 6. continue as for Box #1, but with 2 cm X 2 cm corner cuts.

C. Box #3
   2 - 8. Continue as for Box #2, but with 3 cm X 3 cm corner cuts.

D. Box #4:
   1. Cut a 4 cm X 4 cm square at each corner.
   2. Ask if there is a problem? What is wrong? Have students explain what is wrong in one sentence.

NOTE: as students work, walk around the room to make sure dimensions are correct; help as needed with drawings.
DAY #3

WORK IN GROUPS OF TWO OR THREE STUDENTS. HAVE STUDENTS SHARE TASKS OF MAKING, SKETCHING AND RECORDING RESULTS ON STUDENT WORK SHEET C.

Objectives: to 1) transfer information to the chart, 2) compare shapes to find largest volume and 3) introduce the word and the formula for "volume".

Materials Needed: 1 cm X 1 cm grid paper, Cube Count Sheet A, pen or pencil, scissors, 40 centimeter cubes per group, masking tape, Student Work Sheet C

I. Boxes are constructed from 8 X 8 grid and the results are recorded.

Distribute grid paper, cubes, tape Cube Count Sheet A and Student Work Sheet C

A. Box #1: Ask students to:
1. write 8 and 8 at the top of Cube Count Sheet B.
2. cut a 8 cm X 8 cm square from the grid paper.
3. cut out a 1 cm X 1 cm square at each corner of the rectangle.
4. fold and tape to form a box (as previously done).
5. draw a sketch of the box on Student Work Sheet C, and write dimensions along the edges (as previously shown in fig. 3).
6. fill up the box with cubes so the box is full but the cubes do not stick out of the top.
7. remove cubes from the box and count them. Continue as in Day #2.
8. write the number of cubes that filled the box on the chart at the bottom of Student Answer Sheet C for Groups.

Tell students we will now call the total number of cubes the VOLUME of the box.

B. BOX #2
Continue instructions 2 though 8 as above for Box #1, but with 2 cm X 2 cm corner cuts

C. Box #3
Continue instructions 2 through 8 as above for Box #1 but with 3 cm X 3 cm corner cuts

II. Have students place the three boxes side by side to compare sizes and shapes.

Ask them the following questions:
1. Which box has the most volume? (Ans: box 1)
2. Which box is the shortest? (Ans: box 1)
3. Which box is the tallest? (Ans: box 3)
4. Does the shape of the box tell you anything about the volume? (Ans: the squarish (cuboid) box has the largest volume)

III. Ask how the VOLUME of the boxes could be determined without counting the cubes that filled the box. [allow discussion of possible answers. Ans: count the number of cubes along the length, width and height and multiply]

Ask if we could write an equation to tell volume \[ V = L \times W \times H \].
Then write the equation on the board.
**DAY #4**

**Working alone,** have students continue as above but just **shade corners** (do not cut) and pretend to fold (do not fold) **as described in the following sequence:**

Objectives: 1) to move from concrete to semi-concrete work with boxes; determine box of largest volume and 2) to introduce the word "nibble".

Materials Needed: various sized cubes (sugar cubes, blocks, etc.), pencil, 1 cm X 1 cm grid papers, Student Work Sheet D for Drawn Boxes and Cube Count Sheet B

---

1. **Create Drawings** of boxes and determine volume

Distribute grid papers, Student Work Sheet D for Drawn Boxes and Cube Count Sheet B

A. "Create" 4 boxes from a 9 cm X 9 cm square having the students only shade in the corners and imagine the folded boxes (1-4 below).

1. **Box #1:** 1 cm X 1 cm nibble: Instruct students to:
   a. cut a 9 cm X 9 cm square from the grid paper.
   b. SHADE a 1 cm X 1 cm square in each corner (do not cut out).
   c. remember we are calling the corner cut a "nibble".
   d. PRETEND to fold the rectangle with the nibbled corners to make a box.
   e. sketch the box, on Student Work Sheet D as it would look and label the dimensions on the sketch.
   f. complete the Table on the bottom of the Student Work Sheet D. To determine the volume, ask students how many cubes they think will fill the box and how they could figure it out (i.e., length X height X width). Inform students that we will now just use a sheet to represent that number, or volume. One square cm Cube Count Sheet B will take the place of one cube.
   g. using the number for volume they have shown in their chart on the Cube Count Sheet B, make a graph to show the volume of the box.

2. **Box #2:** 2 cm X 2 cm nibble: Ask student to:
   a. use the SAME 9 cm X 9 cm square.
   b. SHADE a 2 cm X 2 cm square in each corner (shade 2X2 over the old 1X1 shading).
   c. continue with d, e, f and g as above for Box 1

3. **Box #3:** 3 cm X 3 cm nibble. Ask students to:
   a. use the SAME 9 cm X 9 cm square.
   b. SHADE a 3 cm X 3 cm square in each corner (shade 3X3 over the old 2X2 shading).
   c. continue d through g as described in Box #2.

4. **Box #4:** 4 cm X 4 cm nibble. Ask students to:
   a. Use the SAME 9 cm X 9 cm square.
   b. SHADE a 4 cm X 4 cm square in each corner (shade 4X4 over the old 2X2 shading)
c. continue d through g as described in Box #2.

B. Again, have students compare shapes, sizes of boxes and volumes.

C. Ask the following questions:
1. What is the relationship between the size of the corner cut and the height of the box? (Ans: the height is the same as the side of the square corner cut)
2. In comparing two boxes, will the taller box always have the greater volume? (Ans: no......ask why)

D. Discuss how many possible boxes can be made from various grid sizes. For example:
1. from figures with a 3 cm or a 4 cm side only 1 cm square nibbles are possible. You may make as many boxes as desired by expanding the other side: 3 X 3 (1 box), 3 X 4 (1 box), 3 X 5 (1 box), etc.
2. from figures with a 5 cm or 6 cm side, 1 cm square and 2 cm square nibbles are possible and, therefore, a minimum of two boxes is possible. You may make as many boxes as desired by increasing the length of the other side: 5 X 5 (2 boxes), 5 X 6 (2 boxes), 5 X 7 (3 boxes), 5 X 8 (3 boxes), 5 X 9 (4 boxes), etc.
3. from figures with a 7 cm or 8 cm side, 1 and 2 and 3 cm square nibbles are possible and a minimum of three boxes is possible. And, by increasing the length of the box, a large number of boxes is possible.

The main idea for this extension is that by increasing dimensions of the original grid, you can increase the possible number of boxes. Students should be encouraged to note the pattern of increases (1 cm square nibble: 1 box minimum; 2 cm square nibble: 2 boxes, etc)

E. Ask students to define volume; write their definitions on the back of Worksheet D. Allow discussion of definitions. Emphasize the definition as amount of space inside an object.

F. Ask students how they can determine the volume of a box (Ans: length X width X height). Original answers might include: counting the cubes, multiplying sides, etc.

G. Ask if a can of coke, a pyramid, a baseball (and other 3-D objects) have volume. Discuss their answers and how they arrived at their conclusions. Emphasize the three-dimensional aspect of these objects; the dimensions do not have to be straight lines but must not be flat (2 dimensions).

H. Show different cubes (1", sugar, any square object). Ask: could these units be used to measure the volume of a box? (Ans: yes, if box is big enough). Emphasize: a cubic unit is used to measure volume, regardless of the size of a cube.
IV. Oral Word Problems: ask the students to get a piece of scratcher paper.

A. Draw a chart as shown in Figure 5 on the board and ask students to copy it.

B. Tell them you will read some story problems and they must fill in the chart. Fill in the first and second problems as you read them.

C. Problems:

1. I have a box that is 1 centimeter in length, 2 centimeters in width and 3 centimeters in height. What is the volume of my box? Instruct class to fill in the object name (box), length (1 cm), width (2 cm) and height (3 cm). Then figure the volume of the box ($V = L \times W \times H; 1 \times 2 \times 3 = 6$ cubic centimeters, or cm$^3$).

2. My jewelry box is 3 centimeters long, 3 centimeters wide, and 3 centimeters high. What is the volume of my jewelry box? (fill in object (box), length (3 cm), height (3 cm), width (3 cm) and volume (27 cubic centimeters or 27 cm$^3$).

3. I have a basket that is 20 inches long, 10 inches wide and 25 inches high. What is the volume of my basket? (5,000 cubic inches, or in$^3$).

4. A rectangular solid is 15 inches long, 12 inches wide and 10 inches high. What is its volume (1,800 cubic inches or 1,800 in$^3$).

5. My sandbox measures 10 feet long, 5 feet wide and 2 feet high. What is the volume of the sandbox? (100 cubic feet or 100 feet$^3$).
DAY #5

Objective: 1) continue move from concrete to semi-concrete work with boxes; determine box of largest volume as in Day #4 but with students working alone and 2) to move to abstract work with volume problems.

Materials Needed: pencil, 1 cm X 1 cm grid papers, Student Work Sheet F for Drawn Boxes and Cube Count Sheet B

I. Review homework from previous day.

II. Distribute grid papers, Student Work Sheet F for Drawn Boxes and Cube Count Sheet B

A. "Create" 4 boxes from an 11 cm X 14 cm rectangle having the students only shade in the corners and imagine the folded boxes (1-4 below).

1. Box #1: 1 cm X 1 cm nibble: Instruct students to:
   a. cut a 11 cm X 14 cm rectangle from the grid paper.
   b. SHADE a 1 cm X 1 cm square in each corner (do not cut out).
   c. remember we are calling the corner cut a "nibble".
   d. PRETEND to fold the rectangle with the nibbled corners to make a box.
   e. sketch the box, on Student Work Sheet F as it would look and label the dimensions on the sketch.
   f. complete the Table on the bottom of the Student Work Sheet F. To determine the volume, ask students how many cubes they think will fill the box and how they could figure it out (ie. length X height X width).
   g. using the number for volume they have shown in their chart on the Cube Count Sheet B, make a graph to show the volume of the box.

2. Box #2: 2 cm X 2 cm nibble: Ask student to:
   a. use the SAME 11 cm X 14 cm rectangle.
   b. SHADE a 2 cm X 2 cm square in each corner (shade 2X2 over the old 1X1 shading).
   c. continue with d, e, f and g as above for Box #1.

3. Box #3: 3 cm X 3 cm nibble. Ask students to:
   a. use the SAME 11 cm X 14 cm rectangle.
   b. SHADE a 3 cm X 3 cm square in each corner (shade 3X3 over the old 2X2 shading).
   c. continue d through g as described in Box #2.

4. Box #4: 4 cm X 4 cm nibble. Ask students to:
   a. Use the SAME 11 cm X 14 cm rectangle:
   b. SHADE a 4 cm X 4 cm square in each corner (shade 4X4 over the old 3X3 shading).
   c. continue d through g as described in Box #2.
B. Again, have students compare shapes, sizes of boxes and volumes.

C. Ask the following questions:
   1. What is the relationship between the size of the corner cut and the height of the box? (Ans: the height is the same as the side of the square corner cut)
   2. In comparing two boxes, will the taller box always have the greater volume? (Ans: no......ask why)

Handout homework "Volume Problems", which is due the following school day.
Day #6

Objective: Review homework and administer quiz.

Materials: pencil, eraser, test, Activity Sheet #1

I. Have students check their homework answers in groups of 2 or 3 (about 20 minutes). Review the answers in a whole class situation and answer questions.

II. Administer quiz.
   1. Read general instructions aloud. Make sure each student has completed the cover page information.
   2. Administer test (about 20 minutes).

III. When all students have taken the test, review answers. If a student is absent, have him/her take the test upon return and review individually.

IV. Hand out Activity Sheet #1, which will be due in 1 week.
Week Two: One Week Later

I. Review Activity sheet #1; Watch for:
   1) three-dimensional notation (e.g., cm³, in³, etc)
   2) are wrong answers just multiplication or counting errors, or a lack of understanding the formula?

II. Hand out Activity Sheet #2, which will be due in 1 week.

Week Three: One Week Later

I. Review Activity sheet #2; Watch for:
   1) three-dimensional notation

II. Hand out Activity Sheet #3, which will be due in 1 week.

Week Four: One Week Later

I. Review Activity sheet #3

II. Administer final test (the same version as the pre-test).
Materials Needed:
tag board or other firm paper
1 cm square sugar cubes (Cocktail cubes)
student made boxes

Glue student boxes made from one grid size to the tag board
and write the box dimensions along the side

Glue a column of sugar cubes to the tag board above the
boxes, using one cube to represent a volume of one

Continue for all boxes constructed from an identical sized
rectangle or square (i.e., from a 9 x 9 square, boxes
with 1, 2, 3, and 4 cm square corner cuts may be added)

Use original grid size in title (example: Boxes from a 9 x 9
Square)
Homework:
Volume Problems

Name

Volume = length × width × height
V = L × W × H
V = 5 units × 3 units × 2 units
V = 30 cubic units (units³)

1 - 12. Find the volume of each box shown below.

1) 
V = __________

2) 
V = __________

3) 
V = __________

4) 
V = __________

5) 
V = __________

6) 
V = __________

7) 
V = __________

8) 
V = __________

9) 
V = __________

10) 
V = __________

11) 
V = __________

12) 
V = __________
13 - 24. Find the volume of each box described below.

13) \( L = 1 \text{ cm} \)  \( W = 4 \text{ cm} \)  \( H = 5 \text{ cm} \)

14) \( L = 3 \text{ cm} \)  \( W = 6 \text{ cm} \)  \( H = 2 \text{ cm} \)

15) \( L = 3 \text{ cm} \)  \( W = 5 \text{ cm} \)  \( H = 1 \text{ cm} \)

16) \( L = 4 \text{ cm} \)  \( W = 5 \text{ cm} \)  \( H = 3 \text{ cm} \)

17) \( L = 13 \text{ cm} \)  \( W = 7 \text{ cm} \)  \( H = 10 \text{ cm} \)

18) \( L = 25 \text{ cm} \)  \( W = 4 \text{ cm} \)  \( H = 9 \text{ cm} \)

19) \( L = 1 \text{ cm} \)  \( W = 4 \text{ cm} \)  \( H = 5 \text{ cm} \)

20) \( L = 3 \text{ cm} \)  \( W = 6 \text{ cm} \)  \( H = 2 \text{ cm} \)

21) \( L = 3 \text{ cm} \)  \( W = 5 \text{ cm} \)  \( H = 1 \text{ cm} \)

22) \( L = 1 \text{ cm} \)  \( W = 4 \text{ cm} \)  \( H = 5 \text{ cm} \)

23) \( L = 3 \text{ cm} \)  \( W = 6 \text{ cm} \)  \( H = 2 \text{ cm} \)

24) \( L = 3 \text{ cm} \)  \( W = 5 \text{ cm} \)  \( H = 1 \text{ cm} \)

25 - 28. Find the answers to the story problems.

25. A box is 3 inches long, 2 inches wide and 4 inches high. Find the volume?

26. I have a suitcase that is 1 foot long, 3 feet wide and 8 feet high. How many cubic feet of clothes can I put in the suitcase? ______________ cubic feet

27. A safe is 4 feet wide, 6 feet high and 9 feet long. What is the volume of the safe? ______________ cubic feet

28. A swimming pool measures 30 meters long, 15 meters wide and 6 meters deep. How much water will it take to fill the pool? ______________ cubic meters
STUDENT WORK SHEET A

5 cm BY 7 cm Rectangle

Box #1

Box #2

Box #3

1. Can we make a box?

2. Why? Explain your answer in one sentence.
Name __________________________

CUBE COUNT SHEET B

Grid Size: _____ cm BY _____ cm

Box # ______  Box # ______  Box # ______  Box # ______
STUDENT WORK SHEET B
7 cm BY 9 cm Rectangle

Your Sketches

Box #1
Number of cubes needed to fill Box #1

Box #2
Volume of Box #2 cm$^2$
(Number of cubes needed to fill Box #2)

Box #3
Volume of Box #2 cm$^2$
(Number of cubes needed to fill Box #3)

Box #4
1. Is there something wrong with this box?
2. Explain your answer in the space below.
Name ________________________________

CUBE COUNT SHEET A
(Cm size)

Grid Size: _____ cm BY _____ cm

Number of Cubes In My Boxes

<table>
<thead>
<tr>
<th>Box #</th>
<th>Box #</th>
<th>Box #</th>
<th>Box #</th>
</tr>
</thead>
</table>

264
STUDENT WORK SHEET C
FOR GROUPS
8 cm BY 8 cm Square

Your Sketches

Box #1

Box #2

Box #3

<table>
<thead>
<tr>
<th>Box</th>
<th>size of corner cut</th>
<th>box dimensions</th>
<th>Volume (number of cubes to fill the box)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 cm X 1 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 cm X 2 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3 cm X 3 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Name _______________________

STUDENT WORK SHEET D
FOR PRETENDED BOXES
9 cm BY 9 cm Square

Your Sketches

<table>
<thead>
<tr>
<th>Box</th>
<th>Size of corner cut (nibble)</th>
<th>Box dimensions</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 cm X 1 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 cm X 2 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3 cm X 3 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4 cm X 4 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
STUDENT WORK SHEET F
FOR PRETENDED BOXES
9 cm BY 12 cm Rectangle

Your Sketches

<table>
<thead>
<tr>
<th>Box #1</th>
<th>Box #2</th>
<th>Box #3</th>
<th>Box #4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table for 9 cm BY 12 cm Square

<table>
<thead>
<tr>
<th>Box</th>
<th>size of corner cut (nibble)</th>
<th>box dimensions</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 cm X 1 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 cm X 2 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3 cm X 3 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4 cm X 4 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ACTIVITY SHEET 1

Name __________________________

To find the volume of a rectangular solid, multiply the length by the width by the height.

\[ \text{Volume} = L \times W \times H \]

\[ V = 4 \text{ units} \times 2 \text{ units} \times 3 \text{ units} \]

\[ V = 24 \text{ cubic units (or 24 units}^3) \]

1 - 6. Find the volume of the rectangular solids shown below. Write the volume in cubic centimeters (cm\(^3\)).

1. \( (6 \times 3) \times 2 = \) _____

   \[ \text{volume} = \boxed{36 \text{ cm}^3} \]

2. \( (4 \times 3) \times 3 = \) _____

   \[ \text{volume} = \boxed{36 \text{ cm}^3} \]

3. \( (5 \times 3) \times 3 = \) _____

   \[ \text{volume} = \boxed{45 \text{ cm}^3} \]

4. \( (2 \times 2) \times 3 = \) _____

   \[ \text{volume} = \boxed{12 \text{ cm}^3} \]

5. \( (6 \times 3) \times 4 = \) _____

   \[ \text{volume} = \boxed{72 \text{ cm}^3} \]

6. \( (4 \times 3) \times 7 = \) _____

   \[ \text{volume} = \boxed{84 \text{ cm}^3} \]

7 - 16. Find the volume of the rectangular solids whose dimensions are listed below. Write the answer in cubic inches (in\(^3\)).

7. \( L = 5 \text{ inches} \) \( W = 2 \text{ inches} \) \( H = 5 \text{ inches} \)

   \[ \text{Vol} = \boxed{50 \text{ in}^3} \]

8. \( L = 6 \text{ inches} \) \( W = 4 \text{ inches} \) \( H = 10 \text{ inches} \)

   \[ \text{Vol} = \boxed{240 \text{ in}^3} \]

9. \( L = 10 \text{ inches} \) \( W = 3 \text{ inches} \) \( H = 10 \text{ inches} \)

   \[ \text{Vol} = \boxed{300 \text{ in}^3} \]

10. \( L = 8 \text{ inches} \) \( W = 3 \text{ inches} \) \( H = 7 \text{ inches} \)

   \[ \text{Vol} = \boxed{168 \text{ in}^3} \]

11. \( L = 9 \text{ inches} \) \( W = 3 \text{ inches} \) \( H = 4 \text{ inches} \)

   \[ \text{Vol} = \boxed{108 \text{ in}^3} \]

12. \( L = 3 \text{ inches} \) \( W = 3 \text{ inches} \) \( H = 4 \text{ inches} \)

   \[ \text{Vol} = \boxed{36 \text{ in}^3} \]
13 - 16) Word Problems

13. A dog house with a flat roof measures 4 feet long, 4 feet wide and 5 feet high. What is the volume of the dog house?
____________________ cubic feet

14. A rectangular solid is three inches in length, five inches in width and 10 inches in height. What is the volume of the solid?
____________________ cubic inches

15. A stack of gold bars is 2 meters long, 3 meters wide and 1 meter high. What is the volume of the stack?
___________________ cubic meters

16. How much volume is in a box that measures 10 centimeters long, 10 centimeters wide and 20 centimeters high?
__________________ cubic centimeters

18 - 22) Complete the charts below. First shade the corner nibble in the grid. Next draw the box that would be made and label the sides. Then find the volume. See the example in 18.

18. Grid size: 7 BY 5

<table>
<thead>
<tr>
<th>nibble</th>
<th>l</th>
<th>w</th>
<th>h</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 X 1</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>21 units³</td>
</tr>
<tr>
<td>2 X 2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>6 units³</td>
</tr>
</tbody>
</table>

Draw your boxes in the space below

19. Grid Size: 10 BY 7

<table>
<thead>
<tr>
<th>nibble</th>
<th>l</th>
<th>w</th>
<th>h</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 X 1</td>
<td></td>
<td></td>
<td></td>
<td>units³</td>
</tr>
<tr>
<td>2 X 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 X 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw your boxes in the space below
20. Grid Size: 11 BY 14

<table>
<thead>
<tr>
<th>nibble</th>
<th>l</th>
<th>w</th>
<th>h</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 X 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 X 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 X 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 X 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw your boxes in the space below

21. Get a piece of grid paper and cut out a 13 BY 13 grid. How many different boxes can you make from it? To find out, shade the nibbles, sketch the box and label the length, width and height of each box. Then compute the volume.

How many boxes could you make? __________________
ACTIVITY SHEET 2

Name ___________________________

1) Use the pictures of the cubes to complete the chart below.

<table>
<thead>
<tr>
<th>Cube</th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
<th>Number of Cubes</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

2 - 7. Volume Puzzle. What gets wetter the more it dries?

Break the code to find the answer. Match the answers to the letter in the code strip.

2) L = 2 cm
   W = 2 cm
   H = 3 cm
   Vol = ______

3) L = 12 cm
   W = 5 cm
   H = 7 cm
   Vol = ______

4) L = 6 cm
   W = 6 cm
   H = 11 cm
   Vol = ______

Code Strip

[Lists of volume calculations and codes]

Answer: 2 3 4 5 6 7
8 - 9. Work in a group of 2 or 3 students for the next questions. Get a piece of grid paper from your teacher. You will also need a pencil and scissors. Cut out the grid sizes and shade the corner nibbles. Sketch the boxes you would make with the grid paper and label the dimensions. Then answer the questions. You do not have to make all the sketches yourself. Have different people in your group do different boxes.

8) What is the volume of the tiniest box you can make?

volume ________  grid size ________  nibble ________

9) How many different boxes can you make with a 13 X 13 grid? List the nibbles, lengths, widths, heights and volumes in the blanks below. You may not need all of the rows below.

Grid Size: 13 BY 13

nibble ______ length _____ width ____ height ____ volume ______
nibble ______ length _____ width ____ height ____ volume ______
nibble ______ length _____ width ____ height ____ volume ______
nibble ______ length _____ width ____ height ____ volume ______
nibble ______ length _____ width ____ height ____ volume ______

Total number of boxes we made ____________

10 - 12. For each grid listed below, which nibble makes the box with the most volume (circle one):  

<table>
<thead>
<tr>
<th>Nibbles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. with a 6 by 10 grid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. with an 10 by 14 grid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. with a 12 by 14 grid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ACTIVITY SHEET 3

Name ____________________________

1 - 6. Find the volume of the boxes shown below.

1) [Diagram]
   Volume ______

2) [Diagram]
   Volume ______

3) [Diagram]
   Volume ______

4) [Diagram]
   Volume ______

5) [Diagram]
   Volume ______

6) [Diagram]
   Volume ______

7 - 15. Find the volume of the rectangular solids described below:

<table>
<thead>
<tr>
<th>length</th>
<th>width</th>
<th>height</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>7)</td>
<td>3 meters</td>
<td>6 meters</td>
<td>2 meters</td>
</tr>
<tr>
<td>8)</td>
<td>4 cm</td>
<td>3 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>9)</td>
<td>2 meters</td>
<td>3 meters</td>
<td>7 meters</td>
</tr>
<tr>
<td>10)</td>
<td>5 cm</td>
<td>8 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>11)</td>
<td>6 inches</td>
<td>2 inches</td>
<td>2 inches</td>
</tr>
<tr>
<td>12)</td>
<td>1 feet</td>
<td>2 feet</td>
<td>3 feet</td>
</tr>
<tr>
<td>13)</td>
<td>6 feet</td>
<td>7 feet</td>
<td>10 feet</td>
</tr>
<tr>
<td>14)</td>
<td>8 inches</td>
<td>6 inches</td>
<td>4 inches</td>
</tr>
<tr>
<td>15)</td>
<td>4 meters</td>
<td>7 meters</td>
<td>3 meters</td>
</tr>
</tbody>
</table>
16 - 20. Solve the word problems.

16) What is the volume of a box that is 5 inches long, 10 inches wide and 15 inches high? ________

17) A fish tank is 45 centimeters long, 30 centimeters wide, and 35 centimeters high. What is the volume of the fish tank? ________

18) What is the volume of a cube that is 6 inches by 6 inches by 6 inches? ________

19) A brick wall was 12 feet long, 2 feet wide and 9 feet high. What was the volume of the wall? ________

20 - 23. For the next problems, work with a group of 2 or 3 students. Get a piece of grid paper from your teacher. Cut out the grid sizes and shade corner nibbles and sketch the boxes that would be made. Then answer the questions. Assign a different grid size and nibble for each person in your group so that you do not have to make all the sketched boxes yourself.

20) What is the box with the most volume? volume ________

21) How many different boxes can you make with a 10 by 12 grid? Fill in the chart with your findings.

<table>
<thead>
<tr>
<th>nibble</th>
<th>length</th>
<th>width</th>
<th>height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

Total boxes: ________

22) Which box has the most volume? circle A or B or C
   A. a box made from a 10 by 7 grid with a 3 corner nibble
   B. a box made from a 5 by 8 grid with a 2 corner nibble
   C. a box made from a 3 by 10 grid with a 1 corner nibble

23) Which box has the least volume? circle A or B or C
   A. a box made from a 7 by 7 grid with a 3 corner nibble
   B. a box made from a 8 by 8 grid with a 2 corner nibble
   C. a box made from a 3 by 3 grid with a 1 corner nibble
TREATMENT III
TEACHER INSTRUCTIONS TO STUDENTS

DAY #1

I. Before starting, discuss with the class the following concepts and terminology: square, rectangle, area (as two dimensions; how to compute), length (as one dimension). Have students draw a 1" line; then draw a line twice as long. Discuss length as one dimension (length). Have students draw a 1 inch by 2 inch rectangle; then draw a rectangle with twice the area. Discuss area as two dimensions (length and width).

Materials Needed: pencil, paper, scratch paper and ruler for each student, and several sizes cubes, including centimeter cubes for each student, and other sizes for teacher to show: sugar cubes, inch cubes, plastic photograph holder cube, Rubic's cube, other cuboid objects.

II. Three dimensional units

Objective: to become familiar with a three-dimensional object: the cube

A. Distribute centimeter cubes for each student to examine.
   1. ask students to describe the cube (Ans: its length, width and height are all the same).

B. Continue as in A with 1 inch cubes.
   1. ask what is the difference between centimeter and inch cubes (Ans: larger/smaller size)

C. Continue with other cubes.
   1. compare sizes or various cubes
   2. ask how are all the cubes the same (Ans: length, width, height the same)
   3. ask how are all the cubes different (Ans: smaller/larger in size)

D. Have students look around the classroom and list on the board the objects they think are cube shaped. (Note: the students will have the cubes that you have demonstrated with for at least one cube...also the cubes you brought in). After 5 to 10 minutes, ask for their observations and write list on the board. Ask how they know they are cubes (length, width, height are the same).
Day #2

Objective: to introduce a cube as unit of measurement in constructing three-dimensional objects

Materials Needed: scissors, marking or scotch tape, pencil and paper, Cube Construction Sheet

I. Construction of a cube.

A. Distribute tape and Cube Construction Sheet to each student.

B. Construct cube with Cube Construction Sheet
   1. Instruct students to cut along the solid outline.
   2. Fold on the dotted lines.
   3. Tape down the tabs to form a cube.
   4. Measure the length, width and height of the cube.
   5. Ask if the dimensions are the same; what are the dimensions?

C. Make one other cubes (each student will have two).

II. Making figures with the cubes

Objective: make different rectangular solids with cubes

A. In groups of two students each, have students make different rectangular solids by combining the cubes (6 cubes per group) (examples: 1 row of 6; 2 rows of 3, 1 column of 6).

B. Have students sketch the rectangular solids, or boxes, then made. If they have had no experience in three dimensional drawing, you may have to show them how.

C. Ask students to show their results. Draw their constructions on the board as they are shown.

D. Ask how many cubes are in each figure drawn on the board (Ans. 6)

E. If time, construct more cubes and duplicate steps A, B, and C.
Objective: to 1) find volume of a solid figure by counting cubic units, 2) use the cubic centimeter as a standard unit for measuring volume of a rectangular solid, 3) define "volume" and 4) compare shapes to find the largest volume.

Materials: centimeter cubes, inch cubes, several small boxes into which the inch cubes will fit and fill (otherwise make some from construction paper), and six rectangular solids made ahead of time with the following dimensions: A - 1 cube by 1 cube by 10 cubes; B - 2 by 2 by 3; C - 2 by 5 by 1; D - 2 by 10 by 2; E - 3 by 4 by 4; F - 2 by 4 by 5 (see figures 1 and 2 below) (note: I used unifix cubes for figures A - F)

DAY #3

I. Review the following:
1. How are all cubes alike? (Ans: length, width, height same)
2. How can cubes be used to make different figures? (stacked in different positions)
3. How do you know how much space the figure takes up? (Ans: count the cubes that make up the figure)

II. Finding the volume of a rectangular solid by counting cubes that fill a box.
A. Display a box. Write box #1 on the board and roughly draw the shape (remember to hold the box in the same position).
1. Ask a student to fill the box with centimeter cubes
2. Ask another student to remove and count the cubes.
3. While student is removing and counting, have students predict the number to be counted.
4. Write the number of cubes on the board.
B. Explain that the number of cubes is the amount of space inside a figure. We call this amount the Volume of the figure.
C. Write "volume" on the board. Note that it is the same as the number of cubes needed to fill the box.
D. Pass around the centimeter cubes for class to examine.

III. Finding volume of other boxes by filling, removing and counting cubes.
A - B. Continue as above.
IV. Comparing sizes and shapes of pre-made boxes to find the largest volume.

Objective: to demonstrate that a tall or short box may not have largest volume

A. Draw shapes of boxes A, B, and C on the board as shown in figure 1.

B. Show the pre-constructed rectangular solids to the class (be sure they are shown as drawn: tallest, squarish, short).

C. Have three students dismantle the solids and count the cubes to determine the volume. Write the volume next to the drawings.

D. Ask:
1. Which box has the most volume? (Ans: box 2)
2. Which box is the shortest? (Ans: box 3)
3. Which box is the tallest? (Ans: box 1)
4. Does the shape of the box tell you anything about the volume? (Ans: the squarish (cuboid) box has the largest volume)

E. Repeat A through D with the second set of rectangular solids, D, E, F, drawing the shapes as shown in figure 2.
Objective: find volume of rectangular solids using the formula

Materials: Work Sheet A, pencil, paper, pre-made rectangular solids made from Unifix cubes with the following dimensions: 1) 4 X 4 X 2; 2) 5 X 5 X 2; 3) 3 X 3 X 3, 4) 2 X 5 X 5 (note: make sure you make them so you can easily separate the layers)

I. Using the rectangular solids to explain the formula

A. Use the first rectangular solid made with 32 centimeter cubes with the dimensions 4 cm X 4 cm X 2 cm (4 for length and width and 2 for height)

B. Separate the solid into two layers (each layer being 4 X 4)
   1. Ask how many cubes are in one layer (Ans: 16)
   2. Ask how students arrived at their answers. If some say they counted, ask how you could find out without counting (Ans: multiply length by width).
   3. Ask how many cubes are in the other layer (Ans: 16)
   4. Continue as in 2.

C. Put the layers together
   1. Ask how many cubes are in the rectangular solid (Ans: 32)
   2. Ask: how do you know? (2 layers; 16 + 16 = 32)
   3. Ask how could you find out how many cubes are in the rectangular solid without counting (Ans: multiply length, width and height)

D. Explain that this is the way to find out how many cubes are in a rectangular solid. The way to find volume is to multiply length, width and height.

E. Repeat steps A through C with a 5 cm X 5 cm X 2 rectangular solid. Review the formula: V = L X W X H.

F. Repeat steps A through C with a 3 cm X 3 cm X 3 cm rectangular solid. Review formula.

II. Using the formula of a rectangular solid (prism)

A. On the board, draw a chart as in figure 3.

```
<table>
<thead>
<tr>
<th>L</th>
<th>W</th>
<th>H</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Box A | | | |
```

B. Use the 2 X 5 X 5 rectangular solid; call it "Box A".
   1. Ask students to copy the chart on paper.
   2. Ask what is the length of the rectangular solid (Ans: 2)
   3. What is the width (Ans: 5)
   4. What is the height (Ans: 5)
   5. What is the volume (Ans: 50)
C. After 20 minutes, ask students to tell what objects they measured, the length, width and height and volume of the objects.

D. Ask students to define volume; write their definitions on the back of Work Sheet B. Allow discussion of definitions. (emphasize the definition as the amount of space inside an object).

E. Ask students how they can determine the volume of a box (Ans: length X width X height). Original answers might include: counting the cubes, multiplying sides, etc.

F. Ask if a can of coke, a pyramid, a baseball (and other 3-D objects) have volume. Discuss their answers and how they arrived at their conclusions.

G. Show different cubes (1", sugar, any square object). Ask: could these units be used to measure the volume of a box? (Ans: yes, if box is big enough). Emphasize: a cubic unit is used to measure volume, regardless of the size of a cube.

III. Assign Homework "Volume Problems", which is due the following school day.
Objective: to 1) reinforce and practice finding the volume of rectangular solids and 2) use cubic centimeters as a measuring unit

Materials: pencil, ruler, Work Sheet B, Homework "Volume Problems"

1. Review with Oral Word problems. Ask students to get a piece of scratch paper.

A. Draw a chart as shown in Figure 4 on the board and ask students to copy it.

B. Tell them you will read some story problems and they must fill in the chart. Fill in the first and second problems as you read them.

C. Problems:

1. I have a box that is 1 centimeter in length, 2 centimeters in width and 3 centimeters in height. What is the volume of my box? Instruct class to fill in the object name (box), length (1 cm), width (2 cm) and height (3 cm). Then figure the volume of the box (V = L x W x H; 1 x 2 x 3 = 6 cubic centimeters, or cm³).

2. My jewelry box is 3 centimeters long, 3 centimeters wide, and 3 centimeters high. What is the volume of my jewelry box? (fill in object (box), length (3 cm), height (3 cm), width (3 cm) and volume (27 cubic centimeters or 27 cm³).

3. I have a basket that is 20 inches long, 10 inches wide and 25 inches high. What is the volume of my basket? (5,000 cubic inches, or in³).

4. A rectangular solid is 15 inches long, 12 inches wide and 10 inches high. What is its volume (1,800 cubic inches or 1,800 in³).

5. My sandbox measures 10 feet long, 5 feet wide and 2 feet high. What is the volume of the sandbox? (100 cubic feet or 100 feet³).

II. Work in groups of two or three students to find volume.

A. Divide class into groups. Distribute Work Sheet B to each student.

B. Ask students to:

1. Look around the room for small boxes, cubes and other square and/or rectangular objects.
2. Measure length, width and height of several objects, to the nearest centimeter.
3. Record answers on Work Sheet B.
4. Calculate the volume(s).
5. Record the volume(s) on Work Sheet B.
6. ask how they determined the volume (Ans: formula)
7. ask a student to fill in the chart on the board.
8. have students fill in the chart on their paper.

B. Tilt the solid to show a $5 \times 5 \times 2$ solid, "Box B".
   1. continue with steps 1 through 8 as above
   2. ask how the rectangular solids are different (Ans: width and height different dimensions)
   3. how are the solids the same (Ans: same volume)

C. Continue with previously used rectangular solids and fill in
   the chart as in A.

III. Work sheet A for reinforcement as class activity
    A. Distribute Work Sheet A to each student.
    B. Complete Work Sheet A as a class.
Day #6

Objective: Review homework and administer quiz.

Materials: pencil, eraser, test, Activity Sheet #1

I. Have students check their homework answers in groups of 2 or 3 (about 20 minutes). Review the answers in a whole class situation and answer questions.

II. Administer quiz.
   1. Read general instructions aloud. Make sure each student has completed the cover page information.
   2. Administer test (about 20 minutes).

III. When all students have taken the test, review answers. If a student is absent, have him/her take the test upon return and review results individually.

IV. Hand out Activity Sheet #1, which will be due in 1 week.
Week Two: One Week Later

I. Review Activity sheet #1; Watch for:
   1) three-dimensional notation (i.e. cm³ or in³, etc)
   2) are wrong answers simply multiplication or counting errors or
      a lack of understanding of the formula

II. Hand out Activity Sheet #2, which will be due in 1 week.

Week Three: One Week Later

I. Review Activity sheet #2; Watch for:
   1) three-dimensional notation

II. Hand out Activity Sheet #3, which will be due in 1 week.

Week Four: One Week Later

I. Review Activity sheet #3

II. Administer final test (the same version as the pre-test).
INSTRUCTIONS:

1. cut along the solid lines
2. fold along the dotted lines to form a cube
3. glue or tape tabs to make the cube
The number of cubes that fill the inside of a three-dimensional figure is called the VOLUME of the figure.

We can count the cubic units to find the volume.

A cubic centimeter is a standard unit for measuring volume.

To find the volume of a rectangular solid, multiply the length by the width by the height:

\[ V = L \times W \times H \]

Tell the volume in cubic centimeters.

How many cubic units are in Figure A? in Figure B?
Find the volume. Write your answer in cubic centimeters ($cm^3$). See (1) for an example.

1) 
![Image of a 3D shape with dimensions 2 cm x 3 cm x 6 cm]

$$6 \times 2 \times 3 = 36$$

$$\text{volume} = 36 \ \text{cm}^3$$

2) 
![Image of a 3D shape with dimensions 3 cm x 4 cm x 3 cm]

$$4 \times 3 \times 3 = \_ \_ \_ \_

$$

$$\text{volume} = \_ \_ \_ \_ \text{cm}^3$$

3) 
![Image of a 3D shape with dimensions 3 cm x 3 cm x 5 cm]

$$5 \times 3 \times 3 = \_ \_ \_ \_

$$

$$\text{volume} = \_ \_ \_ \_ \text{cm}^3$$

4) 
![Image of a 3D shape with dimensions 3 cm x 2 cm x 2 cm]

$$2 \times 2 \times 3 = \_ \_ \_ \_

$$

$$\text{volume} = \_ \_ \_ \_ \text{cm}^3$$

5) 
![Image of a 3D shape with dimensions 3 cm x 3 cm x 4 cm]

$$6 \times 3 \times 4 = \_ \_ \_ \_

$$

$$\text{volume} = \_ \_ \_ \_ cm^3$$

6) 
![Image of a 3D shape with dimensions 3 cm x 3 cm x 4 cm]

$$4 \times 3 \times 7 = \_ \_ \_ \_

$$

$$\text{volume} = \_ \_ \_ \_ \text{cm}^3$$

7) 
![Image of a 3D shape with dimensions 3 cm x 5 cm x 4 cm]

$$5 \times 4 \times 5 = \_ \_ \_ \_

$$

$$\text{volume} = \_ \_ \_ \_ \text{cm}^3$$

8) 
![Image of a 3D shape with dimensions 3 cm x 4 cm x 5 cm]

$$8 \times 4 \times 5 = \_ \_ \_ \_

$$

$$\text{volume} = \_ \_ \_ \_ \text{cm}^3$$
Student Work Sheet B

Find the Volumes of Square and Rectangular Objects in our Classroom

Name ____________________________
Group's Name ____________________________

Directions: Find seven (7) objects that are square or rectangular shaped objects (boxes, cases, cubes, etc). Measure the sides of each object to the nearest centimeter the sides of each object. Record the length, width, and height of each object in the columns below. Calculate the volume and record your answer.

<table>
<thead>
<tr>
<th>Object</th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
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<td>2)</td>
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<td>4)</td>
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<td>5)</td>
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</tr>
<tr>
<td>6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7)</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Homework:
Volume Problems

Name ____________________________

Volume = length \times width \times height

V = L \times W \times H.

V = 5 \text{ units} \times 3 \text{ units} \times 2 \text{ units}

V = 30 \text{ cubic units (units}^3\text{)}

1 - 12. Find the volume of each box shown below.

1) \hspace{1cm} 2) \hspace{1cm} 3)

\hspace{1.5cm} \hspace{1.5cm} \hspace{1.5cm}

V = \hspace{1cm} V = \hspace{1cm} V =

4) \hspace{1cm} 5) \hspace{1cm} 6)

\hspace{1.5cm} \hspace{1.5cm} \hspace{1.5cm}

V = \hspace{1cm} V = \hspace{1cm} V =

7) \hspace{1cm} 8) \hspace{1cm} 9)

\hspace{1.5cm} \hspace{1.5cm} \hspace{1.5cm}

V = \hspace{1cm} V = \hspace{1cm} V =

10) \hspace{1cm} 11) \hspace{1cm} 12)

\hspace{1.5cm} \hspace{1.5cm} \hspace{1.5cm}

V = \hspace{1cm} V = \hspace{1cm} V =
13 - 24. Find the volume of each box described below.

13) \( L = 1 \text{ cm} \)
\( W = 4 \text{ cm} \)
\( H = 5 \text{ cm} \)
\( V = \) 

14) \( L = 3 \text{ cm} \)
\( W = 6 \text{ cm} \)
\( H = 2 \text{ cm} \)
\( V = \) 

15) \( L = 3 \text{ cm} \)
\( W = 5 \text{ cm} \)
\( H = 1 \text{ cm} \)
\( V = \) 

16) \( L = 4 \text{ cm} \)
\( W = 5 \text{ cm} \)
\( H = 3 \text{ cm} \)
\( V = \) 

17) \( L = 13 \text{ cm} \)
\( W = 7 \text{ cm} \)
\( H = 10 \text{ cm} \)
\( V = \) 

18) \( L = 25 \text{ cm} \)
\( W = 4 \text{ cm} \)
\( H = 9 \text{ cm} \)
\( V = \) 

19) \( L = 1 \text{ cm} \)
\( W = 4 \text{ cm} \)
\( H = 5 \text{ cm} \)
\( V = \) 

20) \( L = 3 \text{ cm} \)
\( W = 6 \text{ cm} \)
\( H = 2 \text{ cm} \)
\( V = \) 

21) \( L = 3 \text{ cm} \)
\( W = 5 \text{ cm} \)
\( H = 1 \text{ cm} \)
\( V = \) 

22) \( L = 1 \text{ cm} \)
\( W = 4 \text{ cm} \)
\( H = 5 \text{ cm} \)
\( V = \) 

23) \( L = 3 \text{ cm} \)
\( W = 6 \text{ cm} \)
\( H = 2 \text{ cm} \)
\( V = \) 

24) \( L = 3 \text{ cm} \)
\( W = 5 \text{ cm} \)
\( H = 1 \text{ cm} \)
\( V = \) 

25 - 28. Find the answers to the story problems.

25. A box is 3 inches long, 2 inches wide and 4 inches high. Find the volume?

26. I have a suitcase that is 1 foot long, 3 feet wide and 8 feet high. How many cubic feet of clothes can I put in the suitcase?

27. A safe is 4 feet wide, 6 feet high and 9 feet long. What is the volume of the safe?

28. A swimming pool measures 30 meters long, 15 meters wide and 6 meters deep. How much water will it take to fill the pool?
To find the volume of a rectangular solid, multiply the length by the width by the height.

Volume = \( L \times W \times H \)

\( V = 4 \) units \( \times 2 \) units \( \times 3 \) units

\( V = 24 \) cubic units (or \( 24 \) units\(^3\))

1 - 6. Find the volume of the rectangular solids shown below. Write the volume in cubic centimeters (cm\(^3\)).

1. \( (6 \times 3) \times 2 = \) 
   \( \text{volume} = \) \( \) cm\(^3\)

2. \( (4 \times 3) \times 3 = \) 
   \( \text{volume} = \) \( \) cm\(^3\)

3. \( (5 \times 3) \times 3 = \) 
   \( \text{volume} = \) \( \) cm\(^3\)

4. \( (2 \times 2) \times 3 = \) 
   \( \text{volume} = \) \( \) cm\(^3\)

5. \( (6 \times 3) \times 4 = \) 
   \( \text{volume} = \) \( \) cm\(^3\)

6. \( (4 \times 3) \times 7 = \) 
   \( \text{volume} = \) \( \) cm\(^3\)

7 - 16. Find the volume of the rectangular solids whose dimensions are listed below. Write the answer in cubic inches (in\(^3\)).

7. \( L = 5 \) inches \( \) \( W = 2 \) inches \( \) \( H = 5 \) inches \( \) \( \text{Vol} = \) \( \) in\(^3\)

8. \( L = 6 \) inches \( \) \( W = 4 \) inches \( \) \( H = 10 \) inches \( \) \( \text{Vol} = \) \( \) in\(^3\)

9. \( L = 10 \) inches \( \) \( W = 3 \) inches \( \) \( H = 10 \) inches \( \) \( \text{Vol} = \) \( \) in\(^3\)

10. \( L = 8 \) inches \( \) \( W = 3 \) inches \( \) \( H = 7 \) inches \( \) \( \text{Vol} = \) \( \) in\(^3\)

11. \( L = 9 \) inches \( \) \( W = 3 \) inches \( \) \( H = 2 \) inches \( \) \( \text{Vol} = \) \( \) in\(^3\)

12. \( L = 3 \) inches \( \) \( W = 3 \) inches \( \) \( H = 4 \) inches \( \) \( \text{Vol} = \) \( \) in\(^3\)
13. $1 = 8 \text{ inches}$
   $w = 6 \text{ inches}$
   $h = 1 \text{ inches}$
   $V = \underline{______}$

14. $1 = 7 \text{ inches}$
   $w = 5 \text{ inches}$
   $h = 2 \text{ inches}$
   $V = \underline{______}$

15. $1 = 4 \text{ inches}$
   $w = 3 \text{ inches}$
   $h = 4 \text{ inches}$
   $V = \underline{______}$

16 - 19. Word Problems

16. A dog house measures 4 feet long, 4 feet wide and 5 feet high. What is the volume of the dog house?
   ____________________ cubic feet

17. A rectangular solid is three inches in length, five inches in width and 10 inches in height. What is the volume of the solid?
   ____________________ cubic inches

18. A stack of gold bars is 2 meters long, 3 meters wide and 1 meter high. What is the volume of the stack?
   ____________________ cubic meters

19. How much volume is in a box that measures 10 centimeters long, 10 centimeters wide and 20 centimeters high?
   ____________________ cubic centimeters

20. Complete the Chart.

<table>
<thead>
<tr>
<th>length</th>
<th>width</th>
<th>height</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 inch</td>
<td>4 inches</td>
<td>8 inches</td>
<td></td>
</tr>
<tr>
<td>1 meter</td>
<td>4 meters</td>
<td>8 meters</td>
<td>32 m$^3$</td>
</tr>
<tr>
<td>6 feet</td>
<td>2 feet</td>
<td>5 feet</td>
<td></td>
</tr>
<tr>
<td>2 feet</td>
<td>5 feet</td>
<td>7 feet</td>
<td></td>
</tr>
<tr>
<td>10 inches</td>
<td>10 inches</td>
<td></td>
<td>1000 in$^3$</td>
</tr>
<tr>
<td>3 yards</td>
<td>4 yards</td>
<td>60 yd$^3$</td>
<td></td>
</tr>
<tr>
<td>3 meters</td>
<td>2 meters</td>
<td></td>
<td>3 m$^3$</td>
</tr>
<tr>
<td>2 inches</td>
<td>3 inches</td>
<td>8 inches</td>
<td></td>
</tr>
<tr>
<td>4 meters</td>
<td>1 meter</td>
<td>20 m$^3$</td>
<td></td>
</tr>
</tbody>
</table>
ACTIVITY SHEET 2

Name ____________________________

1) Use the pictures of the cubes to complete the chart below.

<table>
<thead>
<tr>
<th>Cube</th>
<th>length</th>
<th>width</th>
<th>height</th>
<th>number of cubes</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2 - 7. Volume Puzzle. What gets wetter the more it dries?

Break the code to find the answer. Match the answers to the letter in the code strip.

Code Strip

2) L = 2 cm 3) L = 12 cm 4) L = 6 cm
   W = 2 cm  W = 5 cm  W = 6 cm
   H = 3 cm  H = 7 cm  H = 11 cm

Vol = _____  Vol = _____  Vol = _____  Code Strip

5) L = 8 cm 6) L = 3 cm 7) L = 6 cm
   W = 8 cm  W = 3 cm  W = 5 cm
   H = 10 cm  H = 3 cm  H = 6 cm

Vol = _____  Vol = _____  Vol = _____

Answer: 2 3 4 5 6 7
8 - 14. Volume Puzzle. What keeps looking larger and smaller and larger and smaller?

Break the code to find the answer. Match the answers to the letter in the code strip.

<table>
<thead>
<tr>
<th>Code Strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = 7 cm</td>
</tr>
<tr>
<td>W = 8 cm</td>
</tr>
<tr>
<td>H = 3 cm</td>
</tr>
<tr>
<td>Vol = _____</td>
</tr>
<tr>
<td>E = 6 cm³</td>
</tr>
<tr>
<td>L = 3 cm</td>
</tr>
<tr>
<td>W = 4 cm</td>
</tr>
<tr>
<td>H = 3 cm</td>
</tr>
<tr>
<td>Vol = _____</td>
</tr>
<tr>
<td>H = 3 cm</td>
</tr>
<tr>
<td>W = 3 cm</td>
</tr>
<tr>
<td>H = 1 cm</td>
</tr>
<tr>
<td>Vol = _____</td>
</tr>
<tr>
<td>H = 5 cm</td>
</tr>
<tr>
<td>W = 5 cm</td>
</tr>
<tr>
<td>H = 5 cm</td>
</tr>
<tr>
<td>Vol = _____</td>
</tr>
<tr>
<td>L = 5 cm</td>
</tr>
<tr>
<td>W = 3 cm</td>
</tr>
<tr>
<td>H = 6 cm</td>
</tr>
<tr>
<td>Vol = _____</td>
</tr>
<tr>
<td>L = 6 cm</td>
</tr>
<tr>
<td>W = 3 cm</td>
</tr>
<tr>
<td>H = 6 cm</td>
</tr>
<tr>
<td>Vol = _____</td>
</tr>
</tbody>
</table>

8 9 10 11 12 13 14

15 - 16. For the next problems, work with a group of 2 or 3 students. You will have to use a measuring device to measure in centimeters.

15. What is the cube-shaped object that has the smallest volume of all the cube-shaped objects in your classroom?

- name of the object ______________________________
- length of the object ____________________________
- width of the object _____________________________
- height of the object ____________________________
- volume of the object ____________________________

16. What is the rectangular-shaped object that has the smallest volume of all the rectangular-shaped objects in your classroom?

- name of the object ______________________________
- length of the object ____________________________
- width of the object _____________________________
- height of the object ____________________________
- volume of the object ____________________________
ACTIVITY SHEET 3

Name ____________________________

1 - 6. Find the volume of the boxes shown below.

1) Volume _________________________

2) Volume _________________________

3) Volume _________________________

4) Volume _________________________

5) Volume _________________________

6) Volume _________________________

7 - 15. Find the volume of the rectangular solids described below:

<table>
<thead>
<tr>
<th>length</th>
<th>width</th>
<th>height</th>
<th>volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>7) 3 m</td>
<td>6 m</td>
<td>2 m</td>
<td>______ cm³</td>
</tr>
<tr>
<td>8) 4 cm</td>
<td>3 cm</td>
<td>5 cm</td>
<td>______ cm³</td>
</tr>
<tr>
<td>9) 2 m</td>
<td>3 m</td>
<td>7 m</td>
<td>______ m³</td>
</tr>
<tr>
<td>10) 5 cm</td>
<td>8 cm</td>
<td>3 cm</td>
<td>______ cm³</td>
</tr>
<tr>
<td>11) 6 in</td>
<td>2 in</td>
<td>2 in</td>
<td>______ in³</td>
</tr>
<tr>
<td>12) 1 ft</td>
<td>2 ft</td>
<td>3 ft</td>
<td>______ ft³</td>
</tr>
<tr>
<td>13) 6 ft</td>
<td>7 ft</td>
<td>10 ft</td>
<td>______ ft³</td>
</tr>
<tr>
<td>14) 8 in</td>
<td>6 in</td>
<td>4 in</td>
<td>______ in³</td>
</tr>
<tr>
<td>15) 4 m</td>
<td>7 m</td>
<td>3 m</td>
<td>______ m³</td>
</tr>
</tbody>
</table>
16 - 20. Solve the word problems.

16) What is the volume of a box that is 5 inches long, 10 inches wide and 15 inches high? 

17) A fish tank is 45 centimeters long, 30 centimeters wide, and 35 centimeters high. What is the volume of the fish tank? 

18) What is the volume of a cube that is 6 inches by 6 inches by 6 inches? 

19) A brick wall was 12 feet long, 2 feet wide and 9 feet high. What was the volume of the wall? 

19 - 21. For the next problems, work with a group of 2 or 3 students. You will have to use a measuring device to measure in centimeters.

19) What is the cube-shaped object that has the largest volume of all the cube-shaped objects in your classroom?
   name of the object ________________________________
   length of the object ________________________________
   width of the object ________________________________
   height of the object ________________________________
   volume of the object ________________________________

20) What is the rectangular-shaped object that has the largest volume of all the rectangular-shaped objects in your classroom?
   name of the object ________________________________
   length of the object ________________________________
   width of the object ________________________________
   height of the object ________________________________
   volume of the object ________________________________
APPENDIX E

CHART OF POST MEANS
Table 17. Mean Scores of Post-Instruction and Post-Retention Tests

<table>
<thead>
<tr>
<th>Item Scores</th>
<th>High level</th>
<th>Low level</th>
<th>Total</th>
</tr>
</thead>
</table>

Table 18. Mean Scores of Tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Treatment</th>
<th>Low Level</th>
<th>High Level</th>
<th>Total Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-Instruction</td>
<td>I</td>
<td>8.709</td>
<td>7.258</td>
<td>15.75</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>8.768</td>
<td>7.500</td>
<td>16.23</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>7.764</td>
<td>7.410</td>
<td>15.16</td>
</tr>
<tr>
<td>Post-Retention</td>
<td>I</td>
<td>9.156</td>
<td>8.224</td>
<td>17.36</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>8.972</td>
<td>7.894</td>
<td>16.84</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>8.575</td>
<td>7.319</td>
<td>16.05</td>
</tr>
</tbody>
</table>