Application of a continuum theory of multiphase mixtures to snow on the ground
by Edward Eagan Adams

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering
Montana State University

Abstract:
Snow on the ground is examined as a saturated two phase granular material consisting of small grains of ice and interstitial pores filled by a single vapor. The snow is considered as a continuous mixture comprised of the ice and vapor constituents which are themselves treated as individual but interacting continua.

Snow is of interest, in and of itself as a commonly encountered geologic material. An improved understanding of it, however, is of widespread benefit since it is a thermodynamically active granular material which exists in the natural environment. Processes similar to those involved in snow metamorphism, including phase change, occur in other materials, but usually at more extreme thermodynamic conditions.

Mathematical modeling of the snow is accomplished using a relatively recent continuum theory for mixtures where the individual constituents are physically separate. The approach considers the volume fraction occupied by each constituent as an additional kinematic variable. Therefore, in addition to the balance equations for mass, linear momentum, angular momentum and energy, usually applied in continuum mechanics, an equation which accounts for changes in the volume fraction, called the balance of equilibrated force, is included. Balance equations for each constituent as well as for the mixture are considered.

The immiscible nature of the constituents allows constitutive equations which depend only on variables pertaining to that constituent. Exchange between constituents is accounted for by interaction terms which enter the theory through the balance equations for the constituents. Forms for these interaction terms are developed which guarantee that the entropy inequality is not violated.

The model is used to examine isothermal and temperature gradient conditions for an ideal homogeneous snowpack and an ideal snowpack with layers of varying density. Values are calculated for variables as they change from the transient to the steady state response. Results, with the exception of the vapor velocity, demonstrate an excellent correlation when compared with known and conjectured physical phenomena associated with snow metamorphism.

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APPROVAL

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Edward Eagan Adams

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citation, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Date

Chairperson, Graduate Committee

Approved for the Major Department

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Date

Head, Major Department

Approved for the College of Graduate Studies

7-31-87

Date

Graduate Dean

STATEMENT OF PERMISSION TO USE

To Bill, who encouraged me to attempt it, and without whom the process never would have begun. To Bob, who taught me how, and in the course of it became a friend. And especially to Cat, for always being there.

VITA

Edward Eagan Adams was born on July 29, 1950 at Mercy Hospital in Rockville Centre, New York, the son of BelleRita and Edward Thomas Adams. His received his secondary education from Saint Agnes Cathedral High School in Rockville Centre, graduating in 1968. Mr. Adams graduated from Mount Saint Mary's College, Emmitsburg, Maryland in 1972, obtaining a Bachelor of Arts degree in English with a Philosophy minor. He received a Bachelor of Science degree in Earth Sciences under the Geophysics option in 1979, and a Master of Science in Engineering Mechanics, with a Mathematics minor in 1982, from Montana State University, Bozeman, Montana.

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CHAPTER I

INTRODUCTION

Snow

Snow on the ground is considered as a fine grained granular or porous geologic material with an ice matrix (Figure 1). An improved understanding of snow as a material is of widespread interest, since a large portion of the globe is seasonally or permanently covered with it. The effect of snow on life-style and the environment can be enormous.

It is of obvious interest when considering transportation, watershed management, avalanche hazard, recreational opportunities and the like, but it is also of broad scientific interest, since it is a naturally occurring material which exists at conditions and temperatures conducive to phase change, coupled with heat flux, mass flux, and deformational processes. As a material, snow is very thermodynamically active in its natural environment on the earth's surface, and will undergo a variety of metamorphic processes including the sintering or bonding of grains, complete crystallographic changes of the ice phase, and decomposition through melting or sublimation. Heat and mass flux in snow take place through both the solid and vapor or liquid phases in a manner which may include numerous transfer mechanisms. Since these processes occur under normal atmospheric conditions, it is an excellent model medium which may be applied to other granular geologic and engineering materials where

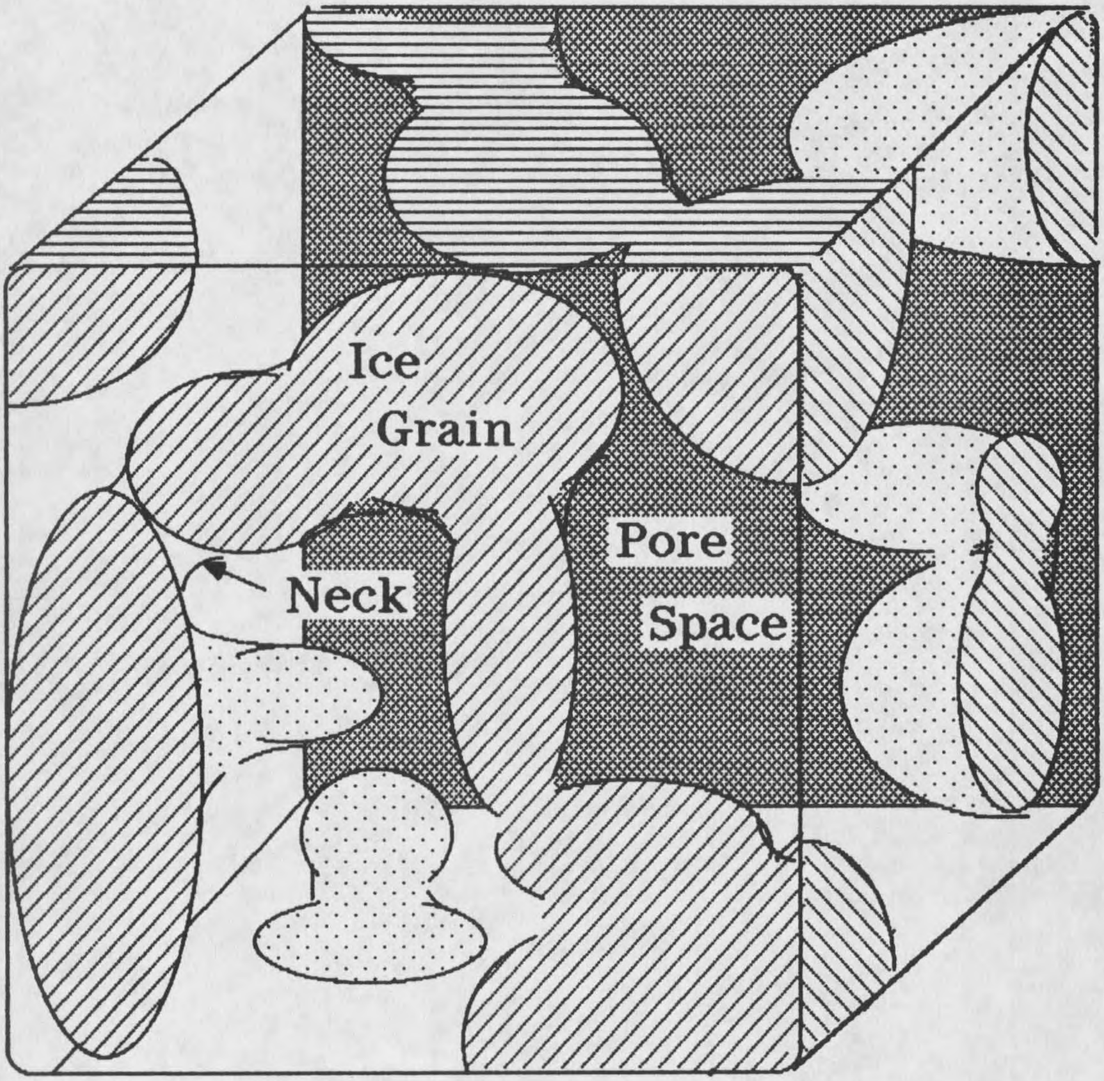

Figure 1. The idealized three dimensional snow on the ground, which is considered as a granular or porous material.

similar processes can take place, but only at more extreme temperatures and pressures.

Metamorphic processes are active in snow from the time of formation in the atmosphere as individual ice crystals, until the ice either evaporates back into the atmosphere or melts as runoff. Atmospheric ice crystals form on condensation nuclei, such as dust, from supercooled water vapor in clouds. Although there are a wide variety of crystal types, the basic crystalline ice structure consists of a basal plane possessing hexagonal symmetry, with an orientation perpendicular to the principal crystallographic axis. Once formed, these intricate crystal forms are in an unstable thermodynamic state and tend to metamorphose toward a more stable configuration after they accumulate as snow on the ground.

In a broad sense there are basically two types of snow on the ground, wet and dry. Snow is considered to be of the wet variety when the interstitial pore spaces between the ice has an obvious liquid water phase present. Wet snow may be liquid-saturated, where the pore is filled with liquid water, or the free water may share the space with a vapor consisting of water vapor and air. In dry snow the liquid phase is not present. The model presented here is concerned with processes which influence dry snow. Dry snow on the ground is generally subcatagorized into two classifications, sometimes based on morphology, but more frequently terms are used which reflect the processes considered to dominate the metamorphic development of the ice forms.

Initially a layer of the fragile atmospherically generated crystals will settle and deteriorate into a more stable configuration, and in the absence of a large temperature gradient will tend toward a rounded shape. Development of this rounded form is variously termed as destructive metamorphism, equi-temperature metamorphism [LaChapelle,1969; Sommerfeld, 1969], or the equilibrium growth form [Colbeck, 1981]. The term destructive, which is no longer in widespread use, is based on the "destruction" of the delicate crystals into the rounded form. Equi-temperature implies an isothermal condition, which may be misleading since this situation seldom, if ever, exists in the natural environment for dry snow. Although the process takes place when the temperature gradient may not be the primary driving mechanism of formation, it is not in general an isothermal situation. In fact, some temperature gradient and heat flow between ice grains is necessary for the rounded grains to form at the rates seen in nature [Perla, 1978, from Colbeck, 1981]. Consequently, the terminology "equilibrium growth form" was introduced.

The development of rounded grains will dominate as a growth form in relatively low temperature gradient conditions, when vapor diffusion is the primary means of metamorphism and growth is slow enough so that the rounded equilibrium form is produced. Development of the equilibrium form involves a reduction in surface energy, approaching a minimum. In this process, the presumed controlling mechanism is the difference in vapor pressure at the ice surfaces due to variations in radius of curvature. Mass will migrate from regions of higher vapor pressure (over ice with a smaller radius of curvature), to lower pressure regions. It is this equilibrium growth which is responsible, as an example, for the deterioration of the common six armed, star shaped ice crystal (stellar dendrite) present in new snow, into the more rounded configuration more typical of an older snow layer (Figure 2).

Due to geothermal heating from below, temperatures at the interface boundary between the ground and the snow usually remain near 0 ^{0}C throughout the winter, while the upper snow surface temperature is predominately governed by the ambient atmospheric temperature at the upper snow boundary. When the snow surface temperature is at or above that at the base, melting will occur. If the situation persists free water will eventually bring the snowpack to an isothermal condition typical of a liquid saturated snowpack.

Frequently, however, the surface temperature will be colder than the underlying snow and a temperature gradient is established across the pack. In the presence of sufficiently large temperature gradients (approximately >10 ^{0}C/m depending on temperature, pore size, grain size etc.), a striated faceted crystal, known as depth hoar, may develop. This type of development is known as constructive metamorphism, temperature gradient metamorphism [LaChapelle, 1969; Sommerfeld, 1969], or the kinetic growth form [Colbeck, 1981]. Temperature gradient magnitudes of 1 to 10 ^{0}C/m are reported to be typical in an alpine snowpack [Yosida, 1963], while gradients of 30 ^{0}C/m are reported annually on the Greenland ice sheet [Benson, 1962] and normal gradients of 100 ^{0}C/m and as high as 200 ^{0}C/m have been reported in central Alaska [Trabant and Benson, 1972].

A pressure gradient associated with thermal gradients will cause a vapor flux from the lower warmer regions toward the upper colder regions. Although conduction of heat is facilitated most readily through the solid

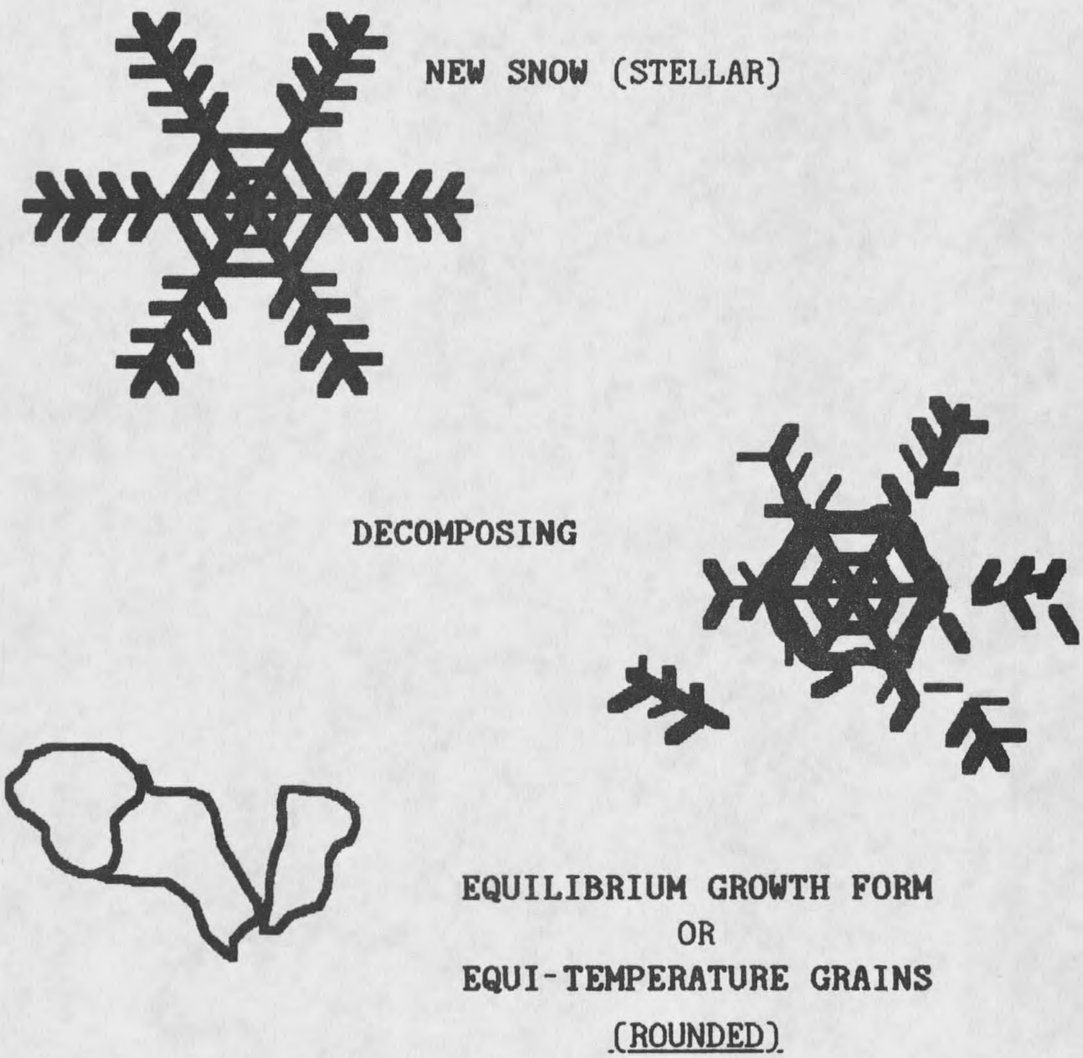

Figure 2. Equilibrium growth or equi-temperature metamorphism, representing the decomposition of unstable crystals into a more stable, rounded configuration.

ice phase, the heat flux will also instigate a mass flux. All of the mass transfer, however, need not take place through the tortuous labyrinth of pores. Mass flux may also proceed through the solid matrix by a "hand to hand" mechanism [Yosida and Kojima, 1950, from Akitaya, 1974], where ice will sublime from the top of a grain (at the bottom of a pore) and the water vapor will be deposited as ice on the bottom of a slightly colder grain above (at the top of the pore) (Figure 3).

A shallow snowpack with differing temperatures at the boundaries will obviously have a larger magnitude of temperature gradient than a thicker pack with the same thermal boundary conditions. As this condition implies, the kinetic growth form of crystal is most pronounced in colder atmospheric environments with shallow snow cover. Low density, poorly bonded snow will also promote faceted crystal growth. Larger pore size associated with lower density snow will allow less restricted vapor movement, encouraging grain growth.

Thermal conductivity in ice is greater than in vapor, so lower density, poorly bonded snow will not facilitate conductive heat flux as readily. Therefore, in a layered stratigraphy typical of a natural snowpack, the magnitude of temperature gradient in lower density, poorly bonded snow will be larger. Thermodynamic processes are generally accelerated at higher temperatures, all else being equal, so growth of depth hoar crystals is most apparent in the lower, warmer, more thermodynamically active regions of the snowpack. Regions of depth hoar crystal development also include small zones above and below crusts [Perla, 1978].

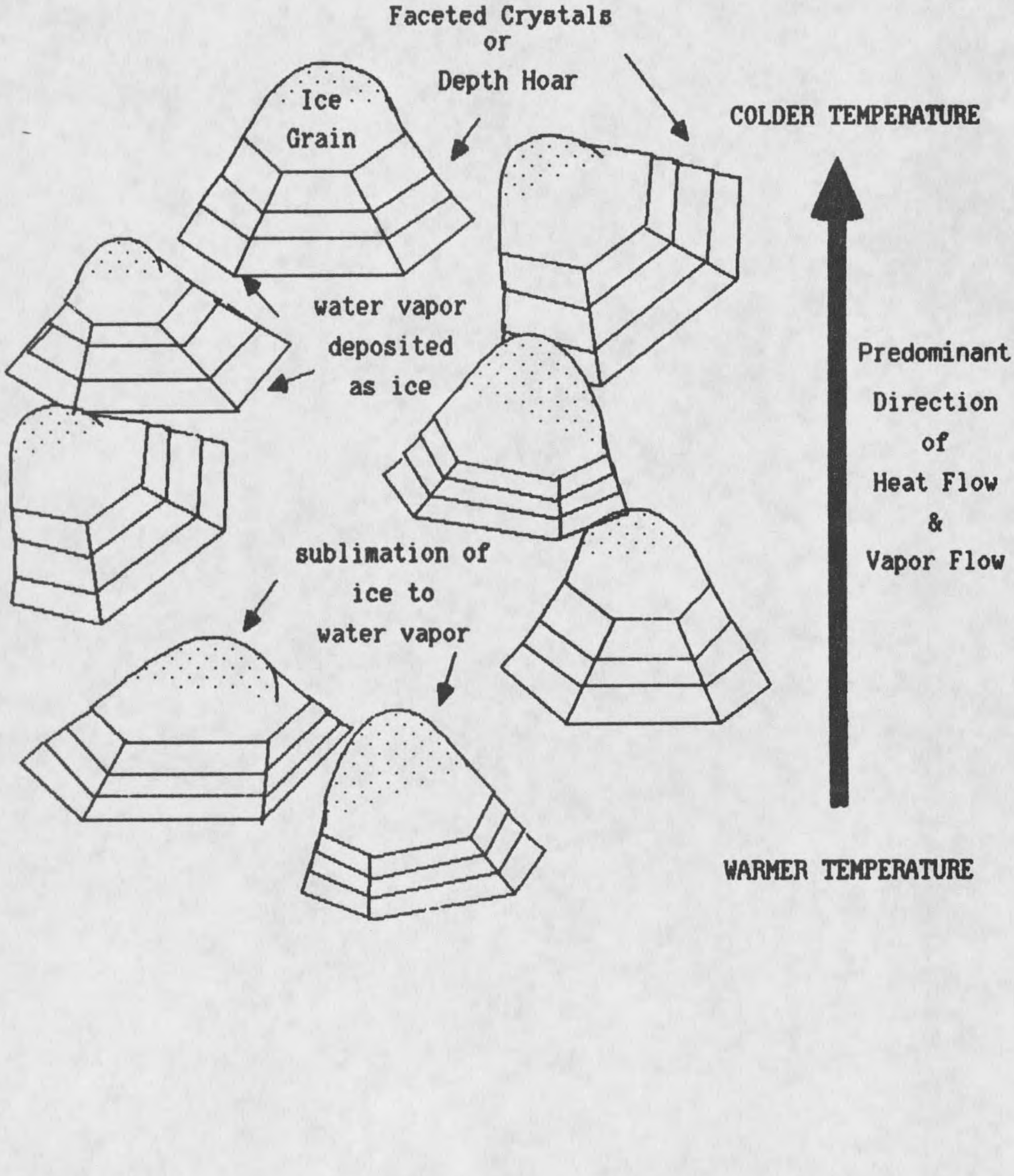

Figure 3. Kinetic growth or temperature gradient metamorphism, showing the direction of vapor flow as well as the orientation of crystal growth.

Importantly, in addition to the form which individual ice grains assume, the overall strength of the snow is affected by the metamorphic process. In the same conditions which dominate the equilibrium growth form, bonds called necks will develop between adjoining ice grains, in a process known as sintering. Presumably, radius of curvature differences will direct the vapor movement toward the lower pressure regions over the concave surfaces which exist when ice grains are brought into contact. It is the deposition of ice at these grain contact points which produce necking. This bonding together of the individual ice grains results in an overall strengthening of the snow. As the process proceeds, necks become larger and the grains lose volume resulting in a fine grained, structurally strong snowpack.

When processes governed by large temperature gradients predominate, a snow layer with little structural integrity develops. This layer consists of large faceted crystals which are poorly bonded together. In snow which has previously been exposed to equilibrium growth form conditions, the rounded grains and necks will disappear, and the noncohesive depth hoar develops. The type of depth hoar discussed here is of the skeleton type, but there also exists a solid type which forms in a lower temperature gradient or at higher snow density [Akitaya, 1974]. This solid type does not show the marked reduction in strength and large crystals typical of the skeleton form although it does have a faceted structure. Fine grained high density snow, greater than 350 kg/m^3, will result in solid type depth hoar being formed [Marbouty, 1980]. On the subject of structural strength of depth hoar, it is worth noting that of the three stages of crystal development, anhedral (early development), subhedral (intermediate development), and euhedral (advanced development), the intermediate subhedral stage is the weakest with a local relative maximum temperature gradient at that point [Bradley, Brown and Williams, 1977; Adams and Brown, 1982].

A natural snowpack is usually developed through a series of events and conditions over a period of time and will in general consist of a complex layered stratigraphy. The type of snow precipitated will vary greatly from event to event and will also vary in the course of any particular storm, depending on a myriad of climatic conditions effecting formation and deposition. In addition, as has been mentioned, varying climatic conditions between precipitation events will greatly alter the structure of the snowpack. The upper snow surface may be affected by wind, rain, sun and temperature causing crusts to develop, which are then buried by subsequent snowfall. Wind acting on the surface will fragment ice grains and result in a dense strong smooth layer. Free water formed at the surface from solar insolation, rain or warm atmospheric temperatures may then refreeze forming a stiff layer. Surface hoar which is deposited as a striated faceted crystal directly from the atmospheric vapor onto the snow surface in a manner similar to depth hoar may form a thin weak layer in the pack after it is buried [Lang, Leo and Brown 1984].

Layering will alter the metamorphic processes due to sudden changes in such things as density, permeability, conductivity, and grain and pore size. Metamorphic development of any particular layer is affected by the structure of the entire snowpack. In addition to weaknesses which may develop or persist as a result of the governing thermodynamic conditions, poor adhesion between layers frequently exist causing structural weaknesses at the interface between layers.

Structural integrity of a snow pack is of significant interest in such areas as over snow vehicle mobility and avalanche hazard evaluation. Failure of a specific zone may be significant. In the case of a vehicle moving over snow it may be significant how deep it will sink and if there is slip between some of the upper layers thereby causing a loss in traction and efficiency. The situation for an avalanche might be quite different since failure at deeper regions will cause a much greater volume of snow to be set into motion. Snow avalanches are extremely difficult to predict, since the medium being dealt with is so complex and thermodynamically active. Most avalanches occur in conjunction with a sudden increase in stress, most frequently caused by deposition of additional snow, or from human involvement such as the the relatively slight additional weight of a skier, climber or vehicle. However, delayed action avalanches also commonly occur, with no apparent warning. It is assumed that the combined processes of metamorphism including deformation due to stress must somehow lead to this final critical instability.

It is apparent that there are many varying conditions and processes which are intricately interwoven and whose interaction must be accounted for in any comprehensive analysis of snow as a material. Snow, and in fact multiphase granular materials in general, have been studied by examining discrete processes which affect its state. The work presented here is an attempt to develop a unified theory for snow in the field. Previous more specific studies such as those involving crystal development may then be used to assist in the interpretation of quantitative results. The continuum theory of multiphase mixtures is well suited to an application in snow. If the theory can be shown to be even modestly successful, the consequences are quite far reaching, since it demonstrates that this potentially powerful theory, although relatively new and with few applications thus far, can be applied to processes as complex as the one under consideration.

Theory of Multiphase Mixtures

The theory of mixtures deals with the stress, deformation and flow of materials, using the basic, but broad approach of continuum mechanics. Versatility, along with the visual and physical clarity of results is the reason the continuum branch of mechanics is so widely used in engineering applications. In fact, the strongest argument in favor of using the method is the "proof of agreement" with the physical processes which have been successfully modeled.

Analysis of a material as a continuous medium implies a disregard for the molecular structure, but instead envisions a continuous body composed of indefinitely divisible, infinitely small particles or points in space having no volume. This concept of a point is akin to the concept of a limit in differential calculus. Conceptually, the idea of a continuum assumes that in the neighborhood of every particle there is material present.

Basic to the assumptions of continuum mechanics is that all mathematical functions which are employed in a theory are continuous, except perhaps at surfaces or boundaries of regions. There are also some specialized situations where functions may be discontinuous such as the stress at a wave front; displacements across cracks, where the cracks are not considered as boundaries; etc.

Malvern [1969] divides the study of the theory of continuum mechanics into three distinct parts. These are,

> "... (1) **general principles**, assumptions, and consequences applicable to all continuous media, (2) **constitutive equations**, defining the particular idealized material, and (3) **specialized theories** of each individual idealized material built on the foundations of the general principles and constitutive equations of that material to the point where boundary-value problems are formulated and solved for application to specific problems.".

Historically, the great majority of continuum models have assumed a single continuous material. Many of these models have been very successful in obtaining the desired information. However, there are large classes of materials for which use of a single material is inappropriate to obtaining some specific information. Fick in 1855 is generally considered to have been the first person to put forth a mathematical theory dealing with the motion of continuous mixtures, in what is now commonly known as "Fick's Law". According to Bedford and Drumheller [1983] it was not until 1957 that the first balance and conservation equations for mixtures as a continuous medium were postulated, by Truesdell [1957].

Consider a continuous body comprised of a number of materials which are intermixed. Let each material itself be considered a continuous body, called a constituent. The entire collection of these constituents taken together is what is known as the mixture. Early work dealing with mixture theory was primarily concerned with mixtures of gases. A basic assumption in this work is that every constituent of the mixture is considered to be simultaneously present in the same differential region of space. Mixtures satisfying this requirement are referred to as "miscible mixtures". One result of this assumption is that in order to properly retain the generality of the theory, it is expected that any state variables relevant to equations which describe the constitution of an individual material (i.e. the constitutive equations), should incorporate these same variables into all constitutive equations for all constituents of the mixture. The assumption of a mixture composed of constituents which are ideally miscible is the foundation for what is considered the Classical Theory of Mixtures.

It is readily apparent that, while the classical theory is pertinent to a number of applications, the requirement that an element of the mixture contain all of the constituents simultaneously is quite limiting to its scope of applications. Immediately, any mixture which contains immiscible constituents is necessarily excluded from a strict application of the classical theory. Examples of such mixtures are fluid and/or vapor flow in porous or granular material, particle suspensions in fluid or vapor, and bubbly liquids.

Given the basis on which the theory of mixtures has developed, materials such as those just described do not even fit the classical definition of a mixture. However, adaptation to include these types of materials in the classification as a mixture have been successfully accomplished. These theories are to a large extent still in the theoretical development stage, and applications are as yet relatively few. Mixtures of this type are alternatively referred to as immiscible mixtures; mixtures