A computer model of the coupled dynamic fluid and electrical interactions within an MHD duct
by Donald James Hammerstrom

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University

Abstract:
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A system of differential and algebraic equations is developed to model the duct. A Lax-Wendroff fluid flow integration scheme for the entire duct is coupled with a circuit representation of the generator channel. FORTRAN computer programs define the finite difference approximations to the three gas dynamic, one dimensional flow conservation equations, integrate these differential equations, and calculate all related algebraic fluid flow and electrical variables.

A number of modeled transient conditions are presented and discussed. Pulse amplitude synthesis and control (PASC), a low harmonic, DC power consolidation and conversion scheme, is then applied to the model to show that the model may be used with somewhat complex switching and control schemes. The model is shown to model the coupled, dynamic gas fluid and electrical behaviors of an MHD channel during a variety of transient conditions, and it may prove very useful for planning future MHD applications. Some future improvements are suggested.

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by

Donald James Hammerstrom

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Dec. 2, 1991
Date

Roy M. Johnson
Chairperson, Graduate Committee

Approved for the Major Department

Dec. 2, 1991
Date

V. Gerez by D. A. Pierre
Head, Major Department

Approved for the College of Graduate Studies

December 17, 1991
Date

Henry L Parsons
Graduate Dean

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Signature Donald J. Hammerstrom

Date November 27, 1991

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LIST OF SYMBOLS

Symbol	Meaning
A	duct cross sectional area (sq. m)
AC	alternating current
B	scalar magnetic flux density (Webers / sq. m)
$\vec{B}$	vector magnetic flux density (Webers / sq. m)
B_z	$\vec{z}$ component of vector magnetic flux density (Webers / sq. m)
C_v	constant volume specific heat (J / K Kg)
CDIF	U. S. component development and integration facility
DC	direct current
E	scalar electric field (V / m)
$\vec{E}$	vector total electric field (V / m)
E_x	$\vec{x}$ component of vector electric field (V / m)
E_y	$\vec{y}$ component of vector electric field (V / m)
f	function multipliers within a state matrix
F	friction term
g	additional functions used in the state formulation
h	channel height (m)
i	index for axial position
I_x	filamentary Hall current (amperes)
I_y	filamentary Faraday current (amperes)

$\vec{j}$	vector total current density (amperes / sq. m)
j_x	Hall component to vector current density (amperes / sq. m)
j_y	Faraday component to vector current density (amperes / sq. m)
k	second index for axial position
M	Mach number
MHD	magnetohydrodynamic
MSE	MHD test facility in Butte, Montana
P	static pressure (Pa)
PASC	pulse amplitude synthesis and control consolidation scheme
Q	heat loss term
R	gas constant (cu. m Pa / K Kg)
R_x	equivalent axial resistance in electrical circuit model (Ohms)
R_y	equivalent resistance to Faraday current in circuit model (Ohms)
t	time (s)
T	static temperature (K)
U	internal energy (sq. m / s^2)
v	scalar velocity--usually the $\vec{x}$ component (m / s)
$\vec{v}$	vector total velocity (m / s)
V_v	equivalent potential due to magnetomotive interactions (V)
V_x	Hall potential across an axial step Δx (V)
V_y	Faraday potential across channel height h_i (V)
V_{Hallx}	axial potential due to Faraday current (V)
V_{Hally}	potential in $\vec{y}$ direction due to Hall current (V)
w	channel width (m)

W	mass flow rate (Kg / s)
x	axial variable of differentiation
$\vec{x}$	x vector direction
Δx	discrete axial step size (m)
$\vec{y}$	y vector direction
z	schematic viscosity term (sq. m / s)
$\vec{z}$	z vector direction
β	Hall parameter
γ	specific heat ratio
ε	permittivity (F / m)
$\in$	stagnation energy state variable (sq. m / s^2)
μ	permeability (H / m)
ρ	density state variable (Kg / cu. m)
$\rho \in$	density-energy product state variable (Kg / s^2 m)
σ	fluid conductivity (Mhos)

ABSTRACT

The introduction of commercial magnetohydrodynamic (MHD) power generation ducts to existing power grids and the associated control problems arising from MHD power take off schemes necessitate an accurate, dynamic model of the coupled gas fluid and electrical behavior of an MHD duct. This thesis presents such a working model and lays the groundwork for further improvements.

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INTRODUCTION

Background on MHD Power Generation

Magnetohydrodynamic (MHD) power generation is the extraction of power from conducting fluids as they flow through a magnetic field. Two types of interactions occur within an MHD duct. The first type of interaction is fluid dynamic, and the second type of interaction is that due to electromagnetic power extraction in a portion of the duct called the channel. The two interactions intimately coexist such that a perturbation of one of the interactions must also affect the other.

An MHD generator cycle may be described as closed or open. Molten metals like sodium are perpetually circulated in a closed cycle; combustion gases from fossil fuel combustion are expelled from an open cycle. One choses open or closed systems to meet economic, environmental, or system constraints.

MHD generators may use any conducting fluid. Some fluids like molten metal naturally conduct, but most open cycle systems require supplementary conducting metallic seed, which may be recovered after use. The word "compressible" describes those fluids for which density is a function of pressure. Compressible fluids propagate sound at finite speed. This consideration of sound speed exacerbates compressible fluid behavior and analysis. Regardless of fluid properties, one investigates MHD fluid behavior as he would investigate many other similar fluid systems like jet propulsions or steam flows.

Generally, MHD generation takes place at high temperature. There are two important benefits to operating gas fluid, open cycle generators at high temperature: The seeded gas fluid conducts much better the higher the temperature, and hot conducting gas fluid--plasma--transports denser internal energy at high temperatures. Therefore, greater energy extraction and system efficiency can occur for hotter combustion processes. Of course, the high temperatures also bring about detrimental corrosion, containment and control problems.

Electromagnetic interactions can be designed to extract power from the fluid within an MHD channel. By the Lorentz force equation, conducting particles deflect from their trajectories in a direction perpendicular to the magnetic field through which they flow. This charge separation is completed on sets of electrodes on the walls of an MHD channel, and the electrodes may be connected in various external circuit configurations. Faraday generator electrode configurations are those for which the majority of power extraction is perpendicular to fluid flow, and Hall configurations are those for which the majority of power extraction is parallel to fluid flow. Diagonal electrode configurations utilize both Hall and Faraday currents. Generally, MHD generators produce direct current (DC) electrical power.

Throughout this thesis the fluid flow and electromagnetic interactions shown in Figure 1 will prevail. Figure 1 shows the conducting fluid flow in the $\vec{x}$ direction and magnetic flux density $\vec{B}$ in the positive $\vec{z}$ direction. Conducting fluid flow in the $\vec{x}$ direction interacts with the given magnetic flux density $\vec{B}$ to produce a motional emf in the negative $\vec{y}$ direction. The resulting electric field is called the Faraday field. If electrodes separated in $\vec{y}$ within this generator are attached to an external load, current flows in the external load, and a current density flows in the negative $\vec{y}$ direction within the channel. This current in the negative $\vec{y}$ direction also interacts with the magnetic flux to produce an electromotive force field in the $\vec{x}$ direction called the Hall field, and electrodes separated in $\vec{x}$ may supply Hall current to external circuitry. Although both Hall and Faraday fields may be present in either the $\vec{x}$ or $\vec{y}$ directions, the $\vec{y}$ fields in MHD literature are called Faraday fields, and those in the axial channel direction are called Hall fields because these are the directions of major Faraday and Hall interaction, respectively.[10]

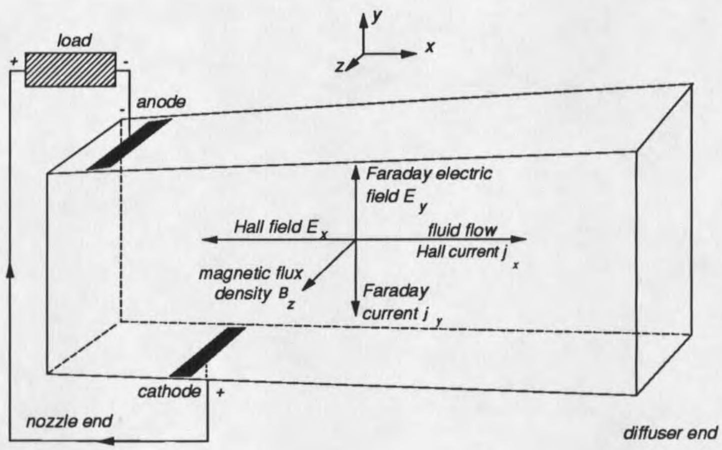

Figure 1
MHD Channel Interactions

Those readers desiring more complete technical background may refer to a book by Rosa[10] or any comprehensive book on plasma dynamics or compressible fluid flows.[9,11,12]

<u>Present State of MHD Research</u>

A number of MHD test facilities have come to operation. However, the addition of an MHD topping cycle to a conventional fossil fuel steam plant is of most interest to introduce MHD into commercial power generation. This design retrofit can fairly easily augment existing power plants, and one may predict a significant increase in overall plant efficiency due to the efficient, high combustion temperatures necessary for an MHD topping cycle.

Presently, the United States Department of Energy's Component Development Integration Facility (CDIF), which is operated by MSE Inc. in Butte, Montana is investigating the technology for coal fired MHD commercial power generation. Considerable success has been demonstrated. However, problems of short electrode life and unpredictable transient behavior still plague the operation. Other countries have shown the viability of clean fuel MHD generation plants, but the retrofitted coal fired plant would be novel in its prolonged use of coal. A retrofit of an MHD coal fired topping cycle to a conventional steam plant is now being planned for possible implementation in the middle 1990s.[14]

Purpose of this Thesis

Most research in MHD power generation and generator design focuses on steady state DC generation. Even DC generation under controlled conditions, however, may encounter unexpected transients, and MHD alternating current (AC) generation may yet have a resurgence of interest. Switching algorithms for power take off and channel optimization also introduce transients. Therefore, the dynamic interaction of the coupled fluid and electrical behavior within an MHD generator should be studied. This thesis presents the formulation for a working dynamic model of the coupled fluid and electromagnetic behavior of an MHD power generation duct. Suggestions for further improvements will be proposed as well.

A good MHD channel model, which correctly models both the steady state and the transient behaviors of the MHD generator from combustor to diffuser end, should greatly enhance the study of control for MHD generation. First, the model would allow fairly accurate extrapolation of small test plant behavior to larger power facilities. Some costly scale-up errors consequently may be avoided. Second, transient behaviors may be investigated to help operators create contingency plans for unplanned transients. One could prepare proper control procedures for electrical transients due to sudden failures and other less severe load fluctuations. Thorough contingency planning can extend component life, maximize operation time, and protect a system from catastrophic failure. Third, switching and control algorithms could be carefully studied before application to a costly test or retrofit facility. Novel control schemes and inversion schemes like pulse amplitude synthesis and control (PASC),[3,4] a source consolidation and inversion scheme, warrant continued investigation.

The pending implementation of a coal fired MHD topping cycle retrofit and the practicality of such an endeavor have biased the formulation and demonstration of the model developed in this thesis. Specifically, the model used in the transient examples represents an open cycle, compressible combustion gas, Faraday power take off system, but the formulation procedure is not inherently limited to MHD generation with any of these attributes. This thesis is intended to emphasize the dynamic capabilities of the developed model, not the abilities of the model to accurately represent any specific MHD duct.

The next section presents the model formulation and the mathematical justification for the formulation. The fluid dynamic and electromagnetic model formulations will be discussed separately. Then the program FORTRAN computer model which models the coupled fluid dynamic and electromagnetic behavior will be presented. The last major section of the thesis presents the results of some dynamic transient experiments as simulated by computer model. The model will be shown to be applicable to fluid transients, electromagnetic transients, and switching transients.

COMPUTER MODEL FORMULATION

Duct Fluid Dynamic Model Formulation

Although the fluid and electromagnetic interactions within an MHD duct are intertwined, it is beneficial to discuss the fluid dynamics apart from the electromagnetic dynamics. Therefore, this section will discuss the mathematical formulation of the fluid MHD behavior only. The model is formulated for compressible, combustion gas behavior, but simple parameter changes would permit the model to be used for general fluids in subsonic or supersonic MHD ducts.

The starting point for the fluid dynamic modeling is a set of three partial differential conservation equations. The three conservation principles are conservation of mass, momentum and energy. Each equation is derived by considering conservation principles applied to an infinitesimal volume. For example, the equation of mass conservation states that the rate of mass flow per volume across an infinitesimal volume's boundaries must be equal to the rate of change in the fluid's density. Similar statements may be made for each of the conservation equations, although the interpretations may not be as obvious. The equation forms shown in Figure 2 come from Rosa.[10]

One makes assumptions from this point to make the given continuity equations tractable. In this formulation it is assumed that the field terms $\frac{\varepsilon E^2}{2}$ and $\frac{B^2}{2\mu}$, the energy storage terms of the system fields, are small or constant. This assumption simplifies the calculation of energy partial differentials. The assumption is reasonable because the applied magnetic field is usually constant in time, and the product of electric field

Mass:

$$\frac{\partial \rho}{\partial t} = -(\rho \nabla \bullet \vec{v} + \vec{v} \bullet \nabla \rho)$$

Momentum:

$$\rho \frac{\partial \vec{v}}{\partial t} = -\rho \vec{v} \bullet \nabla v - \nabla P + \vec{j} \times \vec{B}$$

Energy:

$$\rho \frac{\partial}{\partial t}\left(\frac{v^2}{2} + U + \frac{\varepsilon E^2}{2} + \frac{B^2}{2\mu} \right) = -\rho \vec{v} \bullet \nabla\left(\frac{v^2}{2} + U \right) - \nabla \bullet (\vec{v}P) + \vec{j} \bullet \vec{E}$$

Figure 2
Basic Equations of Continuity

strength and space permittivity ε is quite small. The change of magnetic field in the vicinity of fluctuating conduction currents and current densities may indeed be significant, but this problem is reserved for later models.

The biggest assumption made is that a one dimensional flow model adequately models the behavior of an MHD duct for the studied phenomena. The resulting mathematical simplification is noteworthy in that all gradient operations may then be represented as simple partial derivatives in one variable. The mass and momentum equations, in fact, then contain only partials in respect to $\vec{x}$, the direction of fluid flow. The energy conservation equation will still contain partials in respect to $\vec{z}$ and $\vec{y}$ since energy is indeed exchanged with duct walls!

The consequences of the one dimensional flow assumption are as follows: First, fluid flow occurs primarily in the axial direction down the duct. This assumption somewhat limits duct architectures for which the model is valid. Rapid cross sectional area changes along a duct will prevent accurate solution because at these places expansion causes non-axial fluid velocities, which are not modeled. Inclusion of area gradients in later model formulations may improve model accuracy in this respect. Second, the one dimensional model assumes that physical conditions of pressure, velocity, *etc.* are homogeneous in any duct cross section. This statement is, of course, absurd. Friction slows gases near walls, temperatures differ near walls, and greater compressible fluid densities and pressures occur near the channel anodes. The best one can hope for is a representative example--an average of sorts--for the fluid behavior at any axial position. Other research has successfully developed higher dimension models,[2] but the one dimensional model should provide a good first order representation of dynamic MHD duct behavior for fast control algorithms. The two and three dimensional models are left for later research.

The described simplifications are completed to give a set of equations like those in Figure 3. The formulations shown are those of Ostling,[7] who chose density ρ, mass flow rate W, and the product of density ρ and stagnation energy $\epsilon = U + \frac{v^2}{2}$ as his state variables. He also includes terms for friction and heat losses. The choice of state variables is a natural consequence of the described formulation, but the choice has significant advantage over others because it allows calculation across flow discontinuities like shocks. Regardless, conservation must be maintained. The choice of mass flow rate state variable W also helps assure that mass flow rate will become constant down the duct as steady state is approached. The problem of shocks will be addressed in more detail later.

Mass:

$$\frac{\partial \rho}{\partial t} = -\frac{1}{A}\frac{\partial W}{\partial x}$$

Momentum:

$$\frac{\partial W}{\partial t} = -\frac{\partial W v}{\partial x} - A\frac{\partial P}{\partial x} - F + A(j_y B_z)$$

Energy:

$$\frac{\partial \rho \in}{\partial t} = -\frac{1}{A}\frac{\partial}{\partial x}\left[W\left(\in + \frac{P}{\rho}\right)\right] + Q + \vec{j} \cdot \vec{E}$$

Figure 3
One Dimensional Equations of Continuity

The partial differential equations are then made discrete in axial position by finite difference approximation. The effect of this approximation is that the set of partial differential equations in two independent variables becomes a much larger set of approximated ordinary differential equations in the single independent variable time. The resulting set of equations is fully time dynamic, and the number of equations is three times the number of discrete axial position steps. One's choice of position step size will be determined by his desired accuracy, the importance of the observed region of the duct, and the expected rates of change along the duct due to energy exchange and duct geometry. The axial position steps used in initial examples within this thesis may be considered excessive to some in MHD research.

As an example of the finite difference approximation, the first partial derivative

$$\frac{\partial W}{\partial x}$$

may be approximated by

$$\frac{W_{i+1} - W_{i-1}}{2\Delta x}$$

where Δx is the axial position step size. The second partial derivative

$$\frac{\partial^2 \rho}{\partial x^2}$$

will be approximated by

$$\frac{\rho_{i+1} - 2\rho_i + \rho_{i-1}}{(\Delta x)^2}.$$

Usually the choice of small step size will improve the accuracy of the finite difference approximation.

Following the procedure outlined in Ostling's paper,[7] schematic viscosity terms were then included in the three sets of differential equations. Schematic viscosity is an artificial viscosity which eases mathematical computer integration. Unlike a true fluid viscosity, it is independent of channel surface area and is instead represented by second order combinations of the fluid dynamic variables. The evaluation of a set of schematic viscosity terms may be positive or negative, but it nearly always moderates the rate of change. It may be thought of as a penalty for any nonlinear parameter variation over short axial distances. The conservation equations, equations of first order, then react to the schematic viscosity terms to force compliance with the laws of conservation at each axial position. The net effect is a reduction in oscillation and an increase in stability. The three schematic viscosity additions to the three conservation equations are shown in Figure 4. These additional terms were also made discrete by the finite difference method as described above.

Mass:

$$+\frac{\partial z}{\partial x}\frac{\partial \rho}{\partial x}+z\frac{\partial^2 \rho}{\partial x^2}$$

Momentum:

$$+\frac{\partial z}{\partial x}\frac{\partial W}{\partial x}+z\frac{\partial^2 W}{\partial x^2}+\rho z A\frac{\partial^2 v}{\partial x^2}$$

Energy:

$$+z\frac{\partial^2 \rho \in}{\partial x^2}+\frac{\partial z}{\partial x}\frac{\partial \rho \in}{\partial x}$$

where

$$z=| v |\frac{\Delta x}{2},$$

the schematic viscosity, and all other variables are described elsewhere.

Figure 4
Schematic Viscosity Terms

The state equations could have been formulated to eliminate all non-state variables, but such a formulation would require that inherent assumptions be imbedded within the state equations and would hide some of the more intuitively helpful physical variables like pressure and temperature. Therefore, a two step Lax-Wendroff[5,9] approach is taken to alternately integrate the state equations, then calculate the set of related algebraic equations. The related algebraic equations are briefly listed below:

The most important algebraic equation is the specifying equation, or equation of state. This equation specifies the relationship between density and pressure. For this initial formulation, an ideal gas assumption was made. No gas will behave ideally, but the accuracy provided by the assumption allows good initial dynamic study. Later models must include gas nonlinearities.

$$P = \rho R T$$

Mass flow rate, density, cross sectional duct area and gas velocity are related by

$$W = \rho A v.$$

This equation allows the calculation of duct velocities.

It is assumed that internal energy is a function of temperature only. Consequently, temperatures may be calculated

$$T C_v = \in - \frac{v^2}{2.}$$

And Mach number can be calculated

$$M^2 = \frac{v^2}{R \gamma T}.$$

Here, γ is the ratio of specific heats taken arbitrarily to be 1.4, the specific heat ratio for air at room temperatures. This assumption may be improved in later models, and the ratio may even be calculated dynamically for each axial position.

Only the first and last nodes pose problems to the integration scheme since no upstream or downstream variables exist at these nodes to formulate correct finite difference approximations. Ostling[7] presented a interesting formulation of the input conditions at the back wall of a combustor using the concept of reflection, and his results may be helpful for future models, but the problem is not as well defined at an open input to a nozzle. Therefore, one must define boundary conditions at the first and last nodes, the input to the nozzle and the exit from the diffuser, respectively.

Two possible input boundary conditions were posed. First, since no combustor is included in the model, one may choose to provide the input combustor pressure and stagnation temperature, then calculate mass flow rate. Second, he may provide mass flow rate and stagnation temperature for the combustor and calculate input pressure. The equations for these boundary conditions are those used by Matair.[5] Once either of these two input boundary conditions is accepted, one need only extrapolate the velocity back to the input boundary node and calculate the remaining variables. The MHD operator will most likely have control of mass flow rate rather than combustor pressure. But one should choose the input variable which is least susceptible to change during the transient of interest. Should a combustor be added to the model, the first boundary condition formulation must be chosen and no choice remains.

At the diffuser exit, the most important boundary condition is the requirement that the exit pressure is only slightly higher than atmospheric. This condition is imposed to assure that flow will continue through the bottoming portion of a power plant. Density and mass flow rate are then extrapolated from near nodes. Further, the time differential of the energy-density product state variable is extrapolated to allow its calculation through integration.

These boundary conditions were formulated to allow the greatest possible degree of freedom at the boundaries. Insufficient description, however, gives instability. These boundary conditions may continue to evolve with the computer model. For example, should a combustor be included in the model, the first node will become a reflective wall, and the boundary condition will become relatively well defined as described by Ostling.[7] Such a formulation is especially applicable to current tangential slag and coal input designs.

Before this section is concluded, more must be said about the model's fluid behavior near shocks. A shock front is the boundary between a region which is supersonic and a region which is subsonic. The shock front may be either transient like that which must occur when a supersonic duct is shut off, or the shock front may be static. The speed of sound is primarily dependent upon temperature and the compressible fluid's relationship between pressure and density. Consequently, a shock may result in rather severe discontinuities for temperature, velocity, pressure and other variables on the two sides of any shock. The fortunate thing about the method of formulation used in this thesis is that because the formulation is based on conservation principles, a shock poses no exception. The Rankine-Hugoniot relations still hold at a shock location in the duct. This type of formulation is sometimes called a *through*, *smoothing* or Lax-Wendroff method for the way it treats shocks. The alternative to this formulation would be to shock fit, a method which usually requires that the shock be located in the duct before an accurate curve fit can be made at that location. The shock fit method is quite accurate, but it is not as flexible in dynamic situations as the smoothing method.

The inclusion of schematic viscosity smooths shock fronts slightly, but one gains a startling increase in integration step size, and therefore, one also decreases his program run time.

Channel Electrical Model Formulation

Neither the fluid dynamic model formulation nor the electric model formulation of this thesis are novel, but the combination of fluid dynamic model and circuit representation of the channel interactions is original. The advantage to this union is that the complete model taps both the voluminous experience in fluid dynamics and the extensive available circuit theory of the electrical engineers. This section shall describe the circuit model of the electrical interactions within the MHD channel. The formulation is very similar to that posed by Trung.[13]

The formulation is based upon a fairly complete equation for Ohm's law.[10]

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) - \frac{\beta}{B}(\vec{j} \times \vec{B})$$

The equation states that the current density within the MHD channel is equal to the net electromotive field multiplied by conductivity, less a component due to Hall interaction. The Hall parameter β represents the magnitude of influence perpendicular current density components have on the vector total current density and increases with magnet strength and plasma ion mobilities.[10] The presence of current density on both sides of the equation complicates interpretation, but a few simple cases can be addressed to acquire some intuition about MHD interactions: First, assume that only an axial component to fluid flow is significant. If no electric current flows, it is obvious that the electric field and magnetomotive field vectors are equal in magnitude but opposite in direction. Second, if only Faraday current flows, the magnitude of the magnetomotive field must be somewhat greater than the electric field in the $\vec{y}$ direction, and current flows in the direction $\frac{\vec{v} \times \vec{B}}{|vB|}$. The electric field in the $\vec{x}$ direction divided by conductivity must be approximately equal in magnitude to the Hall interaction term. Similar arguments may be made for the case of conduction in the Hall direction only. These simple cases should provide some insight into the behavior and complexity of the general MHD channel interactions.

The vector components of Ohm's law are separated to begin the formulation of a circuit model for electrical interactions. After initial separation one finds

$$j_x = \sigma E_x - \beta j_y$$

and

$$j_y = \sigma(E_y - vB) + \beta j_x.$$

One can immediately rearrange to get

$$E_x = \frac{j_x + \beta j_y}{\sigma}$$

and

$$E_y = \frac{j_y - \beta j_x}{\sigma} + vB.$$

Assume homogeneity throughout each discrete channel region in the one dimensional, finite difference approximation to obtain current density representations in the filamentary currents I_x and I_y.

$$j_x = \frac{I_x}{wh}$$

$$j_y = \frac{I_y}{w\Delta x}$$

One may then integrate the field equations over Δx, the discrete axial position step, and h, the height of the channel in the $\vec{y}$ direction at an axial position, respectively. The result will be the negative of the potential across the integrated distance.

$$V_x = -\frac{I_x \Delta x}{\sigma w h} - \frac{\beta I_y}{\sigma w}$$

$$V_y = -\frac{I_y h}{\sigma w \Delta x} + \frac{\beta I_x}{\sigma w} - vBh$$

One now begins to see the relationship between the expressions derived and the circuit diagram for a single electrode pair shown in Figure 5. The first terms in the above equations resemble products of currents and resistances; the remaining terms resemble voltage sources. Some notation changes are made to simplify the model.

Let

$$V_x = -I_x R_x + V_{Hallx}$$

and

$$V_y = -I_y R_y + V_{Hally} - V_v,$$

where

$$R_x = \frac{\Delta x}{\sigma w h},$$

$$R_y = \frac{h}{\sigma w \Delta x},$$

$$V_{Hallx} = -\frac{\beta I_y}{\sigma w},$$

$$V_{Hally} = \frac{\beta I_x}{\sigma w},$$

and

$$V_v = vBh.$$

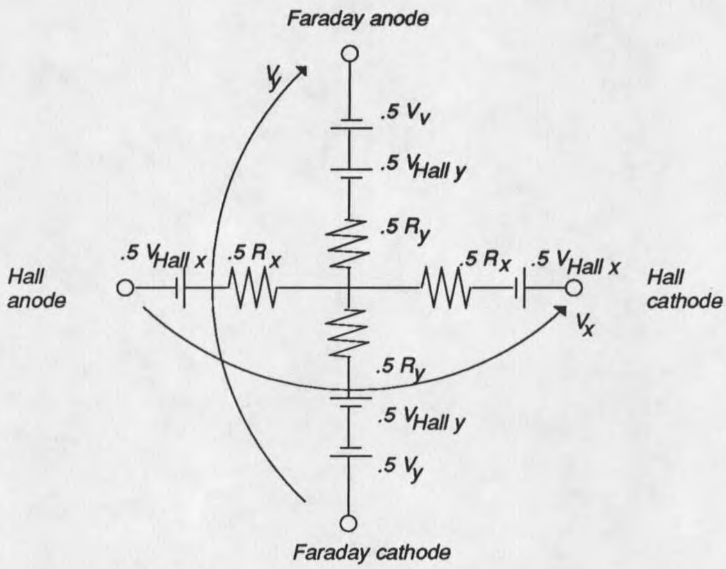

Figure 5
Circuit Model for a Single Electrode Pair

One might notice that the sign of the Hall source V_{Hallx} is carried into its definition. This is done to reflect the fact that the Faraday current is usually in the negative $\vec{y}$ direction, and therefore positive Hall current should flow in the positive $\vec{x}$ direction. The model then avoids negative voltage sources, and the model is quickly and correctly interpreted. Keep in mind that all shown sources and resistances will vary with currents, conductivities, and channel dimensions.

Numerous models may be combined as shown in Figure 6--here shown in a Hall configuration. Each electrode pair should be represented by an integer multiple of models. This suggestion maintains a correspondence between filamentary currents and meaningful channel outputs. It is assumed that no energy storage elements are required