MONTANA STATE UNIVERSITY

Applications of the Green-Ampt infiltration model to watersheds in Montana and Wyoming
by Joseph Alphonse Van Mullem

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University
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Abstract:
Improved prediction of infiltration and runoff volumes from rainfall events will result in improved predictions of peak discharge from watersheds and better designs of hydraulic structures. The adaptability of the Green-Ampt infiltration model is shown by its application to 12 range land and cropland watersheds in Montana and Wyoming. Using soil parameters derived from data in standard USDA soil surveys, 99 rainfall events were modeled. The runoff distributions obtained from the model were then used with a hydrograph model to predict the peak discharge from the watershed. Procedures were developed and applied for adjusting the Green-Ampt infiltration parameters for various cover and condition classes. Suggested values for interception and depression storage were developed for cropland and rangeland. The model was applied to areas of up to 54 square miles with a wide variety of soil and cover conditions. A method for averaging the. Green-Ampt parameters was also developed. The runoff volumes and peak discharges were compared with the measured values and with those predicted by the SCS curve number procedure. The Green-Ampt model predicted both the runoff volume and peak discharge better than the curve number model.

APPLICATIONS OF THE GREEN-AMPT INFILTRATION MODEL TO WATERSHEDS IN MONTANA AND WYOMING

by

Joseph Alphonse Van Mullem Jr.

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This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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TABLE OF CONTENTS

TABLE OF CONTENTS - Continued

LIST OF TABLES

LIST OF TABLES--Continued

LIST OF FIGURES

ABSTRACT

Improved prediction of infiltration and runoff volumes from rainfall events will result in improved predictions of peak discharge from watersheds and better designs of hydraulic structures. The adaptability of the Green-Ampt infiltration model is shown by its application to 12 rangeland and cropland watersheds in Montana and Wyoming. Using soil parameters derived from data in standard USDA soil surveys, 99 rainfall events were modeled. The runoff distributions obtained from the model were then used with a hydrograph model to predict the peak discharge from the watershed. Procedures were developed and applied for adjusting the Green-Ampt infiltration parameters for various cover and condition classes. Suggested values for interception and depression storage were developed for cropland and rangeland. The model was applied to areas of up to 54 square miles with a wide variety of soil and cover conditions. A method for averaging the Green-Ampt parameters was also developed. The runoff volumes and peak discharges were compared with the measured values and with those predicted by the SCS curve number procedure. The Green-Ampt model predicted both the runoff volume and peak discharge better than the curve number model.

CHAPTER 1

INTRODUCTION

The prediction of both the volume and rate of runoff from a watershed from a rainfall event is vital for good design of hydraulic structures. Since the part of the rainfall that infiltrates into the soil is usually greater than the part that runs off, a good estimate of the runoff requires a good estimate of the infiltration.

Green and Ampt(1911) developed their infiltration equation to describe how water entered the soil from a simple application of Darcy's Law. During recent years it has received increased attention as a method for predicting infiltration from rainfall events. Mein and Larson (1971) showed the applicability of the equation for the conditions of constant rainfall intensity and homogeneous soil. Although several empirical equations such as Kostiakov and Horton are popular because of their simplicity and capability of fitting most infiltration data, they contain parameters which are difficult to predict because they have no physical significance. (Mein and Larson, 1973) One-dimensional vertical moisture movement can be described by a second order nonlinear partial differential equation obtained from a combination of Darcy's Law applied to unsaturated flow and the equation of continuity. The resulting equation is sometimes referred to as the Richards equation. (Childs, 1969) It must be solved numerically and requires the relationships of conductivity, and suction versus moisture content to be known. The Green-Ampt equation is both simple and has physically based parameters which can be related to other soil properties.

A method of computing the Green-Ampt parameters from data available in the standard USDA soil surveys (Rawls et al,1982) makes the application of the equation possible wherever soil surveys have been made. The infiltration model has been used successfully to predict infiltration and runoff volumes with laboratory and small plot data. More recently, Rawls and Brakensiek (1986) applied a Green-Ampt model to predict runoff volumes from natural rainfalls on single land use areas of up to 10 acres in size.

For the model to be fully useful there must be methods to easily handle the wide variety of soil and cover conditions found in working-sized watersheds. A method to estimate the Green-Ampt parameters for areas where soil surveys have not been completed is also desired. Prior to this study, peak discharge estimates based on Green-Ampt infiltration have not been reported in the literature.

Goal and Objectives

The goal of this study was to illustrate the advantages of the Green-Ampt infiltration model and to improve its usefulness by relating the models parameters to soil and cover characteristics. The modeling was done for 99 storms and for 12 watersheds in Montana and Wyoming.

Objectives of the study were:

1) To apply the Green-Ampt model to watersheds with areas from less than one to over 50 square miles.

2) To illustrate improved methods of computing Green-Ampt parameters from soils data, and to develop methods of modifying the parameters to reflect cover conditions.

3) To develop methods for averaging the Green-Ampt parameters for the different soils and cover conditions found in a watershed.

4) To relate the Green-Ampt parameters to the SCS Curve Numbers and thereby provide a method of estimating the parameters where soils data are not available.

5) To compare the runoff volumes and the peak discharges obtained from the Green-Ampt model to those obtained by using the SCS curve number model and with observed volumes and peaks.

CHAPTER 2

LITERATURE REVIEW

The Green-Ampt Equation

The Green-Ampt equation was one of the earliest physical infiltration equations developed.(Green-Ampt, 1911) It is based on Darcy's Law which states that the rate of water movement through a soil, v, is proportional to the head loss dh and inversely proportional to the length of the flow path dL.

$$v = K \frac{dh}{dL} \qquad (1)$$

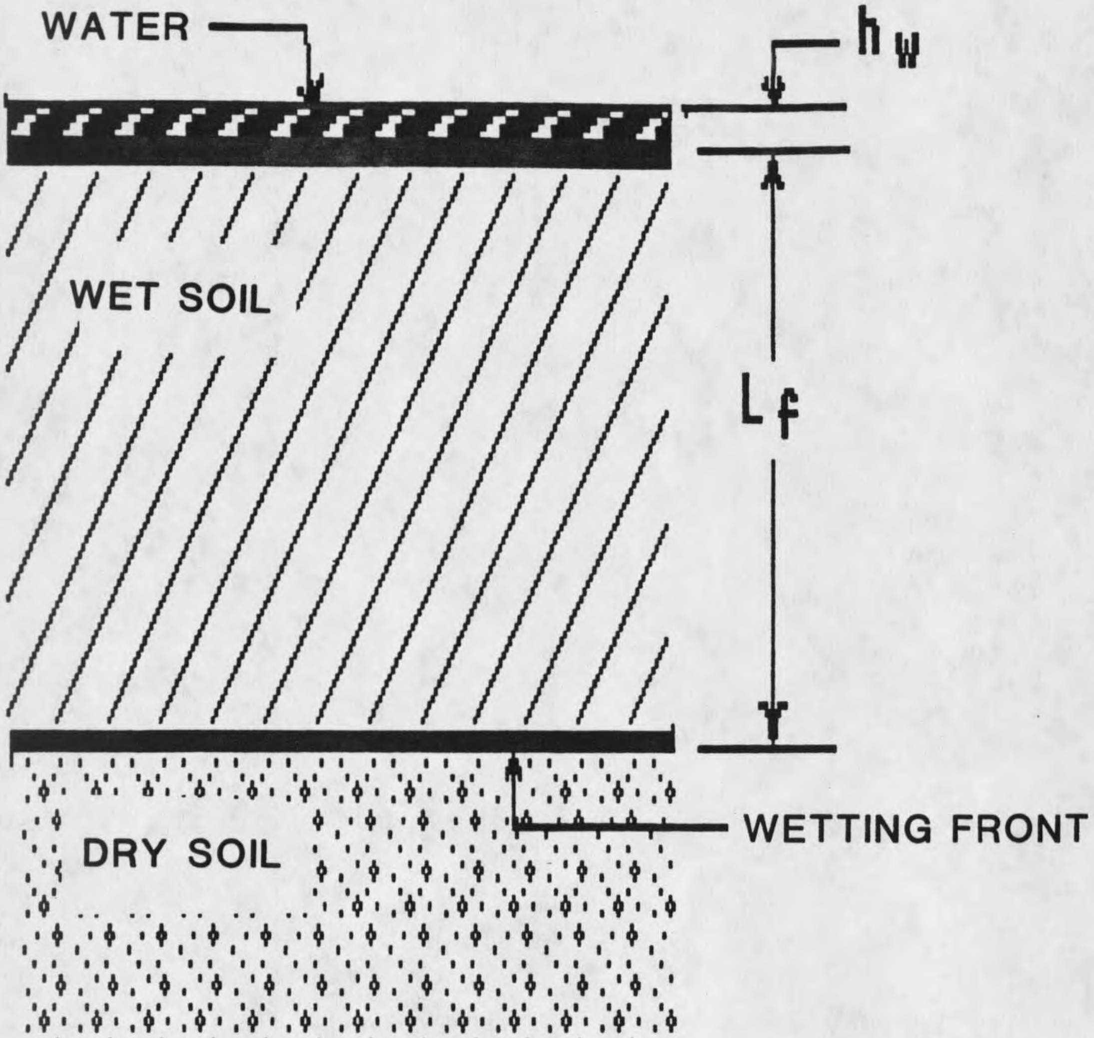
Where K is a proportionality constant defined as the hydraulic conductivity.

Green and Ampt considered the wetting front as an abrupt interface between wetted and nonwetted material. If y_f is the capillary suction or the capillary pressure head at the wetting front, L_f is the depth of the wetting front, and h_w is the depth of the water above the soil, then referring to Figure 1 and writing Darcy's Law

$$v = K \frac{L_f + y_f + h_w}{L_f} \qquad (2)$$

Then with f representing the infiltration rate, F the volume

Figure 1. Definition diagram for Green-Ampt equation



of infiltration per unit area, n the soil moisture deficit, and h_w considered negligible for most runoff conditions;

$$f = v \tag{3}$$

and

$$F = nL_f \tag{4}$$

where $n = \theta_e - \emptyset_i$ (5)

θ_e = effective soil porosity

$\emptyset_i$ = initial water content

combining

$$f = K \frac{(F + ny_f)}{F} \tag{6}$$

The equation is often integrated over time and can be solved for accumulated infiltration F at any time t. That step is not necessary for use in the computer model and the infiltration rate equation shown above is used in this study.

The value of K under conditions of infiltration has been found to be less than K at saturation because of entrapped air which prevents complete saturation. Bouwer (1982) suggests using a value of 1/2 of the saturated hydraulic conductivity for K in the Green-Ampt equation.

The assumption that an abrupt interface occurs between the wetted and nonwetted soil is not correct. Under true infiltration flow the wetted front is more diffuse and the water content as well as the hydraulic conductivity in the wetted zone increase with time. However, the equation has been shown to satisfactorily predict infiltration rates and volumes over time and is therefore useful for practical applications.

The advantage which the Green-Ampt equation has over empirical equations is that the parameters are physically based and can be obtained from measurable properties of the soil.

The Green-Ampt Soil Parameters

The Green-Ampt equation for infiltration rate

$$f = K\frac{(F + ny_f)}{F} \qquad (6)$$

has four unknowns K, F, n, and y_f which must be determined before the infiltration rate at any time can be found. The accumulated infiltration F is equal to zero at the beginning of the storm since all previous rainfall is assumed to be thoroughly mixed and accounted for with the parameter n.

The other parameters are inherent to the soil, the soil cover and the initial soil moisture. When the normal range of each parameter is examined it becomes obvious that the hydraulic conductivity K is by far the most variable and therefore the most important parameter. The effective porosity is normally less than 0.6. Therefore n could vary from some small value to 0.6. The capillary pressure y_f varies from about 1 inch to about 40 inches. K however varies over several orders of magnitude from about .002 to over 2 inches per hour for normal soils.

The general relationship between K and particle size has long been known. It can be shown by dimensional analysis that

$$K = \frac{Cd^2 \gamma}{\mu} \qquad (7)$$

where C = a dimensionless constant

d = a representative pore diameter

γ = specific weight of water

μ = dynamic viscosity of water

The factor d is usually assumed to be proportional to some representative grain diameter and C depends on other factors which affect flow including porosity, packing, and grain size distribution and shape. (Chow, 1964).

A number of formulas have been developed over the years to predict hydraulic conductivity or the related parameter permeability from other soil factors. These include Fair and Hatch (1933), Krumbein and Monk (1942), and Griffith (1955). A brief literature survey on earlier research relating grain size distribution to permeability was done by Masch and Denny (1966).

While past studies have proven statistically significant relationships between some of the properties of porous media and permeability, the application of these formulas has been limited. The renewed interest in the Green-Ampt equation for the purpose of modeling infiltration has spurred an increased interest in finding satisfactory estimates of hydraulic conductivity from other soil parameters.

Transfer of functional relationships developed by investigators in the petroleum industry to applications involved in flow through partially saturated media was made by Brooks and Cory (1964). Their work identified two pertinent soil parameters which relate soil saturation to capillary pressure. These parameters are the pore size distribution index and the bubbling pressure or air entry pressure.

The Brooks and Corey equation is written as

$$S_e = \left(\frac{y_b}{y_c}\right)^{\lambda} \tag{8}$$

where

$$S_e = \text{effective saturation} = \frac{\emptyset - \theta_r}{1 - \theta_r} \tag{9}$$

y_b = bubbling pressure

y_c = capillary pressure

λ = pore-size distribution index

$\emptyset$ = soil water content

θ_r = residual soil water content

Brooks and Corey (1964) describe λ as a factor which characterizes the pore size distribution and y_b as a measure of the maximum pore-size forming a continuous network of flow channels in the soil. More importantly, they showed how both factors can be easily obtained from a curve of the log of the effective saturation and the log of the capillary pressure. On this curve the parameter y_b is the intersect where S_e =

1.0 and the parameter λ is the negative slope of the curve.

Gupta and Larson (1979) used linear regression to relate the soil-water retention curves (i.e. the water content vs. capillary pressure curves) to the particle size distribution, the percentage of organic matter and the bulk density of 43 soils and dredged sediment samples.

Brakensiek and others (1981) used data from 1085 soil-water retention curves from soils with 11 different USDA textures and calculated the Brooks and Corey parameters. They also calculated the Green-Ampt parameters from the estimated Brooks and Corey parameters using an equation (Brutsaert,1967) derived by substituting the Brooks and Cory equation into the Childs, Collis-George permeability integral,

$$K = a \frac{\theta_e^2}{y_b^2} \frac{\lambda^2}{(\lambda+1)(\lambda+2)} \qquad (10)$$

and

$$y_f = \frac{2 + 3\lambda}{1 + 3\lambda} (y_b/2) \qquad (11)$$

where a is a fitted factor and θ_e is effective porosity.

McCuen et al (1981) used statistical analysis to show that the Brooks-Corey and Green-Ampt parameters vary significantly across soil texture classes.

Rawls and Brakensiek (1982) used data from 2543 soil horizons in 18 states to relate the water retention - capillary pressure curves to various soil parameters. They determined that the ability to predict water retention at other levels of capillary pressure was significantly increased by the addition of the -15 and -.33 bar soil water retention values.

Rawls et al. (1982) analyzed 1323 soils and 5350 soil horizons and found the mean values of the Brooks-Corey and Green-Ampt parameters shown in Table 1. Their analysis showed that average values of $y_f = 0.75\ y_b$.

Table 1. Mean Values of Soil Properties by Soil Texture

Texture Class	Effective Porosity θ_e	Bubbling Pressure y_b (inches)	Pore-size Dist.Index	Saturated Hydraulic Conductivity K (in/hr)
Sand	.417	6.29	.694	8.27
Loamy Sand	.401	8.10	.553	2.41
Sandy Loam	.412	11.89	.378	1.02
Loam	.434	15.80	.252	0.52
Silt Loam	.486	20.03	.234	0.27
Sandy Clay Loam	.330	23.39	.319	0.17
Clay Loam	.390	22.22	.242	.091
Silty Clay Loam	.432	27.69	.177	.059
Sandy Clay	.321	31.29	.223	.047
Silty Clay	.423	30.13	.150	.035
Clay	.385	33.70	.165	.024

The average values in Table 1 are a useful contribution to the Green-Ampt modeling process and enable a hydrologist to better estimate the parameters of the model. The parameters are further defined by Rawls et al. (1983) where figures are plotted with the parameters as a function of the percentage of sand and clay and different levels of porosity change.

Of greater use for this study were the prediction equations. (Brakensiek et al 1985). The Fortran program in that paper was modified for this study to a BASIC program with output in a tabular format. The program listing along with an example output file is in Appendix B.

Work has also been done examining the effects of agricultural tillage practices, soil crusting, coarse fragments in the soil, and frozen soil. (Brakensiek and Rawls, 1983) (Brakensiek et al 1985) The use of this information in this study will be described in a subsequent chapter.

The values of the Green-Ampt soil parameters used in this study are those computed using the equations from Brakensiek et al. (1985). The parameters are computed from the percent sand, percent clay, and the soil porosity which are all found from data in a standard soil survey. A more detailed description of the procedure is given in Chapter 3.

The SCS Curve Number Method

The Soil Conservation Service (SCS) developed the runoff curve number procedure as a quick method to estimate direct runoff from a given amount of storm rainfall. A complete discussion of the method is found in Chapter 10 National

Engineering Handbook Section 4, Hydrology (SCS,1972). Rallison (1980) gives some historical background on the equation's origin and development. Victor Mockus, hydrologist with the SCS, is credited with the development of the equation in 1954 based on work done by Andrews and others.

The SCS runoff curve number procedure is probably the most widely used procedure for determining storm runoff volumes. The equation was developed by assuming that

$$\frac{F}{S} = \frac{Q}{P} \qquad (12)$$

where F = actual retention of precipitation during a storm

S = maximum potential retention

Q = direct runoff

P = total precipitation or maximum potential runoff

Since F = P − Q

$$Q = \frac{P^2}{P+S} \qquad (13)$$

The total precipitation is now replaced by the precipitation after runoff begins ($P - I_a$) where I_a is the initial abstraction and represents interception, surface storage, and infiltration that occurs before runoff begins. Substituting $P - I_a$ for P yields

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S} \qquad (14)$$

From field data the empirical relationship $I_a = 0.2\ S$ was determined. When this is substituted into the equation the usual form of the SCS runoff equation results.

$$Q = \frac{(P-0.2S)^2}{P+0.8S} \qquad (15)$$

for all values of P > 0.2 S.

Curve numbers were created for convenience in averaging and interpolating. The parameter S is related to the curve number (CN) as follows:

$$S = \frac{1000}{CN} - 10 \qquad (16)$$

Curve numbers may range from zero to 100. However the normal range for practical purposes is from about 40 to 98. Probably the reason for widespread use of the CN method is the availability of tables which provide CN values for common soil and cover types found around the world.

The SCS has classified all soils into four groups defined as hydrologic soil groups. Soils are grouped based on similar profile characteristics,particularly depth, texture, organic matter content, structure, and the degree of swelling when saturated. The four groups are labeled A, B, C, and D. The A group has the highest infiltration and the lowest runoff while the D group has the lowest infiltration and highest runoff. (SCS,1972) As standard soil series are mapped and named by the USDA they are given a hydrologic soil group classification.

Curve numbers for cover types from forest and rangeland to cropland and urban land can be found for each hydrologic soil group and relative cover condition. (SCS,1972) (SCS,1986)

The Hydrograph Model

The peak discharge can be calculated from the rainfall excess through the use of a hydrograph development model. The model selected for use in this study was the SCS Computer Program for Project Formulation Technical Release Number 20 (SCS, 1982) hereafter referred to as TR-20.

The TR-20 model develops flood hydrographs from runoff and routes the flow through stream channels and reservoirs. It combines the routed hydrographs with those from tributaries and computes the peak discharges. The model performs the analysis of up to 9 different rainfall distributions on up to 200 reaches and 99 structures in one continuous run.

The hydrograph development model used in TR-20 is based on a dimensionless unit hydrograph which has as its ordinate the ratio of q/q_p and as its abscissa values of t/T_p. Where q is the discharge at time t and q_p is the peak discharge at time T_p. The computation procedure is described in Chapter 16 of Section 4 of the National Engineering Handbook (SCS, 1972).

The dimensionless unit hydrograph is converted to an incremental hydrograph for a specific watershed by the application of the equation

$$q_p = \frac{484\ AQ}{T_p} \qquad (17)$$

where q_p = peak discharge of incremental hydrograph in cfs

A = drainage area in square miles

Q = runoff in inches during time increment

T_p = time from the beginning of the runoff increment to the peak discharge in hours

and 484 = a constant determined from the hydrograph shape and a conversion of units

The computation interval in the model, Δt must be less than 0.25 T_p. This interval is also the duration of the incremental runoff. T_p in the model is calculated from the input parameter T_c, the time of concentration of the watershed, and Δt by the equation

$$T_p = \Delta t/2 + 0.6T_c \qquad (18)$$

The flood hydrograph is calculated by the summation of the ordinates of the incremental hydrographs, each of which begins one time unit, Δt, later than the previous one.

The hydrograph model was used with both the Green-Ampt and the Curve Number runoff distributions to predict the peak discharges from the watersheds. The volumes and peak discharges obtained by both methods will be compared to the measured values.

CHAPTER 3

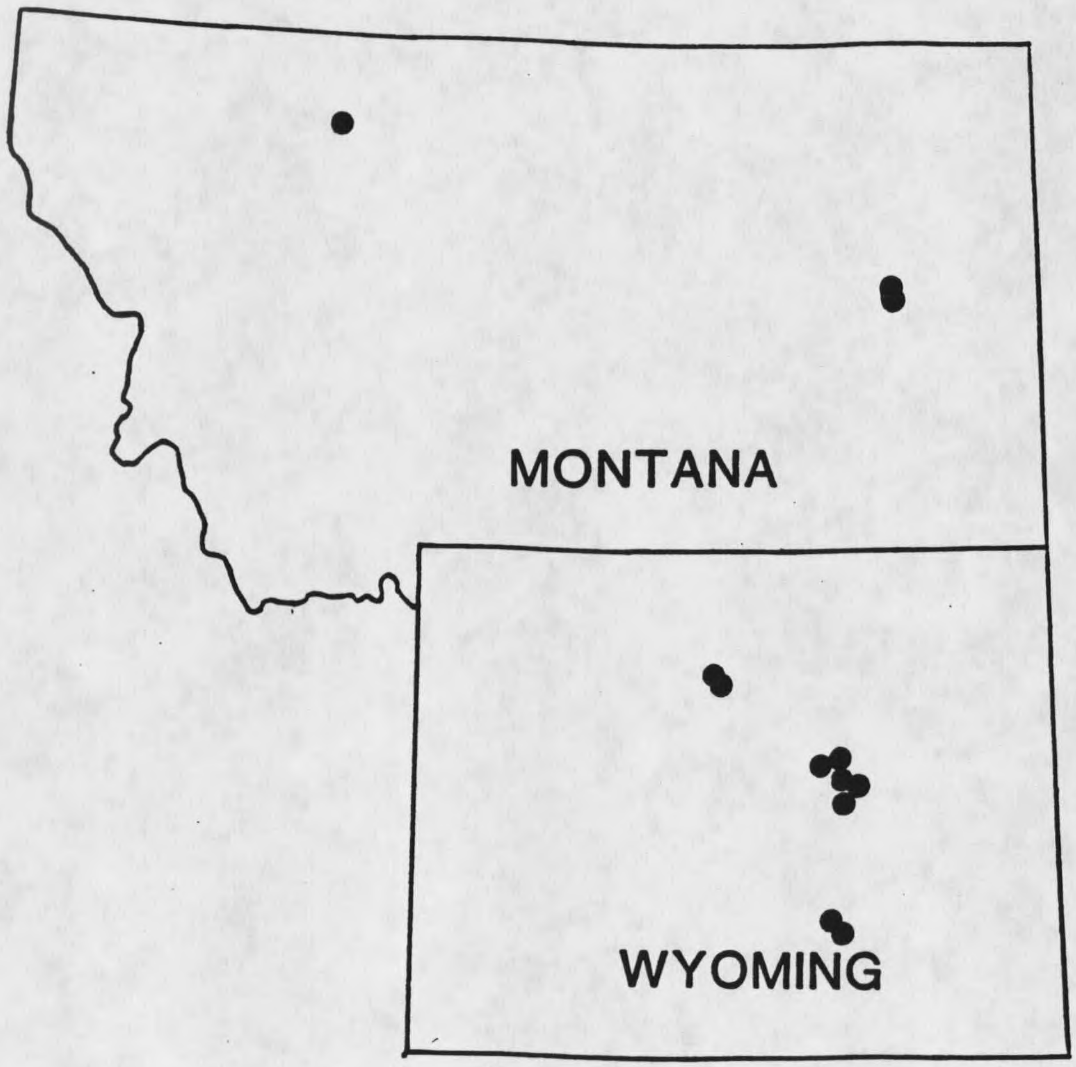
METHODS USED

Data Base

Rainfall and runoff data were obtained from two sources for use in this study. Most of the events which were modeled were from U.S. Geological Survey data published in Rainfall and Runoff Data From Small Basins in Wyoming (Rankl and Barker, 1977). That report contains 392 rainfall and runoff events for 22 small drainage basins in Wyoming. For this study it was necessary to also have a completed USDA soil survey on the drainage area. Nine drainage areas were found which had a completed soil survey. Five of these were in Natrona County, two in Washakie County and two in Carbon County. (See location map, figure 2). The largest of the Wyoming watersheds modeled was 10.8 square miles while the smallest was 0.71 square miles. All of them are in plains areas with rangeland cover conditions and have only ephemeral streamflow.

The rainfall and runoff data was tabulated in 5 minute intervals. The tabulated rainfall was usually the average of the rainfall at two recording gages, one located at the streamgage and one located near the upper end of the basin.

Figure 2. Location of modeled watersheds



Craig and Rankl, (1978) estimate the runoff accuracy as within 10 to 15 percent of the true value and the rainfall to be within 20 to 25 percent of the true value.

Although each of the watersheds in Wyoming had rainfall and runoff data for 15 to 24 events, only the largest 8 to 11 peak discharge events were selected to be modeled from each watershed. This eliminated most of the very small events while still keeping a wide range of events. A total of 84 events were modeled from the Wyoming data.

The other source of rainfall and runoff data was the Final Report - Drainage Correlation Research Project (Williams, 1971). Data for several runoff events on 5 watersheds in Montana are in that report. Three of the watersheds were selected for this study. These were Loneman Coulee in Pondera County and East Duck Creek and Duck Creek in Prairie and McCone Counties. (See location map, Figure 2). Loneman Coulee is a predominately cropland watershed with a drainage area of 14.1 square miles. East Duck Creek is a 13.7 square mile tributary of Duck Creek which drains a total area of 54 square miles. About 20 percent of the Duck Creek and East Duck Creek drainage is cropland with the remaining area being rangeland.

The largest 5 rainfall events were selected for modeling from each of these Montana areas. The watersheds each had two or more recording raingages as well as several non-recording raingages. Small amounts of preceding and following rainfall were deleted from the totals if they were separated by 12 hours or more and if they didn't appear to contribute to the peak runoff. Isohyets were then developed for each of the storms modeled and the average total storm rainfall was calculated from the isohytal map.

One of the recording gages was selected as being the most representative and was used to represent the time distribution of rainfall for the entire watershed. Precipitation at the recording gage was adjusted to the average for the area by multiplying by the ratio of the area average total to the recording gage total.

The rainfall data for the Montana watersheds was only tabulated in one-hour increments. It was observed that the predicted runoff volumes for the larger events were good while the predicted runoff for smaller events with durations of only a few hours were quite low when the one-hour increments were used. An empirical relationship was developed to estimate the maximum 1/2 hour rainfall from the maximum recorded one-hour value. Using 47 precipitation events from recording gages in eastern Montana the following equation was developed:

$$P_{1/2} = .676\ P_1 + .068 \tag{19}$$

Where $P_{1/2}$ = maximum 1/2 hour precipitation

and P_1 = maximum 1 hour precipitation

The correlation coefficient for this relationship is .864.

No improvement was made by including the 3-hour precipitation in the equation.

The greater intensity which resulted when the 1/2 hour rainfall was estimated by this equation increased the runoff volume from the smaller events while not significantly changing the runoff from the larger events. Table 2 shows the results of using the 1/2 hour estimates on the Loneman Coulee data.

Table 2. Effect of using estimated 1/2 hour precipitation on Loneman Coulee runoff.

Storm	Precipitation	Actual Runoff	Predicted Runoff 1-hour	Predicted Runoff 1/2-hour
	inches			
1	5.15	1.67	1.69	1.70
2	2.04	0.31	0.02	0.15
3	4.71	0.42	0.76	0.78
4	1.00	0.11	0.05	0.09
5	2.50	0.17	0.34	0.38

Soils data for the watersheds were found in published reports for McCone County, Montana (SCS 1984) and Washakie County, Wyoming (SCS 1983), and in unpublished reports for the other areas.

Soils maps for each watershed were measured to determine the area of each soil mapping unit in the watershed. The proportion of each soil series was then obtained from the mapping unit description. The required soil characteristics were obtained for each soil series from either the published Soil Survey or from the Soils Interpretations Record (SCS Soils-5 data base). This data is accessed by computer through the Soils Information Retrieval System. (U.S. Army Corps of Engineers, 1983)

The Green-Ampt Infiltration Model

Infiltration Conditions

Infiltration during a rainfall can occur under 3 types of conditions:

1) If the rainfall rate is less than the Green-Ampt saturated hydraulic conductivity then all of the rainfall infiltrates into the soil. If the soil is deep enough runoff will never occur if the rainfall continues at this rate.

2) If the rainfall rate is greater than the Green-Ampt saturated hydraulic conductivity but less than the infiltration capacity then all of the rainfall will infiltrate until the infiltration capacity has reduced to less than the rainfall rate.

3) If the rainfall rate is greater than the infiltration capacity then runoff will occur and the runoff volume will be the difference between the rainfall volume and the infiltration volume.

All three conditions are illustrated in Figure 3 for a