Fresnel diffraction with small apertures
by Andrew David Struckhoff

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Physics
Montana State University

Abstract:
Scalar diffraction theory has been shown to produce accurate results in both the far-field (Fraunhofer) and near-field (Fresnel) regions.7,8,11. The derivation of the theory leads to three possible results of what is known as the obliquity factor. Light from a laser was allowed to shine through a small aperture (circular hole). By investigating the appropriate region of the resulting patterns, it was hoped that the proper obliquity factor could be discerned. Through this study, experimental results have indicated that when the aperture becomes small enough, scalar diffraction theory fails to accurately predict the location of the Fresnel patterns occurring in the (very) near-field region. Rather than verifying the correct obliquity factor, this thesis provides an indication of the region where scalar diffraction theory is no longer valid.

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MONTANA STATE UNIVERSITY
Bozeman, Montana

December 1991

APPROVAL

of a thesis submitted by

Andrew David Struckhoff

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Date

Chairperson, Graduate Committee

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ACKNOWLEDGEMENTS

I would like to thank several people for their assistance throughout this project. first, Dr. Hal Kraus at Idaho National Laboratory, without whose inspiration and support this project would not have been possible. Next, I am deeply indebted to Dr. John Carlsten for his assistance and support throughout this project. His guidance has prompted me to want to learn about physics, not just to learn as needed. Through his experience, I have learned more about the process of experimentation than can be learned in any class. Thank you John. Also, the graduate physics students who provided questions which enabled me to think about the problems which I faced throughout the past two and one half years. I would also like to thank the secretarial staff at the MSU Physics Department for their assistance with the word processor and their invaluable patience. This author would also like to thank the other committee members, Dr. Rufus Cone and Dr. Hugo Schmidt, for their comments and questions regarding the improvement of this thesis.

Final thanks go to my family. To my wife, Ginger, who stood by my side, took care of me, provided a voice of reason in times of turmoil, and simply was present in times of frustration, I am deeply gracious. To my parents, for their support and encouragement. And, a warm thanks to my father, who not only provided a sense of scholarship and importance in education, but whose fight with cancer has taught me more about appreciating life than can be learned out of any textbook.

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ABSTRACT

Scalar diffraction theory has been shown to produce accurate results in both the far-field (Fraunhofer) and near-field (Fresnel) regions.[7,8,11] The derivation of the theory leads to three possible results of what is known as the obliquity factor. Light from a laser was allowed to shine through a small aperture (circular hole). By investigating the appropriate region of the resulting patterns, it was hoped that the proper obliquity factor could be discerned. Through this study, experimental results have indicated that when the aperture becomes small enough, scalar diffraction theory fails to accurately predict the location of the Fresnel patterns occurring in the (very) near-field region. Rather than verifying the correct obliquity factor, this thesis provides an indication of the region where scalar diffraction theory is no longer valid.

CHAPTER 1

INTRODUCTION

The propagation of an electromagnetic wave past an opaque body casts an intricate shadow made up of bright and dark regions quite unlike anything one might expect from the tenets of geometrical optics.[1] This phenomenon of diffraction contradicted the extremely popular belief that light propagated as a ray in a single direction. The first documentation of diffraction was presented by Grimaldi, in a book published two years after his death (1663).[1,2,4] Over the next 100 years, substantial evidence was found supporting Grimaldi's discoveries, however no real explanation of the observed phenomena was advanced.[4] Huygens, the first proponent of the wave theory, seems to have been unaware of Grimaldi's findings, as he would have relied on them to support his ideas.[2] It was not until 1818, when Fresnel applied both Huygens' construction for propagating a wave and the principle of interference to arrive at a possible explanation for diffraction.[2] This combination was given a firm mathematical basis by Kirchhoff in 1882, and ever since, the subject has been investigated by many writers.[1-11]

The phenomenon of diffraction, and the mathematics which support it, has been tested extensively over the past century.[7,8,11] As with most practical problems, though, typical solutions still involve some assumptions and approximations. The resulting mathematics depends solely on what approach one takes to solve the problem and the validity of the assumptions. Kirchhoff's classical theory has been shown to work quite well in describing the diffraction field when relative dimensions of the experiment are much larger than the incident wavelength.[7,8] However, the theory is often criticized, as the solutions to his theory do not recover the assumed boundary conditions at the diffracting aperture.[6] Since Kirchhoff's initial formulation in 1882, there have been two other formulations for describing diffraction theory which do not encounter mathematical difficulties. These are the formulations by Rayleigh and Sommerfeld, involving slight alterations from Kirchhoff's theory. For a majority of the diffraction problems encountered, the Kirchhoff formulation seems to be appropriate, as the mathematical inconsistencies do not contribute to the final solution.[6,7,8]

Chapter 2 gives a summary of the mathematical derivation which Kirchhoff presented, and the subsequent additions given by Rayleigh and Sommerfeld. While the phase relationships of the propagating wave remain the same in all of the derivations, the obliquity factors differ. Recently, Kraus[9,10] has studied the differences in the obliquity factors in order to test the validity of each derivation. This experiment was conceived in an effort to determine which factor is indeed correct. The resulting area of investigation provided interesting results.

Chapter 3 describes the process of the experimental studies. As is often the case, after the initial conceptualization, many refinements had to be made in order to obtain the required data. Throughout the process of development, other related areas of optics were studied. These studies are presented as separate topics and can be found in the appendices.

Chapter 4 presents the results of the experiment. The results go beyond the initial goal of the project. Aside from testing the diffraction integral, an interesting area of diffraction theory was exposed. The experiment was found to probe the region where scalar diffraction theory itself is seemingly no longer valid. This occurs in areas close to the diffracting aperture. In this region, vector diffraction theory is probably needed. Several references appear to indicate that scalar theory will start to fail in the region being probed; however, no definite calculations have been done. The results presented here may give a point of reference as to where scalar diffraction theory will begin to fail.

CHAPTER 2

THEORY

After the discovery of diffraction, the evolution of a quantitative theory did not start until Huygens' principle was combined with wave interference. Christian Huygens hypothesized that as a wave propagates, each point along the wavefront acts as an emitter of secondary sourcelets.[6] The sum of these secondary sourcelets represents the motion of the wave. Figure 1 shows a typical Huygens' construction. By taking relative phases into account, Fresnel was able to accurately predict light distributions using this wave construction method.

The problem of diffraction is that of light impinging on a screen with a specified aperture. On the "shadow" side of the screen, geometrical optics would predict a cone of light, governed by the properties of the light rays which enter the aperture. However, for a circular aperture, physical observations yield a pattern of concentric rings. Kirchhoff's derivation in 1882 was the first true mathematical explanation for these rings. The derivation can be found in almost any basic optics book.[1-3,6] Appendix A reviews the complete derivation of scalar diffraction theory, while the results are simply presented in this chapter. Figure 2 shows the geometry to be used for the problem.

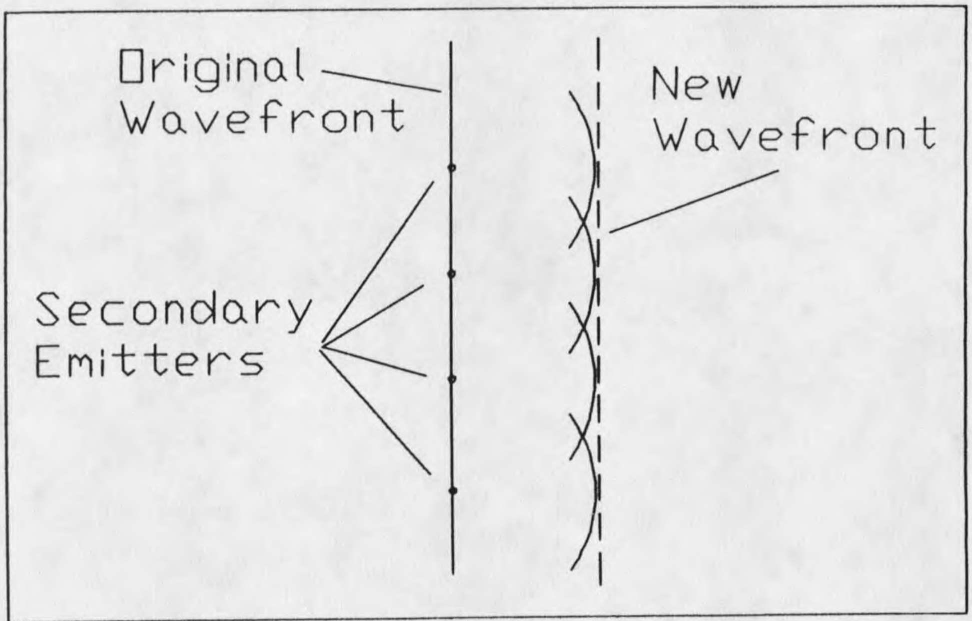

Figure 1 Huygens' construction for propagating a wave

Consider a point source of light located to the left of a screen with an aperture. An accurate description of the light distribution on the right side of the screen is needed. The observation point is denoted as P_0 on the right of the screen.

Using Huygens' construction, the sourcelets are placed in the plane of the aperture. The total distribution at the point P_0 can be attained by adding all of the relative amplitudes and phases of the sourcelets oscillating in the aperture plane. The equation which describes the field at the point P_0 is called the scalar diffraction formula and is written as

$$U(P_0) = \frac{1}{i\lambda} \iint_{\Sigma} \frac{\exp[ik(r_{21} + r_{01})]}{r_{21} r_{01}} \, \wp \, ds \qquad (2:1)$$

The integral occurs over the opening of the aperture Σ, and, r_{01} (r_{21}) is the distance from

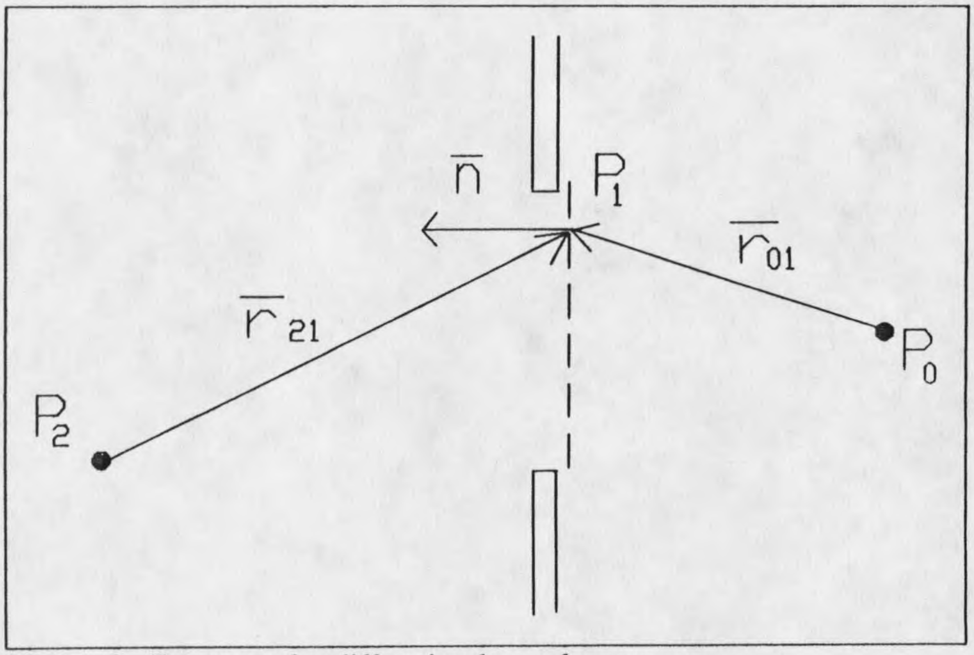

Figure 2 Geometry for diffraction by a plane screen

the observation (source) point to a point in the aperture plane. In essence, the integral adds all the amplitude and phase contributions from a wavelet which originated at the point P_2 and ends up at the point P_0. The difference in pathlength now depends on which point, P_1 in the aperture, is used as the intermediate point. The term $\wp$ is referred to as the *obliquity factor*. The obliquity factor depends upon how equation (2:1) is obtained. In appendix A, it is shown how three possible obliquity factors exist. They are

$$\begin{aligned} \wp &= 1/2[\cos(\bar{n},\bar{r}_{01}) - \cos(\bar{n},\bar{r}_{21})] \quad (\textit{Kirchhoff}) \\ \wp &= \cos(\bar{n},\bar{r}_{01}) \quad (\textit{Rayleigh-Sommerfeld I}) \\ \wp &= \cos(\bar{n},\bar{r}_{21}) \quad (\textit{Rayleigh-Sommerfeld II}) \end{aligned} \tag{2:2}$$

where the term $cos(\bar{n},\bar{r}_{01})$ ($cos(\bar{n},\bar{r}_{21})$) refers to the angle between a)the vector pointing from the observation (source) point to a point in the aperture plane and b)the normal vector of the aperture plane. At this point, an appropriate question would be; "Which obliquity factor is correct?". When the source and observation points are far away from the aperture, the cosine terms in each obliquity factor appear as

$$\cos(\bar{n},\bar{r}_{01}) \approx 1 \qquad \cos(\bar{n},\bar{r}_{21}) \approx 1 \tag{2:3}$$

and all three factors will equal 1. Kraus, however, observed that as the angles increase, the different obliquity factors will produce noticeable differences in the diffraction fields.

Another interesting equation can be derived by confining the points P_0 and P_2 to the axis of symmetry for the problem of a circular aperture. The point P_0 will essentially "see" two different distances to: a) the center of the wave along the axis, and b) the edge of the aperture. The geometry for this situation is depicted in Figure 3.

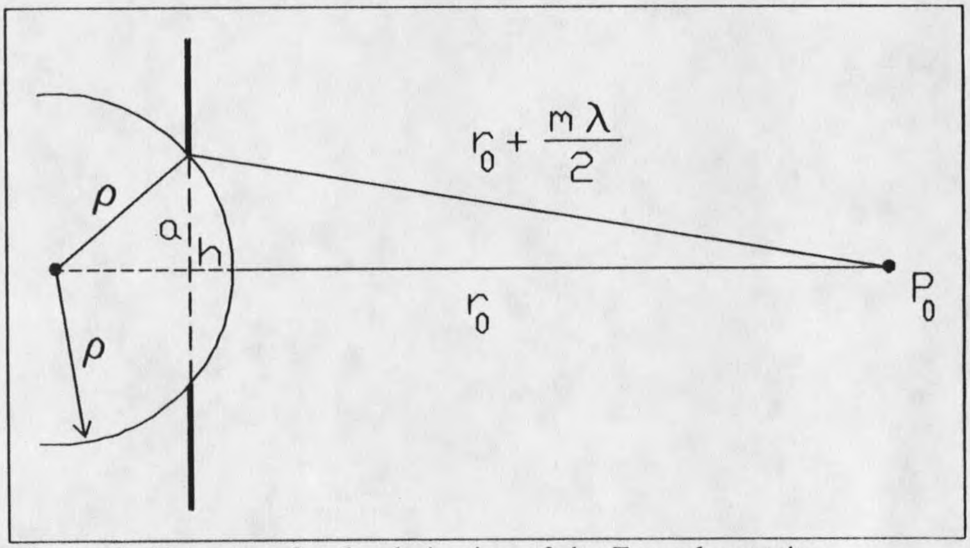

Figure 3 Geometry for the derivation of the Fresnel equation

Notice that the difference in pathlength is what matters here. When the net difference in pathlength is an even number of half-wavelengths, all of the phases across the wavefront cancel out and there is a dark spot along the axis. One can obtain a black spot by shining light through a hole! When the net difference in pathlength is an odd number of half-wavelengths, there will be a bright spot. This effect has been confirmed in several experiments.[1-4,7,8,11] When one of these distinctive patterns occurs, it is known as a Fresnel pattern. The patterns are numbered according to the integer number m in Figure 3. Utilizing the geometry of the problem one arrives at an expression for the location of the Fresnel patterns as a function of the experiment's geometry;

$$r_o(m) = \frac{\lambda^2 m^2/4 - a^2}{a^2/\rho - m\lambda} \qquad (2:4)$$

This is known as the Fresnel equation.

Chapter 3 describes the experiment which was done in an effort to test equations (2:1) and (2:3). Following Kraus' predictions, the appropriate geometry (aperture radius a, beam radius ρ, and observation point r_0) was investigated to find out which obliquity factor in equation (2:1) was indeed correct. Along the way, equation (2:3) was utilized as the first test that scalar diffraction theory was indeed working. Comparing the complicated intensity patterns produced with theory (equation (2:1)) was done at Idaho National Engineering Laboratory. The comparison of the position data was much less involved in terms of computer time, so equation (2:3) served as the initial requirement for testing the theory.

CHAPTER 3

EXPERIMENT

In typical experiments, one usually begins with an initial concept of how the experiment should work, and then the apparatus is refined until the required precision is attained. This experiment progressed in precisely that fashion. Once it was realized what sort of precision was needed, the proper instrumentation was procured and the data was taken. Several additional studies were done in an effort to understand what factors affected the precision of the experiment. These studies included; Spherical Aberrations (Appendices B and C) and Image Formation (Appendices D and E). As the studies were completed, the experiment was altered to accommodate these new considerations.

The basic diffraction experiment is pictured in Figure 4. The nature of the Fresnel diffraction required a coherent light source. A 5 mW helium-neon laser was employed for this purpose. The beam was filtered to remove the unwanted spatial "noise". The beam, prior to the spatial filter is scattered many times from dust particles inside of the laser cavity and in the air itself. These dust particles cause a distinct non-uniformity to occur on the beam. The spatial filter focuses the beam through a small pinhole. The event of focusing separates the desired beam from the unwanted noise.