The effect of high school calculus background on achievement in post-secondary calculus
by William John Swartz, Jr

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University

Abstract:
The problem of this study was to determine, for students at Montana State University in Bozeman and Montana College of Mineral Science & Technology in Butte, the effects of one-semester and two-semester high school calculus courses on achievement in Calculus 1, the first calculus course at the college/university level.

The Calculus 1 student sample from each institution was divided into several subsamples, reflecting each student's freshman or non-freshman status. Data of each subsample and both complete institutional samples were analyzed separately. Students in each group were categorized as having in high school (1) no exposure to calculus, (2) a one semester calculus course, or (3) a year course. Achievement in Calculus 1 was measured by scores of examinations given in the course.

Two main questions occur concerning the merit of calculus as a high school course: (a) What are the effects of advanced placement in college/university mathematics courses? (b) What are the effects on student achievement in Calculus 1 for students who had exposure to calculus in high school but did not receive advanced placement in mathematics at the college/university level? This study addresses the latter question.

Multiple Linear Regression was employed to correlate calculus background and other predictor variables with Calculus 1 achievement. ANOVA and the Newman-Keuls method were used to compare mean achievement among the high school background groups. The Chi Square Test of Independence was used to test for independence of background and achievement.

In several instances, students with a year high school course outperformed either those with no high school background or both other background groups. In some instances, there was no difference in achievement among the three background groups. In no case did the semester group achieve better than the no-calculus group.

There appears to be a positive link between a year of high school calculus background and achievement in Calculus 1. A semester course, however, offers no apparent benefit.

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William John Swartz Jr.

A thesis submitted in partial fulfillment
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APPROVAL

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William John Swartz Jr.

This thesis has been read by each member of the graduate committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Date Sept. 15, 1992

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ABSTRACT

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In several instances, students with a year high school course outperformed either those with no high school background or both other background groups. In some instances, there was no difference in achievement among the three background groups. In no case did the semester group achieve better than the no-calculus group.

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CHAPTER 1

PROBLEM, NEED AND DEFINITIONS OF TERMS

Introduction

Mathematics programs in elementary and secondary schools in the United States underwent considerable revision in the late 1950s and early 1960s. This revamping was one notable manifestation of Sputnik and its ripple effects in American society. The changes that were made in elementary and secondary mathematics curricula are classified by Rash (1977) into two general categories:

1. The use of unifying concepts.
2. Accelerating the content.

The former usually took the form of the so-called "new math" curricula (namely the teaching of number bases in the middle grades and emphasis on set concepts), and the latter led to the introduction of calculus as an advanced course in the high school (Rash). While much discussion and research has centered around the merits and demerits of the "new math," less attention has been devoted to the question of effectiveness of the high school calculus programs and the ramifications for placement in college or university mathematics courses. A recounting of research that has been done in this area will follow in chapter 2.

Statement of the Problem

The problem of this study was to investigate the effects of one-semester and two-semester high school calculus courses on achievement in the traditional first course in calculus for mathematics, science, and engineering majors at the college/university level.

Need of the Study

In the period from 1973 to 1982 in the United States, the number of students enrolled in calculus courses (that is, mathematics courses treating such topics as limits, differentiation, and often integration) in high school grew at a rate of more than 10% annually. In 1982, 234,000 students passed a high school calculus course, with 148,600 of them receiving a grade of B- or better (Carroll, 1984). By 1987 about 300,000 high school students were enrolled annually in some type of calculus course (Steen, 1987).

Concurrently, about 600,000 students enroll annually in first-year Calculus 1 in American colleges and universities (Anderson & Loftsgarden, 1987). Many of these are the same students who had exposure to calculus in high school. However, Calculus 1, the classic introductory calculus course at the college/university level suffers a 35% national failure rate (Douglas, 1988). Even more disturbing is the fact that only 46% of these students completed the first year of calculus (two semesters or three quarters) with a grade average of D or better (Anderson & Loftsgarden). These circumstances clearly warrant concern and raise important questions. As Steen (1986, p. 157) notes,

> Each year about 500,000 students in the United States study [post-secondary] calculus . . . Calculus is big business with far reaching consequences for students, for colleges and universities, for the mathematical community, and for our nation.

While some research efforts have addressed questions relating to the connection between high school calculus and achievement in college/university mathematics, there is a clear need for further investigation in this important area. Studies by Austin (1975), Bergeson (1967), Burton (1989), Dickey (1985), Fry (1973), McKillip (1965), Paul (1970), Pocock (1974), Robinson (1970), Shimizu (1969), Sorge and Wheatley (1977), and Tillotson (1962), each dealing with some aspect of this complex issue, will be discussed in the Review of Literature in chapter 2.

In assessing the worth or benefits of calculus as a high school course, two main issues must be examined:

1. The effects of advanced placement in college/university mathematics courses.

2. The effect on student achievement in Calculus 1 for students who have had exposure to calculus in high school but did not receive advanced placement in mathematics at the college or university.

Studies discussed in chapter 2 provide evidence that advanced placement students who have had calculus as a high school course and have passed either the AB or BC mathematics Advanced Placement Examination generally perform better in the traditional second college/university-level calculus course, and also in subsequent mathematics coursework, than their regular placement counterparts.

Other investigations noted below dealt with the question of whether students receiving two semesters of calculus in high school demonstrated greater achievement in Calculus 1 at the college or university than students without that high school background. However, the studies cited do not satisfactorily resolve this question. Given the considerable and varied changes in secondary and college/university curricula and teaching methods in the last two decades, studies by Paul (1970), Robinson (1970), and Sorge and Wheatley (1977) may be considered outdated. Furthermore, Paul's data were derived only from students at The Ohio State University. Robinson's data came exclusively from the University of Utah, and Sorge and Wheatley's sample was limited to Purdue University. The locations of these studies indicate a need for further investigation which has stronger generalizability to students attending a college or university in Montana or comparable institutions in the Northern Rocky Mountain region. As Robinson notes, "Further studies of this type, conducted at different universities (involving students from other geographic regions) are necessary to determine whether the results can be generalized" (p. 60). This study utilized data from both Montana State University in Bozeman, and Montana College of Mineral Science & Technology (Montana Tech) in Butte. Montana State University is a land grant institution with strong programs in agriculture and engineering, as well as degree programs in various education, fine arts, liberal arts, and science areas. By contrast, Montana Tech is noted for degree programs in engineering areas such as environmental science, geophysics, metallurgy, mining, and petroleum, as well as computer science. The multi-institutional aspect of this study distinguishes it from earlier studies done on this subject.

The manner in which information regarding the content of high school mathematics courses was obtained, directly from school personnel, is unique to the present study.

Another question concerns the achievement in Calculus 1 of students who have had one semester of calculus in high school versus those with little or no exposure to the subject in high school. The literature reviewed below, regarding this issue, is inconclusive indicating a need for further study. Of the five known studies that have been done in this area, one was completed in 1962 and another in 1965, when calculus as a high school subject was in its infancy. The likelihood of change in content, emphasis or teaching methods for such a course in the ensuing decades renders those studies of questionable use today. The three more recent studies, one completed in 1970, and two in 1977, are also not very current. These latter studies have resulted in conflicting conclusions. Paul (1970) found that students with one semester of high school calculus had greater achievement in university Calculus 1 than those students without such a background. Both Rash (1977) and Sorge and Wheatley (1977) found otherwise (no difference in achievement) in their studies. Hence there is a need for further research.

Another question compares the merits of one semester versus two semesters of high school calculus, as measured by achievement in Calculus 1. Again, the findings of previous research fail to resolve the issue. For example, Sorge and Wheatley (1977) note that their data "indicate that the amount of calculus in high school does not seriously affect a student's chances for a particular grade" in university Calculus 1 (p. 644). By contrast, McKillip's results (1965) indicated that a one-semester high school calculus course produced no significant effect on students' grades but that a two-semester high school calculus course did improve the grades in Calculus 1.

Some previous studies in this realm have distinguished between two-semester high school calculus courses offered under the auspices of the Advanced Placement Program and two-semester calculus courses which are not a part of the Advanced Placement Program. The present study did not differentiate between these two categories of courses, since there are very few high school calculus courses offered under the aegis of the Advanced Placement Program represented in the sample of the study.

Another reason for conducting this study was that the potentially interactive effects of several important predictor variables have been ignored by previous research done in this area. Therefore, this study will account for the effects on college mathematics achievement of such predictor variables as student age, student's major, class standing, high school size, and gender. There exists considerable evidence that gender often plays a significant role as a variable in mathematics achievement. Stent (1977) and Tobias (1976) concluded that due to the influence of sex role socialization, mathematics anxiety is more common and more severe among women than among men. Betz (1978) in turn found that higher levels of mathematics anxiety are related to lower mathematics achievement test scores, suggesting that gender should be included as a predictor variable in the present study.

Other included variables may also play significant predictive roles in Calculus 1 achievement. For example, age may correlate closely with mathematics background once attained but forgotten, that is, a "rustiness" factor, particularly for students over traditional age. Perhaps students choosing major areas of study that are strongly mathematics oriented (such as mathematics, engineering, and the physical sciences) achieve at a higher level in Calculus 1 than those who gravitated to majors in subjects that are less mathematical.

Gaining further insight into the effects of one or two semesters of high school calculus on achievement in Calculus 1 is useful for at least five reasons:

1. A common motivation among students for taking a one or two-semester calculus course in high school is the hope that it will enhance subsequent achievement in college/university Calculus 1 (Rash, 1977).

2. If a one or two-semester high school calculus course does not generally demonstrate a positive effect on achievement in Calculus 1, this would be a relevant consideration in deciding whether to discontinue offering calculus as a one or two-semester course at the secondary level (McKillip, 1965; Sorge & Wheatley, 1977). Under such circumstances, a one-semester course could conceivably be eliminated in favor of a two-semester course if the latter showed a significantly greater positive effect on Calculus 1 achievement. Another possibility is that a high school offering a one-semester calculus course may choose to eliminate calculus entirely from its curriculum if budgets, low enrollment, lack of a qualified instructor or other factors make a year-long calculus course unfeasible. This is a plausible scenario in view of both current fiscal problems plaguing many school districts and the plethora of different mathematics courses offered by many high schools in recent years. Whitesitt (1980) notes, for example, that Bozeman Senior High School in Montana offered only four mathematics courses during the 1957-1958 school year, but offered a total of thirteen mathematics courses in the 1977-1978 school year. Similar figures could be noted in high schools throughout the United States. If neither a one or two-semester high school calculus course is shown to enhance achievement in Calculus 1,

that knowledge would be a germane consideration in a decision to eliminate calculus from the high school curriculum.

3. According to data reported by Sorge and Wheatley (1977), students may devote time and effort prematurely studying calculus in high school at the expense of adequately mastering important prerequisite concepts in geometry, algebra and/or trigonometry; thus hampering their efforts in subsequent mathematics coursework. Therefore, the possibility exists that the study of calculus in high school may not only fail to yield justifying benefits, but as Steen (1986) believes, may in fact be counterproductive to the students who pursue further mathematical study at the college/university level.

4. The recently published National Council of Teachers of Mathematics (NCTM) standards suggest that discrete mathematics and probability and statistics be included in the high school curriculum, perhaps in lieu of calculus. The conclusions of this study thus have ramifications for the related question of alternative courses of study for the high school senior mathematics student.

5. Many high schools in predominantly rural areas of Montana and other sparsely populated western states serve relatively few students. In 1989-90 in Montana, one hundred five of the one hundred eighty-four accredited high schools had enrollments of fewer than one hundred students (grades 9 - 12). Of these, more than half had enrollments of fewer than fifty students ("High School Enrollments" 1990). The constraints of small enrollment will necessarily limit the breadth of

courses a high school may offer. Under this circumstance, is the lack of a high school calculus course a deficiency? Is the offering of such a course warranted for very small numbers of students? Among the high schools represented in the student sample of this study, most of those offering a calculus course had enrollments of 100 or more students. However, some very small high schools also provided calculus in their course offerings. For example, Froid High School (enrollment 37 in grades 9 - 12) and Turner High School (enrollment 28 in grades 9 - 12), both in Montana, offered a semester calculus course. Among other Montana schools, Clyde Park High School (enrollment 57 in grades 9 - 12) and Lustre Christian School (enrollment 34 in grades 9 - 12) offered a year-long calculus course.

Addressing the present, Ferrini-Mundy and Gaudard (1992, p. 57) assert, "The research base for understanding [high school calculus] is not well-developed." In summary, Rash (1977, p. 280) observes, "Calculus is an academic and a financial luxury high schools can afford only if it can be shown that some of the outcomes are fruitful." This study sheds needed light on one such outcome - achievement in college/university Calculus 1.

General Questions to be Answered

This study has attempted to answer the following four major questions:

1. What correlation does successful completion of a one-semester/survey calculus course in high school have with achievement in Calculus 1 at the college/university level?

2. What correlation does successful completion of a two-semester calculus course in high school have with achievement in Calculus 1 at the college/university level?

3. Do students in the three categories (1) no high school calculus, (2) one-semester/survey high school calculus, and (3) two-semester high school calculus demonstrate the same mean achievement in college/university Calculus 1?

4. As measured by achievement in college/university Calculus 1, does any one high school calculus option (no high school calculus, one-semester/survey course, or two-semester course) demonstrate superior worth over the other two options?

General Procedures

A copy of the proposed study was presented to the appropriate administrative personnel at Montana State University and Montana Tech. Upon the approval of the proposal, the study was explained to the mathematics department heads and registrars of both schools. Their support and cooperation facilitated gathering most of the needed data.

At both institutions, class rosters and examination scores were obtained from the instructors involved. The Math Anxiety Rating Scale (MARS) instrument was administered to each class section (see Appendix A). Other data such as ACT and SAT scores, gender, minority status, age and class standing were obtained from computerized student records with the assistance of the registrars. Information including high school mathematics courses taken, year of high school graduation, and high school alma mater was obtained by the researcher from direct examination of high school transcript records on file with the registrars' offices. High school enrollment figures were obtained from other sources.

Each student was categorized as a "new" freshman or other undergraduate depending on whether the student graduated from high school the spring preceding the study (1989). Each student was categorized as having taken (1) no high school calculus, (2) a one-semester/survey calculus course, or (3) a two-semester calculus course. The categorization was based on the content of the senior high school mathematics course taken, if any. In numerous instances, the appropriate categorization was not apparent from the high school mathematics course title because of its nebulous nature. Such vague rubrics included "Math IV," "Senior Math," "Advanced Math," and others. In these instances, the high school in question was contacted (usually by mail) and the nature of the course was ascertained by a topic questionnaire. The student's high school calculus background was then categorized based on this information.

The Montana State University and Montana Tech data were analyzed separately. Separate analysis was necessary since Calculus 1 at the two institutions were different courses in some important respects. The scope was different since one was a semester-duration course (Montana Tech) and the other was a quarter-duration course. The two courses also utilized different textbooks. Also, student achievement was measured differently in the two courses.

The student sample from Montana State University was further divided into three different subsamples for purposes of statistical analysis: (1) new freshmen, (2) the complement group, i.e., those other than new freshmen, and (3) the combined sample consisting of both the new freshmen and the complement. The student sample at Montana Tech was divided into three subsamples as described above, and one additional subsample. Subsample definitions are provided in the Definition of Terms later in this chapter.

The data of each subsample were subjected to several analysis techniques, including Multiple Linear Regression, One-way Analysis of Variance, and, for the Montana State University subsample only, Chi Square Test of Independence. The researcher, using these methods, investigated the correlation between high school calculus background and certain other independent variables and achievement in college/university Calculus 1. The researcher also explored the difference in mean achievement among the three high school calculus background categories.

Limitations and Delimitations

The following were limitations and delimitations of this study:

1. The study directly considered only the effects of one and two-semester high school calculus courses and not any other high school mathematics background.

2. The study samples consisted of all students enrolled in Calculus 1 (Math 181) at Montana State University during autumn quarter of the 1989 - 1990 academic year, as well as approximately half of all Calculus 1 (Math 121) students at Montana Tech during fall semester 1989, and all Math 121 students during spring semester 1990.

3. Calculus 1 students for whom no high school transcripts were available (typically transfer students from other colleges or universities) were not included in this study. Those for whom other necessary data were unavailable (such as SAT or ACT scores) were also excluded from the study.

4. The study was supported by the resources available through Montana State University's Renne Library, the registrars' student record files at Montana State University and Montana Tech, and information gathered directly from high schools represented by students in the study samples.

Definition of Terms

The following definitions are provided by the researcher or correspond to those used in other sources as indicated by citation and are to be considered as operational definitions.

Achievement: Achievement in Calculus 1 at Montana State University was measured in this study by the aggregate raw score (500 points possible) attained by the student on three midterm examinations worth 100 points each and the comprehensive final examination given at the conclusion of the course weighted 200 points (see Appendixes B, C, D and E). At Montana Tech achievement was measured by the score (150 points possible) attained by the student on a comprehensive final examination given at the conclusion of the course (see Appendix F).

Advanced Placement (AP): The Advanced Placement Program, a cooperative educational endeavor of the College Entrance Examination Board and secondary and post-secondary schools.

Advanced Placement Calculus AB and Calculus BC Courses: Mathematics courses offered by accredited participating high schools designed to prepare the student for either the Calculus AB examination or the Calculus BC examination, both of which are written, administered, and scored by the Advanced Placement Program. The following course descriptions are provided by the College Entrance Examination Board (1990, p. 5).

Calculus AB: This course is intended for students who have a thorough knowledge of college preparatory mathematics, including algebra, axiomatic geometry, trigonometry, and analytic geometry (rectangular and polar coordinates, equations and graphs, lines, and conics). Calculus AB is a course in introductory calculus with elementary functions. The list of topics for Calculus AB given [in Appendix G] is intended to indicate the scope of the course but not necessarily the order in which the topics are to be taught.

Calculus BC: This course is intended for students who have a thorough knowledge of analytic geometry and elementary functions in addition to college preparatory algebra, geometry, and trigonometry. Calculus BC is considerably more extensive than Calculus AB. All of the calculus topics in Calculus AB are included. The list of topics given [in Appendix G] is intended to indicate the scope of the course but not necessarily the order in which the topics are to be studied.

Calculus 1: The first course in the traditional mathematics analysis sequence, designed for engineering, mathematics and science majors, as defined by the school catalogs of Montana State University and Montana Tech. The course description at Montana State University (Montana State University, 1988, p. 179) is:

Math 181M Calculus and Analytic Geometry 1
Prerequisites: Math 165. [trigonometry]
Functions, graphs of functions, operations on functions, limits of functions, continuity of functions, one sided limits and continuity, limits at infinity, infinite limits, vertical and horizontal asymptotes, derivatives, differentiation formulas, derivatives of trigonometric functions, the chain rule, implicit differentiation, higher derivatives, extrema of functions, the mean value theorem, first and second derivative tests, rectilinear motion, related rates, L'Hopital's

rule, definition and properties of the Riemann integral, the fundamental theorem of calculus, indefinite integrals.

The course description at Montana Tech (Montana College of Mineral Science & Technology, 1988, p. 135) is:

Math 121 Analytic Geometry and Calculus
5 Cr (Hrs:5 lec.)
Covers functions, limits, derivatives, integrals, techniques of integration, exponential, logarithmic and hyperbolic functions. Prerequisite: Math 105 (College Algebra) and Math 106 (College Trigonometry) or their equivalent.

<u>One Semester High School Calculus Course</u>: Any mathematics course of one semester duration offered by an accredited secondary school with content including, but not necessarily limited to, the following topics: functions, graphs of functions, operations on functions, limits of functions, derivatives, differentiation formulas, the chain rule, extrema of functions, first and second derivative tests. The treatment of these topics was usually method-oriented and nontheoretical. For example, epsilon-delta ideas were generally not treated in a one-semester high school calculus course. There was little or no emphasis on proofs of theorems or derivations of formulas. The study of functions, limits and derivatives infrequently extended to trigonometric, exponential or logarithmic functions. Other topics which also were occasionally part of such a course were applications, asymptotes, and an introduction to integration. A two-semester course offering a survey of calculus, as reflected by the same attenuated list of calculus topics and minimal rigor, was classified in this category for purposes of this study.

Subsample 1: Montana State University New Freshmen: Fall quarter 1989 Montana State University students enrolled in Calculus 1 who graduated from high school in spring 1989.

Subsample 2: Montana State University Complement: Fall quarter 1989 Montana State University students enrolled in Calculus 1 who graduated from high school prior to spring 1989.

Subsample 3: Montana State University Combined: All fall quarter 1989 Montana State University students enrolled in Calculus 1 for whom complete data were available.

Subsample 4: Montana Tech New Freshmen: Fall semester 1989 and spring semester 1990 Montana Tech students enrolled in Calculus 1 who had no other college mathematics coursework, and who graduated from high school in spring 1989.

Subsample 5: Montana Tech Freshmen Complement: Spring semester 1990 Montana Tech students enrolled in Calculus 1 who had taken prior college level mathematics coursework (such as trigonometry or college algebra) during fall semester 1989; and who graduated from high school in spring 1989.

Subsample 6: Montana Tech Complement: Fall semester 1989 and spring semester 1990 Montana Tech students enrolled in Calculus 1 who graduated from high school prior to spring 1989.

Subsample 7: Montana Tech Combined: All fall semester 1989 and spring semester 1990 Montana Tech students enrolled in Calculus 1 for whom complete data were available.

Two-Semester High School Calculus Course: Any mathematics course of two semesters duration offered by an accredited secondary school which included (but was not necessarily limited to) most or all of the topics covered in the Advanced Placement Calculus AB course (see Appendix G). Any two-semester high school mathematics course not meeting the content criteria specified by this definition but fulfilling the definitional criteria given for a one-semester/survey high school calculus course was, for purposes of this study, classified in the latter category.

WF Grade: At Montana State University, a grade assigned to a student who was failing at the time of withdrawal from a course. For purposes of computing quarter and cumulative grade point average, a WF grade is treated the same as an F grade (Montana State University Office of Publications and News Services, 1988).

WP Grade: At Montana State University, a grade assigned to a student who was passing at the time of withdrawal from a course. A WP grade does not affect a student's quarter or cumulative grade point average (Montana State University Office of Publications and News Services, 1988).

CHAPTER 2

REVIEW OF RELATED LITERATURE

The question being addressed by this study is "What are the effects of one-semester and two-semester high school calculus courses on achievement in the traditional first course in calculus at the college/university level?" The problem has not been conclusively answered by previous research. This study has considered the possible effects of other potentially intervening variables ignored to a considerable extent, as noted earlier, by other researchers.

The review of literature chapter will first focus on various points of view in favor of and against offering calculus in the high school curriculum. This is followed by a review of studies that investigated the effects of high school calculus on achievement in Calculus 1, Calculus 2, and more advanced post-secondary mathematics coursework. The chapter concludes with an examination of research on the effects of advanced placement in college/university mathematics courses.

High School Calculus: Pro and Con

The Advanced Placement Program in Mathematics, formally adopted by the College Entrance Examination Board (CEEB) in 1955 grew from 368 students taking the Calculus AB or BC examinations in

1956 to 10,675 taking these examinations in 1967 and 14,673 in 1971 (College Entrance Examination Board, 1990). Presently, approximately 31% of American high schools participate, serving about 17% of their college-bound students through advanced placement testing (College Entrance Examination Board, 1987). Acceleration, by moving students to more advanced courses as a means of providing for the needs of academically talented students, has enjoyed considerable support over the last three decades. The Advanced Placement Program "is based on the assumption that some twelfth-grade students can do college freshman work" (College Entrance Examination Board, 1964, p. 24). The assertion of the College Entrance Examination Board is that for colleges not to reward such work completed in high schools would result in needless duplication and, worse, retard the vertical progress of students in a subject field.

There are several reasons commonly proposed in favor of teaching calculus as a high school subject. One purpose is to accelerate the program of the more able students. Having had a college-equivalent calculus course in high school, a student may receive credit for one or more semesters or quarters of college/university calculus. Bypassing one or more college calculus courses, with or without college credit for the high school course, enables the student to take more advanced courses in college, proponents reason.

Another purpose given for teaching calculus in high school is it provides students an additional opportunity to develop problem-solving and logical reasoning skills as well as cultivating the students' ability to deal with abstract concepts.

A third purpose is to give secondary students a "head start" on college/university mathematics. The hope and intent is students with prior high school exposure to calculus will achieve greater success in comparable college/university coursework and/or in more advanced college/university mathematics courses than those without such background.

The validity of the first and third purposes have been investigated by several researchers as discussed below. Relatively little quantitative research appears to have been done regarding the intent to develop problem-solving and logical reasoning skills and cultivate the students' ability to deal with abstraction. Ferrini-Mundy and Gaudard's study (1992), summarized below, explored the question of conceptual learning (as contrasted to procedural competence). Currently, many mathematics educators question whether high school calculus is effective in developing problem-solving and logical reasoning skills and fostering the ability to deal with abstraction. Typical of such views is that of Rash (1977, p. 78), who believes that the skills alluded to earlier

> can be cultivated with good teacher guidance, regardless of the content. When a [high school level] calculus course does not have only the best students, the teacher must proceed at a slower pace and give more explanations (or omit proofs and difficult concepts). Thus, less challenging learning activities are done independently by the majority of students. Students develop a false impression of the

> amount of productive study which is necessary for a college course and an erroneous sense of security in their own knowledge and ability.

Small (1986) concurs with this view and further suggests that students receiving an attenuated calculus course in high school proceed to the college/university level overconfident and possessed with ". . . an unfounded feeling of subject mastery . . . that can lead to serious problems in college calculus courses" (p. 134).

Rash (1977) and Small (1986) go on to assert that there are often other less laudable underlying motives for offering calculus as a high school course: (1) The teachers want to teach calculus. (2) Offering calculus is a status symbol for the school. (3) Students consider calculus a prestigious course. (4) The school is yielding to political pressure from parents, school board officials, counselors and school administrators, as they demonstrate a competitive-type pride in their school's offering of calculus. (5) No one in the school has taken the initiative to consider other alternatives for advanced courses. No data exist to substantiate or disprove these claims. This is not surprising since it would seem difficult to obtain accurate data regarding the issue of subconscious or ulterior motives.

Rash (1977) believes that in lieu of high school calculus, greater emphasis should be placed on precalculus preparation in areas such as algebra, trigonometry and analytic geometry. Sorge and Wheatley (1977) concur, based on statistics from their study which suggest that in the rush to take calculus in high school, the acquisition of adequate background in the aforementioned precalculus areas is being seriously sacrificed. They note from 1962 to 1975 a thirteen year pattern of steady decline nationally in mean mathematics scores on the Scholastic Aptitude Test. In a roughly concurrent observation, Rash (1977) notes that in 1972-73 there were four times as many students studying calculus in high school as there were in 1960. As further evidence, Sorge and Wheatley's data (1977) indicated that in the period 1969-1977 there was a marked decline in the ratio of students who had received analytic geometry in high school. Concurrently, the ratio of students who took a course titled "Precalculus" or "Calculus" increased. According to Sorge and Wheatley, among students in the first course of each of three different freshman calculus sequences at Purdue University (each tailored for student groups with different abilities as measured by such factors as high school grades, high school mathematics courses completed, and SAT mathematics scores), percentages of those who had taken high school trigonometry or analytic geometry decreased, while percentages of those who had taken calculus in high school increased, from 1970 to 1975. Sorge and Wheatley's data (p. 646) are summarized in Table 1. The first courses of each of the three different freshman calculus sequences are denoted with the rubrics MA 155, MA 161 and MA 163 in Table 1. The course MA 155 is the least advanced and MA 163 is the most advanced.

Table 1. Percent of Respondents Who Indicated They Had Taken a Particular Course in High School from the Sorge and Wheatley Study.

Course	Academic Year 1969-70	1974-75	Percent Change
	Trigonometry		
MA 155	90	84	-6
MA 161	96	88	-8
MA 163	98	95	-3
	Analytic Geometry		
MA 155	52	37	-15
MA 161	63	46	-17
MA 163	87	72	-15
	Calculus		
MA 155	15	20	+5
MA 161	28	31	+3
MA 163	47	63	+16

Table 1 indicates a drop in all three groups in the percentage of students who have taken trigonometry in high school and a more pronounced drop in the percentage of those who have taken analytic geometry. The data also shows a concurrent increase in the percentage of high school students who took a calculus course in high school.

Effect on College Performance

The nature of the high school calculus course is itself a variable which must be contended with in designing studies in this area. Obviously, as noted by Burton (1989), Ferrini-Mundy and Gaudard (1992), Small (1986), Steen (1986), and others, the content and level of rigor embodied in the high school calculus courses will vary widely. Hence in considering the questions of effect of high school calculus on college/university-level mathematics performance, several distinctions must be made. Burton and also Ferrini-Mundy and Gaudard based their research on classifying students into four groups:

1. No previous calculus experience.
2. A one-semester course which is a brief, mostly nonrigorous introduction to calculus topics.
3. A full year of non-Advanced Placement high school calculus.
4. A full year of Advanced Placement high school calculus.

For investigative purposes, this study categorized exposure to calculus in high school in a similar manner, except no distinction was made between Advanced Placement and non-Advanced Placement full year high school calculus courses. Grouping separately those students who had an Advanced Placement high school calculus course would have rendered samples too small to be useful for statistical analysis in the present study. Among Subsample 3 (Montana State University Combined), only 14 of 422 students (3.3%) had received such a course. Of the 14 students who had an AP course, 8 were from high schools outside Montana. By contrast, in Ferrini-Mundy and Gaudard's Calculus 1 sample, 103 of 686 students (15%) had taken an AP calculus course. Evidently the Advanced Placement Program is more prevalent in other areas of the country than in Montana. Grouping all students together who had a year of high school calculus, rather than separately categorizing those who had an AP calculus course, is defensible in the present study for two reasons: (1) Since the Advanced Placement Program impacts very few students in the population of this study, it is largely irrelevant to that population. (2) The mean achievement level in Calculus 1 of the few students in this study who had an AP calculus course would likely not differ significantly from the mean achievement level of students who had a full year non-AP calculus course in high school. This contention is supported by the results of Ferrini-Mundy and Gaudard's research, wherein the outcome measures of the two groups were statistically the same.

In some of the research to be cited, each participating student with a high school calculus background was paired with a student of comparable ability (as measured by such yardsticks as class rank if from the same high school or SAT scores) who had no high school calculus background. In this way, it was sought to remove the effects of difference in scholastic ability and thus isolate the effects of high school calculus background on subsequent college mathematics achievement.

Another approach employed in some studies was to use Multiple Linear Regression to derive a predictor of achievement in a given college/university mathematics course without benefit of exposure to calculus in high school. Then the difference between a student's achievement (as measured by an achievement test, in some instances actual examination(s) from the course in question) and his predicted achievement was tested for significance.

Other studies utilized multiple comparison of means methods to compare achievement levels among various high school calculus background groups. In one instance, analysis of covariance, with mathematics aptitude (as measured by mathematics SAT score) as covariate, was used to explore differences in Calculus 1 achievement among several groups of students with different high school mathematics backgrounds.

Investigation of the effects of a high school calculus course on achievement in Calculus 1 and/or Calculus 2 has yielded mixed results.

What is the effect of one semester of calculus in high school on achievement in Calculus 1 at the college/university level? Several studies have addressed this question. Results of the following researchers indicate that students who have had one semester of high school calculus do not achieve significantly better in Calculus 1 at the college/university level: Ferrini-Mundy and Gaudard (1992), McKillip (1965), Rash (1977), Sorge and Wheatley (1977), and Tillotson (1962). Contrary findings were reported by Paul (1970). In his study, students with one semester of high school calculus did outperform their otherwise comparable counterparts who did not have that background.

What is the effect of two semesters (that is, a full year) of calculus in high school on achievement in Calculus 1 at the college/university level? Studies by Burton (1989), Ferrini-Mundy and Gaudard (1992), Paul (1970), Robinson (1970) and Sorge and Wheatley (1977) all rendered the same results: Students who had two semesters of calculus in high school demonstrated greater achievement in Calculus 1 at the college or university than their counterparts who did not have two semesters of calculus in high school. Other data collected by Robinson indicated that students who had two semesters of calculus in high school also outperformed their counterparts without such background in Calculus 2 at the university level.

Austin (1975), in studying the effects of high school calculus on college/university mathematics achievement did not differentiate between those having one semester of calculus and those having two semesters in high school. He tested for achievement with his own instruments in three areas: manipulative skills, problem solving skills and theoretical concepts. Overall achievement was defined to be the sum of these subscores. In his data, there were significant differences between the means of the scores of those students who had studied calculus in high school and those who had not studied the subject there, in overall achievement and in all three subareas of achievement.

Shimizu (1969) compared achievement in Calculus 1 among students who (1) had taken a high school calculus course, (2) had taken an alternate advanced twelfth grade course, namely Analytic Geometry, Survey of Mathematics, or Probability, and (3) had not taken an advanced twelfth grade mathematics course (a control group). She found that students who had taken an advanced course (including calculus) outperformed the control group. However, the calculus students did not achieve better than the students who took the other advanced courses. Shimizu noted that some of the advanced twelfth grade courses were semester courses while others were year courses. The duration of the high school calculus course was not specified.

Two studies are noteworthy by virtue of their currentness. Burton (1989) and also Ferrini-Mundy and Gaudard (1992) used data gathered at the University of New Hampshire. In both studies, students in Calculus 1 were categorized as having either (1) no previous calculus experience, (2) a brief (one semester or less) introduction to calculus, (3) a year-long high school calculus course, or (4) a full year of Advanced Placement high school calculus. From her analysis, Burton claims (p. 351) "The lack of the year of high school calculus [either an AP or non-AP course] can seriously handicap the first year university student" attempting Calculus 1. Burton's data (p. 352) comparing Calculus 1 course grade outcomes of those having a year of high school calculus with those having a semester or less, are reproduced in Table 2.

Table 2. Burton's Calculus 1 Achievement Data.

	A	B	C	D	F
Minimal (or no) previous calculus (370)	9	53	99	94	115
Full year of high school calculus (313)	47	119	95	33	19

Burton notes that overall, the results for students with minimal prior calculus experience is disappointing. Meanwhile, the grades of students with a year of high school calculus preparation concentrate in the B and C brackets, but not in the A bracket. Despite their background, these students do not appear to be overqualified for this so-called introductory course, according to Burton.

Burton (1989) also studied high school calculus background in conjunction with another predictor of Calculus 1 success, a Mathematical Association of America test in algebra and trigonometry, which was given as a calculus pretest during the first week of the Calculus 1 course. Data relating to this is presented in Table 3 (p. 352). Burton makes several observations from these data. (1) Students who pass the pretest are more likely to be successful in Calculus 1. (2) The apparent disadvantage of lack of prior calculus experience is lessened for students who can pass the calculus pretest. (3) The group at greatest risk is those students with little or no calculus background and who could not

Table 3. Burton's Calculus Pretest Results.

		A	B	C	D	F
Minimal (or no) previous calculus:	Failed pretest	0	9	37	31	80
	Passed pretest	8	43	57	57	30
Full year of high school calculus:	Failed pretest	3	20	35	18	12
	Passed pretest	42	96	59	13	6

pass the calculus pretest. (4) Students represented by the second line of the table, while lacking calculus background, nonetheless reflect the stated prerequisites of Calculus 1 by virtue of passing the algebra-trigonometry pretest. Yet, their letter-grade results are disappointingly poor. Burton concludes that students with at least some prior exposure to calculus (even a "year-long but watered-down" course (p. 351)) are at a comparative advantage in Calculus 1 over students for whom the subject is new.

The study of Ferrini-Mundy and Gaudard (1992) investigated the effects of the aforementioned levels of high school calculus on achievement in Calculus 1 in the same sample of students utilized by Burton (1989) at the University of New Hampshire. To control the parallel effects of mathematical aptitude, they employed analysis of covariance in making multiple comparison of achievement means. The covariate measuring mathematics aptitude (as contrasted to achievement) was the student's SAT mathematics score. Achievement was measured on three examinations and a final examination. For each high school background group and each examination, a mean score was calculated. From this, an adjusted mean was calculated adjusting for the effects of the covariate, the SAT mathematics score. No adjusted means were calculated for Examination 1 since the parallelism assumption necessary for analysis of covariance was not confirmed for that measure. The course grade scale corresponds an 11 to A, 10 to A-, 9 to B+, et cetera. Selected data from Ferrini-Mundy and Gaudard's study (p. 62) are given in Table 4.

Table 4. Ferrini-Mundy and Gaudard's Calculus 1 Achievement Data.

High School Calculus Level	Exam 2	Exam 3	Final Exam	Course Grade
No calculus Adjusted M	53.3 ┐	54.8 ┐	15.8	3.2 ┐
Brief introduction Adjusted M	55.6 ┘	58.0 ┘	17.1	3.8 ┘
Year Adjusted M	66.0 ┐	70.6 ┐	20.4 ┐	5.9
Year AP Adjusted M	70.7 ┘	74.0 ┘	21.8 ┘	6.7

Note: Brackets join all groups which did not differ significantly at the 0.05 level.

Examination of the data in Table 4 as well as other data reported in their study led Ferrini-Mundy and Gaudard to several conclusions: Students who had a year of high school calculus (an AP course or otherwise) achieved at a significantly higher level than students who had either no high school calculus experience or a brief introductory course. A brief (typically one semester) introductory course offered very little advantage in comparison to no high school calculus background. Students who had a full year high school calculus course performed significantly better than other groups throughout the Calculus 1 course. This advantage was manifested more strongly in procedural than in conceptual test items. Students with more high school calculus background were more likely to continue into the second semester college calculus course, Calculus 2. However, there were no significant differences in achievement among the four high school calculus groups in Calculus 2. This suggests that any benefits of high school calculus background are relatively short-lived.

What effect does a background of high school calculus have on achievement in advanced mathematics coursework, that is, coursework beyond second semester calculus? Contrary to Ferrini-Mundy and Gaudard's conclusions (1992), Pocock's research (1974) suggests that those who have had calculus in high school are more successful in second year calculus and post-calculus college/university mathematics.

Effects of Advanced Placement

What are the effects of Advanced Placement in college/university mathematics courses concomitant with having had calculus in high school?

Using a self-designed calculus achievement test, Dickey (1986) sought to determine whether students in Advanced Placement high school calculus courses are learning calculus at a level similar to that of college students. He found that the high school students scored as well or better than their college counterparts.

Concerning placement, Fry's study (1973) rendered several relevant results: Advanced Placement students omitting Calculus 1 at the college or university showed greater achievement than regular placement students in Calculus 2 and Calculus 3. Furthermore, the two groups demonstrated equal achievement in a post-calculus linear algebra course. Also, Advanced Placement students omitting both Calculus 1 and Calculus 2 showed equal achievement with their regular placement counterparts in Calculus 3 and significantly greater achievement in Calculus 4 and linear algebra than the regular placement students. These results are mostly consistent with those of Bergeson (1967). In his study, advanced placement of students who had calculus in high school did not adversely affect performance in advanced mathematics courses. The works of Dickey (1986), Fry, and Bergeson all indicate that students who have successfully completed a high school calculus course can (and perhaps should) receive advanced placement in college/university mathematics courses.

NCTM and CUPM Positions

The NCTM has adopted a neutral stance regarding high school calculus courses per se. The NCTM "does *not* advocate the *formal* study of calculus in high school for all students or even for college-intending students" (NCTM, 1989, p. 180). Nor does the NCTM categorically oppose such study. Rather, they advocate systematic inclusion of natural extensions of topics in algebra, geometry, trigonometry and coordinate geometry to motivate the basic ideas of calculus, such as the limit, area under a curve, the rate of change and slope of a tangent line, and infinite sums (series). Such instruction should be exploratory and based on numerical and geometric activities that utilize both the calculator and instructional computer software.

> Although developing a foundation for the future study of calculus remains a goal of the 9 - 12 curriculum for college-intending students, equally important is the development of prerequisite understandings for further study of statistics, probability, and discrete mathematics. [The NCTM] calls for a new balance of skills, concepts and applications in that portion of the curriculum traditionally associated with preparation for calculus. Instead of devoting large blocks of time to developing a mastery of paper-and-pencil manipulative skills, more time and effort should be spent on developing a conceptual understanding of key ideas and their applications. (NCTM, 1989, p. 182)

The Committee on the Undergraduate Program in Mathematics (CUPM) Panel on Calculus Articulation (1987) has recommended that calculus taught in a high school setting should be a year-long course incorporating the content of the Advanced Placement syllabus. In lieu of this, the CUPM suggests that a sound background in algebra, trigonometry and coordinate geometry is preferable to the common one-semester introductory calculus course.

Effects of Mathematics Anxiety and Gender

Richardson and Suinn (1972, p. 551) define mathematics anxiety as

> feelings of tension and anxiety that interfere with manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations.

Studies by Austin-Martin, Waddell, and Kincaid (1980) and Betz (1978) both indicated that mathematics anxiety has an adverse impact on mathematics achievement at the post-secondary level. Therefore, in the present study, data measuring mathematics anxiety was collected for each student to account for the variance in achievement in Calculus 1 attributable to mathematics anxiety. Aichele (1978, p. 36) believes mathematics anxiety is more widespread in women, noting that "Women in our culture have not been expected to excel in mathematics. . . . the 'mathematical mind' has been considered a male attribute." Tobias (1976, p. 57) echoes this view, asserting that mathematics anxiety in women is rooted in

> . . . a culture that makes mathematic ability a masculine attribute, that punishes women for doing well in mathematics, that soothes the slower mathematics learner by telling her she does not have a "mathematic mind."

Data collected by Betz confirmed that women, more than men, suffer from the effects of mathematics anxiety. Since gender has been shown to be a relevant factor in mathematics achievement, it was included as an attribute variable in this study.

CHAPTER 3

PROCEDURES, METHODS, AND ANALYSES OF DATA

Procedures

The purpose of this study was to investigate the effects of one-semester and two-semester high school calculus courses on achievement in the traditional first quarter course in calculus for mathematics, science, and engineering majors at Montana State University and the first semester course in calculus at Montana College of Mineral Science & Technology.

Chapter 3 begins with a description of the population and the sampling procedure used, and is followed by the questions to be answered, expressed in hypothesis form. The investigative categories are defined and measures to control contaminating variables are then discussed. Next, the methods of data collection are outlined along with measures to assure accuracy of the data. Chapter 3 concludes with a list of statistical hypotheses and an explanation of the data analysis techniques that were employed.

Population Description and Sampling Procedure

The population of the Investigative Study consisted of all students in Math 181 (Calculus 1) at Montana State University (including the honors section) during the 1989 autumn quarter, and all students in Math 121 (Calculus 1) at Montana Tech during the 1989-1990 academic year. To be included in the population, the student had to participate in the course to the extent of receiving any one of the following grades for the course: A, B, C, D, F, or WF (withdraw fail). Those receiving a grade of I (incomplete), P (pass) or WP (withdraw pass) were not included in the population. Foreign students and other students completing their high school or equivalent education outside the United States were excluded from the population. Also omitted were students completing their secondary education through the General Educational Development Program (GED). As noted earlier, some students had to be dropped from the sample because of incomplete data.

Montana State University students receiving either an F or WF grade in Math 181 who did not take the final examination were included in the population if they took all three regular term examinations. In these instances, the student's Math 181 achievement score was defined as the student's raw total score on the three regular term examinations (300 points possible) plus a "presumed" final examination score calculated in the manner illustrated: An individual with a missing final examination had a total score on the three midterm examinations that was (for example) 84% of the corresponding mean for the entire sample group. The student's presumed final examination score was then defined to be 84% of the final examination mean for the entire sample group that took the final examination. For a student in the Montana State University population who took the final examination and was missing only one midterm score, a presumed midterm examination score was calculated and used in a like manner to determine an achievement score. Only a small portion of the student sample required the insertion of a presumed score (approximately 20 out of 350). However, it was desirable to include in the sample failing students who did not take the final examination or who missed one midterm examination, since excluding them would bias the sample in a systematic way by excluding only students who were experiencing serious difficulty in Calculus 1. Since the achievement measure for Montana Tech Math 121 students is the final examination score only, F and WF students not taking the final examination were excluded from the population. The sample for this study included all students in the population as it is defined here except for two (of four) autumn Math 121 sections at Montana Tech. Due to administrative difficulties beyond the researcher's control, suitable data from the omitted sections could not be obtained.

Students in the Montana State University sample for whom there were two or three missing examination scores were not assigned an achievement score and were utilized only in the Chi Square Analysis which allows utilization of students whose achievement score data is only nominal in nature. For this purpose, students were grouped by achievement into one of three categories per whether they (1) did not complete the course, as indicated by no achievement score, (2) attained an achievement score in the lower 50th percentile for the sample group as a whole, or (3) achieved in the upper 50th percentile for the sample group as a whole.

Chi Square Analysis to include students with no achievement score was not done for the Montana Tech population, since achievement score was based wholly on the final examination and students who did not take the final examination were typically expunged from the instructor's grade records.

Montana State University, located in Bozeman, Montana, is an accredited, comprehensive land grant institution established in 1893 under the authorization of the Morrill Act of 1862. Bozeman is a small city of approximately 25,000 people. The university offers undergraduate and graduate programs in liberal arts, basic sciences, the professional areas, agriculture, architecture, business, nursing, education and engineering. The student population in 1987 was approximately 9900 (58% men, 42% women), of which 84% were Montana residents; 29% were over 25 years of age. Montana State University has approximately 500 resident faculty members, more than two-thirds of whom hold doctorates. The student faculty ratio is approximately 18:1 (Montana State University Office of Publications and News Services, 1988).

Montana State University operated on the academic quarter system when the study was conducted.

The Department of Mathematical Sciences at Montana State University consists of approximately 25 full-time instructional faculty members, about an equal number of graduate teaching assistants, and several part-time faculty members. Department records indicate that at the conclusion of autumn quarter 1989, there were approximately 407 students enrolled in 14 sections of Math 181. The average class size was therefore approximately 29 students per section. These figures include students who received a grade of WF for the course, but do not include those receiving a grade of I, P, or WP for the course.

Montana Tech, one of six units along with Montana State University which comprise the Montana University System, is located in Butte, Montana, population 35,000. This institution is an accredited engineering, science and technology-based school offering undergraduate and graduate programs with a strong orientation toward business and industry. Established in 1895 as the Montana School of Mines, the college has historically focused on mineral and energy-related professional engineering programs. In a comprehensive, wide-ranging evaluation of college and university programs in 1987, U.S. News & World Report gave Montana Tech the top ranking nationwide for undergraduate science programs in a category of smaller comprehensive colleges. ". . . when presidents of the smaller comprehensive colleges were asked to name top schools in the area of science and technology, Montana Tech was their first choice largely because of a superb program in minerals engineering" (p. 67).

Recently, degree programs have been added in the basic sciences supporting the engineering programs and in fields related to the administration, application and societal impact of the engineering programs. There are approximately 100 academic faculty members. Montana Tech operates on the academic semester system.

In autumn, 1989, there were nine full-time mathematics faculty members. Montana Tech offers B.S. degrees in mathematics and computer science. Student enrollment in Math 121 was approximately 180 in autumn semester 1989 and approximately 100 in spring semester 1990.

Investigative Categories

Investigating the effect of prior high school calculus on achievement in college/university Calculus 1 was the focus of this study. For this purpose, the Montana State University and Montana Tech student samples were subdivided into the seven subsamples defined earlier. These were (a) Subsample 1, Montana State University New Freshmen; (b) Subsample 2, Montana State University Complement; (c) Subsample 3, Montana State University Combined; (d) Subsample 4, Montana Tech New Freshmen; (e) Subsample 5, Montana Tech Freshmen Complement; (f) Subsample 6, Montana Tech Complement; and (g) Subsample 7, Montana Tech Combined. Each student was placed in the appropriate subsample according to the date of high school graduation noted on his high school transcript, and for Montana Tech freshmen, consideration of any college precalculus mathematics courses taken.

Each subsample was divided into three investigative categories:

Category 1: No Calculus: Students who have completed no high school mathematics course meeting the criteria of either Categories 2 or 3.

Category 2: One-Semester Calculus: Students who have completed a one-semester high school calculus course.

Category 3: Two-Semester Calculus: Students who have completed a two-semester high school calculus course.

Each student was placed in an investigative category per his high school calculus background. In many cases, the appropriate categorization for a student was apparent from the mathematics course titles on the student's high school transcript. For example, a transcript listing only mathematics coursework up to "Algebra II" , "Algebra III," "Trigonometry," or "Trig/Analytic Geometry" indicated a placement in Category 1, No Calculus. Transcript course titles such as "Calculus" (sem) or "Intro to Calc" (sem) dictated assignment to Category 2, One-Semester Calculus. Several course titles clearly indicated placement in Category 3, Two-Semester Calculus. These included "Calculus" (yr), "AP Calculus," and "Math 5 Calculus" (yr).

Many course titles were ambiguous with respect to course content, including such rubrics as "Math 4," "Advanced Math," "College Math," and "Senior Math." In these cases the high school in question was contacted by the researcher, in most instances by mail, for specific information regarding course content. This information was solicited in the form of a topic questionnaire (see Appendix H) prepared by the researcher. Topics listed on the questionnaire include the usual assortment of concepts generally included in a first calculus course(s). These are topics in limits, differentiation, and integration of real functions of one variable. Categorization was based on the course content and duration as indicated by the school's response to the topic questionnaire. Eighty-two Montana high schools and sixteen high schools outside Montana were contacted for this information.

Sample sizes and Calculus 1 achievement means of all subsamples and investigative categories thereof are given in Table 5.

Variable Controls

The multiple sections of Calculus 1 at Montana State University are tested under a common hour examination system. All sections of the course are given the same midterm examinations and same final examination at a common evening hour. Each problem is graded on all the examinations by a single designated instructor, with all the course instructors utilized as graders to an approximately equal extent. Under this system, uniformity in grading within and across the sections is greatly enhanced. Because of this uniformity, course examinations were utilized in this study as the measure of student achievement for the

Table 5. Sample Sizes and Achievement Means.

Group		Sample Size	Achievement Mean
Subsample 1	MSU FROSH	277	322.2996
Category 1	NOHSCALC	119	312.0672
Category 2	SEMCALC	74	326.5405
Category 3	YRHSCALC	84	366.0357
Subsample 2	MSU COMPLEMENT	87	317.6092
Category 1	NOHSCALC	54	312.5000
Category 2	SEMCALC	21	303.5238
Category 3	YRHSCALC	12	365.2500
Subsample 3	MSU COMBINED	364	328.7885
Category 1	NOHSCALC	173	312.2023
Category 2	SEMCALC	95	321.4526
Category 3	YRHSCALC	96	365.9375
Subsample 4	MT TECH FROSH	42	103.5238
Category 1	NOHSCALC	14	95.5714
Category 2	SEMCALC	17	101.1176
Category 3	YRHSCALC	11	117.3636
Subsample 5	MT TECH FROSH COMPLEMENT	77	95.7013
Category 1	NOHSCALC	39	88.1538
Category 2	SEMCALC	25	99.6800
Category 3	YRHSCALC	13	110.6923
Subsample 6	MT TECH COMPLEMENT	93	88.9140
Category 1	NOHSCALC	78	89.1410
Category 2	SEMCALC	11	90.0000
Category 3	YRHSCALC	4	81.5000
Subsample 7	MT TECH COMBINED	135	93.4593
Category 1	NOHSCALC	92	90.1196
Category 2	SEMCALC	28	96.7500
Category 3	YRHSCALC	15	107.8000

Montana State University sample. Course examination scores were deemed suitable as the achievement measure for two other reasons: (1) The examinations were designed by experienced faculty specifically for content validity in the Calculus 1 course as it was taught at Montana State University. (2) The examination scores were the main determining factor (83%) of the student's course grade. The examinations are included in Appendixes B, C, D and E. The 500 possible examination points were combined with a 100 point component based on each instructor's choice of such evaluation measures as homework, quizzes, and attendance. Each student's course grade was based on the student's point total according to the scale shown in Table 6. The instructor-designed 100 point component was not used in this study. This omission was to further control the effects of the instructor variable.

Table 6. Grade Scale of Math 181 (Calculus 1) at Montana State University.

Point Total	Grade
540 - 600	A
480 - 539	B
420 - 479	C
360 - 419	D
0 - 359	F

Since there was no provision for common hour testing at Montana Tech, only the final examination was used as the measure of student achievement. A comprehensive final examination was constructed for content validity by mathematics faculty at Montana Tech. Unknown to the students, this same final examination was used in all sections of Calculus 1 constituting the Montana Tech student sample. Like the Montana State University examinations, all final examinations at Montana Tech were group graded in a manner that assured uniform scoring across sections. The Montana Tech Final Examination is shown in Appendix F.

The Math Anxiety Rating Scale (MARS) is a widely used yardstick for measuring mathematics anxiety in post-secondary students (see Appendix A). This instrument is a 98 item self-rating scale that was administered to all subjects in the study sample. Each item on the scale describes a mathematics-oriented situation which may cause anxiety for the student. The student indicates the degree of anxiety aroused by choosing among the alternatives of "not at all," "a little," "a fair amount," "much," or "very much." Once the student has decided the level of anxiety associated with a specific test item, he indicates that choice by marking the appropriate circle on the answer sheet. Directions are included on each test blank for the student to read. Responses are to indicate a student's anxieties as they currently exist. For scoring, each "not at all" response is numerically weighted 1, each "a little" response is weighted 2, and so forth, with the greatest anxiety level, "very much," weighted 5. The raw sum of the numerical weights of a student's responses constitutes his score.

Reliability and validity of the MARS have been established by at least two studies. Richardson and Suinn (1973) tested and seven weeks later retested 35 University of Missouri students. The first testing yielded a mean MARS score of 235.08 (SD = 51.26). The mean score for the second testing was 232.97 (SD = 56.46). The Pearson product-moment correlation compares favorably with the short-term reliabilities of other measures of social and test anxieties cited by the authors. In the same study, an internal consistency reliability coefficient was found to be alpha = 0.97 (N = 397). In essence, this indicates that the average intercorrelation of the items in the test is quite high. It verifies that the test is reliable and shows that the test items are dominated by one single factor, presumably mathematics anxiety. Item-total correlations for all items were also calculated; over half the correlations were greater than 0.50.

Suinn, Edie, Nicoletti and Spinelli (1973) found a test-retest reliability coefficient of .78 after two weeks, significant at $p < 0.001$. This compares favorably to reliabilities of other anxiety scales in common use cited by the authors.

Concerning normative data, the mean score for Richardson and Suinn's (1973) Missouri students (N = 397) was 215.38 with a standard deviation of 65.29. Suinn, Edie, Nicoletti and Spinelli (1973) obtained a mean MARS score of 187.3 and standard deviation of 55.5 from a sample of 119 students. The mean and standard deviation of MARS scores of students in the present study are comparable to these normative results; the mean being 205.507 and the standard deviation being 62.347.

A placement test score was included in the data collected for each student in the sample. This information was included as an independent variable in the Multiple Regression Analysis as a measure of each student's scholastic aptitude. Data procured from the registrars' student records indicates approximately half the students had taken the ACT, half had taken the SAT and some had taken both. For purposes of statistical analysis, these scores were normalized, that is, converted to Z (Standard Normal Distribution) scores. This converts both ACT and SAT scores to a common scale, with a mean of zero and one standard deviation equal to one. Use of both the ACT and SAT are universal in the United States and the validity and reliability of these instruments are well established.

All other variables considered in the study were demographic in nature: age, gender, minority status, class (freshman, sophomore, et cetera), size of graduating high school, academic major, and whether the student took a mathematics course in her senior year of high school. This information was obtained from examination of student college records and high school transcripts.

Method of Collecting Data

The high school transcript of each sample student, obtained from Montana State University or Montana Tech student records, was examined. Students whose transcripts showed no exposure to calculus were placed in Category 1. Those who completed a one-semester calculus course were placed in Category 2, and those who had