Three methods of instruction in high school geometry and the effects they have on achievement, retention, and attitude
by Edward Otis Thompson

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University

Abstract:
This study investigated the effect that three methods of classroom instruction had on (a) achievement of certain geometrical concepts, (b) retention of these concepts, and (c) attitude toward geometry for students enrolled in a high school geometry course. The three methods of instruction included: (I) sma11 cooperative learning groups completing van Hiele phase-based paper and pencil activities, (II) small cooperative learning groups using the computer and accompanying software to complete similar phase-based activities, and (III) whole class instruction based on traditional textbook procedures. Independent variables were methods of instruction, school, van Hiele level, attitude toward mathematics at the start of the geometry course, attitude toward geometry prior to the treatment, pretest achievement scores, gender, age, and socio-economic background. Dependent variables were posttest achievement scores, retention test achievement scores, and attitude toward geometry at the time of the posttest and at the time of the retention test.

The eight-week study was conducted in 14 geometry classrooms at five high schools in Montana. Treatments were randomly assigned to two or three geometry classes at each school. Classroom teachers conducted all instruction and testing activities. The posttest was administered at the completion of four-week unit of instruction; the retention test was administered four weeks after the posttest.

An analysis of covariance found the mean posttest achievement scores on low cognitive level items for treatments I and II were slightly higher than those for treatment III. (No differences among the treatments were detected on mean posttest achievement scores for high cognitive level items). A similar analysis found that the mean retention score for Treatment I students was higher than the mean retention score for Treatment III students with the difference being in the performance on test items which used higher order cognitive skills. There were no differences in student attitude toward geometry among the three treatments.

Based upon the findings of this study, geometry teachers should consider instructional methods using small cooperative learning groups together with phase-based materials as viable alternatives.

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by

Edward Otis Thompson

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of a thesis submitted by

Edward Otis Thompson

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04/16/1992 Lyle Anderson
Date Co-chairperson, Graduate Committee

4/16/92 William D. Hall
Date Co-chairperson, Graduate Committee

Approved for the Major Department

4/16/92 Duane Melling
Date Head, Major Department

Approved for the College of Graduate Studies

4/30/92 Henry L Parsons
Date Graduate Dean

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ABSTRACT

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Based upon the findings of this study, geometry teachers should consider instructional methods using small cooperative learning groups together with phase-based materials as viable alternatives.

CHAPTER 1

INTRODUCTION

Introduction

The role of the teacher in the early American mathematics classroom was to "state a rule, give examples, and provide problems" (NCTM, 1970, p.21); and according to more recent studies (Dossey, 1988; Sirotnik, 1983), mathematics instruction in today's classrooms is much the same. As Sirotnik describes it, "the typical classroom is still didactics, practice, and little else" (p.17). The National Council of Teachers of Mathematics (1989) advocates that a variety of instructional methods (project work, group and individual assignments, and discussion between teachers and students and among students) be used so that students can approach the learning of mathematics both independently and creatively. Venerable and modern theories on how students learn mathematics (e. g., Bruner and discovery learning, Gagne and guided learning, Piaget and constructivism, van Hiele and phase-based instruction) are continually tried and discussed by mathematics educators as they attempt to improve mathematics instruction. In addition, recent improvements in technology (e. g., computers, calculators, video cassette recorders) have given the mathematics educator new tools for instructional purposes.

At the secondary level, geometry has been one of the most controversial courses in the mathematics curriculum. The National Assessments of Educational Progress (Carpenter et. al., 1980; Carpenter, et. al., 1983; Brown, et. al., 1988) indicate that student's performance in and attitude toward geometry are low. Recent research indicates that a student's attitude toward mathematics, and geometry in particular, (Han, 1986) may be related to the method of instruction. The graphing capabilities of the modern personal computer have led to the recent development of computer software which "has the potential to change the ways teachers think about what it means to know geometry" (Lambert, 1988, p. 1). The van Hiele theory on how students learn geometry indicates that the sequencing of instructional materials may need revision (NCTM, 1988). The researcher could only locate four controlled scientific studies (Bobango, 1987; Han, 1986; Lambert, 1988; Yerushalmy, 1986) that partially investigated the effects the method of instruction, computer use in the classroom, and the van Hiele theory have on student achievement and attitude in the secondary geometry course.

Historically, student performance in geometry has been poor. This fact, coupled with the recent developments in geometry learning theory and geometry computer software, necessitates that research involving these variables be conducted to determine if student performance in geometry can improve. Mathematics educators must investigate instructional methods, learning theories, and new technology to determine which combinations of these methods, theories, and tools will have the maximum positive effect on the mathematical education of students. Only a limited number of such scientific studies have been conducted; it is the purpose of this study to add to this base of research data.

Statement of the Problem

The primary purpose of this study was to investigate the effect that three different methods of classroom instruction had on (a) achievement of certain geometrical concepts, (b) retention of these geometrical concepts, and (c) attitude toward mathematics (and geometry in particular) for students enrolled in a high school geometry course. The three methods of instruction included: (a) small cooperative learning groups doing paper and pencil activities following the three phases (information, guided orientation, and explicitation) as described in the van Hiele theory, (b) small cooperative learning groups using the computer and accompanying software together with activities following the three phases of the van Hiele theory, and (c) whole class instruction based on traditional textbook procedures. The dependent variables were scores on the criterion-referenced tests and scores on the attitude opinionnaires administered at the end of the unit and then four weeks later. Independent variables were method of instruction, gender, age, socio-economic background, attitude toward mathematics, attitude toward geometry just prior to treatment application, pretest scores on the criterion-referenced tests, and the van Hiele level of geometrical thinking. Instruction consisted of the teaching of a unit on congruent triangles. The population was the set of high school students enrolled in traditional tenth grade geometry classes in five Montana high schools that took part in this study.

Definitions of Terms

Terms defined for this study:

1. Achievement of certain geometrical concepts is defined as the measure (score) obtained on the criterion referenced test, Geometry Unit Test (Form A or B), constructed by the researcher and given to the students at the completion of the unit of geometry used in this research study (see Appendix A).

2. Attitude toward geometry is defined as a student's self-reported enjoyment, interest, and level of anxiety toward geometry as measured by the modified Aiken-Dreger Geometry Opinionnaire (see Appendix D).

3. Attitude toward mathematics is defined as a student's self-reported enjoyment, interest, and level of anxiety toward mathematics as measured by the Aiken-Dreger Mathematics Opinionnaire (see Appendix D).

4. Computer refers to any one of the Apple(c) II series of microcomputers.

5. Geometrical concepts refer to the topics listed in the Unit Objectives (see Appendix A).

6. Geometry course is the Montana high schools' year-long course taken by those students who have

completed one year of algebra (usually students in the tenth grade). The textbooks used by the students in this study are Rhoad, Milauskas, and Whipple's Geometry For Enjoyment and Challenge (1984) and Jurgensen, Brown, and Jurgensen's Geometry (1985).

7. IMPACT is an acronym for Integrating Mathematics Programs and Computer Technology. IMPACT was a National Science Foundation funded program to train selected mathematics teachers in Montana to provide expertise to other teachers through inservice workshops and math curriculum leadership in districts throughout the state of Montana.

8. Method of instruction refers to one of the three treatments used in this study.

a. Treatment I -- Each concept of the unit on congruent triangles is introduced to the students using worksheets which consist of activities in which the students use paper, pencil, ruler, and protractor. The activities were designed by the researcher based upon the first three phases of the van Hiele theory. The students work in small cooperative learning groups to complete these activities.

b. Treatment II -- Each concept of the unit on congruent triangles is introduced to the students using worksheets which consist of activities in which the students use a computer and the Geometric PreSupposer and Geometric Supposer: Triangles software published by Sunburst Publications. The activities were designed by the researcher based upon the first three phases of the van Hiele theory. The students work in small cooperative learning groups to complete these activities.

c. Treatment III -- Each concept of the unit on congruent triangles is introduced to the students in a class discussion format led by the teacher using only the textbook as a guide. The teacher presents the material in the same sequence as outlined in the textbook, periodically asking random members of the class various questions to check for understanding.

9. Retention of geometrical concepts is defined as the measure (score) obtained on the criterion-referenced test, Geometry Unit Test (Form A or B), constructed by the researcher and given to the students four weeks after the completion of the unit of geometry used in this research study.

10. Small cooperative learning group is defined as a group of three students (or four students when the class size is not a multiple of three) working together to complete an activity to the satisfaction of everyone in this group. As the group works through the activity, the teacher encourages the group to discuss their methods, reasoning strategies, findings and conjectures with one another. The teacher answers only procedural questions and does not give answers to the activity questions. The teacher praises the members of the group for their accomplishments.

11. Socio-economic background is defined in terms of qualification for free or reduced-cost school lunch. Poverty level means the student qualified for free school lunch; low income means the student qualified for reduced-cost school lunch; and all others means the student receive neither free nor reduced-cost school lunch.

12. The van Hiele levels of geometric thought are five levels espoused by P. M. van Hiele and his late wife, Dina van Hiele-Geldof in their model of geometry learning of students.

a. Level 0: Visual or recognition--the level at which a student recognizes geometric figures by appearance alone. The student cannot see the component parts; definitions and class inclusion have no meaning to the student at this level. For example, when asked why a given figure is a triangle, the student at this level would reply, "Because it looks like one!"

b. Level 1: Descriptive or analysis--the level at which a student analyzes the properties of geometric figures and distinguishes figures by their component parts rather than by their appearance. The student at this level neither notices nor perceives relationships between properties or figures, and definitions and class inclusion still have no meaning. For example, when asked why a given figure is an equilateral triangle, the student at this level would reply, "Because all the sides are the same length and all the angles have the same measure." The student would not perceive the relationship between these two facts.

c. Level 2: Logical order--the level at which definitions are meaningful to a student and relationships are perceived between properties and figures by a student. Class inclusion has meaning. A student at this level can give an informal argument, but the student cannot construct a deductive proof.

For example, at this level the student could understand and explain the validity of the statement: "All equilateral triangles are isosceles triangles, but not all isosceles triangles are equilateral triangles."

d. Level 3: Formal logic or deduction--the level at which a student recognizes the need for undefined terms, definitions, and axioms in a particular mathematical system. A student at this level can reason deductively and construct proofs of propositions or theorems. The student at this level cannot compare and contrast different axiomatic systems. For example, at this level the student could construct the proof (based on the Euclidean parallel postulate) that the sum of the measures of the angles of a triangle is 180 degrees, but the student could not understand how the sum of the measures of the angles of a triangle could be less than 180 degrees if the Euclidean parallel postulate were changed.

e. Level 4: Rigor--the level at which a student can compare and contrast different geometric systems. The student at this level can use symbols without referents and can manipulate the symbols according to the laws of formal logic. The student at this level understands the role and necessity of indirect proof and proof by contraposition. For example, a student at this level could understand the proof that the sum of the measures of the angles of a triangle is less than 180 degrees when in Lobachevsky geometry.

13. The van Hiele phases of the instructional process are the stages in the van Hiele model of geometry thought through which a student must pass to move from one van Hiele level of geometric thought to the next.

a. Phase 1: Information--the stage in which a student gets acquainted with the working domain through activities and general conversation with the teacher.

b. Phase 2: Guided Orientation--the stage in which a student explores the topics being considered by doing simple tasks selected by the teacher which are usually accomplished in a specified manner.

c. Phase 3: Explicitation--the stage in which a student becomes conscious of the relations resulting from phase 2 and tries to express them in words with minimal direction from the teacher. A student may use nonstandard terms, but the teacher introduces the appropriate technical language.

d. Phase 4: Free Orientation--the stage in which a student learns to resolve more complicated tasks. During this stage a student may explore his/her own ideas as well as tasks presented by the teacher.

e. Phase 5: Integration--the stage in which a student summarizes all that he/she has learned about the topics, reflects on his/her actions, and obtains an overview of the newly formed concepts and relations.

Need for the Study

The accessibility of the computer in the classroom can have an impact on the method of mathematics instruction. Recent advances in the development of computer software are changing the way some teachers are teaching mathematics. Specifically, the Geometric Supposer, software created by the Education Development Center, Inc. and published by Sunburst Communications, Inc., appears to encourage students to conjecture about and discover geometric concepts successfully (Yerushalmy, et. al., 1987).

Curriculum materials that reflect the van Hiele levels and provide phase-based activities described in this theory are limited (NCTM, 1988).

Although cooperative learning in the mathematics classroom has been investigated by several researchers (Slavin, et. al., 1985), the availability of research data which integrates the computer, the van Hiele theory, and cooperative learning is limited. One purpose of this study is to add to this base of research data.

Five factors were crucial in establishing need for this study. They are:

1. geometry is still a major component of the high school mathematics curriculum;
2. student performance in geometry is poor;
3. even though alternatives to the "traditional" lecture-discussion type of instruction (such as small group work, individual explorations, peer instruction, use of concrete materials and manipulatives) are recommended, these methods are not being used in many mathematics classrooms;
4. instructional materials in geometry using the van Hiele theory are limited; and
5. research regarding the effects of computer assisted instruction in geometry is limited.

The following paragraphs will substantiate why these factors are crucial.

Factor 1: Geometry is still a major component of the high school mathematics curriculum.

Geometry became a standard course for high school students in the United States shortly after the Civil War. Although conferences and committees dealing with how and when geometry should be taught have affected the approaches and content of the high school geometry course, at the present it still remains a year long course in most high schools with the focus on Euclidean geometry (NCTM, 1981). The NCTM Curriculum and Evaluation Standards (1989) includes the study of geometry in two of its fourteen standards in the mathematics curriculum for grades 9-12. The Curriculum and Evaluation Standards further states (1) that "at least three years of mathematical study will be required of all secondary school students" and (2) that "four years of mathematical study will be required of all college-intending students" (NCTM, 1989. pp. 124-125). With the present configuration of high school mathematics programs, most students planning to graduate from high school will take a course in geometry.

Factor 2: Student performance in geometry is poor. Equally disturbing is the fact that nearly half of the high school students do not take geometry.

The Fourth Mathematics Assessment of the National Assessment of Educational Progress reported that about 55 percent of the eleventh-grade students in this study had completed a geometry course (Brown et. al, 1988). This assessment included forty-three geometry items that were given to eleventh-grade students. These items were placed into four categories: identification of figures, properties of figures, visualization tasks, and applications. The items focused on ideas of informal geometry, such as those taught at the elementary or middle school levels. Those students who had taken a high school geometry course did well in the identification of figures category. The students were asked to identify parallel lines, perpendicular lines, a sphere, the diameter of a circle, the radius of a circle, and the endpoints of an arc. On these items, the success rate ranged from 81 percent identifying a radius to 99 percent identifying parallel lines. However, when it came to properties of figures, only 33 percent of the students who had taken a geometry course could correctly choose the set of 3 numbers that could not be lengths of sides of a triangle and 71 percent could correctly choose the set of 3 numbers that could not be measures of angles of a triangle. In evaluating spatial-visualization skills, about two-thirds of the eleventh-grade students responded correctly; those who had taken a geometry course performed only slightly better than those who had not taken geometry. Two of the application problems involved the Pythagorean relationship; for those eleventh-grade students who took geometry, only 48 percent correctly answered the first item and only 30 percent correctly answered the second item.

Factor 3: Even though it is recommended by the National Council of Teachers of Mathematics, a variety of instructional methods is still not being used in the mathematics classroom.

The NCTM Curriculum and Evaluation Standards stresses the need for changes in the roles of both teachers and students in mathematics classes.

> A variety of instructional methods should be used in classrooms in order to cultivate students' abilities to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. In addition to traditional teacher demonstrations and teacher-led discussions, greater opportunities should be provided for small-group work, individual explorations, peer instruction, and whole-class discussions in which the teacher serves as a moderator (NCTM, 1989, p.125).

Currently, a variety of strongly recommended instructional methods is not being used in the typical mathematics classroom. From data collected in the 1986 NAEP (Dossey, 1988), it was found that about three-fourths of the eleventh graders indicated that teacher use of the chalkboard was a daily activity, but only about 26 percent indicated that they themselves frequently work problems at the board. Over half these students reported they never work mathematics problems in small groups; only 57 percent reported that frequent class discussion took place.

In "A Study of Schooling," directed by John Goodlad, similar patterns were found (Sirotnik, 1983). In the secondary mathematics classrooms, this study revealed 79.3 percent of the classroom time was instructional with the teacher talking 57.9 percent of the time and the students talking 16.4 percent of the time.

Factor 4: Instructional materials in geometry using the van Hiele theory are limited.

In 1957, Dutch educators, P. M. van Hiele and his late wife, Dina van Hiele-Geldof, presented a model of geometry learning they had developed. This model identifies five levels of thinking in geometry and phases within each level of thinking. This theory has shown promise in improving student performance when the instruction was provided in accordance with the van Hiele theory (NCTM, 1988). Except for the geometry text by Hoffer (1979), geometrical topics in the text books in the United States are not sequenced according to this theory (NCTM, 1988) and, hence, textbook-guided instruction is not utilizing this promising model.

Factor 5: Research regarding the effects of computer assisted instruction in geometry is limited.

In its An Agenda for Action, the NCTM recommended that "mathematics programs take full advantage of the power of calculators and computers at all grade levels" (NCTM, 1980, p.1). This was reiterated in the NCTM Curriculum and Evaluation Standards when the underlying assumptions included that "a computer will be available at all times in every classroom for demonstration purposes, and all students will have access to computers for individual and group work" (NCTM, 1989, p.124).

Mathematics educators (Fey, 1984; Kantowski, 1981) have pointed out that the graphics and numerical capabilities of computers allow students to manipulate and measure geometric shapes so that they can develop understanding of concepts and theorems of geometry. Several papers have been written describing the use of the computer in the geometry classroom (Lamber, 1988; Yerushalmy, 1986; Yerushalmy, et. al., 1987; Yerushalmy and Houde, 1986); however, the research on the effect of using the computer in the geometry classroom is extremely limited (Suydam, 1985; Suydam, 1987; Suydam, 1988; Suydam, 1989; Suydam and Crocker, 1990).

Questions to be Answered

The questions that this study attempted to answer fall into six categories:

1. Are there differences in achievement of geometric concepts among three groups of students receiving different methods of classroom instruction? Are there any differences in achievement between male and female students? Are there any differences in achievement among the students from different socio-economic status? Are there any differences in achievement among students performing at different van Hiele levels? Is achievement affected because of interactions between or among these independent variables?

2. Are there differences in retention of geometric concepts among three groups of students receiving different methods of classroom instruction? Are there any differences in retention between male and female students? Are there any differences in retention among the students from different socio-economic status? Are there any differences in retention among students performing at different van Hiele levels?

Is retention affected because of interactions between or among these independent variables?

3. Are there differences in attitudes toward geometry among three groups of students receiving different methods of classroom instruction? Are there any differences in attitudes toward geometry between male and female students? Are there any differences in attitudes toward geometry among the students from different socio-economic status? Are there any differences in attitudes toward geometry among students performing at different van Hiele levels? Are attitudes toward geometry affected because of interactions between or among these independent variables?

4. Is there a correlation between the attitudes toward mathematics and achievement by the students who studied geometry in each of the groups defined by the different methods of classroom instruction?

5. Is there a correlation between the attitudes toward mathematics and retention by the students who studied geometry in each of the groups defined by the different methods of classroom instruction?

6. Is there a correlation between achievement and retention by the students who studied geometry in each of the groups defined by the different methods of classroom instruction?

These questions are detailed in null hypotheses form in Chapter 3.

Procedure of the Study

The study took place in Montana. Montana, the fourth largest state in the United States in terms of area, ranks 44th in population. Montana is classified as a rural state with 5.6 people per square mile. Only 196,000 of its 800,000 residents reside in two metropolitan areas (population over 55,000). Principal industries in the state are agriculture, mining, tourism and manufacturing (primarily wood products). Approximately 12% of the population are at the poverty level. Approximately 46,000 students are enrolled in grades 9-12 in the 163 Montana high school districts (U. S. Bureau of the Census, 1989).

This study took place at these high schools: Columbia Falls High School in Columbia Falls, Montana; Cut Bank High School in Cut Bank, Montana; Forsyth High School in Forsyth, Montana; Glasgow High School in Glasgow, Montana; and C. M. Russell High School in Great Falls, Montana. All five high schools contain grades 9-12.

Columbia Falls, population 2,942, is located in northwestern Montana. The primary industries in this community are lumber and wood products and aluminum processing. The high school district, which includes the town and several outlying community areas, has 6,142 residents with a high school enrollment of 632 students.

Cut Bank, population 3,329, is located in northern Montana. The primary industry in this community is agriculture; however, active gas and oil fields are located in the surrounding area. The high school district, which includes Cut Bank itself and the surrounding agricultural area, has 4,223 residents, and the high school population is 255.

Forsyth, population 2,178, is located in southeastern Montana. The primary industry in this community is agriculture. The high school district, with 2,811 residents, has a present enrollment of 210 students.

Glasgow, population 3,572, is located in northeastern Montana. It was once the site of a U. S. Air Force base but now the community lists agriculture as its major industry. The high school district with 4,116 residents has a present enrollment of 306 students.

Great Falls, located in northcentral Montana, is one of two cities in Montana with a population of over 55,000 (population 55,086). An active Air Force base is located adjacent to the city. The city has a diversified economy. Copper, zinc, and aluminum are processed, flour is milled from nearby wheatfields, and there is a large crude-oil refinery. The high school district contains two highs which serve a district of 67,901 residents. CMR High School (the one in this study) has 1,597 students, and GF High School has 1,743 students.

The selection of the high schools involved in this study was based upon these criteria:

1. The geometry teachers at these schools must have recently participated in the IMPACT project or an IMPACT related workshop. Through the IMPACT project or IMPACT workshops these teachers (a) received training in using the computer in the classroom and (b) had experience with cooperative learning techniques.

2. In order to maintain some uniformity in the geometry content, the geometry classes selected at these schools were using one of two identified texts (Jurgensen, et. al., 1985; Rhoad, et. al., 1984) that introduced the unit on congruent triangles (with similar objectives) at approximately the same time in the school year.

3. The administration at the schools must have been willing to cooperate with the researcher by allowing the researcher to collect confidential information about each student.

This study involved three different treatment groups. The first treatment consisted of students working in small cooperative learning groups doing paper and pencil activities designed by the researcher. These activities followed the first three phases (information, guided orientation, and explicitation) as outlined in the van Hiele theory. The last two phases (free orientation and integration) of the van Hiele theory were completed using the exercises in the student text. The second treatment consisted of students working in small cooperative learning groups using the computer and selected software (Geometric PreSupposer and Geometric Supposer: Triangles) together with activities designed by the researcher. These activities followed the first three phases of the van Hiele theory. The last two phases were completed using the exercises in the student text. In the third treatment for this study, the method of instruction was whole class instruction based on traditional textbook procedures.

At four of the five schools involved in the study, one teacher at each school taught all the geometry courses. All four of these teachers were involved in the study. At the fifth school, no mathematics teacher taught more than two sections of geometry so two teachers from this school were chosen to participate since the researcher wanted all three treatments used at a particular school whenever possible. Thus, at two schools, each teacher taught three sections of geometry and each section was randomly assigned one of the treatments. At one school (where computer facilities were limited) only treatments one and three were assigned. At the fourth school, only two sections of geometry were offered, so treatments two and three were assigned to these sections. This was done in order to balance the number of sections using each treatment. At the school where two teachers were involved, one teacher taught one class and the other teacher taught two classes in which each of the treatments was randomly assigned to one of these classes. This selection allowed limited control of the teacher variable.

Since this study was done in a natural school setting where the number of classes available in each school is limited, random assignment of students to treatment groups was impossible. Hence, the quasi-experimental pre-post design based on Campbell and Stanley's (1963) Nonequivalent Control Group Design was the appropriate design to use.

Three instruments were used in this study. (1) The Aiken-Dreger Mathematics Attitude Scale was used to measure the change in the students' attitudes toward geometry as a result of the 4-week treatments received by each of the groups in the study. (2) The CDASSG van Hiele Geometry Test was given prior to this study to determine the van Hiele level of each participant. (3) The Unit Test on Congruent Triangles, designed by the researcher, was used as pretests and posttests to measure achievement and retention of geometric concepts of the unit taught on congruent triangles.

Limitations and Delimitations

The following limitations restrict the generalization of the findings of the study to different classes, situations, and settings:

1. The study was limited to those high school students in selected Montana high schools who were enrolled in a high school geometry course during the 1990-1991 academic year.
2. The study began after the first nine-week period had ended and the treatments were administered four continuous weeks.
3. All classes were taught by teachers having at least three years of teaching experience prior to the study. All five teachers had participated in the IMPACT project or IMPACT workshops and hence may not be representative of average geometry teachers.

4. Each class involved in the study used exactly one geometry text, but texts by two different publishers were used by the schools.

5. Individual teacher's attitude, ability, and familiarity with the materials may have influenced student attitudes and performances.

6. The use of computers was not allowed during the tests.

The following delimitations restrict the generalization of findings of this study to different situations:

1. The study was limited to an eight-week period (four weeks of instruction on a congruent triangles unit with the retention test being administered four weeks later).

2. The phase-based treatments were only for selected, rather than all, topics completed in the unit of geometry used in this research study.

Organization of the Study

Chapter 1 presented an introduction to the problem. With the importance of the problem established, the statement, justification, and purpose of the study were discussed.

Chapter 2 begins with an historical review of geometry in the high school curriculum. It then gives a review of the related literature as it applies to (1) the van Hiele theory on how students learn geometry, (2) the use of computers in the teaching of mathematics, (3) the use of cooperative learning methods in the teaching of mathematics, and (4) the achievement and attitudes toward mathematics of high school students.

Chapter 3 describes the methodology of procedures used in the study. It includes a restatement of the problem, procedures, instruments, and treatment of the data.

Chapter 4 covers the results of the study. It comprises a restatement of the hypotheses, the results of the investigation, and an analysis of those findings.

Chapter 5 presents a summary of the study, the conclusions drawn from the findings, and recommendations based upon these conclusions.

CHAPTER 2

REVIEW OF LITERATURE

Introduction

The review of the literature relative to this study is reported in this chapter. The method of organization of the reviewed literature followed these major topics: (1) an historical review of geometry in the high school curriculum, (2) the van Hiele theory, (3) the use of computers in teaching geometry, (4) cooperative learning methods in geometry classes, and (5) student achievement and attitude with respect to high school geometry.

The research studies reviewed are limited to those which pertain to this investigation and which satisfy the following criteria: (1) they must have been completed within the last 20 years, and (2) they must represent conclusions based on research and not on mere personal opinion.

Historical Review of Geometry in the High School Curriculum

The first high school in the United States, the English High School, was founded in Boston in 1821 and by 1875 several similar high schools were well established in the northeastern part of the United States. Geometry became a high school course when the colleges made it an entrance requirement. Yale first required it in 1865 and Princeton, Michigan, and Cornell began requiring it in 1868. Harvard made it an entrance requirement in 1870 (NCTM, 1970).

Prior to 1920, geometry was essentially a course in logic and demonstration. In the NCTM's Fifth Yearbook entitled The Teaching of Geometry Swenson wrote:

> The sway which Euclid's Elements has held as a textbook for more than two thousand years is without parallel in the history of mathematics. Even the invention of Cartesian geometry in 1637 has not affected the teaching of the so-called Euclidean geometry. An almost unlimited number of textbooks have appeared in modern times but the only way in which they have differed is in the sequence of the theorems. Euclid's treatment has in the main been retained and no modern mathematical methods have been introduced. (NCTM, 1930)

There was some initial movement to change this approach at the turn of the century. John Perry, a professor of mechanics and mathematics at the Royal College of Science in London, advocated the discovery method of teaching. Perry did not approve of the strict adherence to Euclid; he wanted some experimental geometry before the study of formal geometry and he wanted a reduction in the number of theorems and proofs to be memorized. E. H. Moore of the University of Chicago, in his address before the American Mathematical Society in 1902, supported Perry's movement to emphasize the practical sides of mathematics (Moore, 1967). This influence became apparent when the geometry textbooks of the twenties began with an informal, intuitive geometry section rather than beginning with the formal logic and demonstration of the earlier texts.

A second influential component on the high school mathematics curriculum (and hence the geometry course) at this time was the College Entrance Examination Board (CEEB). The CEEB was a product of the committee originally started by Columbia University to set some uniformity in college entrance requirements. By 1903, the CEEB had begun work on syllabi that would become the guide for uniform, nationwide college entrance examinations. Before long, textbooks were designed around these syllabi, and teachers were carefully preparing their students to pass these CEEB examinations.

Then, in 1930, when critics of the mathematics curriculum questioned the rationale and content of the geometry course, the NCTM formed a committee on geometry. An early recommendation of this committee favored the combination of solid and plane geometry (NCTM, 1931). Probably the most influential derivative from this committee was the development of a text which used a metric postulate system for plane geometry. Ralph Beatley, the chairman of this committee, teamed up with George Birkhoff, editor of the NCTM's Fifth Yearbook, to write the high school text, Basic Geometry (Birkhoff, 1941).

In 1935, the Mathematical Association of America teamed with the young and aggressive National Council of Teachers of Mathematics, and together they appointed the Joint Commission consisting of members from the Mathematical Association of America and the National Council of Teachers of Mathematics. In its report, published in 1940, which was the Fifteenth Yearbook of the NCTM, the basic course in geometry was described as "a course that examines somewhat critically Euclidean geometry, and gives brief introductions to projective geometry and non-Euclidean geometry, using synthetic methods" (Joint Commission, 1940).

The next impetus for change occurred in 1955 when the CEEB again thought the time had come for it to take a leadership role in determining the school mathematics program for college-bound students. It appointed the Commission on Mathematics to carry out this function. In its report, the Commission identified three main objectives for the inclusion of geometry in the high school curriculum:

> The first objective is the acquisition of information about geometric figures in the plane and in space.... The second objective is the development of an understanding of the deductive method as a way of thinking, a reasonable skill in applying this method to mathematical situations.... The third important objective of the geometry course is the provision of opportunities for original and creative thinking by students.... (CEEB, 1959).

Many other curricular projects were initiated in the fifties, but one that had a significant impact on mathematics in the sixties and beyond was the School Mathematics Study Group (SMSG). SMSG grew directly out of two National Science Foundation (NSF) sponsored conferences, and SMSG received its first grant from NSF on 7 May 1958. Having had access to preliminary reports of the Commission on Mathematics of the CEEB, SMSG writing teams used these recommendations to design a series of textbooks that were to serve as guidelines to future mathematics textbooks. SMSG geometry used the "ruler and protractor postulates" from Birkhoff and Beatley's <u>Basic Geometry</u>. Suggestions from persons favoring an analytic geometry approach led SMSG to appoint a new writing team to write a text using coordinate geometry.

In the 1970's and early 1980's, the high school geometry curriculum was once again under scrutiny. Mathematicians and mathematics educators could not agree upon what the geometry curriculum should be. One only needs to look at the titles of the chapters in the Thirty-sixth Yearbook of the NCTM to see the variety of approaches that were proposed: "Conventional Approaches Using Synthetic Euclidean Geometry," "Approaches Using Coordinates," "A Transformation Approach to Euclidean Geometry," "An Affine Approach to Euclidean Geometry," "A Vector Approach to Euclidean Geometry," "Geometry in an Integrated Program," and finally "An Eclectic Program in Geometry" (NCTM, 1973).

The van Hiele Theory

With the results from the recent National Assessments of Educational Progress showing that the geometry knowledge of students is rather minimal (Brown et. al., 1988; Carpenter et. al., 1983; Carpenter et. al., 1980), mathematics educators are beginning to look for teaching strategies that are based upon the cognitive processes of geometry students (NCTM, 1987; NCTM, 1988). One such approach to the instruction of geometry that is currently receiving considerable attention from U. S. mathematics educators is a theory espoused by Pierre and Dina van Hiele. These two Dutch educators presented this theory in 1957 and through the efforts of Izaak Wirszup (1976), American mathematics educators became aware of this theory.

The strength of the van Hiele theory is in its use of the detailed description of levels and phases which can be used in designing curriculum and instruction in geometry. The original van Hiele theory identifies five levels of thinking in geometry through which a student progresses when assisted by appropriate instructional experiences. These five levels and their characteristics are defined as follows:

Level 0: Visual or recognition--the level at which a student recognizes geometric figures by appearance alone. The student cannot see the component parts; definitions and class inclusion have no meaning to the student at this level.

Level 1: Descriptive or analysis--the level at which a student analyzes the properties of geometric figures and distinguishes figures by their component parts rather than by their appearance. The student at this level neither notices nor perceives relationships between properties or figures, and definitions and class inclusion still have no meaning.

Level 2: Logical order--the level at which definitions are meaningful to a student and relationships are perceived between properties and figures by a student. Class inclusion has meaning. A student at this level can give an informal argument, but the student cannot construct a deductive proof.

Level 3: Formal logic or deduction--the level at which a student recognizes the need for undefined terms, definitions, and axioms in a particular mathematical system. A student at this level can reason deductively and construct proofs of propositions or theorems. The student at this level cannot compare and contrast different axiomatic systems.

Level 4: Rigor--the level at which a student can compare and contrast different geometric systems. The student at this level can use symbols without referents and can manipulate the symbols according to the laws of formal logic. The student at this level understands the role and necessity of indirect proof and proof by contraposition (Mayberry,1983).

Pierre van Hiele has since revised the model into three rather than five levels of thought (Teppo, 1991). He labels these levels as level 1: visual, level 2: descriptive, and level 3: theoretical. Basically, the original van Hiele level 0 is the new level 1, the original level 1 is the new level 2 and the original levels 2, 3, and 4 have become the new level 3.

According to this theory, using either the original levels or the new levels, (van Hiele, 1986; Teppo, 1991), a student's progress through these levels has certain characteristics. They are:

(1) the levels are sequential;

(2) each level has its own language, set of symbols, and network of relations;

(3) what is implicit at one level becomes explicit at the next level;

(4) material taught to students above their level is subject to reduction of level;

(5) progress from one level to the next is more dependent on instructional experience than on age or maturation, and

(6) a student goes through various "phases" in proceeding from one level to the next.

The five phases that a student goes through as the student proceeds from one level to the next are: information, guided (or bound) orientation, explicitation, free orientation, and integration. These phases are described as follows:

> Phase 1: Information--the stage in which a student gets acquainted with the working domain through activities and general conversation with the teacher.
>
> Phase 2: Guided orientation--the stage in which a student explores the topics being considered by doing simple tasks selected by the teacher which are usually accomplished in a specified manner.
>
> Phase 3: Explicitation--the stage in which a student becomes conscious of the relations resulting from phase 2 and tries to express them in words with minimal direction from the teacher. A student may use nonstandard terms, but the teacher introduces the appropriate technical language.
>
> Phase 4: Free orientation--the stage in which a student learns to resolve more complicated tasks. During this stage a student may explore his/her own ideas as well as tasks presented by the teacher.
>
> Phase 5: Integration--the stage in which a student summarizes all that he/she has learned about the topics, reflects on his/her actions, and

obtains an overview of the newly formed concepts and relations.

Research projects on the van Hiele model have concentrated on determining if the five van Hiele levels can be determined for an individual student and to what extent are achievement in geometry and the van Hiele levels related. Only a limited amount of research is available which considers the implications that van Hiele phase-based instruction has on student achievement in geometry.

The purpose of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project was to "test the ability of the van Hiele theory to describe and predict the performance of students in secondary school geometry" (Usiskin, 1982). Several written tests were developed and used in this study. One of these tests, the Van Hiele Geometry Test, was used to determine the students' van Hiele levels of thought. Based upon the project's criteria for this test for determining a student's van Hiele level, it was found that about 85 percent of the nearly 1600 students in the study could be assigned a van Hiele level using this test at the beginning of the secondary geometry course. At the end of the course, this same test assigned a level to approximately 88 percent of the students. Results from this project also indicated that this test given in the fall was a good predictor of later achievement (r = 0.64).

In a study that used a subsample of the students involved in the CDASSG project, Senk (1983) tested 1520 high school geometry students and found that 30 percent of these students had no competence in proof-writing, 40 percent had some proof-writing skills, and about 30 percent achieved a mastery level of 75 percent (or above) in proof-writing. Senk found that both the student's van Hiele level and non-proof geometry achievement had a high positive correlation with proof-writing achievement.

A second project (known as the Brooklyn College Project) had four main objectives:

1. To develop and document a working model of the van Hiele levels,

2. To characterize the thinking in geometry of sixth and ninth graders in terms of the van Hiele levels,

3. To determine if teachers of grades 6 and 9 can be trained to identify van Hiele levels of geometry thinking of students and to identify van Hiele levels of geometry curriculum materials, and

4. To analyze current geometry curriculum as indicated by American text series to determine the geometry topics taught at each grade level and the

van Hiele level of these topics and to determine if the presentation of the topics was consistent with the van Hiele theory (NCTM, 1988).

Through interview procedures they developed, the researchers in this project concluded that the van Hiele model provides a reasonable structure for describing a student's geometric learning process. They also found, after analyzing three K-8 textbook series, that textbook material provides little opportunity for students to make progress to higher van Hiele levels (Fuys and Geddes, 1984).

Another project, the Oregon Project (Burger and Shaughnessy, 1986), also used clinical interviews to investigate student behaviors on geometric tasks in relation to the van Hiele theory. Specifically, the project investigated the following research questions:

1. Are the van Hiele levels useful in describing students' thinking processes on geometry tasks?
2. Can the levels be characterized operationally by student behaviors?
3. Can an interview procedure be developed to reveal predominant levels of reasoning on specific geometry tasks?

The results of this project indicated that all three questions can be answered in the affirmative. The researchers found that students who were identified as being at different van Hiele levels did in fact perform differently and use different language.

In a study which involved 24 college students, Mayberry (1983) obtained results which confirmed the theory that the van Hiele levels are sequential; i.e., students cannot function adequately at a given level without the ability to think intuitively at each preceding level. This study also found that students do not operate at the same level across all concepts.

Han (1986) conducted one of the first studies in the United States that compared the effects on student achievement and attitude of two geometry textbook programs, one consistent with the van Hiele theory, and the other program being a "traditional" program. The subjects were 478 geometry students from two high schools. One high school used the van Hiele theory based text and the other high school used the traditional text. The study concluded that there were no significant differences between the two groups in van Hiele level, within group correlation between van Hiele level and proof-writing achievement, and attitude toward geometry. The main conclusion of this researcher was that the van Hiele approach did not offer a significant advantage over the traditional one in overcoming the difficulty students have with proof.

Another study investigated the effect van Hiele phase-based instruction had on raising students' van Hiele levels and on their subsequent understanding of geometric facts, concepts, and proofs (Bobango, 1987). The study involved 72 high school students from two regular and two honors geometry classes. The computer programs, Geometric Supposer: Triangles and Geometry Supposer: Quadrilaterals (Educational Development Center, 1985), were used together with researcher-designed lessons with the phase-based treatment group. The students' achievement and van Hiele levels were compared before and after the phase-based instructional treatment. The researcher found that the phase-based instruction had a significant effect on raising regular students' van Hiele levels of thought; however, there was no difference in the achievement in standard content between the treatment and the control groups, nor was there any significant difference in proof-writing success between the two groups.

Bobango's research was the only study located which used van Hiele phase-based instruction to determine the effect on students in a high school geometry course. However, her study does not answer the questions posed and tested in this research.

The Use of Computers in Teaching Geometry

Using the computer in the mathematics classroom with drill and practice programs has demonstrated a positive effect on the acquisition of computational skills of students (Burns and Bozeman, 1981; Hartley, 1977; Kulik, et. al., 1983). A similar positive effect for mathematical comprehension skills has been recorded when the computer and tutorial programs that test knowledge of concepts and terminology and the ability to reason have been used with students (Bridges, 1985; Burns and Bozeman, 1981; Dugdale and Kibbey, 1983; Henderson et. al., 1983; Kulik et. al., 1983). However, the use of computer simulations that could aid in the application and analysis of geometrical concepts has been limited because of lack of software. Newly developed software tools such as The Geometric Supposer: Triangles (Educational Development Center, Inc., 1985) and Geometry One: Foundations (IBM EDUCATIONAL SYSTEMS System, Inc., 1987) are now available for classroom use and the effect that these programs have on students is just beginning to be studied.

The Geometric Supposer: Triangles is one of four in a series of geometry computer programs. The user first selects a type of triangle or constructs a triangle of a particular size and shape when using this program. Then the user can construct and label points, segments, angle bisectors, medians, altitudes, circles, parallels, perpendiculars, and extensions of line segments. The user can also measure distances, angles, and areas for the figures constructed. These constructions and measurements are all completed very easily by choosing from an appropriate menu. The authors of this program identify the strengths of this program to be its ability to easily provide the user with a wealth of visual and numerical data in order to test conjectures or to find counterexamples (Yerushalmy and Houde, 1986; Schwartz and Yerushalmy, 1987).

In 1984-1985, Yerushalmy (1986) used The Geometric Supposer to investigate (1) the inductive process that takes place with high school students as they generalize conjectures from empirical data, and (2) the reasoning process that takes place in developing mathematical arguments to support these conjectures. The study compared two groups of students in the Weston, Massachusetts, high school. One group (45 students) learned geometry inductively, using The Geometric Supposer, as a learning tool, and the other group (40 students) learned traditional, deductive geometry. The results of this study showed that (1) the students in the inductive group were more willing to consider nonconventional methods of analysis, (2) the students in the traditional group showed signs of a decrease in the ability to generalize and in the use of induction whereas the opposite was true for students in the inductive group, (3) the students in the traditional group were less motivated to think about richer ideas or to change what were assumed to be key features, and (3) the students in the traditional group had a better performance in skills related to the requirements of traditional school mathematics.

The following year, a similar project was conducted in three Boston area suburbs (Yerushalmy, 1987). This study examined student learning and the issues involved in the implementation of a guided inquiry approach using The Geometric Supposer to teach high school geometry. The experimental group that was taught using the guided inquiry approach consisted of 39 students, and the comparison group that was taught in the traditional method had 30 students. Pretests and posttests were developed to assess students' abilities (1) to make generalizations given data or a description of a geometric situation and (2) to produce proofs. Two important and statistically significant performance differences were found between the experimental and comparison groups with regard to the ability to make generalizations. First, the experimental group produced higher level generalizations on two out of three questions on the posttest. Second, students in the experimental group produced more arguments on the posttest abstract question, even though no arguments were requested. As for the ability to produce proofs, there was no significant difference between the two groups.

Bobango (1987) used the Geometric Supposer: Triangles in her study which investigated the question of whether students who had phase-based instruction (as per the van Hiele theory) using the computer as a tool achieved significantly higher measures of van Hiele levels or higher test achievement scores when compared to a control group. She concluded that the phase-based instruction had a significant effect on raising students' van Hiele levels of thought; however, the experiment led to the conclusion that there was no difference in achievement in standard content or proof-writing success in geometry. Since there was not a treatment group using phase-based instruction without the computer, no conclusions could be made regarding the effectiveness of the computer as a tool.

Cooperative Learning in Geometry Classes

In general terms, cooperative learning involves a small group of learners who work together as a team to accomplish a common goal. Even though there are many cooperative learning structures that researchers have developed and studied, all of them have certain elements in common. Artzt and Newman (1990) identified these elements as follows:

> First, the members of a group must perceive that they are part of a team and that they all have a common goal.
> Second, group members must realize that the problem they are to solve is a group problem and the success or failure of the group will be shared by all of the members of the group.
> Third, to accomplish the group's goal, all students must talk with one another--to engage in discussion of all problems.
> Finally, it must be clear to all that each member's individual work has a direct effect on the group's success.

The results of the use of cooperative learning methods in mathematics classrooms have generally produced favorable results. Davidson (1985) reviewed over seventy studies in mathematics which compared student achievement in cooperative learning versus whole-class traditional instruction. In over 40 percent of these research studies, students in the cooperative learning groups significantly outscored the students in the whole-class traditional groups on individual mathematical achievement. In only two studies (Loomer, 1976; Johnson, et al., 1978) did the whole-class traditional students perform better, and Davidson indicated that "both of these studies had irregularities in design" (Davidson, 1990).

Only one study (Cox, et al., 1989) was located which compared students taking high school geometry using small cooperative learning groups versus whole-class traditional instruction. Two classes of high school honors geometry at a private, preparatory school in Hawaii were used in this study. One class was taught using cooperative learning methods, and the other was taught by the same teacher using traditional methods. There was no significant difference in achievement on tests between the two groups; however, the responses from the students in the cooperative learning groups indicated that this method of instruction had positive effects on their relationships with their peers and mastery of the subject and allowed a closer relationship with the teacher.

Student Achievement and Attitude with Respect to High School Geometry

The research on student attitudes toward mathematics (in general) is quite abundant and the findings have been summarized in the writings of Aiken (1970; 1976), Kulm (1980), and Suydam (1984). These summaries of the research indicate that:

1. there is a low correlation between attitude towards mathematics and achievement scores in mathematics;

2. generally, students of all levels recognize the value of studying mathematics;

3. attitudes toward mathematics are formed primarily in grades 4-8 and have a slow but steady decline during the high school years;

4. there is evidence that male and female attitudes toward mathematics are different and that the changes in their attitudes over the grades differ in some ways; and

5. attitudes toward mathematics are probably formed and affected by many variables (In the early grades, home and classroom variables play an important role. In junior high and high school, classroom variables become less important and social interaction becomes more important. At the college level, variables relating to utility and employment opportunities play a major role.)

All of the studies located that involved student achievement and attitudes with respect to high school geometry involved comparing innovative courses with conventional ones. In general, there were no significant differences in student attitudes toward the innovative courses and the conventional courses.

Harbeck (1973) did a comparative study on students' attitudes toward geometry and proof between a group of students instructed by the flow-diagram format and a group instructed by the statement-reason format. She found no significant difference in attitudes toward geometry and proof, but the students using the flow-diagram format had significantly more favorable attitudes toward that format.

Wood (1976) compared students' attitudes for two groups of students--one group an informal geometry course and the other a formal geometry course. No difference in attitude toward geometry and no difference in attitude changes were found between the two groups.

In a study that investigated student responses to a transformational geometry course for low achieving high school students, Herot (1976) found no significant change in attitude. However, she did find that a positive attitude toward mathematics correlated with achievement for these students.

In his research project determining the effects of various levels of teacher/student verbal interaction on geometry students, Dittmer (1978) found some differences in attitude and achievement among three treatment groups. In the first treatment group, students were not permitted to talk. In the second group, the teacher/student talk ratio was 2/1. In the third group, this ratio was 1/1. The second and third groups' attitude scores toward mathematics were significantly higher than the first group's attitude scores. Also, the second and third treatment groups scored significantly higher that the first treatment group on an achievement test of abstract geometric concepts.

In eight classes of average and below average students enrolled in a "novel" informal high school geometry program, Decovsky (1978) found a slight tendency for the students to change their attitudes in the positive direction, although the change was statistically significant ($p < 0.05$) in only one class.

Cox (1979) compared students' attitudes and achievement in an informal geometry course and those in a conventional course. The results showed that the informal group expressed positive attitudes toward geometry, and the attitudes of the informal group rose slightly during the course, although the attitudes of the two groups were not significantly different. The conventional geometry group performed significantly better on a 40-item standardized test; however, the informal group's pretest/posttest achievement gains on the author's 42-item test (which excluded proof) were significantly better than the conventional group's gains.

Prince (1982) investigated the relationship among attitude, achievement, and the personality dimensions of introversion-extraversion among high school students enrolled in traditional and transformational geometry classes. The extroverts with positive attitudes were assigned to the transformational geometry class, and the introverts with positive attitudes were assigned to the traditional class. No significant differences in achievement and attitude between the two groups were found. A positive correlation between attitude and achievement was found.

In a study comparing the effects on student achievement and attitude of two textbook programs, one consistent with the van Hiele theory and the other a standard textbook, Han (1986) found no significant difference in attitude between the two groups. Significant differences between the two groups were found in favor of the traditional group in proof-writing achievement and attitude toward proof. Han also found that the combined group students' attitudes toward geometry declined in the last half of the course.

Summary

Ever since geometry has become a standard course in the high school mathematics curriculum, it has been a course that has received special consideration. Several approaches for teaching high school geometry have been tried, but none seems to have provided any change in student success.

Fey and Good (1985) commented,

> Geometry has been a troubled strand of the curriculum for many years. Despite bold proposals for new approaches the standard experience of most students is still limited. (p.44)

Cox (1985) went on to say,

> To have any chance for success in achieving some degree of universal geometric competence, we cannot merely offer "more of the same." The population of students is diverse and many have little or no chance of achieving success in a traditional proof-oriented course on plane geometry. (p.404)

The van Hiele theory with its phase-based instruction may provide students with an instructional mode that would increase their competence in geometry, but little research with this instructional method has been published.

Computer software for geometry that makes use of the graphic capabilities of the computer is now available, but again, research data on how this software effects the students' abilities to learn geometry is somewhat limited.

According to the literature, students who were taught in a cooperative learning classroom environment had a (statistically) significantly higher achievement level than those taught in a classical classroom environment.

A review of the literature indicates a trend toward a low, but positive, relationship between attitude and achievement. When comparing attitudes of two or more treatment groups using a variety of instructional modes in high school geometry, there were no significant differences in the attitudes of the students toward geometry among the treatment groups.

CHAPTER 3

PROCEDURES

Introduction

The primary purpose of this study was to determine the effect that three different methods of classroom instruction had for students enrolled in a high school geometry course on (a) achievement of certain geometrical concepts, (b) retention of these geometric concepts, and (c) attitude toward mathematics (and geometry, in particular). One method of instruction involved the students working in small cooperative groups doing paper and pencil activities which follow the first three phases (free orientation, guided orientation, and explicitation) of the van Hiele theory. The second method of instruction involved the students working in small cooperative groups using the computer and accompanying software together with activities which follow the first three phases of the van Hiele theory. The third method of instruction used the students' textbooks in a traditional whole-class format.