



An investigation of the adequacy of recontoured spoil sampling regulations using geostatistics  
by Charles Kinsey Hardy

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Land Rehabilitation

Montana State University

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**Abstract:**

Surface coal mines in the western United States are required to sample regraded spoils to determine if spoil handling procedures have been effective, to screen the material for adverse physicochemical properties, and to determine how adequately premining overburden information can predict the nature of this spoil material. At the Absaloka Mine in southeast Montana the surface 2.44 meters of regraded spoils were sampled in two depth increments, 0-1.22 meters and 1.22-2.44 meters, and at a sampling interval of approximately 69 meters as required by the Montana Department of State Lands. Geostatistical procedures were used to more objectively characterize the spoil material. The purpose of this study is to show how geostatistical techniques can be used to develop a more efficient approach to sampling regraded spoil.

The objectives of this study were to determine if significant differences in physicochemical properties exist between the two spoil sampling zones, to quantify the spatial aspects of spoil physicochemical properties using the semi-variogram, and demonstrate how geostatistical techniques can be used to develop sampling strategies for characterization of the regraded spoil.

Semi-variograms were computed from 240 samples for pH, EC (mmhos/cm), saturation percentage, SAR, ESP, and percent clay and sand. The semi-variograms had large nugget variances and ranges of influence that varied between 244 meters and 366 meters. Semi-variograms were fitted with spherical models and validated using jackknifing techniques. Block kriging was used to map the spoil properties and delineate areas of spoil that are potentially phytotoxic.

To characterize the spoils a two phase sampling strategy is proposed. Spoils are first sampled at a fixed sample spacing based on semi-variogram properties and kriging techniques. At a square grid sample spacing of 140 meters, kriging estimates could be made anywhere within the sample region for all the spoil properties investigated. Second phase sampling is implemented if problem areas of spoil are found during first phase sampling. To better characterize problem areas of spoil, the sample intensity is increased over the 140 meter sample intensity set during phase one sampling. Second phase sampling is based on the average estimation variance associated with a particular sample spacing. Curves were developed to help guide this additional sampling.

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## TABLE OF CONTENTS

	Page
LIST OF TABLES . . . . .	vi
LIST OF FIGURES . . . . .	vii
ABSTRACT . . . . .	xi
INTRODUCTION . . . . .	1
THEORY . . . . .	4
LITERATURE REVIEW . . . . .	10
Anisotropies . . . . .	10
Kriging Procedures . . . . .	12
Sampling Strategies . . . . .	15
MATERIALS AND METHODS . . . . .	18
Site Description . . . . .	18
Data Set . . . . .	18
Laboratory Analysis . . . . .	20
Statistical Analysis . . . . .	22
RESULTS AND DISCUSSION . . . . .	28
Comparing Differences Between Sampling Zones . . . . .	28
Semi-variograms and Anisotropies . . . . .	32
Validation . . . . .	34
Comparison of the Sampling Zones Using Spacial Properties . . . . .	36
Block Kriging . . . . .	40
Sampling Strategy . . . . .	43
SUMMARY AND CONCLUSIONS . . . . .	58
LITERATURE CITED . . . . .	61
APPENDICES . . . . .	64
APPENDIX A HISTOGRAMS . . . . .	65
APPENDIX B DIRECTIONAL SEMI-VARIOGRAMS . . . . .	72
APPENDIX C HISTOGRAMS OF ERRORS FROM JACKKNIFING . . . . .	79
APPENDIX D SURFACE PLOTS OF KRIGED VALUES . . . . .	83
APPENDIX E SAMPLE SPACING vs. ESTIMATION VARIANCE CURVES . . . . .	90

TABLE OF CONTENTS—Continued

	Page
APPENDIX F MAXIMUM BLOCK VALUE vs. SAMPLE SPACING . . . . .	94

## LIST OF TABLES

Table	Page
1. Unsuitability criteria for regraded spoil. . . . .	23
2. Summary statistics. . . . .	29
3. Paired t-test results. . . . .	30
4. Regression results, using the surface zone to predict values for the subsurface zone. . . . .	32
5. Validated semi-variogram models. . . . .	35
6. Summary statistics of kriging errors from semi-variogram validation . . . . .	37
7. Maximum and minimum block kriged values. Data taken from block kriged estimates made on both zones of spoil. . . . .	42
8. Corner value estimation results. . . . .	46
9. Area (hectares) at the study site meeting phase II sampling criteria. . . . .	53

## LIST OF FIGURES

Figure	Page
1. Idealized spherical semi-variogram model . . . . .	5
2. Histogram of errors from jackknifing for the spoil parameter percent clay. . . . .	9
3. Site location of the Absaloka Mine . . . . .	19
4. Location of regraded spoil sample sites at the Absaloka Mine. . . . .	21
5. Histogram for percent clay using data from the surface zone. . . . .	31
6. Histogram for percent clay using data from the subsurface zone. . . . .	31
7. Directional semi-variograms for percent clay, viewing the 90°, 67.5°, 45°, and 22.5° angles. . . . .	33
8. Directional semi-variogram for percent clay, viewing the 0°, -22.5°, -45°, and -67.5° angles. . . . .	33
9. Histogram of kriging errors for percent clay using data from both zones of spoil. . . . .	38
10. Surface plot for the spoil parameter percent clay. . . . .	41
11. Sample orientation within the estimation region for percent clay. . . . .	49
12. Surface plot of estimation variances . . . . .	50
13. Curve showing the relationship between average estimation variance and sample spacing for percent clay. . . . .	51
14. Maximum block value curve for percent clay. . . . .	52
15. Surface plot of areas where average estimation variance curves can be applied for percent clay. . . . .	55
16. Surface plot of areas where maximum block value curves are applicable for percent clay. . . . .	56
17. Histogram for EC using data from the surface zone. . . . .	66

LIST OF FIGURES—Continued

Figure	Page
18. Histogram for EC using data from the subsurface zone. . . .	66
19. Histogram for saturation percentage using data from the surface zone. . . . .	67
20. Histogram for saturation percentage using data from the subsurface zone. . . . .	67
21. Histogram for SAR using data from the surface zone. . . . .	68
22. Histogram for SAR using data from the subsurface zone. . .	68
23. Histogram for ESP using data from the surface zone. . . . .	69
24. Histogram for ESP using data from the subsurface zone. . .	69
25. Histogram for percent sand using data from the surface zone. . . . .	70
26. Histogram for percent sand using data from the subsurface zone. . . . .	70
27. Histogram for pH using data from the surface zone. . . . .	71
28. Histogram for pH using data from the subsurface zone. . . . .	71
29. Directional semi-variograms for EC, viewing the 90°, 67.5°, 45°, and 22.5° angles. . . . .	73
30. Directional semi-variograms for EC, viewing the 0°, -22.5°, -45°, and -67.5° angles. . . . .	73
31. Directional semi-variograms for saturation percentage, viewing the 90°, 67.5°, 45°, and 22.5° angles. . . . .	74
32. Directional semi-variograms for saturation percentage, viewing the 0°, -22.5°, -45° and the -67.5° angles. . . . .	74
33. Directional semi-variograms for ESP, viewing the 90°, 67.5°, 45°, and 22.5° angles. . . . .	75
34. Directional semi-variograms for ESP, viewing the 0°, -22.5°, -45° and the -67.5° angles. . . . .	75
35. Directional semi-variograms for SAR, viewing the 90°, 67.5°, 45°, and 22.5° angles. . . . .	76

LIST OF FIGURES—Continued

Figure	Page
36. Directional semi-variograms for SAR, viewing the 0°, -22.5°, -45° and the -67.5° angles. . . . .	76
37. Directional semi-variograms for pH, viewing the 90°, 67.5°, 45°, and 22.5° angles. . . . .	77
38. Directional semi-variograms for pH, viewing the 0°, -22.5°, -45° and the -67.5° angles. . . . .	77
39. Directional semi-variograms for percent sand, viewing the 90°, 67.5°, 45°, and 22.5° angles. . . . .	78
40. Directional semi-variograms for percent sand, viewing the 0°, -22.5°, -45° and the -67.5° angles. . . . .	78
41. Histogram of errors from jackknifing for EC using data from both zones of recontoured spoil. . . . .	80
42. Histogram of errors from jackknifing for saturation percentage using data from both zones of recontoured spoil. . . . .	80
43. Histogram of errors from jackknifing for SAR using data from both zones of recontoured spoil. . . . .	81
44. Histogram of errors from jackknifing for ESP using data from both zones of recontoured spoil. . . . .	81
45. Histogram of errors from jackknifing for pH using data from both zones of recontoured spoil. . . . .	82
46. Histogram of errors from jackknifing for percent sand using data from both zones of recontoured spoil. . . . .	82
47. Surface plot for the spoil parameter EC. . . . .	84
48. Surface plot for the spoil parameter saturation percentage. . . . .	85
49. Surface plot for the spoil parameter SAR. . . . .	86
50. Surface plot for the spoil parameter ESP. . . . .	87
51. Surface plot for the spoil parameter pH. . . . .	88
52. Surface plot for the spoil parameter percent sand. . . . .	89

LIST OF FIGURES—Continued

Figure	Page
53. Average estimation variance from block kriging as a function of sample spacing for pH. . . . .	91
54. Average estimation variance from block kriging as a function of sample spacing for electrical conductivity. . .	91
55. Average estimation variance from block kriging as a function of sample spacing for saturation percentage. . . .	92
56. Average estimation variance from block kriging as a function of sample spacing for SAR. . . . .	92
57. Average estimation variance from block kriging as a function of sample spacing for ESP. . . . .	93
58. Average estimation variance from block kriging as a function of sample spacing for percent sand. . . . .	93
59. Maximum block value as a function of sample spacing for pH.	95
60. Maximum block value as a function of sample spacing for electrical conductivity. . . . .	95
61. Maximum block value as a function of sample spacing for electrical conductivity. . . . .	96
62. Maximum block value as a function of sample spacing for saturation percentage. . . . .	96
63. Maximum block value as a function of sample spacing for SAR. . . . .	97
64. Maximum block value as a function of sample spacing for ESP. . . . .	97
65. Maximum block value as a function of sample spacing for percent clay. . . . .	98
66. Maximum block value as a function of sample spacing for percent sand. . . . .	98

**ABSTRACT**

Surface coal mines in the western United States are required to sample regraded spoils to determine if spoil handling procedures have been effective, to screen the material for adverse physicochemical properties, and to determine how adequately premining overburden information can predict the nature of this spoil material. At the Absaloka Mine in southeast Montana the surface 2.44 meters of regraded spoils were sampled in two depth increments, 0-1.22 meters and 1.22-2.44 meters, and at a sampling interval of approximately 69 meters as required by the Montana Department of State Lands. Geostatistical procedures were used to more objectively characterize the spoil material. The purpose of this study is to show how geostatistical techniques can be used to develop a more efficient approach to sampling regraded spoil.

The objectives of this study were to determine if significant differences in physicochemical properties exist between the two spoil sampling zones, to quantify the spatial aspects of spoil physicochemical properties using the semi-variogram, and demonstrate how geostatistical techniques can be used to develop sampling strategies for characterization of the regraded spoil.

Semi-variograms were computed from 240 samples for pH, EC (mmhos/cm), saturation percentage, SAR, ESP, and percent clay and sand. The semi-variograms had large nugget variances and ranges of influence that varied between 244 meters and 366 meters. Semi-variograms were fitted with spherical models and validated using jackknifing techniques. Block kriging was used to map the spoil properties and delineate areas of spoil that are potentially phytotoxic.

To characterize the spoils a two phase sampling strategy is proposed. Spoils are first sampled at a fixed sample spacing based on semi-variogram properties and kriging techniques. At a square grid sample spacing of 140 meters, kriging estimates could be made anywhere within the sample region for all the spoil properties investigated. Second phase sampling is implemented if problem areas of spoil are found during first phase sampling. To better characterize problem areas of spoil, the sample intensity is increased over the 140 meter sample intensity set during phase one sampling. Second phase sampling is based on the average estimation variance associated with a particular sample spacing. Curves were developed to help guide this additional sampling.

## INTRODUCTION

Sampling and analysis of regraded spoils is a regulatory requirement for surface coal mines in the western United States. As part of the premine baseline data collection process overburden is drilled and sampled for hazardous materials. One consideration of the sampling and testing program is to screen the regraded material for deleterious chemical and physical properties. This is done to determine if spoil handling procedures have been effective. Sampling regraded spoils serves to determine how adequately this overburden information can predict the nature of the regraded spoils. Regraded spoils information will also be reviewed during the bond release process.

Data for this study come from Westmoreland Resources' Absaloka Mine in southeast Montana. These data were taken from an existing data base kept on file by the Montana Department of State Lands (DSL), Reclamation Division. Of the coal mines in Montana the Absaloka Mine has the most intensively sampled regraded spoil. The mine has been operating since 1974, therefore sufficient data exist to make a geostatistical analysis.

Appropriate sampling intensity and parameters to be analyzed are determined on a site-specific basis according to mine permitting agreement. Due to this flexibility, sampling and testing requirements can vary significantly between mines. The guidelines for sampling regraded spoils require samples to be taken prior to topsoiling and

revegetation activities (Montana Department of State Lands, 1983). The spoil sampling should be conducted to a depth of 2.44 meters (8 feet) with samples taken from the surface 0-1.22 meters (0-4 feet) of spoil and the subsurface 1.22-2.44 meters (4-8 feet) of spoil. Spoils should also be sampled on approximately 91 meter (300 foot) centers. Parameters to be analyzed include: saturation percentage, pH, conductivity, SAR and/or ESP, particle size distribution and bulk density.

Geostatistics is a relatively new statistical technique developed mainly by Matheron (1963) and Krige (1966) for the estimation of ore reserves in mining. There are fundamental differences between geostatistics and classical statistics according to Matheron; classical methods are unable to adequately treat the spacial aspect of data, and neighboring samples may not be independent of each other and, as a result, samples taken close together tend to be more similar than those that are far apart. By taking into account spacial dependence in the data, geostatistics and particularly kriging procedures, will yield unbiased estimates of spoil properties and minimize the variance associated with the estimate. In the sense that it minimizes the variance associated with an estimate, kriging is an optimum means of interpolation.

Mining companies have been collecting regraded spoil data for years with little attempt to analyze these data. For many mines sufficient data have been collected to make a geostatistical analysis feasible. With this "new" analytical tool a more critical assessment of a regraded spoil sampling program can be made. It is the intent of

this study to use a geostatistical approach to assess the adequacy of regraded spoil sample spacing requirements. \*

The objectives of this study were too:

- 1) Determine whether significant differences exist between the two spoil sampling zones. Spoils are sampled from 0-1.22 meters (0-4 feet) and from 1.22-2.44 meters (4-8 feet) below the surface.
- 2) Determine if spacial dependencies in regraded spoil physical and chemical properties exist at the Absaloka Mine.
- 3) Demonstrate that geostatistical techniques can be used to improve sampling strategies for characterization of the regraded spoil.

### THEORY

Geostatistics is based on the theory of regionalized variables. A regionalized variable is a variable distributed in space and is in part dependent on the spacial position of the variable. A regionalized variable possesses two fundamental characteristics: (i) a local, random erratic component similar to that of a random variable; (ii) a general structural aspect which can be described by a samples spacial relationship to neighboring samples. Unlike most classical statistics, the assumption of independence is not made. A comprehensive review of the practical application of geostatistics to the problems of ore reserve estimation is given by Journel and Huijbregts (1978).

The semi-variogram is the basic tool of geostatistics. All other geostatistical techniques are contingent on the semi-variogram (i.e. kriging, determination of estimation variance). Semi-variance can be defined by the following equation:

$$\gamma(h) = \frac{1}{2} E[ \{ Z(x+h) - Z(x) \}^2 ] \quad [1]$$

$Z(x)$  and  $Z(x+h)$  are two numerical values separated by a vector  $h$ . The vector  $h$  has a distance and direction component and semi-variance is calculated for as many different distances as possible. The semi-variogram is a plot of semi-variance on the vertical axis and distance between sample pairs, or lag on the horizontal axis (Figure 1).

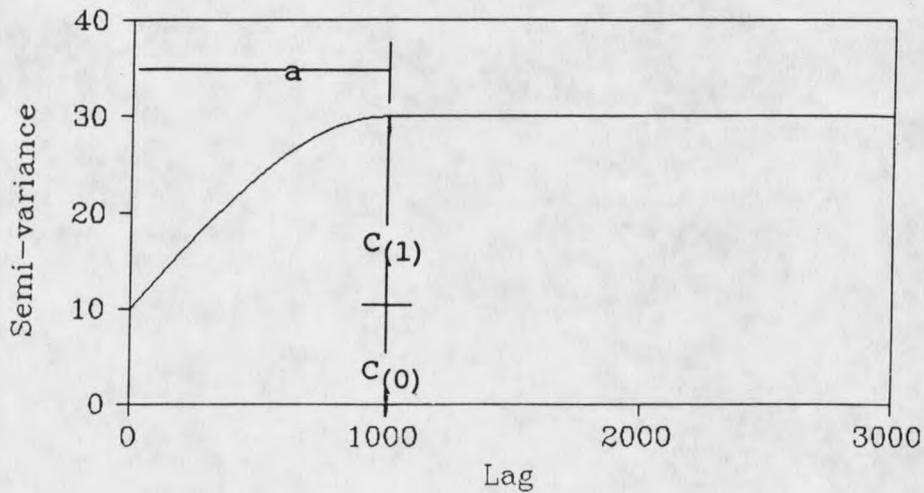


Figure 1. Idealized spherical semi-variogram model with nugget variance,  $C_{(0)}=10$ ; sill,  $C_{(0)}+C_{(1)}=30$ ; and range,  $a=1000$ .

Figure 1 shows an example of a semi-variogram model. Models are fitted to experimental semi-variograms to describe the spatial character of a regionalized variable. Common models used are spherical, exponential and linear. The model shown is a spherical model with a range of influence ( $a$ ) of 1000, a sill value ( $C_{(0)}+C_{(1)}$ ) of 30 and a nugget value ( $C_{(0)}$ ) of 10. The spherical model is described by the equations:

$$\begin{cases} \tau(h) = C_{(0)} + C_{(1)} \{1.5(h/a) - 0.5(h/a)^3\} & \text{for } 0 < h \leq a \\ \tau(h) = C_{(0)} + C_{(1)} & \text{for } h > a. \end{cases} \quad [2]$$

In these equations "h" is the separating distance between points, or lag. The nugget value represents a random component to the semi-variance. The nugget value  $C_{(0)}$  can't be explained by a samples spatial location and is due to errors in measurement and micro-

variabilities of the parameter of interest. The range of influence (a) represents the distance within which samples are spatially correlated. Beyond this range samples are independent of each other. The value at which  $\tau(h)$  levels out is called the sill. It consists of the nugget variance,  $C(0)$ , plus a component,  $C(1)$ , which represents that portion of the semi-variance which is due to spacial dependence in the data.

Kriging uses the principle of weighted local averaging which can give estimates of spoil properties at unknown locations and is therefore a type of interpolation. Kriging provides the best linear unbiased estimate of the unknown characteristic with an associated estimation variance. Kriging estimates were made using a data set of spoil parameters and the semi-variogram model describing the spacial variability in the studied zone. An excellent review of kriging, that used soil properties is given by Burgess and Webster (1980 a,b,c) and McBratney and Webster (1983).

The kriging system predicts values at unknown sites by appropriately weighing adjacent known values through the use of the semi-variogram. Each estimated value,  $Z^*(x_0)$ , at some unobserved location,  $x_0$ , is determined from a linear combination of the known values  $Z(x_i)$ ,  $i=1,2,3,\dots, n$ . Thus,

$$Z^*(x_0) = \sum_{i=1}^n \Gamma_i Z(x_i) \quad [3]$$

where  $\Gamma_i$  are the weights applied to each sample in the kriging neighborhood. The  $\Gamma_i$  associated with each  $Z(x_i)$  are chosen with the constraint that

$$\sum_{i=1}^n \Gamma_i = 1 \quad [4]$$

which insures unbiasedness (Tabor et al. 1984).

Minimizing the error variance associated with each estimate involves solving a linear system of equations. For punctual kriging this involves finding the partial derivatives with respect to each  $\Gamma_i$  and introduces a Lagrange parameter  $\mu$ . The weights,  $\Gamma_i$ ,  $i=1,2,3,\dots,n$ , and the Lagrangian multiplier,  $\mu$ , were obtained by solving the linear system

$$[A] \begin{bmatrix} \Gamma \\ \mu \end{bmatrix} = [b]. \quad [5]$$

The  $n+1$  by  $n+1$  matrix A contains the covariances between all  $n$  points within the estimation neighborhood. The  $n+1$  vector b contains the average covariances between the observed points in the estimation neighborhood and the point to be estimated. The solution vector  $\begin{bmatrix} \Gamma \\ \mu \end{bmatrix}$  contains the weights and Lagrangian multiplier. Covariances are determined from the semi-variogram model. The weights are then applied to equation [3] to obtain the punctually kriged value at each location (Trangmar 1986).

The minimum estimation variance,  $\sigma_E^2$  for each point was obtained by solving

$$\sigma_E^2 = [b]^T \begin{bmatrix} \Gamma \\ \mu \end{bmatrix}. \quad [6]$$

The weights used in the kriging system take account of the known spatial dependencies expressed in the semi-variogram and the geometric relationships among the observed points. In general, near points carry more weight than distant points, points that occur in clusters carry less weight than lone points, and points lying between the point to be

interpolated and more distant points screen the distant points so that the latter have less weight than they would otherwise (Burgess and Webster 1980a).

The mathematical models fitted to the semi-variograms are used in subsequent applications, e.g. kriging. To date, there is no foolproof, purely objective method for fitting models to sample semi-variograms. A method which is commonly used to cross-validate semi-variogram models is jackknifing or leave-one-out validation (Morkoc et al. 1987). When jackknifing, a data point is eliminated and a local estimate of the eliminated point is made, using kriging procedures, from remaining data. This process is carried out for all samples in the area of interest. The statistics of the errors between the measured value,  $Z(x)$  and the estimated value,  $Z^*(x)$  are analyzed to see if the model is acceptable.

For a model to be considered valid a number of requirements are necessary. Since kriging is an exact interpolator, an estimated value should be equal to the measured value, i.e.,  $[Z^*(x)=Z(x)]$ . The observed distribution of errors,  $[Z^*(x)-Z(x)]$  should have a mean equal to zero and the variance of actual errors should equal the kriging variance (Journel and Huijbregts 1978). The variance of actual errors, and the kriging variance have the same mathematical expectation ( $E\{[Z(x)-Z^*(x)]^2\}$ ). Also, 95% of the observed errors should fall within  $\pm 2\sigma_E$  of the mean.

As an example the histogram of errors for the spoil parameter clay percentage is shown in Figure 2. For this particular semi-variogram model clay percentage values from both spoil sampling zones are used

and 456 samples are used in the validation process. The model used is spherical with a nugget variance of 22, a sill of 45, and a range of influence of 366 meters (1200 feet). The errors are approximately normally distributed with a mean equal to  $-0.0092\%$  clay, a variance of errors equal to 37.49 and a kriging variance equal to 37.44. Also, 94.52% of these data fall within two standard deviations of the mean. This model is unbiased since the mean of the errors is essentially zero when compared to a mean percent clay of 26.8. The standard normal confidence interval ( $\pm 2\sigma_E$ ) correctly estimates the 95% confidence interval and the kriging variance is approximately equal to the error variance.

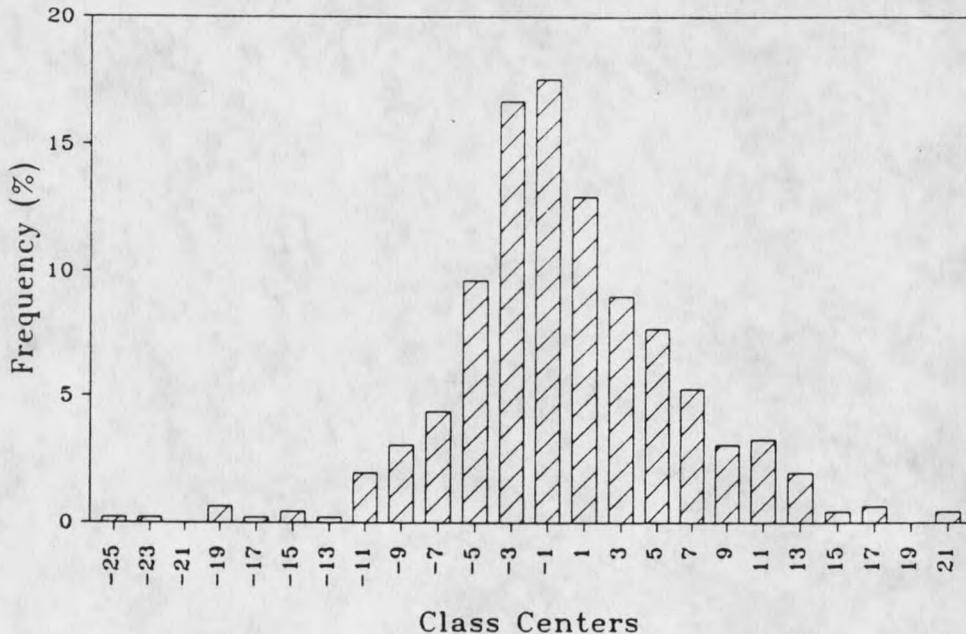


Figure 2. Histogram of errors from jackknifing for the spoil parameter percent clay.

## LITERATURE REVIEW

Geostatistics or the Theory of Regionalized Variables was first applied to problems in mining and geology. The technique is firmly established in the mining industry for the estimation of ore reserves. However, geostatistical techniques can be used whenever a sample value is expected to be affected by its position and its relation to its neighbors. Recently, geostatistics has been applied to spatially related data by plant, soil, and environmental scientists.

An early study examining the rates of spacial variability of soil properties using geostatistical methods was performed by Campbell (1978). Spacial variation of sand and pH measurements were analyzed using the semi-variogram on contiguous delineations of two soil series in Kansas. The soil property pH had random variation within both areas implying no spacial correlation between the data. In the soil series derived from glacial till, sand showed a range of influence of at least 40 m. The soil series derived from fine textured sediments produced a range of influence equal to 30 m. It was concluded that this information can be used to select optimum spacing of samples when designing soil sampling plans. \*

### Anisotropies

Anisotropies are variations in spatially related properties with direction. Anisotropies are analyzed by computing semi-variograms for different directions. The properties of the directional semi-

variograms are usually accounted for when modeling the spacial variation of the property of interest. Including anisotropies in the model will generally improve the quality of the estimates made when kriging.

Burgess and Webster (1980a) found anisotropic variation in soil stone content at the Welsh Plant Breeding Station. An anisotropic linear semi-variogram model was used. Mode of deposition of the soil was shown to have an influence on the spacial structure of the semi-variograms. The soil was fluviially deposited and a general trend in stone content was present over the study area. Semi-variograms along the strike were more similar and had a lower slope than semi-variograms in the direction of the dip. Klusman (1985) found anisotropic variation in soil molybdenum and also used a linear anisotropic semi-variogram model.

Tabor et al. (1984) compared isotropic and anisotropic semi-variogram models of nitrate in irrigated cotton petioles. Petiole sampling and analysis is useful in monitoring the nitrogen status of cotton fields. The models were compared using the jackknifing technique. The anisotropic model produced a lower error variance indicating a better fit than the isotropic model. The anisotropic model portrayed the strong influence cultural practices such as direction of rows and irrigation has on the variability of petiole nitrate.

Wollum and Cassel (1984) used semi-variograms to analyze the spacial distribution of Rhizobium japonicum in two cultivated North Carolina soils. The variance structure was found to be directional.

It was different in the direction "parallel to the row" compared to the direction "perpendicular to the row". Possible contributors to the changes in rhizobia populations observed were root densities, inherent soil properties, seasonality, and management practices.

Quantitative analysis of anisotropic spacial dependence can aid interpretation of soil genesis. Trangmar et al. (1986a) used anisotropic spacial dependencies of particle size fractions, pH, and 25% HCL-extractable P to analyze differences in the main soil-forming factors in Sitiung, West Sumatra. Directions of maximum variation coincided with the main axis of volcanic tuff fallout, deposition of alluvium and the general sequence of soil weathering in the region.

#### Kriging Procedures

Kriging procedures are used to make estimates of spatially related properties at unrecorded places with a known and minimum variance. Kriging is generally used to create maps of regionalized variables, also variances associated with the kriged estimates can be mapped. A variety of kriging techniques have been used in the environmental sciences including: ordinary kriging (punctual or block), universal kriging and co-kriging.

Burgess and Webster (1980a) used ordinary punctual kriging to map sodium content, stoniness, and cover-loam thickness in Central Wales and Norfolk. As the name implies punctual kriging is used to make point estimates of soil properties. The maps produced by punctual kriging showed intricate isarithms and substantial short range variation, caused by semi-variogram models with large nugget variances.

Another paper by Burgess and Webster (1980b) used ordinary block kriging, instead of punctual kriging, to map the above soil properties. Block kriging produces average estimates of soil properties over areas rather than point estimates. Estimation variances for block kriging are much smaller and the maps produced are considerably smoother than the punctually kriged maps, showing a more distinct and purposeful regional pattern. The large nugget variances for these soil properties make the largest contribution to the estimation variances. Maps produced by punctual kriging are erratic and have marked discontinuities near the data points. The authors concluded that block kriging is likely to prove more appropriate than punctual kriging for estimating values of soil properties. This is because land managers will usually be interested in average values over areas rather than point values.

Another kriging technique demonstrated by Webster and Burgess (1980c) is universal kriging. Universal kriging is a form of interpolation that takes account of local trends, or drift in the data, which are identified by structural analysis. Ordinary kriging operates under the assumption that drift is not present in the data. Universal kriging was applied to measurements of electrical resistivity made in the soil at 1 m intervals at Bekesbourne, Kent, United Kingdom. Structural analysis showed the data could be represented as a series of linear drifts over distances of 4 m to 8 m. Semi-variograms of residuals from drift showed negligible nugget variance, and were used to kriging missing values at the site.











































































































































































