Two elementary student teachers understanding of mathematical power and related pedagogy  
by Susan J Adams Phillips

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education  
Montana State University  

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TWO ELEMENTARY STUDENT TEACHERS' UNDERSTANDING OF
MATHEMATICAL POWER AND RELATED PEDAGOGY

by

Susan J. Adams Phillips

A thesis submitted in partial fulfillment
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APPROVAL

of a thesis submitted by

Susan J. Adams Phillips

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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November 30, 1995
This thesis is dedicated to my children, Jessica and Jacob, because they thought it would be cool -- and because I really, really like them.
I wish to express my appreciation first and foremost to the four teachers who so generously and unselfconsciously participated in this project. I am also grateful to Elisabeth Charron and Lyle Andersen, codirectors of the STEP Project, for the support I received during the course of this research; to Bill Hall and Glenn Allinger for their careful and helpful reviews of this work; and especially to Carol Thoresen and Kay Adams, who kept saying, "You can and you will."
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CHAPTER 1

STATEMENT OF THE PROBLEM AND
REVIEW OF THE LITERATURE

Introduction

Since the publication of the National Council of Teachers of Mathematics (NCTM)'s Curriculum and Evaluation Standards in 1989, members of the mathematics education community across the nation have seized with enthusiasm the spirit of reform, and are making efforts to “implement the Standards.” These Standards call for the wide scale adoption of a new vision of mathematics teaching and learning in which the development of mathematical power is the central goal (NCTM 1989, 5; NCTM 1991, 1; NCTM 1995, 83). The traditional overemphasis on computational accuracy and the memorization of formulas will not adequately prepare students for today’s and tomorrow’s society. In contrast, mathematically powerful students can explore, conjecture, and reason logically; solve “messy,” unfamiliar problems; connect and communicate mathematical ideas; and are flexible, persistent, curious, and confident in doing mathematics (NCTM 1989, NCTM 1991, NCTM 1995). The Standards do not provide a prescription, only a vision, for the kind of teaching that will nurture mathematical power. It is presumed the Standards can be implemented in many
and various ways. But it is not possible to truly implement the spirit of the Standards if students are not becoming empowered mathematically. As we prepare new teachers for our mathematics classrooms, and provide ongoing staff development and support to help practicing teachers implement change, an understanding of mathematical power must be the central goal. This research study examined the cooperating teacher/student teacher dyad as a locus for change. It describes the classrooms of two elementary teachers who are trying to implement the Curriculum Standards, and how the student teachers assigned to these classrooms grew in their understanding of mathematical power.

Purpose of the Study

Research on student teaching indicates that the quality of the student teaching experience depends greatly on specific classroom sites (Guyton and McIntyre 1990). The purpose of this study was to examine how placement in a Standards-based classroom can strengthen a student teacher's understanding of the concept of mathematical power and the instructional decisions and strategies which support the development of mathematical power. The study took place in the classrooms of two elementary teachers from the local school district who had been actively involved in the mathematics curriculum revision process, who were knowledgeable about and committed to the vision of
mathematics reform espoused in the Standards, and who had taken on the additional responsibility of supervising a student teacher. It was presumed that all four participants' beliefs, attitudes, and prior mathematics experiences would affect their understanding of mathematical power; and that the manner in which the Curriculum Standards were being implemented would affect the developing mathematical beliefs and pedagogy of the student teachers.

**Research Questions**

1. What prior mathematical experiences, and what attitudes and beliefs about mathematics teaching and learning, do the student teachers and cooperating teachers bring to the classroom setting?

2. What knowledge, beliefs, and criteria do the cooperating teachers use as they enact a mathematics curriculum based on the NCTM Standards, and how are these conveyed to the student teachers?

   a.) What is the nature of the tasks in which the students are engaged each day, and how are these selected?

   b.) What is the nature of the discourse among students and teachers that occurs during the mathematics lesson?

   c.) What are the features of the classroom environment associated with the mathematical empowerment of students?

   d.) How do the cooperating teacher and student teacher reflect on (analyze) the effectiveness of the program components -- the tasks, the discourse, the environment, the pedagogy -- in the empowerment of students?
3. How does placement in a Standards-based classroom influence these student teachers' understanding of mathematical power and related pedagogy?

The NCTM Standards Documents

The theoretical framework for this research study is derived from the two Standards documents: the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) and the Professional Standards for Teaching Mathematics (NCTM 1991). The first document presents a vision of a mathematics curriculum grounded in a cognitive / constructivist theory of learning which focuses on the development of mathematical power. The second document presents a guide to teachers (and other educators and policymakers) as they are considering ways to enact this vision of empowering students mathematically. The first is more concerned with the content, the second with instruction, although there is overlap. The goals of both documents include promoting the development of mathematical power in all students. The recently published Assessment Standards for School Mathematics (NCTM 1995) also calls for a shift from mastering isolated skills, to aligning assessment practices with the goal of mathematical power.
The Curriculum and Evaluation Standards

The NCTM Curriculum and Evaluation Standards (hereafter called the Curriculum Standards in this paper) were created in response to the many calls for mathematics education reform during the 1980s (e.g., A Nation at Risk 1983; The Underachieving Curriculum 1987). These publications noted that many mathematics classrooms today still reflect the mathematics that was needed for an industrial age. As a result, a broad-based task force was commissioned to arrive at a consensus concerning the kind of mathematical skills and abilities needed for the 21st century. According to the task force, new societal goals must include the development of: mathematically literate workers, opportunities for all people to learn substantive mathematics, an informed electorate, and citizens with the problem solving skills needed to be lifelong learners (NCTM 1989, 3-4). School mathematics, then, must reflect and address the importance of mathematical literacy. To this end, the Curriculum Standards describe a coherent vision of mathematical teaching and learning that will empower students in the following ways. Students will: learn to value the role of mathematics in the "real world"; develop the confidence and flexible thinking needed to solve problems they have not seen before; and learn to reason and communicate mathematically (NCTM 1989, 5). The traditional view of mathematics — as a collection of procedures that can be transmitted to students to enable them to arrive at the correct answers to a limited variety of (usually) computational problems — must give way to an understanding of mathematics as
a sense-making process. Mathematics must be seen as a collection of attitudes, problem solving habits, and exercises in active thinking and reasoning, rather than a collection of procedures and algorithms (NCTM 1989, 7).

At the elementary level, the new emphasis is on a curriculum that will:

Be conceptually oriented, emphasizing the development of understanding.

Actively involve children in doing mathematics, offering classrooms equipped with a wide variety of physical materials and supplies.

Emphasize the development of children's mathematical thinking and reasoning abilities, by building appropriate problem solving experiences into the ongoing curriculum.

Include a broad range of content, including measurement, geometry, statistics, probability, and pre-algebra, which have significant and growing applications in many disciplines and occupations, and provide contexts for the use of computation skills.

Make appropriate and ongoing use of calculators and computers. Although calculators do not replace the need to learn basic facts, compute mentally, or do reasonable pencil/paper computation, they unquestionably free both teachers and students to focus a more appropriate share of instructional time on real problem solving.

Provide instruction that honors the unique developmental characteristics of its learners, and incorporates strategies such as cooperative learning and writing which have been shown to enhance learning across all curriculum areas.

Include multiple techniques for assessing students which are integral to the instructional process (NCTM 1989, 16-18).

Thus, the Curriculum Standards provide a vision for the kind of classroom mathematics program that will enable K-12 students to develop
mathematical power. These standards define a broad range of content, including mathematical applications which utilize technology, that should be included in such a curriculum; and specify the expected student outcomes associated with each standard, along with some examples of the kinds of problems and tasks which encourage powerful mathematical thinking. The first four standards (Problem Solving; Communication; Reasoning; and Mathematical Connections) are considered process standards, and are intended to provide a foundation for, or be present in, activities from the remaining content standards. (A list of the K-4 Standards areas is provided in Appendix A). It is these Curriculum Standards from which the cooperating teachers in this study are creating Standards-based instruction.

The Professional Standards for Teaching Mathematics

The enactment of the Curriculum Standards requires the creation of a curriculum and an environment for teaching and learning that are much different from predominant past and current practice, which was and often still is characterized by an overemphasis on didactic instruction, rote learning, and pencil-paper drill and practice. In 1991, the Professional Standards for Teaching Mathematics was published “to provide guidance and direction to those involved in changing mathematics teaching . . . on how to teach mathematics to enhance the development of mathematical power.” This document (hereafter called the Professional Standards in this paper) rests on
the assumptions “that teachers are the key figures in change and that such changes require long-term support and adequate resources.” (NCTM 1991, 2).

Five major shifts in the environment of mathematics classrooms are needed to move from current practice to mathematics teaching for the empowerment of students. We need to shift —

toward classrooms as mathematical communities -- away from classrooms as simply a collection of individuals

toward logic and mathematical evidence as verification -- away from the teacher as the sole authority for right answers

toward mathematical reasoning -- away from merely memorizing procedures

toward conjecturing, inventing, and problem solving -- away from an emphasis on mechanistic answer-finding

toward connecting mathematics, its ideas, and its applications -- away from treating mathematics as a body of isolated concepts and procedures (NCTM 1991, 3).

To facilitate change in these directions, the Professional Standards offer guidelines to teachers, evaluators of teachers, university educators, and policymakers.

Section One of the Professional Standards, called the “Professional Teaching Standards,” provided the constructs which enabled the researcher to relate instructional practice in the research classrooms to the idea of mathematical power. These Standards develop:

a vision of what a teacher ... must know and be able to do to teach mathematics as envisioned by the NCTM Curriculum and Evaluation Standards for School Mathematics and the Professional Standards for Teaching Mathematics. They are organized around a framework
emphasizing the important decisions that a teacher makes in teaching:

Setting goals and selecting or creating mathematical *tasks* to help students achieve these goals;

Stimulating and managing classroom *discourse* so that both the students and the teacher are clearer about what is being learned;

Creating a classroom *environment* to support teaching and learning mathematics;

*Analyzing* student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions. (NCTM 1991, 5).

These Standards provided the framework for Research Question #2 on page 3. Research related to these four areas: task, discourse, environment, and analysis, and research related to teacher beliefs, supports the notion that these are indeed key areas for the development of mathematical power in students.

**Review of the Literature**

**Research Relating to Beliefs**

The first research question explored the beliefs and attitudes about mathematics teaching and learning which participants brought to the research setting, their own prior experiences in learning mathematics, and their entry interpretations of mathematical power. It was important to establish this frame of reference, because research has shown that teachers' beliefs, attitudes, and experiences influence how they teach, (e.g., Thompson 1984), and what their students learn, in turn, will be integrally connected to how they learn it (Bruner
Beliefs about Mathematical Power. Two prominent belief systems about teaching and learning have influenced the development of mathematics education.

The first is the behaviorist orientation, whose proponents conceive of mathematical knowledge as a body of information and skills to be transmitted to the learner, and learning as a series of behavioral objectives to mastered (Post 1988, 2-4). Therefore, a student who was mathematically powerful would presumably be armed with as many skills, procedures, and formulas as possible.

The second belief system is the cognitive orientation. As defined by von Glasersfeld (1989), cognitive theorists believe that meaningful learning cannot be transmitted from one person to another; rather, it must develop from within, as the learner takes new ideas and new experiences and relates them to his own previous experiences and prior knowledge, integrating and adjusting and processing them until they make sense to him (Cooney 1994). No matter how well a teacher "explains" a concept or problem in a way that makes sense to the teacher, no matter how many tricks or devises the teacher offers the student to help him "get the answer," meaningful learning does not occur until the student does the personal and social work that enables him to make sense of the situation within his own framework. According to this belief system, mathematical
power would consist of the processes and the attitudes that enable a learner to actively construct meanings and knowledge: the abilities to reason, communicate, make connections and form relationships, persevere, and solve problems from scratch by figuring them out rather than remembering a formula, all of which build confidence in one's own ability to do mathematics. This is the belief system, and the interpretation of mathematical power, upon which the NCTM Standards documents are based (NCTM 1991, 1-2).

A survey by Post (1977) determined that 96% of mathematics educators described their own primary philosophy as cognitive. Since traditional textbook-dominated classrooms are still the norm in mathematics education, such a statistic could indicate that a number of educators' beliefs and practices are not completely aligned, or perhaps that teachers use practices from both schools of thought. The Standards documents endeavor to help teachers align their practice with cognitive beliefs.

**Background Experience with Mathematical Power.** Another source of influence on teaching practice is prior mathematics experience. A teacher's own experience shapes her conception of mathematics as well as her conception of self as a doer of mathematics. It is commonly accepted that teachers tend to teach as they've been taught. Therefore, teachers whose own prior educational experience contributed to feelings of mathematical power and confidence would be more likely to teach for mathematical power in their classrooms. However, work by Thompson (1992) would indicate that it is possible for teachers from
traditional educational backgrounds to change from a behaviorist perspective of mathematical power to a cognitive perspective of mathematical power, when exposed to new ideas and experiences. Thompson's review of the research linking conceptions to practice, concluded that teachers' “belief systems are dynamic, permeable mental structures, susceptible to change in light of experience.” (Thompson 92, 140).

Research Relating to Task

In the behaviorist tradition of presenting mathematics as a set of skills, lecture and worksheets predominate as the tasks of choice. This pattern was confirmed by Porter et al. (1988) in a five year study of content determinants in elementary school mathematics. This research indicated that individual teachers have enormous power relating to content: they determine how much time is allotted to mathematics; what topics will be taught and to which students; when and in what order the topics are taught; and to what standards of achievement a topic is taught. Together these decisions influence a student's opportunity to learn mathematics, which in turn is a major influence on student achievement. Conventional wisdom would suggest that fourth grade teachers, for instance, simply teach a standard fourth grade curriculum, and that there would be little variation in the decisions cited above. However, using a three-dimensional taxonomy to determine the content of four 4th grade mathematics texts and five standardized achievement tests, minimal commonality of topics
was found -- only six topics out of 385 total were common to all nine resources -- dispelling the idea of a hidden national curriculum. Computation was found to dominate the content of the classrooms studied, comprising an average of 75% of the allotted time, yet the range varied from 55% to 80% with individual teachers. The allotted time itself varied by a factor of 1.5 -- 9,000 minutes vs. 6,000 minutes across a full school year. During the remaining 25% of the time not devoted to computation, teachers covered a huge number of topics very thinly, and the choice of topics varied widely. This lack of balance among the teaching of concepts, skills, and applications is reflected in the textbook emphasis on computation, but not in the content of standardized tests, which usually contain a more balanced presentation of computation, concepts, and applications. This situation concerned the researchers, who point out that "applications are both more important and more difficult to learn than are skills, and conceptual understanding is . . . of more lasting value than either skills or applications." (Porter et al., 106). They also expressed concern that students were always given problems to solve, and rarely asked to formulate problems.

Teachers were also found to vary greatly in their knowledge of mathematics, their interest and enjoyment of mathematics, their beliefs about its importance, and their expectations for their students. What then influenced the teachers' choice of content and tasks? The teachers' own convictions and repertoires were found to be the largest determinant. "They will teach what they have taught before, what they feel comfortable with, and what they deem
appropriate for their students." (Porter et al., 98). School policies such as mandated textbooks, district objectives or curriculum guide, and tests were also found to have some effect on teachers' decisions. Advice from external sources (higher grade teachers, parents, university staff) was a minor influence.

The authors point out that "only if instruction centers on important content does it have potential for being worthwhile" (Porter et al., 96) and conclude by suggesting a need for research that "starts with a judgment as to appropriate content and then seeks to design an environment that will encourage and support the teaching of that content." (Porter et al., 107).

The Professional Standards call on teachers to offer worthwhile mathematical tasks that are based on:

- sound and significant mathematics . . . and that engage students' intellect; develop students' mathematical understandings and skills; stimulate students to make connections; call for problem formulation, problem solving, and mathematical reasoning; promote communication about mathematics; represent mathematics as an ongoing human activity; display sensitivity to, and draw on, students' diverse background experiences and dispositions; and promote the development of all students' dispositions to do mathematics. (NCTM 1991, 25).

Although textbooks can be a useful resource, teachers who implement the Standards are expected to adapt or depart from texts to find tasks that will encourage the development of mathematical power.

Problem Solving Develops Mathematical Power. Problem solving is the preferred mode of mathematics learning in a Standards-based classroom. In the Agenda for Action (1980), the National Council of Teachers of Mathematics
targeted problem solving as the focus for the 1980s. Schroeder and Lester (1989) describe the evolution of problem solving since that time. At first, there was a rush to get on the bandwagon and teach “about” problem solving. This involved presenting heuristics, such as Polya’s “steps” and various strategies for solving nonroutine problems. Another approach was to teach “for” problem solving; that is, a concept such as division would be introduced, perhaps with manipulatives, practiced, and then students would be ready to apply the skill or concept in a problem solving setting. The current emphasis has shifted from making problem solving the focus of instruction, to making understanding the focus of instruction (Schroeder and Lester, 39). Understanding involves the ability to create relationships among mathematical ideas. Problem solving becomes the vehicle for, not just the goal of, learning. Students are given a problem at the outset, and they apply prior knowledge to construct new knowledge by solving, or working on, the problem. Teachers are encouraged to move from teaching about or for problem solving, to teaching “via” problem solving (Schroeder and Lester 1989).

Tasks Providing Real World Connections Develop Mathematical Power.

Bebout and Carpenter (1989) stress the importance of providing context, or realistic settings, for problems. It is important for students to build on previous understanding in order to connect students' informal experiences with mathematics to the more formal mathematics of the classroom. Kamii (1990) reinforces these ideas. She points out that while story problems are usually
presented after computational exercises, “the sequence should be reversed because children construct logico-mathematical knowledge out of daily living. Computation with numbers, which do not involve contexts, should come after a great deal of problem solving with real-life contexts.” (Kamii 1990, 29).

Lampert also argues for teachers to better align school math with real math (Lampert 1991). She is referring not only to real world applications and contexts, but also to the way real mathematicians go about doing mathematics. Doing math in schools too often means learning the rules, and truth is determined by the teacher’s determination of whether the answer is right or wrong. Doing math in the discipline of mathematics means testing assertions in a reasoned argument, and truth is determined by the reasoning process and the agreements that the community comes to share through these arguments (Lampert 1991, 124). An important role of the teacher, then, is to select “good problems,” ones that provide learning environments that are both safe and productive. By safe, Lampert means the problem is presented in a familiar context which allows the student to call upon, reveal, and use their current knowledge about mathematical structures. By productive, she means the same problem has the potential to lead students into unfamiliar mathematical territory, allowing them to create new knowledge. An example of such a problem would be: Find a way to make $1.00 with 19 coins (Lampert 1991, 128). Cobb and Merkel also advocate problems “that can be solved in a variety of ways that make sense to pupils at different conceptual levels.” By selecting such tasks,
"the issue of individual differences is addressed as students use methods which
make sense to them, and at the same time become more curious about math
and interested in others’ approaches." (Cobb and Merkel 1989, 72).

The Role of Computation in Developing Mathematical Power.

Mathematical understanding is essential to the development of competence
(Hiebert 1990). With the availability and incorporation of calculators and
computers into current life, rote mastery of procedures is no longer essential to
the development of competence. Should routine procedures play any kind of
role in the selection of tasks? According to Hiebert, two distinct kinds of
cognitive processes can be used in executing routine procedure: automatization
and reflection. Automatization can free up mental effort, which can then be used
in pursuit of understanding. Understanding comes from constructing or
recognizing relationships. Reflecting on procedures means learning them, or
learning about them (e.g., the two digit by two digit multiplication algorithm) in
such a way that conscious consideration is given to the patterns and
relationships that emerge. "Recognizing patterns does much more than help
students make sense of the algorithm; it helps students make sense of the
system." (Hiebert 1990, 38). He goes on to make a recommendation regarding
which routines to automatize:

If calculators are readily available, very few calculation skills would need
to be automatized. A plausible suggestion is that for the sake of
convenience, the whole-number arithmetic facts along with the base-ten
notation rules for combining larger and smaller numbers (e.g., 200 x 40 =
8000; 0.2 x 0.4 = 0.08) should be automatized. These facts and
procedures provide benchmarks that permit access to numerous sums
and products and enable useful estimation skills to develop. (Hiebert, 35).
Coburn also discusses the extent to which computation should be considered a worthwhile task (Coburn 1989). According to Coburn, the thrust of current curricular reform is not to reduce the importance of computation, but to reduce the overemphasis of written computation, and to broaden the definition of computation. Computation skill continues to be useful for learning other topics, in daily life, and in most occupations. But computation modes far more useful than written algorithms, are mental math and estimation and the use of calculators to solve problems. These are the computation roads to mathematical power.

The Use of Manipulatives in Developing Mathematical Power. A final area of interest regarding task selection has to do with the use of manipulatives, long considered to be a hallmark of nontraditional classrooms. Wheatley (1992) notes that manipulatives are often used to help make abstract concepts more “apparent” or comprehensible to the student. For example, the regrouping procedure is often modeled with base ten blocks. However, the teacher is still beginning with the abstraction. Although the materials are concrete, in the sense that they are seeable and touchable objects, this does not necessarily make the concept meaningful for many students. From some constructivists’ point of view, this use of manipulatives does not foster mathematical power. The manipulatives foster the teacher’s intention to help students make sense, but if 2 flats, 4 longs, and 6 ones is the teacher’s representation of 246 which she is trying to convey, then it is not necessarily the student’s construction. Students
can and often do learn to manipulate the materials just like the teacher, and still not have a personally meaningful understanding of place value (Wheatley 1992; Cobb 1991).

Kamii’s approach is to have students invent their own algorithms and procedures. In the classrooms where she does research, an algorithm is never directly taught by the teacher; rather, student created procedures and strategies are shared with one another and checked and considered through lively discourse. Kamii notes that many students, allowed to do their own thinking, will adopt a left to right procedure for addition regrouping. When students receive a steady diet of externally imposed procedures, they gradually lose their inherent desire to make sense; and their purpose becomes to follow the teacher’s directions and try to remember procedures (Kamii 1989).

Research Relating to Classroom Discourse

The Professional Standards highlight classroom discourse as a primary vehicle for helping students become mathematically powerful. What do we mean by mathematical discourse? The Professional Standards talk about discourse as all the ways of representing, thinking, talking, agreeing, and disagreeing, for the purpose of making sense of mathematics. Worthwhile discourse is taking place when students are explaining their thinking; listening and responding to one another; using a variety of tools (computers, calculators, diagrams, graphs, tables, metaphors and stories, concrete models, terms and
symbols) to solve problems and communicate their solutions; making conjectures and exploring examples and counterexamples; and relying on mathematical evidence to determine validity and truth rather than the teacher or the answer key. Good mathematical discourse requires engaging tasks and problems, and a learning environment in which each student’s ideas and thinking is respected, and all contributions are valued.

It is the teacher’s responsibility to select tasks that promote thinking and reasoning and to provide a supportive environment for discourse. The teacher “orchestrates” discourse by doing less talking, less modeling, and less explaining herself, and encouraging and expecting all students to do more; by making decisions regarding which ideas to explore further so as to keep the discussion reasonably focused; by asking provocative or clarifying questions; and by deciding when to let students struggle with an idea and when to provide directed input. The student’s ability to reason and communicate mathematically within a community of people collaborating to make sense of ideas becomes the central focus of the daily math lesson (NCTM 1991, 34-54).

Whitin describes an investigation from a 6th grade classroom which, although abbreviated here, illustrates the flavor of empowering discourse (Whitin 1989). Whitin had recently read that if you took a 4 digit number, reversed the digits, and subtracted, the difference would always be 6174. He tried it a few times and it worked, so he decided to share this idea with the students as a motivating way to practice subtraction. As students worked on their own
problems, they quickly found some nonexamples, and a spontaneous investigation followed. They decided to keep track of their findings on the board (Numbers that Work and Numbers that Don’t Work), and students added to the lists as they tested other numbers. Some began working with partners or small groups, and Mr. Whitin facilitated communication as students began raising their own questions and making their own conjectures. They explored the ideas that the sum of the digits must be an even number; that the digits must be all even or all odd; that the thousands digit must be larger than the ones digit. Good questions were being asked: Can you repeat a digit? (Yes, they agreed). Has anyone found a number that works that contains a zero? (No one had but someone did). Each conjecture uncovered more information. Persistence was the mode. One student reasoned that 6 was the only number that could be paired with zero to work (6 _ _ 0), because other combinations would produce numbers too large or too small. She displayed this finding in a chart, and other students formulated a list of 6 _ _ 0 numbers that worked. They then discovered that the difference between the middle numbers was always two; and eventually conjectured that the difference between the first and last numbers should be six. From there they eventually generated an organized list of 32 four digit numbers that “worked.” This investigation thoroughly engaged the class for two straight hours.

Whitin noted several “lessons” suggested by this lesson (Whitin, 189-190): 1) Value the process of mathematizing. One student said, “We started by
bungling along; then we were getting somewhere." Whitin thinks schools should promote more bungling. 2) Allow students to solve problems in their own way. He had no preconceived notion about a strategy, but supported the process through open ended questions, recognition of efforts, and trust in the students to devise meaningful solutions. 3) Recognize the power of learning as a social event. Students learn from each other by sharing, asking, challenging, and building on one another's ideas. 4) Teachers are learners too. Modeling the process of investigation and discovery can be as enlightening to students as guiding the process.

These ideas about empowering students through discourse stem from constructivist theories of learning which maintain that "learning is a process in which students actively construct mathematical knowledge as they strive to make sense of their worlds" (Cobb, Yackel, and Wood 1992, 6).

Constructivist theory has its modern roots in Piagetian developmental psychology (Steffe and Kieren 1994). Piaget postulated that children construct their own understandings of the world and how it works as they manipulate objects and interact with their environment. The mind develops in stages of increasing ability to abstract, and moves through these levels as the child is confronted with situations that no longer make sense to him from his old framework. From this state of disequilibrium he begins to form new mental structures which can better accommodate and relate new information and experiences.
The Representational Point of View. Many mathematics educators took Piaget's ideas to mean that students need "hands on" learning activities and manipulatives to facilitate their development and understanding of new concepts. The mathematics was seen to be located in the materials; for example, place value concepts could be represented by Deines blocks; fraction concepts by pattern blocks. The instructional issue was simply a matter of how explicit to be. Does a teacher just put out the materials and hope the student happens to discover or construct the mathematical idea the teacher has in mind; or should the teacher clearly demonstrate (map) the relationship between the materials and the concept? (Cobb, Yackel, and Wood 1992).

The Constructivist Point of View. Cobb contrasts the current constructivist perspective with the "representational" point of view described above. He and other constructivists maintain that there is no mathematics "out there"; rather, the mathematics is in the child. The instructional dilemma described above is resolved when mathematics is no longer seen as something to transmit, show, or be discovered (Cobb, Yackel, and Wood 1992, 27-28). Instead, the mathematics will be constructed by each learner, as students actively relate existing knowledge and ways of thinking to new ideas, and the channel through which this process takes place is discourse about the task, whether or not that task is manipulative in nature. (Yackel et al. 1991).

In the constructivist classroom, the teacher plays an important role in helping students to construct their own understandings through discourse:
the teachers' role in initiating and guiding mathematical negotiations is a highly complex activity that includes highlighting conflicts between alternative interpretations or solutions, helping students develop productive small-group collaborative relationships, facilitating mathematical dialogue between students, implicitly legitimizing selected aspects of contributions, redescribing students' explanations in more sophisticated terms that are none the less comprehensible to students, and guiding the development of taken-to-be-shared interpretations when particular representational systems are established. (Cobb et al. 1991, 7).

Vygotsky's Social Constructivism. These ideas reflect "a subtle movement taking place in many quarters of the mathematics education research community from a Piagetian cognitive perspective to a Vygotskian social interactionist perspective -- a theory that ascribes greater weight to the role of social processes in the construction of knowledge." (Kieran 1994, 602).

Although Vygotsky's theories are similar to Piaget's regarding the intrinsic and developmental nature of learning, a main point of difference is Vygotsky's principle that learners reach higher levels of thinking and understanding through social interaction, and then refine and internalize their new concepts and ways of thinking individually. Jones and Thornton (1993) describe the application of Vygotsky's ideas in the classroom setting. A student has an "actual" developmental level, which is his level of performance in a problem solving situation without help, and a "potential" developmental level, at which the student can function when interacting with a teacher or more capable peer. The "zone of proximal development" is the distance between these two points, and the target area for instruction. Teachers need to provide rich, interactive settings, in which modeling of higher level thought processes helps students to
bridge their zone of proximal development. This modeling is not the same as direct instruction or passive learning; rather the teacher has a strategy for the problem which may not fall within the child's zone, and the child has a strategy which obviously will fall within the zone. If the teacher's strategy is imposed, no meaningful learning will take place. Therefore, the two negotiate a meaning for the task somewhere in between -- which requires a great deal of flexible thinking and on-the-spot assessment from the teacher, and active engagement and hard work on the part of the student. Intersubjectivity is established when the two parties are in tune with each other, when "both parties are able to recognize, examine, negotiate or mutually adopt each other's perspectives." (Jones and Thornton 1993, 21). The teacher and the peer group facilitate children's learning through modeling, appropriate dialogue, feedback, and other forms of "scaffolding." (Jones and Thornton 1993).

A number of social constructivist classrooms have been described in the literature (e.g., Wheatley 1992; Kamii 1989; Yackel et al. 1990; Bauersfeld 1992; Pirie and Kieren 1992). They have in common a structure or routine in which a problem is presented, students work toward a solution strategy alone or with a small group, and then solution strategies are shared and discussed as a lively discourse among the whole class unfolds. Social norms for discussion have been established which stress sense making as the primary purpose, and respectful consideration of all points of view as the posture.

This current thinking on how meaningful learning takes place, reflected
strongly in the Standards, depicts the role of the teacher not as the dispenser of knowledge and the giver of tests; nor as one who simply arranges, facilitates, and observes as learning unfolds; but as one who makes professional decisions before and during instruction about the content of the lesson, and is an active agent in guiding his or her students to construct their individual and shared mathematical knowledge through empowering discourse. (E.g., Etchberger and Shaw 1992; Knapp and Peterson 1993).

Research Relating to Environment

The Professional Standards states that the environment:

is foundational to what students learn. More than just a physical setting with desks, bulletin boards, and posters, the classroom environment forms a hidden curriculum with messages about what counts in learning and doing mathematics: Neatness? Speed? Accuracy? Listening well? Being able to justify a solution? Working independently? If we want students to learn to make conjectures, experiment with alternative approaches to solving problems, and construct and respond to others’ mathematical arguments, then creating an environment that fosters these kinds of activities is essential. (NCTM 1991, 56)

Higher Level Thinking. Peterson (1988) reports on research relating to the identification of environmental factors that seem to support and encourage the development of higher-order thinking in mathematics. Higher-order thinking, or problem solving, is one integral component of mathematical power. She points to three dimensions that are influential in facilitating higher-order thinking. These are:
1) Rote Learning versus Meaning and Understanding. Peterson found that while direct instruction improved performance in lower-order thinking skills (defined as the National Assessment of Educational Progress categories of knowledge and skill), it was insufficient to promote improvement in higher-order thinking (defined as the NAEP categories of understanding and application), and that approaches supporting children's active construction of knowledge were more effective.

2) Teaching Higher-Level Executive Processes and Strategies for Mathematics Learning. Peterson found a positive relationship between the use of specific cognitive processes during higher-order thinking tasks (such as checking answers, applying information, reworking problems, rereading directions, relating new information to prior knowledge, asking for help, using aides, and using memory strategies) and achievement. Successful students often reported specific steps or strategies, described sequences in their thinking, and verbalized insights about the nature of the task. Explicit instruction and modeling of these cognitive processes was recommended.

3) Teacher Control and Direction versus Student Autonomy and Independence. Studies undertaken at the University of Wisconsin during the Cognitively Guided Instruction Project (Carpenter and Fennema 1988) document the importance of persistence and motivation to successful higher-order thinking. The author recommends small group cooperative learning experiences to support the development of student autonomy (Peterson 1988).
Cooperative Learning. Classroom environments that nurture the development of mathematical power are collaborative communities. Cooperative learning is a powerful tool for increasing students' self-confidence (Davidson 1990). One meta-analysis of 80 studies comparing students' mathematical achievement in cooperative groups versus traditional instructional settings, found that the students in small group approaches significantly outscored control students in over 40% of the studies. In studies where the experimental teacher was not as familiar with cooperative learning methods, there was no significant difference. In only two studies did control students significantly outscore cooperative learning students (Davidson 1985). Furthermore, attitude surveys from both teachers and students consistently yield these responses regarding the advantages of cooperative learning:

(Students) learn to cooperate... improve social skills, ... communicate mathematically... The classroom atmosphere tends to be relaxed and informal, help is readily available, questions are freely asked and answered, and misconceptions become quickly apparent and are readily resolved... Students become friends with their group members... usual disciplinary problems of talking and moving around are eliminated... students maintain a high level of interest... many like math more... students have an opportunity to pursue the more challenging and creative aspects of mathematics and to become more confident problem solvers while acquiring at least as much information and skill as when they are taught with more traditional approaches. (Davidson 1990, 60)

Johnson and Johnson have conducted extensive research in the area of cooperative learning, and advocate its use in mathematics classes for these reasons (Johnson and Johnson 1989):

First, mathematical concepts and skills are best learned as a dynamic process with the active engagement of students... Active learning
requires intellectual challenge and curiosity, which are best aroused in discussions with other students.

Second, mathematical problem solving is an interpersonal enterprise. . . . Students have more chances to explain their reasoning . . . in small groups.

Third, mathematics learning groups have to be structured cooperatively to communicate effectively. Within competitive and individualistic structures, students will not engage in the intellectual interchange required for learning mathematics.

Fourth, cooperation promotes higher achievement in mathematics than competitive and individualistic efforts. (Cites results of meta-analyses).

Fifth, by working cooperatively, students gain confidence in their individual mathematical abilities.

Sixth, choices of which mathematics courses to take and what careers to consider are heavily influenced by peers. (Johnson and Johnson, 236-237)

However, simply having students work in groups does not promote better understanding or improved communication and reasoning unless teachers ensure that key conditions are in place: the teacher clearly promotes positive interdependence in each group; students engage in promotive (assisting, supporting, encouraging) interaction during assignments; students are individually accountable; students learn and use small group skills; teachers help groups engage in regular group processing (Johnson and Johnson, 238).

Risk Taking. An environment which supports risk taking during whole group discussions is also important. In a constructivist classroom, the teacher's attitude is crucial to the development of a positive problem solving environment.
Every time a child contributes, the teacher assumes that the contribution is meaningful to that child. "By allowing a child to proceed with an explanation even when the answer is wrong, the teacher fosters a belief that the teacher is not the sole authority in the classroom to whom children have to appeal to find out if their answers are right or wrong." (Yackel et al. 1990, 18). Children learn to think for themselves, and listen to their own and others' thinking. The teacher's role is to assist, if and when appropriate, and to establish these expectations for communication: students cooperate, reach a consensus, explain their thinking, try to understand the other's thinking, and persist in trying to figure things out. Helping one's peers must be a central concern rather than a marginal activity (Yackel et al. 1990).

Research Relating to Analysis

The Professional Standards charge teachers to regularly consider what effects the tasks, discourse, and environment are having on their students' mathematical power.

What do students seem to understand well, what only partially? What connections do they seem to be making? What mathematical dispositions do they seem to be developing? How does the group work together as a learning community making sense of mathematics? (NCTM 1991, 62)

As teachers make changes in their content, their methods, and their purposes, a dilemma arises regarding assessment (NCTM 1995, 3). Traditional assessment practices are not necessarily consistent with the new goals of mathematics. Teachers must find ways to assess their students' conceptual and
procedural understandings, their ability to reason mathematically, and their dispositions. Attending to students during whole group discussions, observing them in small group discussions, conducting interviews, reading math journals, observing performances, and using traditional methods are possible means of assessing students' growth in mathematical power. Teachers also need to ask themselves questions about their selection of tasks, conduct of discourse, and features of the environment in order to better "understand the links between these and what is happening with their students" and make adaptations as needed. (NCTM 1991, 64). Thus the Standard on Analysis refers both to student assessment and to reflection about instruction.

**Analysis of Students' Mathematical Power.** Regarding student assessment, Wheatley (1992) reports on the assessment practices of teachers in the Math Learning Project, which are aligned with the suggestions cited above:

Teachers assess pupils using informed professional judgment. Grades are not given for daily work and tests are not administered except for the required state and national assessments. In fact, at no time does the teacher communicate a judgment of the students' mathematics. She does not collect worksheets, mark the ones that are wrong, or have students correct their 'mistakes.' Instead, the teacher keeps notes of students' activities in which she considers their persistence, confidence, cooperation, communication, and the quality of their mathematical constructions. (Wheatley, 531)

On the state and national assessments, after two years in this Standards-based, constructivist program, students scored at grade level in computation, and well above grade level on concepts and problem solving. Prior to the program,
scores had been below grade level on all subsections. "Although some importance is given to this data, it is doubtful that a standardized test is an accurate indicator of the scope of mathematical power developing in many students." (Wheatley, 532).

In the Problem Centered Instruction classrooms involved in the Purdue research project (where mathematical argumentation was emphasized and procedural instruction de-emphasized), second grade project students’ conceptual understanding and problem solving abilities were superior to non-project peers on state-mandated standardized tests, and this remained true even after the next year when students were back in traditional classrooms. The Purdue studies also measured and compared beliefs and motivations, with project students placing significantly greater value on effort and understanding as reasons for success (Cobb et al. 1992).

**Analysis as Reflective Practice.** Is classroom instruction helping students develop mathematical power? The ideas of self-reflection and self-reliance, of teaching without an external prescription or script, permeate the Curriculum Standards and the Professional Standards and are firmly grounded in literature about the "reflective practitioner," a concept that has its roots in the work of Dewey. According to Dewey, reflective action is the "active, persistent, and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it and the consequences to which it leads," while routine action "is guided primarily by tradition, external authority, and circumstance."
Zeichner and Liston describe the elementary student-teaching program at the University of Wisconsin:

(The program) emphasizes the preparation of teachers who are both willing and able to reflect on the origins, purposes, and consequences of their actions, as well as on the material and ideological constraints and encouragements embedded in the classroom, school, and societal contexts in which they work. The goals of the program are directed toward enabling student teachers to develop the pedagogical habits and skills necessary for self-directed growth and toward preparing them, individually and collectively, to participate as full partners in the making of educational policies. (Zeichner and Liston 1987, 23)

Smyth offers a four step process to help teachers “see” their own ideologies, and think about the structural conditions that are operational in their own classrooms, schools, or districts from a perspective of possible change:

1. Describe -- What do I do? (Or what is being done?)
2. Inform -- What does it mean?
3. Confront -- How did I (or things) come to be like this?
4. Reconstruct -- How might I do things differently? (Or how could things be done differently?) (Smyth, 1989)

Armaline and Hoover reinforce the importance of reflectivity, arguing that our knowledge structures are the lenses through which we make our assumptions and our interpretations of truth. Rigid knowledge structures ensure the preservation of the status quo, whether it is “working” or not. They claim that “the degree to which field experiences foster dynamic, intellectual, critical perspectives that empower teachers to empower their own students is the measure of success of teacher education.” (Armaline and Hoover 1989, 42).
An Historical Perspective on Student Teaching

There are over a half million preservice teachers enrolled in teacher education programs at over 1,200 institutions each year (Doyle 1990) and each and every one of them will presumably culminate his or her experience with that ubiquitous grande finale called student teaching. However widely teacher education programs may vary in their conceptual underpinnings and goals, student teaching is common to all, and the student teaching experience is the one aspect of their teacher preparation programs that teachers consistently rate the most influential (Tom 1991; Colburn 1993; Guyton and McIntyre 1990). Yet the quality of these experiences is very diverse.

Doyle (1990) has described five possible purposes or goals to which teacher education programs might subscribe.

1) Some programs train the “good employee” by socializing the student into prevailing practices, and emphasizing the technical and experiential aspects of teaching. An effective teacher is considered one who can do it like it always has been done. School administrators and experienced teachers support this model of teacher preparation.

2) Other institutions strive to develop the “junior professor” by providing a strong academic emphasis. A successful candidate would have a strong knowledge base in the academic disciplines which they can then impart to their future students. Academics and legislators find this model appealing.
3) A third paradigm, whose proponents are often from psychology, counseling, and elementary education, values the “fully functioning person,” and provides a program for students whose goal is personal understanding, clarification of values, a sense of satisfaction and purpose from teaching, and knowledge of human development and ways to promote growth. The desirable outcome of this program is psychological maturity, and an understanding of the nature of student learning and growth.

4) The fourth paradigm focuses on the “innovator,” and hopes to develop new teachers who will be sources of renewal and change in schools. The emphasis of this program is training in the latest theories and practices of education; therefore, field experiences in conventional classrooms might be undesirable because students would be at risk for indoctrination into traditional practice -- lab settings would be ideal. Behavioral and social scientists, researchers, and teacher education professionals are often among the supporters of this model.

5) The final conceptual framework for teacher education is that of the “reflective professional.” This model is one of the current hot topics in research, and its proponents hope to develop teachers who think critically about their work and workplace and are committed to ongoing change and improvement. The program consists of training in observation, analysis, interpretation, and decision-making. An effective teacher from this paradigm would be perpetually questioning, fine tuning, and adjusting his or her personal and professional
practice to become ever more effective (Doyle 1990, 5-6).

Do programs of teacher preparation really have such well-defined identities and mission? According to Howey (1989) 65% of faculty and students from a large number of teacher education programs surveyed identified a discernable dominant conception of teaching at their institution. Howey’s categories for orientation were: skill or competency-based; clinical or problem solving; humanistic and person-oriented; inquiring and reflective; and liberal education. However, respondents from the same institution often responded differently, and it would seem that most teacher preparation programs would be likely to embrace elements of several paradigms.

The “Professional Development Standards” (Section 3 of the NCTM Professional Standards 1991) state that it is imperative for preservice and practicing teachers to understand the notion of mathematical power. This document advocates a broad based reform effort in the preparation of teachers which speaks to goals from each of the five frameworks cited above: pedagogical knowledge, content knowledge, knowledge of students as learners, socialization into renewal and reform, and the ability to reflect on and analyze one’s own beliefs and practice (NCTM 1991, 123-173).

Whether or not a teacher education program has a clearly conceived and communicated sense of mission and direction, and whatever the nature of that direction, it is certainly the case that within most university / public school pairings for field service, the arena of control regarding the nature of the student
teaching experience shifts at that point to the public school, or more specifically to the cooperating teacher.

This "medieval apprenticeship training model" has been the modus operandi since 1700 when the first normal school was established in France (Guyton and McIntyre, 514-515). A departure from this induction format took place in the early part of this century, when Dewey's educational theories led to the development of campus lab schools to serve as research and field experience sites. These lab schools were short-lived, enjoying a brief resurgence again in the 1960s as sites for experimenting with discovery learning, but never seriously challenging the public schools' apprenticeship approach to teacher induction. In this decade there have again been resounding calls for reform and restructuring of teacher education, with the concept of Professional Development Schools emerging as the reform model of greatest current interest.

First posited by the Holmes Group (1986), this and related reform ideas were developed at great length by Goodlad, whose extensive research convinced him of the "need to provide exemplary practice settings. . . . Some of the most unacceptable shortcomings in the settings we studied were found in this part of the program, although both students and faculty members rated student teaching highest among program components for impact." (Goodlad 1990, 280). Among Goodlad's many recommendations are 1) the placement of elementary student teachers in schools, not classrooms, so that they are
exposed to a wider variety of practices and develop a broader perspective, and 2) their active involvement in the experience of renewal and reform rather than the mastery of craft.

The Research Focus

This study describes the experiences of two elementary student teachers who were placed with cooperating teachers actively involved in renewal and reform of their mathematics programs, in a school district also supportive of mathematical reform. The research project was specifically designed to investigate these student teachers’ existing and developing understanding of mathematical power, and how to teach for mathematical power.
Schoenfeld (1994) provides a concise overview of the gradual shift in methodological approaches to mathematics research that has taken place over the last two decades. By the 1960s, education research, including mathematics education research, had aligned itself firmly with statistical methodology and empirical design in an effort to lend "scientific" credibility to the field. One research model, for instance, utilized a factor analysis design, where attempts were made to find correlations between certain discrete abilities (e.g. spatial perception, computation, verbal) and performance on mathematical tasks, such as problem solving. In another typical model, researchers set up experimental designs with control groups to test the effects of various treatments or interventions, usually created by the researcher or some other source external to the actual educational environment. This treatment or intervention was usually derived from a theory about which instructional components were likely to result in more effective outcomes (e.g., higher test scores). Problematic to such designs was the fact that real classroom settings did not lend themselves well to
isolating factors and controlling variables. Fragmented behaviors and isolated contexts have very little to do with real teaching and learning, which are necessarily complex and holistic and dynamic and varied — not easily "controllable" in the scientific sense. "In short, many factors other than the ones in the statistics model — the variables of record -- could and often did account for important aspects of the situation being modeled." (Schoenfeld 1994, 701).

During this time, the behaviorist school of psychology undergirded many of the assumptions about what constituted rigorous and acceptable research: behavior and effects must be overtly observable; introspective data about what was occurring, whether from the subjects or from the researcher, was off limits, and statistical significance was what counted (so to speak).

By the late 1970s, the mathematics education community, and educational research generally, was realizing the limitations of process/product research methodology, and looking to other disciplines for new approaches. The influences of both Piaget's work and information-processing models of learning caused the clinical interview and observational methodology to gain a foothold, as the internal processes of thinking and learning once again became legitimate areas of inquiry. Examples of such research would be the constructivist teaching experiment, in which a researcher would pose a problem to a student, and then interact with the learner as he or she struggled with the problem, observing closely the thinking processes that unfolded (e.g. Cobb and Steffe 1983). This type of research has produced much valuable knowledge
about learning, which was utilized by writers of the NCTM Standards.

Kilpatrick discusses the purposes of mathematics education research and their appropriate methodologies. When the goal is to explain, predict, or control — then the empirical/analytical tradition from the natural sciences is useful. When the goal is to help students and teachers gain greater freedom and autonomy in their work, or to improve practice and involve the participants in that improvement — then action research, introduced in sociology, can be applied. But when the goal is to understand the meanings that learning and teaching mathematics have for those engaged in the activity — then the interpretive framework from anthropology works well. Kilpatrick reiterates that during the last decade there has been movement away from the empirical/analytical designs and toward fieldwork and interpretation, but he points out that even when empirical methods were dominant, the predominant motive for such research was more often understanding than prediction and control, which indicates that historically there may have been a frequent “mismatch” between the goals of mathematics education research and the methodology utilized (Kilpatrick 1992).

The Research Design

A qualitative research design was selected as the most appropriate method for gaining insight into the research questions posed in this study. All
participants were involved in the processes of learning, of change, and of reform. According to Patton, “Qualitative inquiry is highly appropriate in studying process because depicting process requires detailed description; the experience of process typically varies for different people; process is fluid and dynamic; and participants’ perceptions are a key process consideration.” Further, process studies “look not only at formal activities and anticipated outcomes but they also investigate informal patterns and unanticipated interactions.” (Patton 1990, 95).

**Documentation of a Need for This Type of Study**

A prevalent theme in current reviews of the literature is the need for naturalistic, qualitative inquiry in both student teaching and mathematics education. In the section “An Emerging Paradigm for Research on Field Experiences,” from their chapter on student teaching in the *Handbook of Research on Teacher Education*, Guyton and McIntyre state:

The phenomenon of the atheoretical nature of field experiences that has existed for many years can be attributed partly to the imposition of a scientific research paradigm on situations that are not compatible with the methods, purposes, philosophy, epistemology, or assumptions of the paradigm. This paradigm adopted the scientific methods of the natural sciences. A major breakthrough in the 1980s was the emergence of a new paradigm for research on field experiences, a naturalistic approach. This new paradigm has generated more meaningful results than the old and, if continued, could lead to substantive and procedural changes in the way field experiences are conceptualized and structured...
Naturalistic inquiry regards field experience as a process rather than as a variable. This systemic approach acknowledges the complexity of field experiences. Also, more naturalistic inquiry has included subjects' frames of reference. Traditionally, studies have ignored the meanings actors bring to the experience. Research has moved from the more restrictive pretest-posttest design studying predetermined variables and from descriptive survey data to a discovery mode in which concepts and categories emerge from the data. Field experiences are abstruse processes, making it difficult to identify a priori the important variables, particularly given the limited knowledge base (Guyton and McIntyre 1990, 529).

Likewise Brown and Borko echo the research implications and recommendations from many mathematics education articles and reviews: “It is important that researchers focus more on teachers’ classroom actions. . . . If a goal of research on becoming a mathematics teacher is to help teachers become better able to teach mathematics as envisioned in NCTM documents, then it is important to study teacher actions as well as cognitions, as well as to identify conditions under which changes in teacher cognition are likely to be accompanied by compatible changes in classroom actions.” (Brown and Borko 1992, 236).

**Sequence of Events Before and During the Study**

The study took place in a northwestern university community of 30,000. The local school district has seven elementary (K-5) schools, and district efforts toward mathematics curriculum revision and reform had been ongoing since 1986. During 1986, the elementary teachers on the mathematics curriculum
revision committee, including the researcher, took a course on current Elementary Mathematics Methods and reviewed the latest documents and publications pertaining to mathematics education, including A Nation at Risk (1983) and the NCTM publication An Agenda for Action (1980). The committee then affirmed problem solving as the cornerstone for a “new” elementary mathematics curriculum, and adopted a textbook series that the committee felt best helped teachers facilitate this goal.

In 1992, the curriculum revision cycle began again, and a new committee was formed. The cooperating teachers from this study as well as the researcher were active members of this committee, planning and participating in all curriculum processes and activities. A summary of these activities provides background information for the context of the current study.

The committee once again spent the first year immersed in the study of the current thinking in mathematics education, including the newly published Curriculum and Evaluation Standards for School Mathematics, and Professional Standards for Teaching Mathematics (NCTM 1989, NCTM 1991). During that year the committee wrote a district philosophy which reflected a rationale for mathematics education in keeping with the Standards and decided:

that no textbook series currently on the market could adequately “teach to the Standards”;

that we would reaffirm our existing textbook series as an adequate resource and encourage the purchase and use of a wide variety of additional
that we wanted our district’s teachers, not textbooks or commercial programs, to be in charge of the management of their mathematics curriculum.

In 1993, the Math Committee goals were:

1) to provide all K-5 teachers with inservice about the Curriculum and Evaluation Standards

2) to develop a Scope and Sequence aligned with the Standards

3) to develop and implement a needs assessment and materials ordering process, and

4) to maintain communication regarding the Standards and the curriculum revision process.

To this end, all 100 K-5 district teachers were provided an overview of the content and spirit of the Curriculum Standards during two half-day release days in the fall of 1993, and attended a number of grade level meetings throughout the 1993-1994 school year. A needs assessment instrument was prepared to determine what materials and further inservice teachers felt they would need to begin (or continue) to implement a Standards-based curriculum in their classrooms. Meanwhile the committee began work on the collection of ten alternative assessment tasks to be used at each grade level. The thinking was that since “assessment drives curriculum,” teachers who had a repertoire of rich tasks to utilize in assessing their students’ mathematical understandings would want to “teach to these new tests” by planning similarly rich tasks for their
students on a regular basis. Simultaneously, the committee created a curriculum
guide in which each standard was analyzed in terms of learning outcomes for
each K-5 grade level. In the spring of 1994, each teacher was given a budget
roughly equivalent to that of a textbook adoption with which to order resource
books and manipulatives. These were to be used along with, or instead of, their
existing textbooks and manual to implement Standards-based instruction in their
classrooms. This ordering process followed guidelines and instructions
established by the District Math Committee. Presentations were made to the
School Board to familiarize them with our goals, process, and progress; and
meetings were held with the principals, who also attended the teachers'
inservice sessions.

In the fall of 1994, “implementation” began, or in many cases continued,
and it was during the 1994/95 school year that this study took place. Math
Committee goals for the 94/95 school year included:

1) to help teachers begin/continue implementing a Standards-based
curriculum in their classrooms, using the new Curriculum Guide (grade level
learner results), their new (and old) resource books and materials, and their
Alternative Assessment Packets;

2) to provide appropriate inservice opportunities and supervision to
support curricular change; and

3) to create a plan for Program Assessment (i.e., evaluation of the new
K-5 math curriculum at the district level).

To achieve these goals, all K-5 teachers participated in a half-day
workshop in September, 1994 to become familiar with the new Curriculum Guide
and Alternative Assessment Packet for their grade level. An additional half-day inservice in November, 1994 was custom-designed by each grade level according to their expressed needs -- math and literature, math journals, and geometry activities were some of the topics explored. Each grade level met periodically to discuss the Alternative Assessment Packets. These were considered working documents, and the expectation was for teachers to try the assessment tasks, keep notes, and discuss adaptations and needed changes. The seven elementary principals attended two mornings of inservice regarding their role in facilitating and supporting change, and also attended both rounds of teacher inservices.

Finally, data was collected in April, 1995 in the form of teacher questionnaires and principals' surveys regarding aspects of program implementation. The results indicated that the vast majority of elementary teachers in this district were very actively involved in mathematics reform efforts. Teachers differed in which aspects of reform they were emphasizing (remember, the Standards are not prescriptive), as well as the stage or level of reform in which they were involved. The classrooms of two of these teachers provide the context and the data for this study.
Participant selection

Two teachers were selected for the study who had been actively involved in their district mathematics committee's curriculum revision process for three years, and who accepted the additional responsibility of supervising a student teacher during the Spring 1995 semester. Consideration was given to selecting the participating teachers in a random fashion to represent a "typical" level of understanding and commitment to program goals. However, this study's goals focused on the influence and impact upon the student teachers of their placement in a classroom actively involved in mathematical reform, and so the two cooperating teachers were selected specifically for their high level of understanding of a Standards-based curriculum and their commitment to its implementation in their classrooms.

The two student teachers were seniors at the local university completing their B.S. degrees in Elementary Education. Both had completed two semesters of paraprofessional work during the methods blocks prior to student teaching. During the paraprofessional blocks, each student completed 4 four week half-day school practicums. Both had taken a two-semester math content course for prospective elementary teachers, and both had taken the Math Methods course during the fall 1994 semester. Both were 21 years old. These two preservice teachers had been randomly assigned to classrooms selected for the research study, and graciously consented to participate.
Data Collection

Field Notes and Observation

The researcher attended and observed the math classes in each classroom throughout the 13 weeks of the student teaching assignment (January through April, 1995), and observed a number of planning and debriefing sessions as well. In Classroom A (3rd grade), math class was held 54 days, and the researcher observed 44 of those days. In Classroom B (4th grade), there were 57 math periods, and the researcher observed 47 of those days. These observations constituted a total of 160 hours of observational fieldwork.

Fieldnotes were collected using the categories described in “Standards for Teaching Mathematics” (Section One in the Professional Standards) as a framework to guide observation, so that particular attention was paid to the nature of the tasks, the nature of the discourse, the learning environment, and the self-reflection and analysis of the teaching and learning that took place by the teacher and student teacher. These categories are thoroughly described in Chapter One, and the Observation Template can be found in Appendix B.

Extensive notes were recorded during each daily math session, and the teacher and student dialogue was scripted. Lessons were audiotaped or videotaped four times in Classroom A and seven times in Classroom B for later analysis of teacher exposition and classroom discourse. Additional notes were collected during the teachers' and student teachers' planning and evaluation sessions.
There were 12 such sessions in Classroom A and 31 sessions in Classroom B. Notes were also recorded of participants' informal observations and statements.

**Interviews**

Three formal interviews were conducted with each of the four primary subjects. The entry interview elicited their various backgrounds, goals, and belief systems regarding mathematics education; the midterm interview probed the student teachers' growing understanding of the concept of mathematical power; and the final interview elicited participants' perspectives on what the student teachers' learned about teaching mathematics in a Standards-based classroom. The questions were formulated according to the general interview guide approach as described by Patton (1990). Interview protocols are included in Appendix C. Informal conversational interviews took place throughout the research period with cooperating teachers, student teachers, and students, to supplement direct observation regarding conditions that foster mathematical power.

**Journals**

Each participant was asked to keep a journal in which she recorded one feature or aspect of the daily lesson that stood out in her mind. The format was one of forced brevity, both to protect the informants from unreasonable intrusion into their time, and because this format was intended to bring to light the
informant's interpretation of the most salient features of the lesson. Carter and Gonzalez (1993) used the construct of the "well remembered event" in a study of student teachers' socialization processes. They claimed that "by analyzing how novices interpret events they consider especially salient, we may gain insights into what they know and how their knowledge changes with additional experiences in watching and doing teaching." Sample journal pages are included as Appendix D. One aspect of this methodology deserves a word. Although there was no treatment or intervention in this study, the keeping of a daily journal, brief though it was, imposed on the participants an overt element of "analysis" or self-reflection that may or may not have otherwise taken place.

Document File

The researcher collected copies of teachers' and student teachers' lesson plans; samples of students' products from daily lessons; and data from student assessments. This information provided valuable support and reinforcement for the descriptions of classroom activity and mentorship processes.

The schedule for data collection is included as Appendix E.

Data Analysis

Inductive techniques suggested by Guba (1978) and others were used to find patterns and develop categories. Guba describes analysis as the process
of dealing simultaneously with convergence (figuring out what things fit together) by looking for recurring regularities, establishing priorities, and testing the categories for completeness; and divergence (fleshing out the patterns and categories) by extension, bridging, and surfacing. In this study, field notes were analyzed for patterns relating to the categories of "task," "discourse," "environment," and "analysis" (meaning reflective practice on the part of the subjects). These sensitizing constructs were provided by the Professional Standards. The constant comparison method of Glaser and Strauss was also used (1967, Chapter 5). This strategy also focuses on identifying categories and coding data to discover relationships and patterns. In this study, the researcher analyzed data for strategies, behaviors, processes, and effects relating to the themes of the mathematical empowerment of students, and the enactment of mathematical reform as guided by the Standards documents. The interpretations of these themes which the participants brought to the study was an integral component of data analysis, as is common in naturalistic studies.

In empirical studies, when an experiment is repeated in the same manner, under the same conditions, and results match, reliability is established. The experimental model is well suited to natural sciences, but in educational settings it is impossible to replicate research in a strict sense. Likewise, validity in empirical studies concerns the ability of the results to be generalized to larger or other populations, and depends on such factors as a large sample.

In a qualitative study, reliability and validity are also important
components of the research process, but take on different meanings. Reliability and validity are established through the depth, breadth, and detail of description and documentation which the researcher is able to convey, as well as through the multiplicity of data points. Freudenthal suggests that to establish reliability, research must be "reported on so candidly that . . . the experience can be transmitted to others to become like their own experience. (Freudenthal 1991, 161). Regarding validity in qualitative research, Gravemeijer claims that "a differentiated generalizability is more important; the question is how certain elements of the results will apply to other situations." Further objectivity is gained through the processes of "participation-dissociation balance, (where) the researcher must find the middle course between too much dissociation and too much involvement, (and) triangulation, (the principle that) two sources will tell more about a certain phenomenon." (Gravemeijer 1994, 455). In this study, field notes and researcher observations, participant interviews, and journal entries were compared each with the others for corroboration or contrast. Some initial observations, descriptions, and interpretations were shared with other researchers and participants for their response, as a further point of triangulation.

Patton's description of data analysis is quite clear:

The qualitative analyst's effort at uncovering patterns, themes, and categories is a creative process that requires making carefully considered judgments about what is really significant and meaningful in the data. Because qualitative analysts do not have statistical tests to tell them when an observation or pattern is significant, they must rely on their own intelligence, experience, and judgment. This sometimes leads to the making of the qualitative analyst's equivalent of Type I and Type II errors
from statistics. The evaluator-analyst may decide that something is not significant when in fact it is; or, conversely, the analyst may attribute significance to something that is meaningless. (Patton 1990, 406)

In this study, researcher error was controlled and validity enhanced by using multiple means of data collection as described above to provide triangulation; by disclosing and bracketing personal bias and preconceptions; by embedding the study in a framework of related research; and by augmenting inductive methods with tools of logical analysis, such as enumerations or matrices. (Patton 1990, Chapter 8; Goetz and LeCompte 1984, Chapter 6).

The Researcher’s Role and Bias

This researcher’s interest in mathematics reform began twenty five years ago when I was hired to teach in a Follow Through classroom of twenty-two bilingual, bicultural, third graders with no textbook to use and no curriculum guide to follow. I was given some Nuffield resource guides that emanated from the British Primary School System, lots of hands on materials, and these instructions from my supervisors based on Piaget’s theories: Mathematics involves the processes of classification, seriation, and forming spatial and temporal relationships. If you help your students develop in these areas, you will be helping them learn mathematics. From this formative (and fun) experience I grew to understand that the mathematics (the process of mathematizing) resides in the student, not the worksheet or the materials; and
instruction (the responsibility for generating mathematizing) resides in the teacher, not the manual or the materials.

In subsequent years I served as a mathematics specialist, directing a math lab and problem solving approach to learning, and participated in a National Science Foundation project (EMME: Excellence for Montana Mathematics Education 1986-1987) designed to educate teacher-leaders to adopt and disseminate reformed approaches to mathematics education. The EMME group was asked to review initial drafts of the Curriculum Standards.

Interest in conducting this study arose from my involvement in two major projects: the local school district’s Mathematics Curriculum Revision Committee, which I chaired 1992-1995, and the National Science Foundation funded STEP (Systemic Teacher Education Preparation) Project, for which I worked as a graduate research assistant at Montana State University 1994-1995. The local school district, under the guidance of the Math Committee, had begun implementation of a revised elementary mathematics curriculum aligned with the Curriculum and Evaluation Standards during the year of this study. The STEP Project, among other things, placed student teachers in “model school sites” around the state, with lead teachers selected for their ability and willingness to provide their student teachers with an exemplary experience in mathematics and/or science education. The goal of both projects was to strengthen the teaching and learning of mathematics in Montana, by providing teachers and teacher candidates with exposure to, training in, and support for a
mathematics curriculum grounded in the NCTM Standards.

In the current research study, I assumed two research roles during the course of data collection: observer, and interviewer. My role as observer was to watch and record the teaching and learning, and the mentoring and modeling, that occurred in each classroom. The purposeful selection of Standards-based cooperating teachers served as a sort of "control," to ensure that the student teachers would have exposure to Standards-based classroom instruction. However, no treatment or intervention was planned for the study. The study was descriptive and interpretive, the purpose being to faithfully record what the tasks, discourse, analysis, and environment looked like in two classrooms where instruction was tied to the Curriculum Standards, and to document and interpret what the student teachers learned about mathematical empowerment from such a classroom. Participants quickly accepted that I was not there to evaluate, offer suggestions or advice, or actively participate in planning or teaching, and they were very considerate about not putting me on the spot in that regard. As interviewer, my role was to gather information about each participant's beliefs and attitudes, and to engage them in dialogue centering around empowerment, and their evolving interpretation and understanding of an empowering mathematics program. I was especially interested in the cooperating teachers' reflections on how best to guide a student teacher in that direction. At all times I was interested in eliciting their ideas on mathematical empowerment, without imposing my own.
CHAPTER 3

PRESENTATION OF THE DATA

Introduction

This chapter offers a detailed description of mathematics reform-in-action in two elementary classrooms, and how the student teachers assigned to these classrooms grew in their understanding of what empowering mathematics instruction is all about. Pseudonyms were chosen to help the reader distinguish between student teachers (Miss) and cooperating teachers (Mrs.); and between the 3rd grade class (Miss A___ or Mrs. A___) and the 4th grade class (Miss B___ or Mrs. B____).

The chapter begins with a profile of each participant, to illustrate the various background experiences and perspectives each person brought to the study. These are the findings for Research Question 1: What prior mathematical experiences, and what attitudes and beliefs about mathematics teaching and learning, do the student teachers and cooperating teachers bring to the classroom setting?

Following these descriptions, the chapter contains two sections which describe the instructional activities in each classroom from the framework of the four Professional Teaching Standards: Task, Discourse, Environment, and
Analysis (NCTM 1991). These two sections, labeled Classroom A and Classroom B, constitute findings for Research Question 2: What knowledge, beliefs, and criteria do the cooperating teachers use as they enact a mathematics curriculum based on the NCTM Standards, and how are these conveyed to the student teachers? Specifically,

2a. What is the nature of the tasks in which the students are engaged each day, and how are these selected?
2b. What is the nature of the discourse among students and teachers that occurs during the mathematics lesson?
2c. What are the features of the classroom environment associated with the mathematical empowerment of students?
2d. How do the cooperating teacher and student teacher reflect on (analyze) the effectiveness of the program components -- the tasks, the discourse, the environment, the pedagogy -- in the empowerment of students?

Findings for Research Question 3: (How did placement in a Standards-based classroom influence these student teachers' understanding of mathematical power and related pedagogy?) and interpretation of the data will take place in Chapter 4.

Profiles

Mrs. Alder

Emma Alder is a 3rd grade teacher with fourteen years of various teaching experiences. She took eight years off to start a family. She remembers being good at mathematics, and feeling confident. “They just taught you the algorithm and you practiced it over and over and over again.” In high school: they started ability grouping, so you had all the supposedly high kids in
one or two sections. I ended up in the high one... I had excellent teachers in junior high and high school -- excellent! People who explained things and just made me really like math... They also had high expectations... The one thing that sticks out in my mind was in Algebra One. I always performed well and it was nice, but I can remember literally the day when I went, "That's why!" I could always do it before, but suddenly I could figure out why.

In college Emma took two quarters of Trigonometry instead of Math for Elementary Teachers because it was “basically an introduction to modern math and I’d already had that.”

Mrs. Alder believes that the purpose of school is “to help each child become a functioning, productive member of society. Someone who finds their strength and feels confident of what they can contribute not only on an individual level but to society as a whole.” And a good teacher is “a person who lets the child know that he or she is valuable, important, you have things you can do well, you are important to you and to the rest of us.” This idea came up again when I asked Mrs. Alder what it means to empower students mathematically.

It means to give students the skills and thinking strategies, problem solving strategies that they’re going to need to function in today’s society. And to feel confident in math.

Mrs. Alder herself conveyed a sense of confidence and depth of understanding when she spoke about mathematics:

Math has to do with quantity. Quantity of anything -- money, length, time, distance -- how they relate to one another and how they relate to you and the way you operate your life... I always used to think of math as ‘Here it is. This is how you do it. End of discussion. This is when you add; this is when you subtract... I didn’t think of it as... what’s the word I’m looking for... I thought of it as more concrete (fixed), rather than as a growing, changing body of knowledge.
What are the most important things for students to learn in mathematics?

A sense of correctness. When they're working with these numbers that represent quantities of things, they need to have an idea of what reasonable is, and then given any situation they need to know when you need to be exact and when you don't need to be exact, and have the tools so that when you need to be exact, how to do it, in a multitude of ways -- maybe it will be calculations on a piece of paper, maybe you need your computer, maybe you need your calculator. But a sense of correctness and reasonableness.

Regarding the best ways to learn and teach math, Mrs. Alder said, "You have to make it real and meaningful. You have to make it as important to them as you can ... they have to see a need for it." A high quality mathematics program, according to Mrs. Alder, has "students (who) are involved; real life problems; good assessment -- by that I mean the teacher knowing a lot about what each child knows and what needs to be learned." What are some ways to know if your students understand? "You need to talk to them, observe when they solve a problem or whatever it is you're giving them to do, you need to have communication with them to really know."

Mrs. Alder's mathematics instruction has changed over the years. A Math Their Way class nine or ten years ago, followed by a Math Labs in the Classroom summer course, "really started me thinking." She worked with a parent group at a rural school to purchase math lab materials with a grant, and that "opened my eyes to all the different ways you could use manipulatives in the classroom." Many colleagues have influenced her. But the biggest change, Mrs. Alder says, has come about from her work on the District Math
The Math Committee has changed what I think about math teaching and given me the opportunity to learn it." She goes on to say:

Although I use the book (textbook) as a bottom line and I read it over and refer to it, I do not go through the book one page at a time. I look at the big ideas, the scope and sequence from the Math Committee, what is the big thing I need to get across to the kids and how can I do that? I no longer say, ‘Well turn the page’ Another change is manipulatives -- all the things in my classroom -- even when we first started getting them, you would look at them and go, ‘I’m not really sure how this is going to work’ so that has changed a lot too. The confidence, the experience, and the tools -- how do you get everybody participating and watch for mathematical learning -- using the tools more effectively.

Miss Aragon

Kelly Aragon was a young senior in her final semester, eager to complete student teaching and have a classroom of her own. Her mother is an elementary school secretary, so Kelly grew up hanging out in various classrooms, "volunteering." In high school she taught dance and baton lessons to young children and “that's basically what got me into kids.”

I just like how kids learn and how they get excited when they can learn something. I guess I've always known that's what I would do . . . This is a lot from my personal experience, but it seems to me kids learn better when they discover. Instead of the teacher standing in front and saying, “2 + 2 = 4” (you should) have them count it out, things like that. Kids can learn facts and they get excited about ‘Oh I know this and I know this’, but it's not as meaningful as when they discover it on their own. You know, mostly hands on as much as possible. Granted there's a lot of things you can't do hands on, there's some things you just have to know, like facts, but for the most part discovery learning on their own, and the teacher as a facilitator rather than a . . . what's the word I'm thinking of . . . teller or lecturer. Refiltering the learning. I'm very much against lecturing.
Kelly had these thoughts about what makes a good teacher:

Not a certain personality because there are so many different kinds of teachers out there that you can’t really pinpoint it, but genuine concern for the students and I think a teacher has to love to learn themself. If you don’t like school and don’t love to learn yourself, then maybe you shouldn’t try to teach learning.

Kelly was a strong math student herself.

In high school I took pre-calculus and trigonometry, and liked those a lot. And my physics class in high school was a tutorial; there was no teacher-student instruction. Even if we needed help we were pretty much on our own, or just go to other students. So it was difficult, but I did very well because of my math skills.

Kelly was a peer tutor for that physics class. In college she took precalculus.

After that she wanted to take calculus.

I like working with those big numbers and equations, but I found out that it didn’t count for elementary education, so I had to go back and take 130 and 131 (Math for Elementary Teachers).

I enjoyed 131 far more than 130 but it had to do with the teacher. We used a lot of manipulatives. He would bring in things to do the probability with. So instead of doing it on paper, we would draw a picture or use Cuisenaire rods. The reason I think I enjoyed it so much is that I wasn’t in the education blocks yet and this was the first thing I had for teaching. He’d say, ‘You’re going to use some of this when you’re teaching,’ and that would get me all excited and I’d think, ‘Oh my gosh finally something for my degree!’

What does it mean to be a good student in an elementary classroom?

For one thing, involved. And lots of wonder, lots of questions. Not necessarily the right answer because a lot of times there’s still thinking going on so that’s not really indicative.
Kelly described mathematical power in this way: “Well, I would say it’s letting students figure it out, by using reasoning, critical thinking, and manipulatives.”

Kelly felt prepared by her methods classes and para experiences to begin student teaching. She talked about what she was feeling, and her expectations for student teaching.

Excitement, but a little anxious. A little nervous about the solo. I know I can do it. Like last night when I went home I thought, “Ok, I wish tomorrow I could just go and just start soloing, do it all by myself,” so I’m excited to do that, yet I’m still nervous. This is my most important semester yet, so I expect a lot of confidence out of myself. I’m a very confident person; yet the only thing that makes me not confident is myself at times. But I do expect it; that during student teaching I’ll become confident enough to start teaching, subbing, whatever. I guess what happens happens, you know, everything’s going to turn out fine.

And what happened was, this young lady grew in confidence and pedagogical skill each day, as her beliefs and intuitions were reinforced in a Standards-based environment, and as she confronted the challenge of channeling the energies of active third graders into positive learning experiences.

Mrs. Birch

Lillie Birch is a 16 year veteran 4th grade teacher who loves to teach math. Yet as a student herself, she claimed math was “the subject I felt least comfortable with.” She remembers feeling anxious about math beginning in junior high:

I didn’t do well in high school algebra. I had a teacher who was not helpful, if you didn’t get it you didn’t get it. I got a C. From then on it was the old proverbial, ‘Oh -- math.’ ‘Math -- I’m a girl.’ . . . As I look back now and I teach kids I wish that there had been more concrete
experiences for me. I was definitely a student that needed more of that. Sometimes I would do OK, but it was never a subject where I had a level of confidence. I would always question my performance in that area. Geometry I enjoyed more. But I never really pursued higher level math courses. I took what was required and that was all. Then as I started teaching math it was almost like relearning things that I hadn't learned before. Simple things that I might teach in 4th grade, but I was seeing it in a different light, or a different way, so it was reliving those experiences and I really changed my thinking about math. It's my favorite subject to teach now, and probably the course that I've pursued the most as a teacher, trying to enrich what I know about math. Still, upper, more complex math I really don't have a background in, and I've not pursued it in that way, but I want to know better ways for me to teach kids math.

Mrs. Birch started college after she started a family, and a strong personal desire to learn was reflected in many comments throughout the research study. For instance, a good teacher was “someone that values learning, not only for children but as an adult. I think it's the desire to help kids achieve and learn, to be flexible and to know that there are different ways that people learn and work.” I asked where her ideas about teaching math have come from.

They are just continually evolving and developing. I wouldn't always state it the same from year to year. As I grow as a teacher and a person, I know that what I would have said when I first started teaching would be different from what I'd say now. It's probably more broad, more global, because in 16 years I see a great change in the world and it changes my philosophy of education . . . When I started 16 years ago math was very book oriented, the way I taught math, and I taught it the way I learned it. There was a page in the book that you did, and whatever the story problem was, that’s what you did. It didn't relate to things that were going on in my life. It's the subject I see totally moving away from a textbook as it deals with hands on materials, as it deals with real life experiences that you can relate math to and see a need for math. It involves a lot more problem solving now. Sixteen years ago I thought problem solving was a story problem in a book and now it's just enlarged to a great degree. We teach strategies, and I see kids -- it's amazing to me what I teach at fourth grade now, that I probably had in junior high, and the teaching and
learning going on now -- kids are very bright, and it's opened a lot of pathways for me to work with kids. That's why I continue to think of math as my most favorite subject to teach, because it has been such a growing experience for me.

Mrs. Birch cited a class at the local university nine years ago as the first influence on her changing teaching style. From that class she learned how to extend her math program with games and hands on activities. Then a math lab was added at her school, with a full time math lab teacher, and that opened opportunities to broaden the curriculum with strategy problem solving and peer planning. She was selected to serve on the District Mathematics Curriculum Committee where she learned about the NCTM Standards. A Marilyn Burns course the previous summer concluded Mrs. Birch's list of influences on her teaching.

Asked what she considered most important for students to learn in mathematics she said,

Problem solving, because it fits everything together and utilizes those tools. Problem solving is what life experiences are, so I know I can use computation, I can use calculators at times, there might be a variety of ways that I need to look at a problem.

What is the best way to learn and teach mathematics?

Well there are a lot of ways you can teach math and there are a lot of learning styles out there. Important components would be problem solving groups. Being able to work cooperatively in a group is an important part of math. I use manipulatives a lot more now than I used to, and tools, like the calculator, protractor, being able to put those into use. There are times when a pencil and paper approach is important. I think teaching persistence (is important), that problems are not always solved in a ten minute setting or one math period. Being able to encourage students to problem solve, listening to other ideas in small groups,
learning from others, listening to peers tell you what they think about this problem and how they solved it is more effective than saying, 'This is how you do it.'

And what does it mean to be a good student in mathematics?

Feeling you have the capability and confidence to approach any problem. That you have the necessary strategies, that you know what to do. A teacher supports a student by providing the opportunities and successful experiences and meaningful lessons. Being a good student means wanting to learn, being able to utilize the things that are ongoing in the classroom.

Mrs. Birch was asked to talk specifically about mathematical empowerment, and what she understood the concept to mean:

(Researcher) I want to ask you about mathematical empowerment, because that's the subject of my research -- following these student teachers and how they learn to pick up on the concept of mathematical empowerment. What is your interpretation of mathematical empowerment?

When I think of empowerment, what comes to mind is that I didn't feel empowered in math. So it's something I think I can transfer to kids through being able to use a lot of strategies. There are a lot of ways to work a problem, to come to a solution. Empower kids by learning how to do that and being able to put it to use in what we do with math. Knowing there are a lot of different ways to problem solve. We talk about that a lot in class as we work collaboratively. And when I first started out, I taught like "This is the way to do it," and now that's totally turned around. I think empowerment enables kids to know that there's a lot of tools such as group work, calculators, resources that empower you mathematically. And math isn't just computation, it isn't just isolated, it's very much a part of your everyday world.
Miss Barnaby

Shayne Barnaby was a university senior of traditional age, engaged to be married shortly after graduation. Her mother is a French and Spanish teacher; her grandmother a special education teacher. She said:

I wanted to do lots of things in high school -- I wanted to be a nuclear physicist and I wanted to be a translator and a lawyer and my dad just said, 'I don't think you're cut out for that; why don't you go into teaching?'

So Shayne decided to major in French with a teaching option, but while on exchange at another university, she was required to spend thirty hours in an elementary classroom "and ended up spending ninety. I had so much fun. It was a third grade classroom in a Catholic school." She switched majors to elementary education.

Thinking back on her own experiences as a mathematics student, Shayne said it was "very successful." Junior high mathematics was "exciting and challenging." In high school she took advanced mathematics courses:

... algebra to pre-calc, three years with the same teacher and I was frustrated. I would go in and ask him for help and he would just repeat the lecture and it didn't help. From that experience I realized that you can't explain it one way over and over because some students won't understand. You need to have them look at math in a new way each time you explain it because there might be something that clicks with that student.

In college Shayne took Business Calculus because she wanted a challenge. But then she took Math for Elementary Teachers out of sequence and felt frustrated because "I was the youngest person in class and really intimidated. I did one
computer tesselation assignment three times and never got it right -- I didn't know what I was doing with computers." I asked if that was a different experience for her and she replied, "Yes because (previously) everything was just concrete, worksheets, and book problems, and this was something I had to come up with on my own. It was a creativity, problem solving math assignment and I was frustrated with it. I haven't touched Logo since." Shayne had a similarly mixed experience with mathematics tests:

Tests were usually easy, I could just whip through them and finish them, and I was always told use your first guess, but there was one question and you had to figure out fractions looking at a pie graph and I remember I could not figure out whether the number was bigger or smaller and how you could figure that out.

Shayne described her philosophy of education this way:

I think that kids really need to have a creative type of classroom. I don't believe in just sitting in desks. Sometimes that can stifle creativity because some kids aren't those kinds of learners; there are kids who learn by moving, and kids who learn randomly, and kids who learn orderly, and I know there are names for all these, but I think if you had kids sitting in desks all the time and not interacting with each other, that it can stifle learning. Having them interact with each other can teach them. It's not just the social skills; it's how you can get your idea across to another person. That's something I don't think I did a lot of when I was a student. I know in high school I had trouble conveying my ideas. I think that's really important in elementary school, to convey your ideas and do a lot of problem solving.

Had Shayne's understanding of what mathematics is changed over the years?

Yes it has; especially after this last math class (Math Methods). When I thought about math I just thought about adding, subtracting, multiplying, and dividing, and the dreaded story problems. But after taking this math class -- we used the Curriculum Standards -- it was an eye opener. I didn't know how many more -- categories I guess the word is -- (there are) for math. I didn't realize communication was part of it, reasoning, logic,
problem solving. Problem solving was strange because it isn't just story problems. So my idea of math has broadened to incorporate a lot of different ideas.

What does it mean to be a good student in mathematics?

To work hard. Not necessarily get the right answer but come up with strategies and check them. Do they work, do they not work? Why? To analyze how you're doing, and to try. I think that's the biggest thing.

(Researcher) The subject of my research is how student teachers learn to teach for the empowerment of students. Does the term mathematical power mean anything to you? Is that a concept you're familiar with?

No it's not a concept I'm familiar with, but...

(Researcher) What would it mean to you at this point in time? To teach for math power? For students to be empowered mathematically?

To be able to analyze, convey, communicate ideas that have to do with math, to use problem solving in everyday life, and to not just look at it as a subject but something that involves their life, and how they can use it.

Shayne felt "nervous and motivated and excited and a little stressed, but mostly excited" at the beginning of student teaching. Her expectations for student teaching:

Well, the first one -- management skills, to improve those... My knowledge of student behaviors and how to not necessarily, I don't want to use the word control, but...manage, that's a poor point, but I want to work on it... I have a hard time with the whole group and then trying to get three students to come back in and focus, it's going to take time. (And) to really become comfortable with being a teacher. I know I have all these ideas, but are they going to work? And how am I going to make these ideas match the cognitive level of the students -- is it going to be too low: too high -- how do I adjust my ideas to fit that, and are they really realistic? Is it going to be too much money, or am I going to be able to get hold of materials if there's really something they need to learn, is this a good experience? I need to come down to a practical level.
In this third grade classroom, math time was at the end of the day. Students returned from recess at 2:00 and listened to a story or had a sharing time. At 2:15 math began. At 3:15 the students were dismissed, and it took ten minutes or so for the dust to settle. Occasionally Mrs. Alder and Miss Aragon would sit down then and have an evaluation or planning session which I would script; in any case I usually stayed for a while to listen or chat. During math I sat at a back table, or in a missing student’s desk where I could see better, and scripted whenever anyone was “teaching.” During periods when students were working alone or in groups, I floated and noted what they were doing, occasionally asking a question about their process or thinking, if it wasn’t clear from what I could see and hear. Although the students were at first mildly interested in what I was doing, I quickly became part of the furniture. Miss Aragon’s student teaching assignment lasted 13 weeks (62 school days). Math was taught 54 of those days; I observed 44 lessons.

The math topics presented during this semester included subtraction and place value (begun before Christmas) for 2 weeks; time and money for 4 weeks; and multiplication and division for 7 weeks. During the initial interview, I asked if Mrs. Alder could give me an overview of her math plans for the period of student teaching.

We’ll be concentrating on time and money for two or three weeks depending on what we find out about what the children know and don’t know. Then we’ll be going on to the concept of multiplication, and doing a
lot of concept development of multiplication.

(Researcher) What are your mathematical goals for your students during these next thirteen weeks?

To be able to tell time to the minute, to give change; by the end of the multiplication unit I'll want them to be able to express multiplication factors and products through 6 x 6, but even if they don't have that I want them to have the framework for building the answer. And we go beyond that, too.

(Researcher) How will you know if they've met these goals?

Oh, lots of different ways. I'll listen to them; I'll walk around and observe them; they'll do writing about their thinking; I'll talk to them one on one, and so will Miss Aragon and the Chapter teacher; and we have a record keeping system for learning the facts. Like a driver's license with their picture on it, laminated, and the principal signs it...they love it.

The Nature of the Tasks

The first day of student teaching began with a problem solving activity at the overhead. Miss Aragon watched as Mrs. Alder told the students she was thinking of two two-digit numbers whose sum was 49; what might they be? A lively discussion took place. Then Mrs. Alder explained the guessing game “Pico, Fermi, Bagel”, where pico means one digit is correct but in the wrong place, fermi means one digit is correct and in the right place, and bagel means no digits are correct. They played this game for several rounds, and students were frequently asked to explain the logic behind their guesses. On the last number, which was 47, Mrs. Alder segued into the next activity: “Now 47 is an important number to me; it's my husband's age. I'm 43. How much older is he
than me?" and, "If my son is 20 (and he is) how old was I when he was born?"
Student responses and their strategies were discussed and recorded on the
overhead. Then an overhead containing a number of items for sale and their
price tags was projected, and Mrs. Alder posed a variety of problems calling for
estimation, mental math, and/or computation. "If Bill bought two pencils (31
cents each), how much change did he get from $1.00?" Students worked on
their think pads (a scratch pad kept handy for math time). They had learned the
common "regrouping" algorithm for subtraction before Christmas, and this was
often used, as were other strategies such as counting up, or compensation (60
from a dollar is 40; count down two more is 38 cents).

Finally Mrs. Alder engaged students in a few minutes of oral drill on
subtraction problems such as 70 - 20 and 120 - 50; then they were given a
worksheet to complete with similar problems. About 7 minutes of class time
remained. She instructed students who finished early to "turn your paper over
and write as many equations for 18 as you can. I want to see complex thinking.
No 20 - 2. I want to see different operations, and perhaps some pretty big
numbers."

When the students had gone, Mrs. Alder shook her head mumbling, "too
long; too much talking" (referring to herself) as she and Miss Aragon sat down to
debrief.

The format of this math period followed a pattern that was predictable and
dependable in this class:
a problem solver 15 - 20 minutes

a lesson to develop concepts, provide information, or practice skills 20 - 30 minutes

an activity in which students apply concepts and/or skills 10 - 20 minutes.

Each of these lesson components was consistently hands on and/or minds on.

Problem solving occurred at the beginning of every lesson. The problem nearly always related either to a theme the class was studying or to a real life situation. After the problem was introduced, usually on the overhead and often with overhead manipulatives, students hauled out their “think paper” (scratch pad) to record strategies and solutions. Mrs. Alder or Miss Aragon meandered through the aisles, conversing, inquiring, assessing, complimenting, or challenging for several minutes; and then returned to the overhead to guide whole class discussion of the task. Sample problems are listed below to illustrate the simple but rich nature of this 10 - 15 minute daily activity.

1. I have five coins in this purse. How much money do you think I have?

2. The sum of four consecutive numbers is 86. What are they? (Much discussion about the meaning of consecutive.)

3. Here is a pancake recipe that makes 12 pancakes: 2C mix 1C water and 1 egg. Write a recipe for 36 pancakes.

4. What year was Lincoln elected President? Here is some data: Roosevelt was elected in 1933; Wilson 20 years before that; Lincoln 52 years before that.

5. Sven, you are doing research on dolphins. A dolphin swims 70 miles per hour. How far can a dolphin swim in 3 hours? 9 hours? (Students use various strategies; Miss Aragon eventually models making a table).
The next part of the lesson provided the teacher with an opportunity to present new concepts, provide practice, informally assess student learning, or explain directions to a game or activity. Direct instruction was often used. For example, a large Judy clock enabled Mrs. Alder to teach time to the minute and measurement of elapsed time, and overhead coins were used to model and practice counting back change. These lessons always elicited and utilized prior knowledge, and required lively participation and attention from the students. The lessons usually provided a lead-in to an independent or group activity or project. Again, sample lessons / projects are given.

1. After modeling a Thursday school schedule on the overhead, students used blank clock stamps to create personal charts of a Thursday, with labels, showing important times in their day (e.g. get up, leave for school, lunch, t.v. show, bedtime)

2. Students created a pattern block design using 6 - 12 pattern blocks. They then created a chart to show how many pattern blocks would be required to add layers to the design — up to 12 layers. (This during the multiplication unit.)

3. To develop division concepts, groups of four shared bags of chocolate chips, raisins, pretzels, and peanuts to create a recipe (expressed in appropriate equations) for gorp.

4. Miss Aragon read *Sea Squares*, after which students drew ocean scenes illustrating a square number (e.g. eight octopuses with eight legs each, or four shells containing four pearls each)

A major class project during the money unit was the creation of a store. Students met in small groups to build miniature products (cardboard cutouts), set prices, and plan learning activities. On store days, some students were given an amount of play money and a shopping list (record sheet); other students served as cashiers.
Tasks from the forty-four observed lessons are grouped in Table 1 by type and frequency. There is overlap (e.g., one journal activity was also an assessment) but no omissions.

<table>
<thead>
<tr>
<th>TYPE OF TASK</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving</td>
<td>40</td>
</tr>
<tr>
<td>Teacher directed lessons</td>
<td>28</td>
</tr>
<tr>
<td>Projects</td>
<td>23</td>
</tr>
<tr>
<td>Assessment</td>
<td>9</td>
</tr>
<tr>
<td>Games</td>
<td>7</td>
</tr>
<tr>
<td>Writing/journaling</td>
<td>6</td>
</tr>
<tr>
<td>Literature based activity</td>
<td>4</td>
</tr>
</tbody>
</table>

The textbook or related worksheets were never used with students. The District Curriculum Guide, which listed 3rd grade learner outcomes for each Standard, and the textbook teacher’s manual were the primary resources for guiding Mrs. Alder’s decisions about content. To select or design specific activities for each day, Mrs. Alder relied on Marilyn Burns materials (designated MB in this study), a TOPS Problem Solving deck, Family Math, her own ideas, and ideas from the textbook teacher’s manual, as well as other resources. Facts and computation skills were developed and practiced in the context of the activities listed in Table 1; only three times were facts or computation practiced in isolation, and then for five minutes. The multiplication unit, begun in February, stressed the development of conceptual understanding; there were no
timed tests and other drills. However, in April Mrs. Alder began giving students opportunities to earn their “driver’s license”, which indicated automaticity with facts through $6 \times 6$, and a star on the license for mastery of facts through $9 \times 9$. She expected all students to earn their license, and by mid May all but four had done so. Students were assessed during this period using two textbook chapter tests, five alternative assessment tasks, and two teacher made skill assessments. More will be said about student achievement in a later section.

How did Miss Aragon fit into the selection and delivery of these tasks?

Table 2 indicates the rate at which Miss Aragon began participating in lesson planning and delivery of the 44 observed lessons.

Observed only = Mrs. Alder taught the whole lesson.
Problem solving only = Miss A taught only the first part of the math period and observed the lesson during the second part.
Led group or lesson only = Miss A taught only the lesson (second part) and observed while Mrs. Alder led the problem solving opener.
Full responsibility = Miss A planned and led both the problem solving activity and the lesson activity.

TABLE 2: Schedule of Induction for Miss Aragon - Number of Days

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Observed only</th>
<th>Problem solving only</th>
<th>Led group or lesson only</th>
<th>Full responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>8</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4-6</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>7-9</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>10-13</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11</strong></td>
<td><strong>21 (Shared a lesson)</strong></td>
<td><strong>12</strong></td>
<td></td>
</tr>
</tbody>
</table>
This pattern of induction turned out to be quite powerful in terms of balancing the learning to be gained from observing an experienced teacher model mathematical reform, with the learning to be gained from getting in there and trying it. Because the two teachers shared 21 math periods, Miss Aragon was able to benefit from both teaching and observing more frequently than if she simply taught some days and observed others. The predictable flow of the lesson format in this classroom allowed Miss Aragon to take on “pieces” of the teaching responsibility. In particular, the daily problem solving (15 minute) session proved invaluable in helping Miss Aragon tune into some important components of mathematical empowerment. During shared days, very rarely did Miss Aragon and Mrs. Alder teach simultaneously, so when Miss Aragon was not leading a problem solving session or teaching a lesson, she was observing. It was her pattern to sit or stand within the class grouping and attend to the lesson. She did not do other work, nor did she become involved with individual students while a whole group lesson was being conducted by Mrs. Alder. When the students transitioned into an independent or group activity at the end, she would float and interact. Because Miss Aragon observed intermittently this way until the final few weeks, she was able to make productive use of her observations to compare the effect of her own teaching on students’ empowerment with that of Mrs. Alder, to fine tune her planning and delivery, and to continue to gain new awarenesses about mathematics teaching and learning, as evidenced from interview and journal data.
Miss Aragon began conducting problem solving sessions early on, and performed this task during 29 of the 44 observed lessons. She quickly gravitated to good resources for these problems, and part way through the semester began adapting or making up her own problems to integrate with a unit being studied.

Miss Aragon also planned unit activities during 20 of the 44 observed days. Modeled after Mrs. Alder’s activities, these were always hands-on and interactive. During the time and money unit, Miss Aragon created menus, and groups of four assumed roles and used play money to explore ordering, tallying, paying, and receiving change at a restaurant. She extended learning from this activity with “coupons” the next day. During multiplication, both teachers drew from Marilyn Burns resources for philosophical support and specific activities.

Miss Aragon’s lessons included:

- designing candy boxes (making arrays with tiles)
- multiples — students created their own data set (e.g. four legs on a dog); made a table to show the relationship between number of dogs and number of legs; wrote equations next to each term in their table (e.g. 5 x 4 = 20); and then mapped this pattern on a hundred square.
- how many cubes in a handful? in which handfuls were tallied on a large chart, followed by a discussion of range, mode, total cubes grabbed, and mean handful.
-division activities such as Corrals and Horses (MB), Leftovers (MB) and an adaptation of Gorp (Family Math)
-lessons deriving from the stories Sea Squares, Anno's Multiplying Jar, The Doorbell Rang, and My Cousin Has Eight Legs.

The Standards describe good tasks as “ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students’ curiosity, and that invite them to speculate and to pursue their hunches.” (NCTM 1991, 25) The combination of mentally engaging problem solvers, as Miss Aragon referred to them, and activity based lessons which allowed students to further their understanding of time, money, multiplication, division, and a multitude of other embedded mathematical topics in meaningful ways, provided students in this class with a daily diet of worthwhile tasks, and provided the student teacher with valuable practice in task selection. Here is what Miss Aragon said midway through her student teaching about the type of tasks Mrs. Alder chooses:

She chooses ones that she thinks are really going to teach their mind to think — make them really think to find the answer. And she looks for variety... I like the problem solving introductions, and the store, because it's actual experience... I like the way she has actual things for them to touch and feel, because I think some of the ones in there are visual learners and they'll be able to do it that way.

And at the end of her student teaching experience, when asked what she will do in her own class to be sure her students are empowered mathematically, Miss Aragon said:
One thing I’m definitely going to do is problem solving. I think that’s such an important component in math that I never did see before in the past. Not in my paras, maybe because I often wasn’t there for math. And the math I had known from school just didn’t involve problem solving. It was ‘this is how you do it and you use this formula and don’t forget this formula because you’re going to need it.’ But what if I don’t understand that formula? So these kids are doing math so they know why they’re doing it, they know how to do it, so if they understand it 100% they can take it out into the world and use it.

Classroom Discourse

Worthwhile tasks are a crucial component of the vision of mathematics teaching and learning contained in the Standards, but the element having the most potential to affect the empowerment of students is the discourse that takes place, or doesn’t take place, around those tasks. Cobb, for instance, discusses “the crucial role that genuine communication about mathematics plays in children’s learning.” (Cobb 1991). Wheatley says that activity based learning is not enough; that the regular opportunity to reflect on one’s own and others’ thinking is paramount to the process of constructing knowledge. (Wheatley 1992).

This section will look closely at the development of several lessons which are prototypical of the daily discourse in Classroom A.

January 11. (Week 2) Mrs. Alder holds up a wallet, and opens it up to describe and show all the parts (driver’s license, photos, credit cards, bill section, checkbook).
Mrs. A: How many pennies do you think are in my change purse? (She takes several estimates, recording them on the overhead.) Why didn’t anyone guess 2,000? Why wouldn’t that be a reasonable estimate? Melissa: Because you would have traded them in before now. Tiffany: It would be too heavy.
Collin: That would be $20 - way too many to fit.
(Here Mrs. A. guides exploration of how you "trade them in" and whether 2,000 pennies really makes $20. She then empties the contents onto the overhead and they count 28 cents.)
Mrs. A: Who had hot lunch today? (She counts hands and records 9 on the overhead) How many people would you estimate had hot lunch in our whole school today?
Amie: 109 (records on overhead)
Mrs. A: What are you thinking, Amie?
Amie: Just guessing
Mrs. A: I'll give you some more information. There are about 300 children at our school. Other estimates?
Dan: 130 (records)
Mrs. A: What are you thinking, Dan?
Dan: I looked around at lunch and there were a lot more colds.
Mrs. A: How many colds were in our class?
Dan: (takes a long time . . . ) 12
Mrs. A: And how did you get that?
Dan: I counted up from 9.
Mrs. A: Can anyone think of an equation for this?
Arthur: 21 - 9 = 12
Mrs. A: Did more than half or less than half take hot lunch in our class?
Several students: Less.
Mrs. A: How do you know?
Sven: Because 11 + 11 = 22 (there are 21 in the class) and 9 is less than 11.
Mrs. A: How could this help you make a good estimate?
Melissa: Maybe half the kids had hot lunch.
Sven: Maybe less than half.
Mrs. A: I'll take some more estimates. (Records 145, 200, 192) The exact hot lunch count today was 175.
Amy: But that's not half.
Mrs. A: So, is the rest of the school like our room?
Ross: No, it's more.
Mrs. A: I wonder why.
Ross: Lots of people like cheese sandwiches.
Arthur: But you said about 300. Maybe there is more.
Mrs. A: Oh - I see what you're thinking. 175 is half of what, Art?
Arthur: (pause) 250
Mrs. A: How did you get that?
Arthur: 175 + 175
Mrs. A: Let's do it together (writes on overhead; he talks her through)
Arthur: I mean 350
Mrs. A: So if the exact total of students is really 350 or more . . .
Arthur: Then it's not half.
Mrs. A: So in that case, the rest of the school would be more like our room. Good thinking. We will collect some more data about school lunches tomorrow.

This problem solving opener took about eight minutes. Important concepts included reasonable estimates; how context affects what is reasonable; establishing 1/2 as a referent; proportional thinking; and number as a part of our daily lives. The class also reviewed some computation skills. But the part of the discourse having a cumulative effect on students' mathematical power (because they heard and responded to it every day) was, "Explain your thinking," "How did you get that?" and "How do you know?" Mrs. Alder was adept at choosing which of these responses to develop further, either to develop her target concept (e.g., "I saw more cold lunches"), or to take quick little side trips (e.g. trading 2,000 pennies in).

By mid semester, Miss Aragon was planning problem solvers (her term) and practicing the orchestration of discourse around these problems.

February 10 (Week 6) On the overhead Miss Aragon writes:

\[ 3 \ 5 \ 4 \ 6 \ = \ 1 + \]

Miss A: Use all these numbers and symbols just once each to make a true number sentence. What do I mean by a true number sentence?
Sam: \[ 2 + 2 = 4 \] (She records).
Miss A: Good example. And \[ 2 + 2 - 1 = 3 \] is a true number sentence. I'd like to see everyone working on their think pads. (She floats a minute.) Something I know already -- there's more than one way, that's for sure. (A few minutes later) I wonder how many ways there could possibly be?
(This cued students to keep going, and fast workers to search for a systematic approach.)

After four minutes, the lists I surveyed as I floated contained from two to nine true number sentences, and evidence of many "false starts." All students were on task. Back at the overhead, Miss A. collects number sentences from students and records them.

Miss A: How many found two ways? three? four? five? I saw a lot of good thinking. Did anyone have an approach they used?

Kelsey: I did one and then switched the numbers.

Amy: I started with the big numbers to subtract and then added the other one to try to get the other one.

Bill: I did that too -- you can get negative numbers.

Miss Aragon was becoming quite adept at orchestrating discourse in which students were expected to share their thinking. She was not yet as skilled at translating student responses into language that would make that student's thought process accessible to everyone. Mrs. Alder indicated later that she would have asked Amy and perhaps Bill to explain further, and / or paraphrased their thinking and illustrated it on the overhead.

Mrs. Alder took over at this point. (As mentioned earlier, they frequently shared the lesson time, with one teacher doing the opening problem solver, and the other doing the lesson. This was extremely effective in socializing Miss Aragon into the existing routine, and alerting her in a concrete way to important features of mathematical empowerment.) The class had recently begun their unit on multiplication. The Marilyn Burns Math By All Means book was a frequent resource. This day the students were making "Circles and Stars" booklets.
Mrs. A: I’m going to be watching like a hawk to be sure I have good listening because you’re going to need it. I want to fold this paper into eighths. What does that mean?
Dustin: Eight parts.
Mrs. A: Yes, and eighths means eight equal parts. (Folds once) How many equal parts?
Tiffany: Two.
Mrs. A: (Folds again) How many equal parts do you think I will have now?
Melissa: Four.
(Mrs. A starts a vertical chart on the overhead.) One fold, two parts. Two folds, four parts. Three folds — how many parts do you think there will be?
Several students say: Six.
Some say: Eight.
Mrs. A: What are you thinking, Sven?
Sven: It doubles the amount each time, not the other side.
Mrs. A: Sven thinks eight because the parts double.
Sven: People think six because they’re counting by twos. That’s not the pattern.
Mrs. A: Let’s open it. How many Josie?
Josie: Eight.
Bill: Sixteen, both sides.
Mrs. A: Yes, we’ll just look at one side. What shape do we have?
Tiffany: A square.
Mrs. A: Tell me about a square, Tiffany.
Tiffany: Four sides.
Mrs. A: Tell me more.
Tiffany: Four equal sides.
Mrs. A: Tiffany? Does it? You measure, and we’ll come back to you.
(At this point, all students fold their papers, while Tiffany measures Mrs. A’s paper.
Mrs. A: Tiffany, you changed your mind. Explain.
Tiffany: It’s a rectangle. The sides aren’t the same.
(Now Mrs. A gives directions while demonstrating.) Fold in half. Pinch the fold. Cut from the fold at the middle, to the middle. Open. Fold down to the long skinny rectangle. Squish it together into a diamond. Push together until you have a plus sign. Fold it to make a booklet. (Pauses and repeats as needed.)
Mrs. A: Now we need a title. (Writes on overhead: Cir____) What could it be?
Sam: Circle!
Mrs. A: We need the plural.
Amy: Circles.
(Mrs. A writes: Circles and St____) What could it be?
Josie: Stamps.
Kevin: Stripes.
Mrs. A: (Writes Sta___) I start mine with a 7.
Sven: Stars.
Mrs. A: OK. You will roll a die to see how many circles to put on the first page. Then your partner does the same. After that you roll again to see how many stars to put in each circle. Your partner does the same. The object is to see who gets the most stars on each page. At the end of the book, you may figure out how many stars are in your whole book.

Students go to work. While I am floating, Bill says, "I wonder what the most stars possible would be? (How could you find out?, I inquire.) There's 36 for a page. (How many pages?) Eight. It would be over 400. No, over 300. (How do you know?) Well, that's 72 + 72 + 72 + 72. That's 280, 282, 284, 286, 288. That's the most you could get." Bill, by the way, is a boy who has a hard time paying attention.

Features of this lesson common to many of Mrs. Alder's lessons included connections and integration (geometry, language); and the use of riddles and conjectures.

Students went on in this multiplication unit to build strong concepts of multiples (sets of three, sets of seven, etc.), and multiplication as repeated addition. Facts were never practiced or drilled in isolation. Many activities incorporated the need for students to build tables for multiples, and problem solvers often used large numbers. For example:

Jan. 30 (Week 5) Miss Aragon begins with problem solving.
Miss A: What's my number? It is less than 50. You say it when you count by twos, threes, and fives.
(Students work on think pads. Both teachers float and probe with individuals. Miss A. returns to overhead, students share solutions and strategies, and then she puts up an OH Hundred Square)

Miss A: Let's look at this problem this way. Melissa, when I count by twos, what numbers do I circle? We'll stop at 50.
(Melissa names them. At this point Mrs. Alder interjects a line of thinking)
Mrs. A: Could anyone find a number between 90 and 100 that I would say if I count by twos?
Dustin: 96.
Mrs. A: How do you know so quick?
Dustin: Because they're all in rows.
Kelsey: I keep 2, 4, 6, 8, 10 in my head.
Ben: They all have 2, 4, 6, 8, 10.
Mrs. A: So they all have 2, 4, 6, 8, 0 in the ones place?
Kara: They're all even
Mrs. A: What does it look like?
Matt: Columns.

Back to Miss Aragon, who gets help from students counting by threes and fives and circles the multiples in different colors on the Hundred Square.

Miss A: Now, Amy, could you explain your strategy once again?
Amy: Since it's fives, it has to be fives or tens. And since it's twos, it has to be the tens. And so then you say what tens will count by threes?
(Miss A illustrates Amy's thinking on the Hundred Square as Amy talks).
Miss A: So which number is circled with red, blue, and green?
Terry: 30. (The solution Amy had offered.)
Ben: If you went to 100, the next would be 60, then 90.
Mrs. Alder: Did you keep going on in the Hundreds Square to figure that out?
Ben: Well, sometimes you count by threes, so 30, 60, 90.
Mrs. Alder: Tomorrow we'll try to prove your theory.

Miss Aragon and the class received support from Mrs. Alder in this activity to extend and clarify thinking. A month later, Miss Aragon was inventing her own problem solvers, which usually integrated with the Oceans Unit she was teaching.
February 22 (Week 8)

Miss A: How much do you think a whale weighs? (takes guesses) A grown whale weighs 16,000 pounds. How much would eight whales weigh?

(Worktime)
Miss A (at the overhead): How did you do this? (She records all responses on the overhead)
Matt: I added up four 16's in my head and got 64,000. Then I added another 64,000 and got 128,000.
Amy: I added four 16's and got 64, then added four more and got 128. Then I put on three zeros.
Miss A: Why?
Amy: Because a thousand is three zeroes.
Miss A: Good thinking. Are there any other ways?
Sven: I started with two 16's, doubled it to 32, doubled it to 64 and doubled that to 128 and added the zeros for the thousands.
Kara: I added 16 eight times.
Miss A: In a column? (Miss A makes a column of eight 16's and Kara comes up)

\[
\begin{array}{c}
16 \\
16 \\
16 \\
16 \\
16 \\
16 \\
128 \\
\end{array}
\]

Kara: And then I put on the three zeroes.

Miss Aragon often reflected on the problem solving abilities of the students in this class.

It amazes me how they get so many ideas. I'll think of a problem solver and think, 'This is probably what they'll come up with,' and they'll come up with fourteen different ways and they're all saying, 'Oh yeah, I hadn't
thought of that; oh yeah, you could do it that way too,' and they all want to
share, and they all have a different way of coming up with it.

Manipulatives were often used to help students build new concepts.

March 20 (Week 11) Miss Aragon has finished the problem solver. Mrs.
Alder leads this lesson.

Mrs. Alder builds this design on the overhead using pattern blocks.
Mrs. A: What kind of shape is this?
Dustin: Quadrilateral
Mrs. A: What does my design look like to you?
Several responses: Flower and petals; bird’s eye view of the room;
candy dish; well; cup with the yellow piece as the bottom.

Mrs. A: Now what does it look like?
Matt: A flower with leaves.
Mrs. A: Today is my anniversary and I’m going to call it my anniversary
flower.

Records: 1 anniversary flower with 11 blocks is 11 blocks altogether.
2 ____________ with __ blocks is __ blocks altogether.

Cody: Looks like you’re going to make us work.
Mrs. A: Yup, I am. That’s my job. What multiplication equation goes with
this story?
Melissa: 11 x 11
(Mrs. A guides her through to see the equation 1 x 11 = 11, then stacks up
another layer to make 2 flowers)
Mrs. A: You must use the same kind of pieces every time. But you may
get to the point where you don’t need to build each layer to know how
many blocks it would take. (She goes through 2 x 11 and 3 x 11, with
input from Melissa) Your job is to use between 5 and 13 pattern blocks to
make a design.
Mrs. A: So how many blocks can you use?
Kevin: 6, 7, 8, 9, 10, 11, or 12.
Mrs. A: Perhaps you can make a sea creature.
Sven: But do you have to keep to that one thing?
Mrs. A: Yes -- but you may not need to build every layer.
Kelsey: I see the pattern for 11 -- 11, 22, 33, 44, 55.
Mrs. A: I hope you will try the patterns for 7, 8, or 9. Because we’re working on multiplication facts and patterns, so if you need to work on sevens, maybe you could build a shape that would use seven blocks. Oh, and you must also trace and color it so we will know what your pattern looked like.

Thirty minutes of pattern block design and table building ensued. Miss Aragon’s journal entry from this day:

Pattern Block Multiplication. “Build anything you want.” I like the freedom she gave them. They explored their equations so well because it was something they constructed - it was their own creation. Such a FUN way to learn multiplication facts.

Empowering discourse was an integral part of the daily mathematics routine in this classroom. At midterm, Miss Aragon was asked what she noticed about Mrs. Alder’s teaching that she (Miss Aragon) connected to the idea of mathematical empowerment:

The way she asks questions, like probing questions for more answers, and not one answer’s right. There might be five different ways you could do a couple of them -- that kind of thing. ‘How did you do it? Ok that’s the way he did it too. What are the differences?’ Or when they’re solving a problem she says, ‘Ok, what’s the next step?’ She doesn’t do it for them; they do it. They have to think in their mind, ‘Ok, how am I going to do this?’ instead of watching it done the first time. (In this classroom) it doesn’t matter if you get the right answer; it’s more the questions you ask and how you express yourself, and they know that, and I think that’s why they try so hard and actually do the problem solving and get excited about it, because they know it’s not just about a right answer.
The Learning Environment

If you had dropped into this classroom during math time on any given day, you would have seen one of the episodes described earlier or others that were very similar. You would have seen 23 third graders actively engaged in constructing meaningful mathematics, a teacher orchestrating discourse and constantly analyzing the mathematizing of her students, and a student teacher in tune with the pulse of the classroom routine. However, if you were there every day, you would also know that there were at least four students in this class who were perpetually demanding and potentially disruptive, and you would have become aware of the important role that management plays in creating a classroom environment conducive to developing the mathematical power of all students.

Management seems to be the area of greatest concern for most student teachers. The Standards speak of the classroom environment as the interplay of all the intellectual, social, and physical goings-on that determine what behaviors and interactions are encouraged and expected. (NCTM 1991, 20) Although "management" is not referred to directly in the Standards documents, management skill and style would seem to provide the foundation for an environment in which mathematical power is the focus.

Miss Aragon began student teaching with an upbeat, sparkly, but also very quiet sort of demeanor. I could hear her plainly during our initial interview, but when I replayed the audiotape to transcribe, I could barely hear her at the
During the fourth week, Mrs. Alder said:

Kelly is doing a really good job. She has solid skills. She's very shy and quiet, which is not a weakness, and she's also very young. She needs to gain confidence. She has lots of good ideas and plans. I need to draw her out more. Her quietness can be a problem only in that it can be a bit harder to communicate, but it can also be an asset. She has solid skills in dealing with children.

These solid skills were tested early on when Miss Aragon's small group for planning the class store contained three of the aforementioned "challenges", who provided her with the traditional trial by fire that most student teachers seem to have to face. During that twenty minutes she practiced inviting, ignoring, and delivering consequences. It was a lost cause. Nevertheless, there was never a voice raised, only modulated, and what Mrs. Alder seemed to mean by Miss Aragon's "solid skills" was the gift of friendly perseverance. Miss Aragon's journal entry:

January 18 Store planning (talents). Just when I feel I'm ready for something -- POW! What stands out in my mind is chaos! Wow! Somehow, with all the excitement of the day wrapped inside Kevin -- he was literally out of control! The planning was good but it was hard to let them plan it when I already had an idea of what they should do! It was very difficult (to say the least) to manage Kevin, Bill and Dan all at one time. I wish we could have got a lot more done but I guess that is how it goes sometimes! I felt like screaming at the end of the day, that's for sure!

Miss Aragon maintained a consistently friendly, respectful, and nondefensive approach in her interactions with students, and by the end of the
semester this was paired with a self-assurance which conveyed to students clear expectations for their positive behavior and high expectations for their learning. During Week 5, Mrs. Alder was at a workshop one day, leaving Miss Aragon in charge. The math time went smoothly, although Kevin was sent to the hall for purposefully distracting behavior. Afterward, Miss Aragon said she felt good about the day despite the consequences she had to deliver.

I said, ‘It’s your choice whether you follow the rules, no matter who is here.’ I said to them, ‘you get mad at me, but you should get mad at yourselves.’ Bill was a pill before music. He started with his noises. We got everything done today, though. I expect them to behave and they don’t scare me. I told Kevin, ‘I care about you and I won’t put up with a disrespectful tongue.’

Remember the speaker is a small, soft spoken young lady with a permanent twinkle in her eye. During her “solo” week Miss Aragon was able to manage this difficult class very effectively, and during the exit interview she speculated about Mrs. Alder’s role in this transformation.

Sometimes I would feel frustrated with a management issue, because it was kind of a tough class, and I would struggle and she would let me struggle. She wouldn’t go, ‘Oh I’ll handle it; I’ll talk to the kids; I’ll do it for you,’ or whatever. She made me struggle and that made me frustrated but it made me learn how to handle it and I felt it was beneficial for me to (1) have a class like that and (2) have a teacher that let me figure it out on my own. Because now I can say that I feel my management is strong. On the other hand if I were to be in that class where if I struggled she would take over, I’m sure that I wouldn’t know if I could handle it. But I was forced to come up with my own style of discipline and handle it.

Furthermore, Miss Aragon’s use of voice had changed dramatically. She was more clear and concise, used more inflection, and her voice was stronger in quality (although I still had to crank up the volume on the exit interview tape).
I describe this management component for two reasons. First, a casual observer would have noticed no “discipline problems” during math time, and yet the expectation and communication of appropriate behaviors certainly was an ongoing concern, as it is in any real world classroom. I believe this speaks to the power of an environment that encourages meaning and sense making to mediate and influence behavior in productive ways. This is not to say that Standards-based instruction or a constructivist framework prevents discipline or management problems, but rather that an environment where students’ sense-making and productive communication is the central concern supports effective, embedded management. Secondly, the development of mathematics pedagogy cannot be separated from general pedagogy, including management.

I explored this idea with Mrs. Alder at the end of the semester, asking what obstacles or hurdles Miss Aragon had to overcome on her journey to becoming an elementary mathematics teacher who teaches for mathematical power.

Mrs. Alder: Working with the children themselves, behavior management. She had a solid concept of what needed to be done, and what the changing focus of mathematics was, but there are always kids who are difficult to deal with, so you don’t always get done what you want to because of that.

Researcher: So what role does classroom management play when we’re trying to learn to teach for mathematical empowerment?

Mrs. Alder: It’s a big factor. Because you need to instill that desire to want to problem solve, the habits of responding and taking risks in students, so if you don’t have the behavior management skills, some of the things you do go nowhere.

Researcher: So what are the interactions that take place between you and a student teacher who might be struggling with management? What
do we do to help student teachers integrate the management with learning to teach for math power?

Mrs. Alder: One, I think you have to give opportunities to self-assess. ‘Ok, what were your objectives again? How do you feel you achieved them? What could you have done to make it better?’ And then you have to take their answers, because I’ve had ones whose answers didn’t match reality, and then you need to say, ‘Well I noticed that children weren’t responding, or paying attention,’ and then I would give suggestions, and if that didn’t work, I would tell them what they needed to do and see if that would work. But I think you have to start from letting them assess.

Beyond management, the essence of an environment conducive to developing mathematical power in this or any classroom is the attitudes that are conveyed. Does the teacher look forward to math time? Does she integrate mathematical thinking into other parts of the day? Does the teacher have a genuine interest in and respect for students’ ideas and points of view? Are all students expected and helped to contribute ideas to the learning community? (NCTM 1991, 56-61). Here are some of Miss Aragon’s journal reflections from the middle and later part of the semester. (Prompt: What are you noticing about how Mrs. Alder teaches for mathematical power?)

They don’t realize they are doing multiplication — she does it in such a way as to have them use their own personal knowledge to figure it all out. ‘Did you get all the ways?’ ‘You tell me.’ ‘I like the way you are thinking.’ She makes them figure out if they are right or wrong. While the others are figuring something out she has those without a job work on something else, or asks probing questions such as ‘What could you buy with this much money?’ . . . They are so active. I like this the best. Students’ involvement and activity is the reason for learning. She presents it all in a way that seems enough of a balance between fun and challenge that they grab and seize the problem. When they solve it, they don’t realize it but they’ve constructed their own learning. This way of teaching makes so much more sense to me than rote memorization. I never realized it completely until I could see ‘their’ learning — WOW!
Analysis of Teaching and Learning

Analysis of instruction recognizes the intimate relationship between teaching and assessment...teachers must continuously monitor students' capacity and inclination to analyze situations, solve problems, and make sense...Teachers should use such information about students to assess not just how students are doing, but also to appraise how well the tasks, discourse, and environment are working together to foster students' mathematical power and to adapt their instruction in response. (NCTM 1991, 67)

Analysis as Student Assessment. In Classroom A assessment was an ongoing, integral part of the routine. In the after school "recaps" of the lessons which I happened to script, teachers always mentioned specific student responses, performances, or contributions. This data often influenced future lessons.

An example is taken from one of the early store days. (The store was a class project occurring as part of the time and money unit in January.) During several prior lessons, students had practiced counting back change from $1.00. On this day, Mrs. Alder witnessed an exchange between a cashier and a customer in which the total was $11.59, and the customer gave the cashier $11.75. The cashier counted back change to $12.00. Mrs. Alder related this problematic incident as they were discussing how the store lesson had gone. The next day, Mrs. Alder had the two students re-create their situation in front of the class, using overhead coins and bills. (She said, "Let's help the class learn what you learned.") They role played, sharing their original thinking and their new conceptions as the whole class became involved in a discussion about:
What does making change really mean? Is there more than one way to do it? Are there right ways and wrong ways to do it? What are the important things about giving change? How much money should the customer give? These are some of the ideas that emerged from the discussion:

1. The order of coins doesn’t matter (although there were some arguments about there being easier ways: e.g., pennies to get to a multiple of five or ten; fewer coins saves time; etc.)

2. The customer can give any amount over the cost -- but might get the same coins right back if she gives “too much.” (One student said, “Just give the next highest dollar because that’s the easiest.” Another said, “Give whatever is just past the cost if you have it.” (E.g., for $11.29 just give $11 and a quarter and a nickel.)

3. What’s important is to count from the cost up to the amount given, carefully. Some students seemed to firmly grasp the idea that the cost and the change should equal the amount given, some did not.

Miss Aragon was actively coached by Mrs. Alder regarding the use of student response data to adapt further lessons, after a Marilyn Burns multiplication activity Miss Aragon had planned on “candy box designs.” Together they determined that Miss Aragon had spent too long modeling designs with four tiles. (Miss A: “It blows my mind that they can’t just see it and say, ‘no more ways. ok.’ They have to keep trying to find another way.”) Miss Aragon’s ability to conduct effective discourse was being shaped: the teacher
does not have to listen to every idea or comment that students feel like contributing; she can direct, extend, and even cut off discourse to manage the flow of the lesson in productive ways. They discussed three students' responses, and decided to extend the lesson the next day with some pacing and logistical modifications. Mrs. Alder also asked for an accountability factor to be planned in -- they decided to use 36 (3 dozen) tiles as an assessment later, with no partners.

During these thirteen weeks, nine formal assessments were given and scored. They consisted of one pretest (telling time); one mental math / place value dictation (e.g. 800 + 500: write the answer only); two traditional textbook chapter tests (one on subtraction and one on multiplication facts and concepts); and five alternative assessments (one time and money; one place value; two multiplication; and one division). The class mean on the chapter tests were 84% (subtraction) and 94% (multiplication).

The alternative assessment for time and money was a performance task in which students were asked to draw hands on a clock face, write time from a clock, measure elapsed time, and show several ways to make 75 cents. (See Appendix F). The assessment was scored with a 10 point rubric based on the successful completion of the above tasks. Ten students scored 9 or 10; nine students scored 7 or 8; and two students scored 5 or 6. But these raw scores do not convey the same kind of message as a percentage; in fact, they convey nothing in isolation. Rather, the teachers obtained very specific information
about students' developed and undeveloped skills, and/or their misconceptions. Miss Aragon said, "I figure this is really the only way to assess if they have the concept of time and money! At my group, I could have had all of them draw any time in the world on their clock and they probably would have got it right! The thing they have problems with is: what time will it be in 37 minutes, etc. On a paper pencil test I would never have known these discrepancies."

Analysis as Reflective Practice. Formal analysis of Miss Aragon's (or for that matter Mrs. Alder's) lessons was infrequent. Because I didn't personally see or hear many such debriefing or evaluation or planning sessions as described above, I asked several times if they took place at other times. The answer seemed to be that although the two "always knew what the other was doing," Mrs. Alder did not spend much time checking, discussing, or evaluating Miss Aragon's plans or lessons, either beforehand or afterward. At midterm, Miss Aragon said:

... when I plan a lesson I don't write out the objectives, but I know what they are, and I do them, and we talk about, 'Ok, well I think that reinforces their counting back money skills' -- I don't know; we don't really have that much time to talk about every lesson... but after a lesson, we'll often say, 'Wow, this person doesn't have it, or this person really was amazing,' and that's the kind of stuff we talk about. Then we can say, 'Ok, perhaps we should go this way and stress this for a while,' or 'Gosh they're not getting that zero place holder still; we'd better work on that.' So in that way we do analysis, but not the all-over objectives.

At the end of the semester, when asked if she felt there was anything lacking in her student teaching experience, Miss Aragon said:

Mrs. Alder did give me feedback once in a while, but I would have liked more feedback. I know she was happy with what I did, and I guess I
should have asked for her feedback and I didn’t but there were certain times when I thought, ‘How could I have done that differently?’

Miss Aragon’s early journal entries revealed a steadily growing ability to analyze lessons both in terms of student mathematizing, and in noticing elements of mathematical power that she observed in Mrs. Alder’s lessons. Her later entries contained lots of self-analysis.

1/4 She uses teachable moments --like figuring out ages -- kids respond well to this.

1/9 Some of them confuse big and little hands. 3:00 would be read as 12:15.

1/11 The estimating hot lunch thing ties in great to their real world.

1/17 I like the probing questions Mrs. Alder uses to keep them thinking on and on.

1/24 (Store) It seemed like it was chaotic, but it was really running smoothly! When I was helping some of them, they seemed to not want to do the rounding part of it because they were afraid of going over on their spending -- they wanted to be exact.

1/25 restaurant (working w. money) My Video Taped Lesson. It went OK. There needs to be more for each of them to do. It was good that each of them had a chance to be the customer, cashier, and waiter, but there needs to be something for them to do the entire time. Maybe work in groups of two rather than groups of four -- also -- take a note to do informal assessments!!!

2/2 I loved these problems. The last time problem was interesting in that they came up with 2 hours and 85 minutes! HMMM I wonder what 85 minutes on the clock would look like!

2/9 I thought it was such a good idea that they wrote in their journal “What is multiplication?” Some of them know a lot about multiplication and really are doing it and they just don’t realize it. The chopsticks problem is a GREAT way to introduce multiplication.
3/27  Anno’s went ok — the story part went really well I thought, but I
guess I felt like I was explaining things a little too much. They got it (the
concept of factorial) and I didn’t expect them to right away — if I was to do
it again, I would let them discover more without so much teacher
modeling.

4/6  Gorp. I think things went great! I loved the activity! It was fun. I
think it helped them in understanding and picturing division!

Did the students in Classroom A grow in mathematical power? The most
outstanding feature of this class was the high value placed on students’ thought
processes. The daily problem solving task was always one in which all students
could engage at their own level of mathematical development; one which was
made challenging for each student. Every day students heard, “Who would like
to share a strategy?” “That’s one good way. Who has another way?” “Please
tell us what you’re thinking.” “Can you explain why you chose _____?” It
became perfectly clear to the students that their thinking, not this or that answer,
was the important factor in this class; and students beamed just as radiantly
whether they were reinforced for self-correcting in the middle of an idea, for
changing their mind and agreeing with someone else’s thinking, or for being able
to explain an elaborate thought process. In this climate, all students took risks,
all students stayed busy trying to figure it out, and all students evidenced the
confidence and enthusiasm of successful problem solvers.
Across town in Mrs. Birch's fourth grade classroom, mathematics began first thing in the morning. The bell rang at 8:30; during attendance and lunch count students started right in with a warm up activity on the board or occasionally on a worksheet. By 8:45 math time began, and lasted one hour. Following math, students went to music or library, which gave me an opportunity to eavesdrop on planning and evaluation sessions which sometimes occurred afterward. As in Classroom A, I sat in the back and took notes, scripting all teaching sequences and exchanges of discourse. When students were working independently or in small groups I would make note of Miss B's and Mrs. B's activities, and then move among students, observing or casually interacting. They were courteous and accepting of my interest. I think they considered me just slightly more animated than a chair. Miss Barnaby's student teaching assignment lasted thirteen weeks (62 school days), with an additional week at the end for observation in other classrooms. Math was taught 57 of those days; I observed 47 lessons.

The Nature of the Tasks

The mathematics activities for this thirteen week period fell into three main units: multiplication, division, and fractions. Students had been reviewing and practicing multiplication facts before Christmas. The content of the first two
weeks of student teaching centered around building conceptual understanding of multiples, factors, and primes. From there the group explored "long" multiplication through estimation, computation practice, and applications. A week was spent with "two digit times two digit" multiplication as an enrichment, because the new curriculum guide had resequenced all computation topics K-5, moving this topic to fifth grade. By Week 5 there was a transition into division, with an introduction to division vocabulary and exploring the relationship of division to multiplication through fact families, followed by several weeks with the long division algorithm, and finally application of division skills in problem solving situations. Week 11 heralded the beginning of a fraction unit, which Miss Barnaby had been asked to plan and deliver on her own, with consultation from Mrs. Birch.

The warmup was a daily, predictable activity. Sometimes it was problem solving in nature, sometimes it was a multiplication “timing” or a quick computation review; in any case its purpose was to practice skills related to the current mathematics unit and to get the students focused for the day. Some warmups were written, and the warmup would either be on the desks when they came in, or students would use scratch paper if the warmup was on the board. Others were oral. Examples of warmups include:

1. Word problems are on the board. Students decide which operation to use.
2. Find two numbers whose sum is 7 and product is 12.
3. Write the steps for long division.
4. Estimate, and then find the average for these numbers: 28, 19, 39, 37
5. Timings (students practiced one set of facts, say the “times sevens,” until mastered and Mrs. Birch kept a record). This was the warmup on nine of the days.

6. __ __ __ X __ __ = 1090  Create a problem with this product.

7. I have 8 coins which make 50 cents. What coins could they be?

8. (Worksheet: 70 x 50; 5 x 200; 40 x 600; etc.) There were a number of mental math worksheets like this which the students finished very quickly.

9. Four completed computation problems are on the board. One is addition, one subtraction, one long multiplication, and one long division. Students copy the problems and find and circle the error in each one.

10. Six division problems are on the board. Students tell if the quotient will be a two digit or three digit number.

Of the 39 warmup activities coded, 12 were problem solving in nature (like #1, 2, 6, 7 above) and 27 were designed for students to practice or review estimation, mental math, or computation skills (like # 3-5 and 8-10 above). As mentioned before, these activities were completed before math time officially began. For the problem solving type warmups, students would share solution strategies and there would be a brief discussion; otherwise the class simply checked solutions. The warmups were sometimes collected.

Except for the warmup activity, the routine in this classroom followed more of a cyclic pattern than a daily one. Students would spend several days developing a concept or procedure, often using manipulatives, and then apply the new skill or concept in problem solving settings. Students often worked in groups during the multiplication and division units. The nature of the tasks in
this class fell into several categories, described in Table 3 by type and frequency. There were 47 observed math periods; many periods contained several tasks, and many tasks fell into more than one category. For instance, the game Product Squares was coded both as a game and a problem solving lesson. Use of manipulatives, use of calculators, and use of cooperative groups also occurred either during a lesson or a game. A problem solving lesson was defined by the researcher as one in which the students were expected to figure out what to do themselves. A procedural lesson was considered to be one where students were told or shown what to do, or guided in a fairly directed fashion. A lesson that used manipulatives to model a procedure was considered procedural; likewise, a problem solving activity where a certain strategy was modeled by the teacher and then practiced by the students was also coded as procedural. Warmup activities are not included in this table.

Table 3: The Nature of the Tasks - Classroom B

<table>
<thead>
<tr>
<th>TYPE OF TASK</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem solving lessons</td>
<td>25</td>
</tr>
<tr>
<td>Procedural lessons</td>
<td>27</td>
</tr>
<tr>
<td>Games</td>
<td>8</td>
</tr>
<tr>
<td>Writing or literature</td>
<td>4</td>
</tr>
<tr>
<td>Use of manipulatives</td>
<td>17</td>
</tr>
<tr>
<td>Use of calculators</td>
<td>5</td>
</tr>
<tr>
<td>Use of cooperative pairs or groups</td>
<td>11</td>
</tr>
<tr>
<td>Problem solving worksheets</td>
<td>13</td>
</tr>
<tr>
<td>Computation worksheets or practice</td>
<td>8</td>
</tr>
<tr>
<td>Assessments (formal)</td>
<td>7</td>
</tr>
</tbody>
</table>
During the first week Mrs. Birch oriented Miss Barnaby to her planning process.

Jan. 5 ... I used the district learner outcomes to orient myself to what to teach, then I looked at the old textbook because I'm familiar with it and chose activities I like and I used the Marilyn Burns book and Lane County for other ideas for activities, and a few other resources too for the multiplication unit. ...Today I'm looking at assessments, and I'd like you to look over Taxman and maybe prepare that. ...it's from our Alternative Assessment packet. ...and I'll help you. ...next week we'll be finishing facts and going into one digit by two and three digit multiplication.

Miss Barnaby taught her first lesson, Taxman, at the end of Week 2. Because it was also an alternative assessment task, this lesson will be described in the section on Analysis. Although a few math periods were shared, with each teacher leading a part of the lesson, usually it was one or the other in charge. Whenever Miss Barnaby was teaching, Mrs. Birch was in the classroom, attending to the lesson, except for one day during Week 5 and also during Week 13 -- the "solo" week. The pattern for Miss Barnaby's induction into the mathematics teaching routine (for the 47 observed periods) is given in Table 4. (It might be helpful to remember that elementary student teachers are assuming gradual responsibility for all subjects. This is the pattern for math only.) Notice that Miss Barnaby had "Full Responsibility" for twice as many lessons as Miss Aragon (24 compared to 12). This meant that Miss Barnaby did not have as many opportunities to observe Mrs. Birch directly modeling empowering instruction. Her primary source of information about empowering pedagogy was Mrs. Birch's coaching.
TABLE 4: Schedule of Induction for Miss Barnaby - Number of Days

<table>
<thead>
<tr>
<th></th>
<th>Observed only</th>
<th>Shared a lesson</th>
<th>Full responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weeks 1-3</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Weeks 4-6</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Weeks 7-10</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Weeks 11-13</td>
<td>0</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>16</strong></td>
<td><strong>7</strong></td>
<td><strong>24</strong></td>
</tr>
</tbody>
</table>

The induction pattern in this classroom unfolded in this way. During the first five weeks, Mrs. Birch did much of the teaching, modeling strong pedagogy with somewhat of a constructivist orientation. Miss Barnaby observed much of this period, but often was involved with an individual in a tutoring role while Mrs. Birch was teaching the rest of the class. Her journal entries substantiate a tendency to focus on individual students and their learning processes. Some samples:

I noticed a boy who could not relate $9 \times 4$ and $4 \times 9$. He was also working on memorizing his sevens but didn’t seem to remember any previous factors.

Factoring Numbers 1-20. I noticed that students tended to miss at least one set of factors for 20. I think they need more practice with factoring and prime numbers.

Triple TicTacToe and How Many in the Box. I noticed that some students were multiplying correctly but using incorrect numbers. On the Triple TicTacToe, some students could come up with a problem but they didn’t try to block Mrs. Birch who was playing against the class. I wonder why?

Problem Solving — Writing Question. I worked with a group who chose to write about rollerblades. They knew what they wanted to ask but had a difficult time with adding too much information. By the time they had
written down the information, they had forgotten the question. Does this activity need to be more structured or more direction?

Two Digit Multiplication. I was frustrated! I worked with a student who did not want to attempt this task because he thought it was too difficult. I think that he didn't want to concentrate. I know that he can do the problems when he uses base ten blocks. I think that he can do the problems without the blocks but I'm not sure. I know he became extremely frustrated. We worked with base ten blocks to relieve that frustration. He could do the problem but he needed to use concrete materials. Graphing was an entirely too abstract way to approach this task of multiplying large numbers.

Missing Numbers. I worked with Paul. I only said "What number makes this product?" He did the whole thing easily.

Estimating. Most students seemed to have a firm grasp of estimating, although Cameron tends to round 342 to 200 instead of 300. We've worked on this before. He did an excellent job knowing without instruction multiplying. He does transpose numbers a lot.

These entries are an eloquent description of a beginning teacher constructing her own knowledge about how fourth graders learn mathematics. During these early weeks, Miss Barnaby spent much of her nonteaching time working with individual students, no doubt a worthwhile endeavor for her; however, it also meant she was not able to observe as holistically, or notice nuances of Mrs. Birch's pedagogy which might provide cues about embedded management, pacing, and the conduct of discourse. These areas became problematic later.

During the middle five weeks, Mrs. Birch's goals were to help students grow in their conceptual understanding of multiplication and division, to help students acquire algorithmic competence, and to offer problem solving settings
in which to apply these skills. To this end, tasks involved:

1. the frequent use of estimation, mental math, and calculators.

2. the use of manipulatives (e.g. base ten blocks, graph paper, color tiles) to model aspects of long division and long multiplication procedures such as regrouping, partial products, partitioning, and remainders.

3. numerous applications, such as order forms, finding the dimensions of a room on a floor plan, determining unit prices. These activities were usually worksheets -- the students did not, for instance, use catalogues, measure the area of an actual room, or use real prices from newspaper ads. A classroom data collection project on averages took place during this period as well.

4. games and problem solving groups as formats for exploration and student to student discourse.

During this middle period, Mrs. Birch was turning over more of the teaching responsibility to Miss Barnaby. Mrs. Birch retained the responsibility for procedural instruction (long multiplication and long division algorithms), while Miss Barnaby experimented with problem solving lessons. Remember that while Mrs. Birch had been modeling strong problem solving pedagogy during the previous month, Miss Barnaby's concern was more with observing student learning than observing discourse and sense-making. Now Miss Barnaby was beginning to attend to Mrs. Birch's methods (which had become largely procedural), while struggling at the same time with some of the issues involved in developing a pedagogy for problem solving.
The last three weeks constituted Miss Barnaby’s unit on fractions. Mrs. Birch had targeted this unit, her personal favorite, as a wonderful way for Miss Barnaby to “have a lot of latitude for planning and see how using manipulatives and small groups can work for her.” When teaching multiplication and division algorithms, Mrs. Birch used manipulatives to “show” students the procedures. However, when helping students learn about fractions, Mrs. Birch had always offered students a variety of manipulatives which they could use to develop their own understandings about fraction concepts. Thus Mrs. Birch had distinctly different instructional approaches for these two topics, and used manipulatives in two completely different ways. Mrs. Birch expected and hoped, of course, that Miss Barnaby would create an approach to fractions that was empowering to students, one that encouraged discovery, reasoning, communication, and problem solving. Instead, Miss Barnaby did indeed use lots of manipulatives—but to model procedures, as she had observed Mrs. Birch do with algorithmic instruction during multiplication and division!

In the next section we will look at sample lessons from each teacher to help us investigate patterns of discourse in this classroom, and then look more closely at Miss Barnaby’s final fraction unit.

Classroom Discourse

In our initial interview, Mrs. Birch had identified improvement in conducting classroom discourse as a professional goal.
Another area (I'm working on) is classroom discussions. I'm interested in approaching it differently than I have in the past, going more into depth, I'd really like to learn the strong points of leading a good classroom discussion -- I'm really interested in that and seeing that modeled. I'm not to where I want to be.

During Miss Barnaby's initial interview, she had identified “management skills”, and “making my ideas match the cognitive level of the students” as two of her goals for student teaching. These themes appeared regularly during the thirteen weeks.

This day, Mrs. Birch was leading an investigation of prime numbers. The class had done lots of work with multiples and factors, and some beginning work with primes. Mrs. Birch had a large 100 Square taped to the chalkboard.

January 6 (Week 1)

Mrs. B: Today let's see if we can find all the prime numbers from 1 - 100. Who knows one?
Ashley: Seven
(Mrs. B circles) Why is seven a prime number?
Ashley: Because it only has one and itself.
Mrs. B: Its only factors are one and itself. Another one?
Greg: Three (She circles)
Lauren: Five (She circles)
Mike: Eleven.
Mrs. B: Are you sure? How do you know?
Mike: Because it only has 1 and 11.
Mrs. B: But how do you know that?
Mike: umm.
Mrs. B: Ashley?
Ashley: Well, nothing else works.
Mrs. B: What does Ashley mean nothing else works?
(Pause)
Ashley: If you count by anything, you won't get there.
Mrs. B: OK - if you make groups of two, how close will you get?
Ashley: 10
Mrs. B: How about groups of three? How close will you get if you count by threes?
Ashley: 12. No, 9.
Mrs. B: Do you see what she means? Do you know any more primes?
Emily: One.
Mrs. B: Well, one has itself and one, I suppose, but do you remember we said mathematicians don’t consider one a prime. But how about two?
Beau: No, because you can count by twos.
Emily: Yes, because it just has one and itself.
Beau: Ok
Mrs. B: One and itself for factors. (She circles two) I’d like to find a way to find all the primes efficiently, without having to just guess. Does anyone have an idea?
Emily: Five is prime. Maybe all the numbers with five in the ones place are prime.
Mrs. B: Ah - let’s check it.
(There are many comments and mumblings -- they decide not)
Paul: So far, all of them are odd.
Mike: Not two.
Paul: But the others.
Mrs. B: Let’s check it.
(Several counterexamples are offered)
Mrs. B: We’ve been talking about multiples. Can you use that information in any way to find numbers that are prime and numbers that are not?
(Several other conjectures are made)
Nathan: I know one -- 37.
Mrs. B: How do you know that one, Nathan?
Nathan: Because it’s not on the chart. (He was referring to the 10 x 10 Multiplication Table posted high on the wall.)
Mrs. B: Oh -- would that work? Try 65. Is it there?
Nathan: No
Mrs. B: Is it prime?
Nathan: No, it’s a multiple of 5.
Mrs. B: But what are you saying about the numbers that are on the chart?
(Pause)
Mrs. B: Are any of the products on the chart primes?
(They discuss this awhile as individual 100 Squares are being distributed.)
Mrs. B: Ok, who knows some numbers on here that definitely are not primes?
Wesley: All the multiples of two.
Ashley: Except two.
Mrs. B: Ok, why don't we cross out all the multiples of two up to 100.
Now, see if you can find all the primes between 1 and 100.

After the class had gone to music, I mentioned that I thought it was an
interesting discussion. Mrs. Birch said, “I got bogged down. I feel like I can only
go so far, and then it seems like things are getting confused. I want to be able to
break through to the next step.” She was aware of the wonderful mathematizing
that was going on, and very sensitive to what constitutes enough guidance to
help students articulate and explore a conjecture, and what constitutes so much
guidance that it stops independent thinking. This is difficult terrain to negotiate.

During this same class discussion on primes, Miss Barnaby had worked with two
students individually. Her journal entry for this day:

Prime numbers. I noticed that the students needed quite a bit of coaching
to understand how to find prime numbers. I worked with Dan who asked
me what prime numbers were. I went through the numbers 1 - 7. I think
he started to understand.

Miss Barnaby’s interpretation of this discourse seemed to be that Mrs. Birch had
something in mind and the students just weren’t getting it. Expertise in which the
teacher is concerned with helping students develop and extend their own
thinking, rather than guiding and directing their thinking toward a predetermined
outcome, requires a subtle shift in perspective. One says, “Let’s share and
extend what we know about prime numbers,” the other says, “Let’s learn this
about prime numbers.” Both are concerned with student understanding as
opposed to superficial or rote “learning,” but the first is more concerned with
mathematical empowerment.

Mrs. Birch continued to experiment occasionally with ways to orchestrate empowering discourse during the multiplication unit, but as her instructional focus began to shift toward teaching procedures for long multiplication and long division (long standing fourth grade topics), the predominant mode of discourse tended to be the traditional question-recitation format to elicit and maintain student participation in achieving behavioral objectives. This is not to indicate, however, that math time in this class even remotely resembled the typically traditional picture of teacher lecturing, students doing one computation exercise after another. Rather, Mrs. Birch followed what might be called the "traditionally progressive" instructional sequence of building understanding of math concepts and procedures through creative, hands on experiences, practicing those skills and procedures (often in a game format), and then applying the new concepts and skills in a problem solving setting.

To introduce the idea of long multiplication (multiplying a two or three digit number by a one digit number), Mrs. Birch began with a visualization activity from Mathland. On the overhead, she had secretly arranged base ten blocks in a pattern. As she uncovered the pattern, students wrote what they saw, and then discussed their visualization and thinking strategies.
January 17 (Week 3):
Overhead problems:

1.

Problem 1
Stephanie: I looked at the bottom - six - so $6 + 6 = 12$. Then I added $12 + 12$.
Joe: I saw 6, 6, 6, 6, and I knew $4 \times 6 = 24$.

Problem 2
Dan: Four tens are 40 and I counted the ones by twos, equals 48.
Nate: I first recognized $10 + 2 = 12$. Then $4 \times 12 = 48$.

Problem 3
Paul: I counted by twenties: 20, 40, 60, 80. Two groups of eight = 16. So $80 + 16 = 96$.
Beau: $4 \times 20 = 80$ and $4 \times 4 = 16$ and $80 + 16 = 96$.

Problem 4
Lisa: $2 \times 6 = 12$, and $1 + 1 = 2$, so it's 22. (Mrs. B probes). I added one from the 16 and one from the 12 together. (She was actually adding the ten from the 12 and the ten from the 16 to get the twenty in 22.) At that point Mrs. Birch said, "Did anyone do it a different way?"

Brian: $6 \times 2 = 12$ and $2 \times 1 = 2$, put it here. Then add those up, that makes 42. (Mrs. B tried to dialogue with him, then said, "You're on the right track. Let's call on someone to add to your thinking.")

Emily: I knew $8 + 8 = 16$ and there's two sixteens, so that's $8 + 8 + 8 + 8$. And $4 \times 8 = 32$.

Mike: $2 \times 10 = 20$ and add $12 = 32$.

Sarah: I added two tens and pretended the 6 wasn't there for now, that made 20. Sixteen has one ten and six ones, so now add the six twice -- that's twelve -- and add it on. That's 32.
It's clear that all students evidenced strong thinking on problems #1 - 3. On problem #4, Lisa and Brian seemed to have a vague notion of the traditional regrouping algorithm and were trying to explain it, while Emily problem solved, and Mike and Sarah offered conceptual thinking which constituted their own invented algorithm. Mrs. Birch struggled with how to incorporate Lisa and Brian's ideas into a discussion which could illuminate and extend their thinking, but couldn't orchestrate this effectively. The net result was that five people contributed five separate strategies, and Mrs. Birch did not try to help the class discuss and compare the strategies nor connect the thinking on problems #1, #2, and #3 with the thinking on #4. Afterwards, Mrs. Birch said, "I haven't ever approached it in this way before so I am just feeling my way. Marilyn Burns suggests not teaching the algorithm at all, but I probably will."

In the next lesson, Mrs. Birch introduced graph paper to help students visualize partial products, which she tied to direct instruction of the traditional algorithm. She noted in her journal entry that she wasn't satisfied that students were making the connection, so in the next lesson she returned to base ten blocks, linking this visual representation to the algorithm. Students then practiced the traditional algorithm without blocks with Mrs. Birch, who modeled the steps on the overhead as students provided short answer input.

The division unit progressed in a similar fashion. The algorithm was taught directly and modeled with base ten blocks, practiced, and then applied in real world contexts such as finding unit prices, mileage, and averages.
textbook was used only twice all semester; worksheets tended to be problem solving or contextual in their orientation; and algorithmic practice exercises were usually copied from the board or overhead, perhaps six or eight problems so that no more than ten minutes at a time was spent practicing procedures.

The dilemma Mrs. Birch faced as she struggled with how much instructional time to allot to the acquisition of procedural skill is one she shares with all teachers involved in mathematical reform. When should students be encouraged to construct their own algorithms, grounding them in a deeper understanding of the structure of mathematics? When should algorithms be modeled and taught directly, providing students with an efficient tool for more advanced problem solving? At what point should computational instruction defer to the use of calculators? Constructivism is not just a learning theory for practicing teachers; it raises a management issue as well.

At a second grade level, Kamii (1989) and others have contributed much to our knowledge about the exciting sense making that can occur when students who are just beginning to understand units of ten explore ways to combine and recombine these units, and create their own procedures for doing so. Classrooms where teachers see their primary mission as one of inculcating procedures and overseeing the daily practice of those procedures to the exclusion or reduction of more empowering aspects of mathematics, are depriving students of opportunities to become more powerful and more proactive in their own sense making. Yet it is not sufficient to dismiss the teaching of
algorithms and procedural instruction out of hand. Teachers know that procedures can be empowering tools (Hiebert 1990), and there is no right way or best way to resolve the dilemma.

Meanwhile, as Mrs. Birch retained the responsibility for teaching multiplication and division procedures, she delegated to Miss Barnaby what Mrs. Birch considered the “fun stuff” — the responsibility for finding and presenting problem solving tasks. Many of the problems selected for these lessons were rich and potentially empowering, in that they contained the possibility for creative thinking, reasoning, communication, and real world and mathematical connections. However, the manner of presentation and the format for working on problems is crucial to the facilitation of meaningful learning. Miss Barnaby consistently struggled with unclear directions at the beginning of the lesson, management of group and individual work, and closing “discussions” that simply modeled one teacher-selected procedure. These group problem solving lessons often resulted in confusion. To compensate, she began to turn many “problem solving” lessons into a didactic presentation of how to do the problem, followed by whole group practice as they worked problems together, such that little time or opportunity for real problem solving or creative thinking was left for the students. This pattern of “overteaching,” (Mrs. Birch’s phrase) adversely affected the pacing of most of Miss Barnaby’s lessons. A description of Miss Barnaby’s lesson on Unit Pricing, and the lesson after it on Pyramids, will illustrate some of these patterns.
February 21 (Week 8) Unit Pricing worksheet (Appendix G)

Miss Barnaby begins by reading the directions to the class.
Miss B: So if apples are three for 50 cents, how do we find the price of one? (She models this computation on the board, asking recall questions for each step of the algorithm.)
Miss B: Ok, work together in your groups and everyone must record. The leader will help you stay on task.

Students gather in their groups of three. Miss Barnaby had not mentioned estimating (suggested in #1 and #2), and students simply did six division problems. Questions arose about rounding. Miss B told students to follow the rule; if the remainder is less than 1/2, round down. One student pointed out that the directions said to round up. Finally, by the time groups were on #3, confusion reigned. Miss B moved from group to group trying to translate the poorly written directions, but most groups were stymied and off task. Miss B gathered the class together again.

Miss B: (reads #3) Let’s take apples (3 for 50 cents.) We know how many three cost. How can we find out how many two cost?
Steph: 16 + 16
Miss B: Another way?
Matt: 16 x 2
Miss B: So two apples cost 32 cents. How about pens? How much did one pen cost?
Brody: 18 cents
Miss B: Ok; because they’re four for 72 cents. Now we want a different number of pens. Ok, say four cost 72 cents; how much will eight pens cost?
Steph: 18 x 8
Miss B: Cards. Six for 77 cents. We want to change the group. How many in our new group?
Greg: Seven.
Miss B: How much will seven cost?
(No answer)
Miss B: How much did one card cost?
Joe: 13 cents
Miss B: So now we have seven. How much?
Joe: 13 x 7
Miss B: How much is that? (She models the algorithm while Greg tells her the steps.)
As Miss B goes through this didactic presentation with input from the students, she creates a wonderful table on the board to organize the information.

Miss B: How can we come up with a rule? So they have the same unit price they began with?
(No answer. She waits. No answer)
Miss B: The rule is, multiply the unit price times the number of items you have.

When the students left for music, Miss Barnaby sat down shaking her head and said, "That was horrible! I'm so disappointed in myself! I did such a poor job. I tried to explain it to each group and I got better as I went along until I thought, 'Why didn't I explain it like this the first time?' " Mrs. Birch guided the reflection about this lesson in her usual calm, reassuring, and insightful manner. Through their dialoguing process these ideas came up: engaging students at the beginning is important -- let students know here's another use for division -- relate it to real experiences -- don't be locked into the worksheet -- it can be used as an idea for a lesson rather than an isolated activity -- maybe bring in newspaper ads -- what happens in a real store about pricing? about rounding? -- follow up can include 'which is a better buy?' -- when I planned I wanted to see how many kids would use a table -- relate it -- help students find their own 'rule'.

Miss Barnaby's analysis of the video of this lesson, an assignment from her university supervisor, is offered in Appendix G and is a testimonial to the hard work of novices as they struggle to create a pedagogy that comes so naturally to experienced teachers.

In Miss Barnaby's next lesson, the Pyramid Lesson, she began with a
large map showing Egypt, and led a brief discussion about pyramids which also
included a review of perimeter. She then set up a table on the board with a
column for the names of the seven great pyramids, a column for the perimeter of
the base, and a column for the length of each side of the base. Miss Barnaby
filled in the first two columns for the students to copy; finding the length of the
sides at the base of each pyramid was the task.

February 23 (Week 8)

Miss B: We want to find the length of the sides. We only have one
number. How can we do that?
(No response)
Miss B: Each pyramid is a square . . .
Ashley: Divide by four.
Miss B: Let’s set up the first problem. 920 divided by four. Will it be a
three digit or two digit quotient? (She goes sequentially through the
algorithm on the board, with student input.)
Miss B: So let’s try the next one. (The class helps her “solve” the next
two problems by coaching her through the algorithm. Approximately 20
minutes has passed.)
Miss B: I notice there’s no remainder on any of these problems. I wonder
why?
Joe: It’s a square, each side is equal. There won’t be a remainder.

Students complete the rest of the exercises on their own. Mrs. Birch
points out that many students are finishing. Miss Barnaby puts more
practice problems on the board.

This lesson had an anticipatory set, a suggestion from the previous discussion
which Miss Barnaby took to heart. This lesson had structure and clarity; there
was no chance of confusion. In fact, there was no challenge at all. This lesson
did not use groups. In their desks, the students were attentive, if not excited,
during this highly sequenced lesson, which was probably reinforcing to Miss
Barnaby's sense of control. A case could be made that this lesson was technically an improvement from the Unit Price lesson, but neither lesson involved real problem solving, and neither was empowering to students.

By contrast, the very next week Miss Barnaby began the Averaging Project, an activity that was mathematically empowering to all. Mrs. Birch had seen the need to help Miss Barnaby develop a more meaningful context for her lessons, and so encouraged her to plan this project in which student groups chose a question to investigate, collected data, presented the data in graph form with written interpretation, and then presented their findings to the whole class.

This activity took approximately one week.

February 27 (Week 9) Miss Barnaby began by eliciting prior knowledge.
Miss B: What's an average?
Mike: Like in basketball, how many point per game.
(The class discusses averages in sports for a while.)
Miss B: We're going to do a project. You can find averages for everything. Let's say Nathan's a good shooter and he makes 6 baskets one game, 4 baskets the next game, and 5 baskets the next game. We can figure out his average number of baskets for each game. (She demonstrates this procedure on the board.)
Miss B: Or let's say Cameron has 20 pogs, Samantha has 23, Jessie only has 5, Matt has 17, and Ashley has 15. (There's lots of interest and response to this idea.) We're going to find the average. So let's add them up. (Records 80). Now divide. What do we divide by?
Lauren: 80 divided by 5
Miss B: And what is that?
Rhett: Sixteen. (Miss B does it on the board.)
Miss B: This weekend, I wanted to find the average number of letters in a last name. I used this class for a sample. (Miss Barnaby refers to a large, colorful chart she has made which displays her data, a graph of her data, and the steps for completing their project. She refers to this chart as she explains the process she went through.)
Miss B: I want you to find the average number of letters in the first names of the people in your row. Then we'll find the average for the
class. You can use calculators. Everybody in the row needs to do it.

After they complete this practice exercise, Miss Barnaby begins a discussion of the group projects.

Miss B: Ok, let’s say I want to find the average number of cold lunches at our school each day. How can we?

They discuss this, including sources of information and ways to keep track of information. Then they select topics.

Topics for this project included finding the average number of:

- students eating hot lunch
- pets per student
- siblings
- students absent each day
- students tardy each day
- books read since September
- fiction books checked out each day
- nonfiction books checked out each day

Miss Barnaby guided the students through the various stages of this project, gathering data, graphing data, writing reports, interpreting data. As they worked in their groups she moved efficiently and happily among them, troubleshooting, advising, and keeping everyone on task. As she passed me on her way to another group she said, “Did you hear what they just said? ‘We’re each writing a sentence.’ This is good! This is exciting!” Another morning she announced that she had put this activity on America Online and ten people had already downloaded!

Thus, Miss Barnaby did not accommodate this researcher with a nice, neat linear pattern of growth. She toggled back and forth between difficulties and
successes, between frustrations and feelings of accomplishment. Miss Barnaby continued to try to adjust her lessons in response to her own sense of the difficulties and Mrs. Birch's suggestions, and maintained a positive and upbeat demeanor throughout the semester. And so, before launching into a description of Miss Barnaby's Fraction Unit, a word about perspective is in order. This research study concentrated on two student teachers' acquisition of an understanding of the characteristics and pedagogy associated with mathematical power. As such, a highly focused lens was operating. There are many criteria with which to view mathematics pedagogy besides that of mathematical power, and the description and conclusions in this regard do not constitute an evaluation of Miss Barnaby's performance or potential. We are looking at her understanding of mathematical power. She was not a poor student teacher. In fact, her own reflection on her student teaching experience was a very positive, enthusiastic, and appreciative one, and Mrs. Birch's evaluation highlighted Miss Barnaby's many strengths.

The three week Fraction Unit which culminated the student teaching experience was not an empowering one for fourth graders, but provided a wealth of continued learning opportunities for Miss Barnaby. Each day she presented a lesson which would spring a leak or two. Mrs. Birch would make suggestions for how to fix the leak, sometimes even providing the patching materials, and then the next day's lesson would spring a new leak. Here is a partial list of leaks to look for as we reconstruct some of the lessons and the flow of this unit:
1. Holes in content knowledge. Miss Barnaby did not convey the sense that she understood what constituted the “big ideas” in beginning rational number concepts; rather she focused on narrow definitions and specific procedures for finding equivalents, fractions of a number, and “converting” improper fractions to mixed numbers. The resulting discourse was correspondingly narrow, with students simply searching for the answer Miss Barnaby was looking for.

2. Lack of knowledge about how students learn: in particular, that to be meaningful, learning must be constructed by the students as they wrestle with challenging problems and share their thinking. Miss Barnaby clearly and consistently conveyed the intention to help students understand. However, to her this meant getting them to see it her way. She did not elicit prior or current knowledge or have students relate fraction knowledge to real experiences. She did not engage students in discussions about fraction concepts or applications, but stayed focused on the specific idea she wanted to teach and her one route for getting there. All questions called for a very short, right answer. She never used a problem solving context from which students might discover some ideas about fractions. (Miss Barnaby was excited one day that students were discovering ideas about fractions from pattern blocks, and another day from fruit loops. These were indeed more engaging lessons; however, in both instances she guided them narrowly down a predetermined path; the students simply did what they were told to do with the fruit loops -- to their credit, I might add.) Miss
Barnaby did not use cooperative groups or pairs, a learning mode frequently used prior to this unit. Nor did she plan ways to collect data about how students were doing, until finally Mrs. Birch specifically instructed her to do so.

3. Inability to find and target the appropriate cognitive level of fourth graders. (Miss Barnaby had initially identified this as an area in which she wanted to improve). Despite her use each day of hands on materials and activities, the lessons were either too abstract, such as an algorithmic approach to generating equivalent fractions, or too easy, such as coloring activities which were fun but lacked mathematical substance. Discourse was therefore constrained, either by confusion or by lack of something interesting to discuss.

4. Inappropriate pacing. Miss Barnaby continually repeated herself. She stayed too long on one problem, asking the same question over and over and waiting indefinitely until she thought most students knew the answer; then repeated the same type of problem, instead of moving to another level; and even repeated the previous day's lesson the next day as a "review" that would go on too long. Pacing is integral to orchestrating effective discourse.

5. All of the above, combined with her previously noted inability to observe holistically and "take the temperature of the class", as Mrs. Birch would say, affected classroom management and student engagement. Although they were reasonably polite and there was no downright mutiny, students learned how to pay just enough attention to be able to copy what Miss Barnaby did; otherwise their attention often did not appear to be on mathematics.
March 20. The fraction unit began with sets of brightly colored paper cutouts displayed on the board (e.g., four blue butterflies and one red butterfly) which were used to develop fraction language (e.g., four-fifths of the butterflies are blue), display the symbol 4/5, and drill on numerator / denominator definitions. Then the set of butterflies was “represented” on graph paper. Students were instructed to outline a strip of five squares and color four of them, and then label it “4/5 butterflies.” Following this, students were given Fraction Strips and told to cut them into pieces. (Please see Appendix H).

Miss B: How can we use these strips to represent our fractions? How about 2/3?
Steph: The number on the bottom says 1/3, 2/3, 3/3.
Miss B: Can we use just the strip that says 2/3?
Mike: No, we need two 1/3 strips.
Miss B: Ok, for 2/6? What will we use?
Aaron: Use two 1/6s.
Ashley (confused): I have three 1/3s.
Miss B: Yes, that’s great. What will we use to show butterflies? 4/5?
Brody, what will you put on your desk?

(Miss B. wanders off, Brody murmurs, “I don’t get this stuff.” As she floats, she sees many students don’t know what she’s getting at. They are no doubt thinking, why would she want us to just put four of those 1/5 things on our desk? That’s so easy. What’s it supposed to mean? Butterflies???) Miss B returns to the board.

Miss B: Ok, look at the sailboats. We used 1/3 and 1/3 to make 2/3. And the balloons – 2/5 – what does this 5 mean? It tells us which strips to use. And the 2? It tells us how many. (She outlines these numerals in colored chalk). So butterflies. 4/5. What will we use? Paul?

Paul: 1/5?
Miss B: How many?
Paul: (no answer)
Miss B: Joe?
Joe: Four.
Miss B: Why?
Joe: We have 4/5.
Miss B: Ok. Shoes. 1/4. Which strip?
Matt: This one (holds up 1/4 piece)
Miss B: How many?
Matt: One?

The lesson continued in this way for quite a while. The “whole” strip was never used as a referent for the small pieces. Students were not able to make sense of the proceedings, and in fact the proceedings were nonsense. The four little strips each labeled 1/5 were being used for some unknown reason to stand for four butterflies, but they were not representing 4/5 of anything. During the debriefing, Mrs. Birch pointed out the importance of keeping each strip intact so that relationships could be seen and comparisons made. She suggested that Miss Barnaby plan another activity that would relate fractional parts to the whole.

March 21. The next day, Miss Barnaby repeated the same lesson as a review (i.e., five balloons on the board; two fifths are yellow; “graph it” by outlining a strip of five squares and coloring two; review definitions of numerator and denominator). Then she used five volunteers to come to the front and be a set.

Miss B: What can we say about these students? How can we make a fraction?
Greg: Two are boys.
Miss B: How can we say it as a fraction?
Greg: 2/5 are boys.
Miss B: Ok, put it on your paper, with squares.
Rhett: Can we draw people?
Miss B: No, we’re going to use squares. What else can we say?
Steph: 3/5?
Miss B: 3/5 what?
Steph: 3/5 are girls.

This activity continued for a while, with new sets of volunteers, and was a fun way for students to become familiar with fraction language in a meaningful way. The lesson ended with students making their own posters of balloon or butterfly cutouts and labeling them with a fraction sentence, e.g., “2/3 of the balloons are striped.”

March 22. On this day, students made their own sets of circle fractions; then Miss Barnaby led a guided “discovery” of eighths, fourths, halves, etc. as the students practiced showing the fractions she dictated. In this way they explored fractions of regions, and ventured into equivalents: “Take your yellow piece and lay it on top of the two blues. Are they the same?” (yes) “So does 2/4 = 1/2? (yes) “So we have two ways to say the same thing.” The lesson ended with students designing a flag. Miss Barnaby gave these directions: “Your flag has to be 1/3 white, 1/3 red, and 1/3 blue. It could go like this (draws a rectangle with horizontal thirds) or like this (draws a rectangle showing vertical thirds). Does it have to be a rectangle? No, it could be like this (she draws a circle partitioned in thirds).”

Miss Barnaby felt very positive about this lesson. Her comments afterward:

We did discovery today! It was exciting. I know some kids were bored with parts of it, but some of them really needed it. I think they understood the “whole” better. And Mike came up with: ‘Well, blues are smaller” so
that's why there's eight blue and only six pink.' From here Miss Barnaby went to a lesson on equivalents. The students referred briefly to the circle fractions, and then she basically taught the algorithm of multiplying the numerator and denominator by the same number. The students made their own “charts” of equivalents for 1/2, 1/3, 1/4, 2/3, 3/4, etc. which they kept in their desks and used any time they needed to know an equivalent, or compare fractions, or whatever. Thus the “answer” was on the chart rather than in their heads; that is, these activities were not allowing students to construct a personal meaning for fractions. No real world context was presented for problems, and when asked an isolated question, e.g. 4/6 = ?, many students had no strategy but to look at their chart for the answer.

For the topic of comparing fractions, Miss Barnaby used a number line on the board, plotted unit fractions on the line (inaccurately; again she did not use the referent of the whole to decide where to plot, say, 1/3 or 1/8), and then listed all the equivalents underneath the unit fractions. With this visual referent, students could then respond very accurately to her questions about which fraction was larger or smaller, while engaged in other tasks at the same time: Matt and Brody were talking, Rhett was drawing, Samantha was sleeping, Paul was daydreaming and folding, Bryce was folding, and Mike was facing backwards, talking. Ten minutes later, on a worksheet asking them to color code fractions that were greater than, less than, or equal to 1/2, they had no clue how to figure this out. So Miss Barnaby told them to look at the number line on the board. Again Miss Barnaby was modeling the belief that in
mathematics, the answer lies with the teacher, or (in this case) on the board, or in the answer section at the back of the book, or in the algorithm — anywhere but within the student.

The debriefing from this activity included Mrs. Birch’s observation that Miss Barnaby so far had no way of assessing what she was teaching, “nothing to show if they understand or not. You will need to build that in. And you will want to assess them without using the chart.” Mrs. B. added, “Don’t feel tied to the overhead. Use it, but assess by walking around.”

The rest of the unit proceeded in a similar fashion. A sample of Mrs. Birch’s comments throughout the unit reveals her frustration and puzzlement about the didactic style which Miss Barnaby seemed to be stuck in, and the dilemma she felt about whether she should intervene more directly.

I wish I had chosen something else for her to plan . . . this would have been a good unit to plan and teach together . . . it’s a pretty big jump to do a whole unit from scratch and not follow a text . . . she has a different way of teaching than I would ever have expected . . . I was expecting more creativity and variety . . . she doesn’t take the temperature of the class . . . she’s very rote in what she’s doing . . . I’m stymied . . . but I think she needs to have the latitude to do her own thinking and not be led through . . . I thought this would be a good unit for her to use what she’s learned about groups and problem solving and activity based learning . . . I think she isn’t using groups because of concerns about management . . .

Before Miss Barnaby’s “solo” week, Mrs. Birch insisted she plan some problem solving and real-world applications. Most of the week was typically procedural: Miss Barnaby used pattern blocks to model mixed numbers and improper fractions in a didactic manner, while teaching the conversion algorithm.
However, at the end of the week we had these refreshing problems:

April 7 (Week 13)

Miss B: If you are an average student, this is your day. (She draws a circle graph on the overhead). You spend 1/4 of your day in school. How much time is that?
Ashley: 6 hours.
Miss B: How do you know?
Ashley: 24 divided by 4 = 6.
Miss B: Ok, you spend 1/8 of your day eating, 3/8 sleeping, 1/6 of the day working, playing, watching t.v. or travelling, and 2 hours doing homework or other things. How much time do you spend doing these activities?

Students then worked in pairs to solve this problem and the next one:

Here is a recipe for fruit salad: 1/2 C apples; 3/4 C bananas; 3/8 C pineapple; 1 C oranges; 1/4 C walnuts. The problem is, I only have an eighth cup measure. How many scoops of each ingredient will I need for this recipe?

During this lesson, her final one, students were visibly more mentally and physically engaged in creative thinking than in previous lesson. I asked Miss Barnaby where she had found these problems. She had adapted the circle graph from the textbook manual; and had invented the recipe problem!

Thus, throughout most of the semester, Miss Barnaby was not able to orchestrate powerful discourse by encouraging students to develop, share, and discuss their own and others' strategies for solving problems. During the Fraction Unit no problems were presented, only procedures, until the last day.
The Learning Environment

In Classroom A we explored the role of management in affecting an environment that fosters mathematical power. Likewise in Classroom B, management emerged as a discriminating component: Mrs. Birch's embedded management skills contributed to the empowerment of her students, while Miss Barnaby's inexperience with classroom control constrained opportunities for the development of mathematical power. For example, Mrs. Birch valued collaborative learning and had negotiated with her class the social and behavioral expectations which would lead to productive group work and accountability for learning. Group problem solving under her direction resulted in increased mathematical understandings. Miss Barnaby often did not consider the logistical aspects of planning a lesson, and group work was less organized and more confused during her lessons. With Mrs. Birch's coaching, Miss Barnaby made efforts to improve in this area. For instance, although Mrs. Birch provided guidance and support for the Averaging Project, it was Miss Barnaby who successfully planned and delivered this activity, which was empowering to herself as well as her students. And yet, in her final solo unit, Miss Barnaby abandoned the group format and fell back into patterns of classroom control with which she felt more comfortable, but which were less empowering for students.

Another theme that emerged in both Classrooms A and B was that of valuing student contributions and sense making. Again, "environment" as used in the Professional Standards refers to the messages that are conveyed about
what is important. Some of the dimensions of the learning environment which foster mathematical power that are highlighted in the Professional Standards include the expectation that students contribute their thinking in a shared community of learners, respect for students' ideas, and valuing the reasoning and sense making process (NCTM 1991, 58). Thus, creating a learning environment that fosters mathematical power is closely tied to classroom discourse.

Much of student discourse sounds like babble to the casual observer, and perhaps to the casual reader of lesson scripts such as those included in this study. However, there is often, but not always, a remarkable eloquence and sophistication beneath or within the babble. Processing this babble requires a high level of thinking and engagement on the part of the teacher. Student contributions can be easily dismissed or discounted if teachers do not recognize the mathematizing inherent in the babble, or if they acknowledge it but are wary of their ability to interpret it.

In a traditionally organized lesson, if a great deal of effort goes into the planning stage, then the lesson itself is apt to run smoothly and predictably. Although pedagogical expertise is certainly required in the delivery of the lesson, the hardest work in producing a successful traditional lesson is arguably at the front end. Effective discourse, by contrast, cannot be pre-planned in this same way. Teachers must think on their feet in perhaps a more intense way. Directions and themes for mathematical learning can and should be planned for
and anticipated ahead of time, but during the actual discourse, teachers must work hard to

1) listen for the mathematizing and sense making beneath students' use of primitive language and syntax,
2) reflect and interpret this sense making to the classroom community in language that is more mathematical, or guide others in this interpretive process, and
3) determine when and how to keep the discussion “on task” relative to the mathematical goals in the mind of the teacher; and when and how to “go with the flow” when potentially fruitful directions are presented by students.

Many teachers are uncomfortable with the uncertainty inherent in conducting effective discourse. Mrs. Birch recognized the potential of discourse to develop mathematical power, and was determined to practice this new skill despite her discomfort. Miss Barnaby was not yet able to perceive the subtleties involved in allowing students to create or construct their own mathematical connections and meanings.

Analysis of Teaching and Learning

Analysis as Student Assessment. In Classroom B, as in Classroom A, Mrs. Birch placed a high priority on the need to monitor student progress and keep tabs on how students were doing, while in the process of teaching. This was a difficult undertaking for Miss Barnaby. Because she was focused closely on the content and goals of the lesson she was trying to teach (what do I need to do next?), she tended to be less aware of the students as individuals, or even as a group (how are they responding?). Miss Barnaby took her cues as to what
should happen next from her lesson plan, rather than from student responses. However, she knew very well which students were likely to be struggling (these were the ones she worked with individually while Mrs. Birch taught), and paced her lessons to these students. Her pattern was to ask a question, wait for hands, repeat the question several more times while waiting for more hands, and when she had enough hands, move on. Since most students tended to be only semi-engaged in the lesson, it often took a while for enough hands to be raised to move on. She did ask questions, but usually to keep the lesson going (e.g., "What equivalent fraction would come next?" after telling them a way to generate equivalent fractions) or to assess at a low cognitive level (e.g., "So what does the numerator tell us?" after writing the definition for numerator on the board).

Miss Barnaby did not plan lessons to generate and simultaneously assess mathematical power. The type of lesson Mrs. Birch was hoping to see, (for example, asking students to find and draw as many representations for 1/2 as possible using their fraction circle pieces), would have induced students to build their own understanding of equivalent fractions. These representations could then be shared with the class, and labeled with a number sentence (e.g., 4/8 = 1/2; 3/6 = 1/2). Finally, the drawings could be examined to discover, "Where is the numerator in this picture?" and the symbols discussed to determine, "So what is the numerator really trying to tell us in this fraction symbol?"
But Mrs. Birch’s ongoing concern was that even within the fairly traditional format of Miss Barnaby’s lessons, she was having trouble monitoring the level of student engagement and understanding. A raising of hands did not mean that students were either actively attending or understanding. As in Classroom A, Mrs. Birch often asked Miss Barnaby to include some element of informal assessment that would be a concrete indication of student understanding. Some videotaped lessons seemed especially helpful in presenting this idea more concretely. By the end of the semester Miss Barnaby had this to say:

Informal assessments, you do that every day. When I taught math, it was like going back and looking at how they behaved. One time they were really noisy and off task, and when I went back and looked at that, it was because what they were doing was too hard for them. Or had too many variables for them to juggle. (And there were) other things they came up with really quick and they wanted more and more and more, and that to me showed they could do the computation. They might not have understood the concept but they could do the computation and it was easy for them. So every time you have a lesson it’s just not looking at what they got wrong, it’s also how they behave, whether they’re in their desks, and whether they’re focusing.

Eight formal assessments were taken during the 13 weeks. Four of these were Mrs. Birch’s adaptations of the pencil paper chapter tests on multiplication facts; long multiplication; long division; and fractions. The class means for the adapted chapter tests were 97%, 93%, 95%, and 93% respectively. One teacher-made assessment was taken on division computation, and three alternative assessments from the Math Committee’s Alternative Assessment packet were given (all by Miss Barnaby) and scored. One of these was Taxman (Appendix J).
Taxman was the first lesson Miss Barnaby taught, and it was challenging and fun. Taxman can be played with any list of consecutive numbers. It is played in this way (my directions):

The numbers 1-10 (for instance) are on the board. There is a column for my points, and a column for the Taxman. When I choose a number, I cross it out on the board and move it to my column. The Taxman immediately gets all the factors of that number placed in his column. I must choose a number that still has factors on the board. When there are no such numbers left, the Taxman gets the remaining numbers.

January 12. Miss Barnaby began by explaining the rules and modeling a game on the board.

Miss B: What number shall I take?
Samantha: Five.
Miss B: Then the Taxman gets?
Samantha: One.
Miss B: What number next?
Mike: Nine.
Miss B: And the Taxman?
Mike: Three.
Miss B: Next?
Lauren: Seven.
Miss B: You can't take seven. There are no factors of seven left. What else?
Lauren: Eight?
Miss B: What will the Taxman get?
Emily: Four.
Miss B: What else?
Emily: Two.

At this point I was amazed that students seemed to have the concept of factors down cold and seemed to understand the game, but I felt skeptical.

Miss B: Ok, we have six, seven, and ten left. None of them have factors so the Taxman gets them. So let's add this up.
Students help total the columns. The score is:
Miss Barnaby - 22; Taxman - 33

Miss B: I didn’t beat the Taxman. You’re going to play three games and try to beat the Taxman.

Some students began. Many were confused. Because this activity was a required one from the Alternative Assessment packet, Mrs. B. was concerned that the students hadn’t been well enough prepared. She asked for the attention of the class; then engaged Miss B in a dialogue to extend the directions.

Mrs. B: So, Miss Barnaby, in my next game should I start with five again?
Miss B: No, you can start with any number. It might be a good idea to start with prime numbers.
Mrs. B: And will I need to explain my thinking in the next part of this activity?
Miss B: Yes.
Mrs. B: So this sounds like a game of strategy.
Miss B: Yes. Try to find a way that you can beat the Taxman, and get as high a score as you can.

Students played for a while, as the teachers floated, observed, and assisted. Then Miss Barnaby returned to the front of the class.

Miss B: How many found a strategy to help you win?
Daniel: Start with seven.
Miss B: What kind of number is seven?
Emily: Prime.
Miss B: So, a prime number and the Taxman gets?
Mike: One.
Miss B: What other kind of numbers?
Nathan: Nine.
Miss B: Why are we picking these high numbers?
Paul: So they will add to a higher number than the Taxman.
Mike: Who is the Taxman?
(Miss B looks beseechingly at Mrs. B)
Mrs. B: The Taxman is someone who thinks he has a right to part of everything you earn. So believe me, you want to try to pay as little tax as possible.
Nathan: I think it’s impossible to get over 34.
Miss B: Why?
Nathan: Because the Taxman always gets six or seven.
Miss B: Let's try again to find a way to use six.

Miss B starts a new game on the board. They project -- what if? -- at each step, and more thinking begins to be shared. Finally Rhett is very excited. He has found a way to use six and shares it. Now the score is Us - 40 Taxman - 15.

Miss B: Can there be a higher score than 40?
Samantha: No, because all the highest numbers are on your side, and all the lowest numbers are on the Taxman's side.

After the lesson that day, Mrs. B and Miss B had a lively dialogue about this activity.

Miss B: That was really hard to explain. They didn't understand you could only use a number once. And that if there's no factors left the Taxman gets it. And choosing high numbers was their only strategy.

Mrs. B: It became an activity more that an assessment. We need to find ways to get kids into thinking about their thinking.

Miss B: Tomorrow I was going to do it with 1 - 20. Bryce played it today with Paul as the Taxman. Should we go over the strategies?

Mrs. B: Could you devise a warm up or activity to review the concepts of factors and primes?

Miss B: Maybe we could list all the factors for numbers through twenty.

Mrs. B: What do you think about 20 being too much?

Miss B: Some would do fine but some are still confused.

Mrs. B: How could we handle that? Do they all need to work on the same task?

Miss B: They could all start out with 1 - 15, and then have some go on to 1 - 20.
The next day Miss Barnaby began with another game modeled on the board with the numbers 1 - 12. Then she said, "A strategy that would help you play this game is to make a list of the factors for 1 - 15." She started the list on the board, with input from the students. She circled the primes as they came to them. "If you make a list like this, it will help you to make your decision about how to get the highest possible score. So I want you to write 1 - 15 across the top of your paper and make a list of factors for each number. Then you may play Taxman."

The students were thoroughly engaged for the rest of the period, and the active mental involvement was evident during the closing discourse. Afterward, the teachers analyzed the lesson:

Mrs. B: What a difference from yesterday!

Miss B: Cameron was able to use the visual support of the factor list.

Mrs. B: For many kids there was a gap yesterday. But today they could really relate what was going on. They were excited. The discussion I heard between students was great. You were much more confident today. You really had the vision to see where you wanted to go with this.

Mrs. Birch's journal entry on this day: "The different levels of involvement were very apparent. I now see why this is a good alternative assessment activity."

Many teachers in this district, all of whom were experimenting with the same Alternative Assessment Packets as Mrs. Birch and Mrs. Alder, seemed to have difficulty at first with the idea that an authentic assessment, which is an activity rather than a test per se, provides a completely different kind, and
quality, of information about a student's learning than a skills survey. The idea of formative assessment is one that teachers use comfortably and confidently every day as they interact with their students (NCTM 1995, 29) but to elevate this ongoing informal assessment to a more formal status somehow doesn't seem right to many teachers, at least at first. Many seem to believe that if it's an activity, it can't be an assessment, because activities are supposed to get students ready for assessments. In other words, developmental knowledge doesn't seem like a legitimate end in itself; knowledge about mastery of skills is the proper end, and knowledge about students' development is merely a means to that end. But the Assessment Standards (NCTM 1995) point out that new assessment methods are needed from which teachers can identify and evaluate not just the product but also the process of doing mathematics. How else will teachers be able to monitor student progress in aspects of mathematical power such as persistence, confidence, connecting ideas, and disposition? Mrs. Birch and Mrs. Alder were both working out ways to use alternative assessment methods to enhance their new ways of teaching, as requested by their District Mathematics Committee, during this first official year of "implementing the Standards."

To score Taxman, a developmental rubric based on protorubrics for a high (5), medium (3), and low (1) response was used (Appendix J). A frequency distribution of scores is given:
Although presented numerically, these scores represent qualitative information about each student — levels and types of understanding — rather than answers, and as such are not particularly useful without the assessor’s interpretation and the task or rubrick itself.

**Analysis as Reflective Practice** To Mrs. Birch, reflecting was like breathing. Already an accomplished professional and master teacher, she seemed to truly enjoy the challenge of constantly stretching. Perhaps because she so fluently considered and analyzed her own teaching, she was very skilled at helping Miss Barnaby analyze her lessons. At the beginning of the semester she described her responsibility to her student teacher as “being a mentor, imparting what I know and my philosophy of how a classroom works. I think it’s a shared experience and I always look forward to learning from and working with a colleague.” At midterm, when asked what she hoped Miss Barnaby would notice from observing her mathematics lessons, Mrs. Birch said:

The interactions between teacher and kids, and between kids and kids. How to provide guidance, give good directions, and help kids find answers for themselves. Ways to produce confidence and help kids learn from one another. That there are a variety of ways to solve a problem. You know, I
like to share what I’ve learned and what I’m learning; but I also learn from her -- it’s an exchange -- I don’t have all the answers.

Throughout the semester, Mrs. Birch was concerned that Miss Barnaby could notice and talk about these elements of mathematical power, but had difficulty translating them into practice. Several times she expressed a desire for some kind of tool or checklist that might help Miss Barnaby bridge the gap:

I keep asking, Did I overestimate (her understanding of Standards-based instruction) or did I really not have a true sense ... because when I look back as a supervising teacher and see where the holes are, I’m wondering if my assessment practices with a student teacher need to change. ... specifically having student teachers look for things that are Standards-based and talking about it -- even working from a sheet -- and be thinking about empowerment and the Standards and having them look for those in lessons that I do and lessons they do really will help to direct that.

Miss Barnaby in turn seemed to have an accurate sense of her own learning process. At midterm she said:

Yesterday, when I had the whole class to myself, I saw how it all went and it was a good experience too. I think I can handle a whole day, but ... I still need help assessing whether I should go ahead with the lesson or pull back and do something that might help them with the lesson. I can see it afterward, it’s just at the moment I have a hard time seeing it. But as far as just going through the motions of the day, I think I can do that, but I need help planning the day.

Miss Barnaby also had good intuition about her areas of difficulty. For example in the fifth week she said, “It seems to me that the worksheets I’ve come up with have either been too difficult or too easy for them. So I need to revise them and try and guess at what level.” And at the end of the semester: “Some lessons I
felt like I went too fast and they didn’t get it; some lessons I felt like I went too slow and it just drug on and on and on.”

In the exit interview, Miss Barnaby said this about her student teaching experience:

Oh boy, it has met and gone above! (her expectations at the beginning.) I thought I could just jump in and it would be a piece of cake. I knew I’d have to go home and do a lot of preparing and . . . it’s been a lot of work. Overwhelming at times. I think the first four years are going to be hard . . . but as far as the emotional expectations, that has just gone above what I expected. I think there have only been a few days where I went home and said, ‘I can’t do this; I’m not going to be a good teacher; I’m just going to be so horrible.’ Most of the time I go home and say, ‘Guess what I did in school today!’ It has made me feel so good!

Despite the hard work and sporadic difficulties, Miss Barnaby never seemed discouraged on the outside; however, journal entries indicated mixed feelings on the inside. Statements from her journal entries were analyzed to determine if the tone reflected a positive self-statement or affect (e.g., liked, interesting, exciting, went well); a negative self-statement, evaluation, or affect (e.g., frustrated, disappointed, should have, horrible); or was neutral.

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<th>TABLE 6: Miss Barnaby’s Journal Entries (43 Total Statements)</th>
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<td>Positive tone or affect</td>
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Miss Barnaby worked hard throughout the semester to plan and deliver lessons where students would understand. It was an explicit goal of hers; yet she was continually frustrated because the students often did not “understand.” But her consistent, energetic, and sincere commitment to the goal of understanding — both her students’ mathematical understanding and her own understanding of her students’ understanding — is the hallmark of the reflective practitioner. This willingness to examine and work at her own pedagogy, and a belief system that is evolving toward a Standards-based philosophy even though in discordance with her current level of pedagogy, will serve her in good stead as she grows through experience.
CHAPTER 4

INTERPRETATIONS, IMPLICATIONS, RECOMMENDATIONS

This study documented the experiences of two student teachers who were placed in elementary classrooms with teachers who were committed to implementing the NCTM Curriculum and Evaluation Standards (1989). Mrs. Alder was the cooperating 3rd grade teacher; her student teacher was Miss Aragon. Mrs. Birch was the cooperating 4th grade teacher; her student teacher was Miss Barnaby. The setting was a northwestern university town of 30,000. The study examined how placement in a Standards-based classroom can strengthen a student teacher’s understanding of the concept of mathematical power and the instructional decisions and strategies which support the development of mathematical power.

Both student teachers were given positive evaluations; and both have the potential to become effective teachers. However, when a part of the student teaching day was examined under the highly focused lens of “teaching for mathematical power,” differences were documented in the abilities of these two student teachers to assimilate and utilize modeling and mentorship practices related to the mathematical empowerment of students.

Qualitative methods, including extensive observation, interviews, and journal entries, were used to record and interpret the data regarding possible
reasons for these differences. The implications of the results will be discussed, and recommendations which could strengthen preservice teachers' awareness and understanding of mathematical reform efforts will be suggested.

Interpretation of the Findings

Research Question 1:

What prior mathematical experiences, and what attitudes and beliefs about mathematics teaching and learning, do the cooperating teachers and student teachers bring to the classroom setting? To answer this question, interviews with each participant were recorded at the onset of the study.

Miss Aragon, the student teacher in Classroom A, had herself experienced innovative curricula and instruction which was mathematically empowering when she was a student. Thus she was able to “teach as she had been taught” with an emphasis on understanding and personally constructed meaning. Miss Aragon expressed beliefs about learning and teaching that were consistent with the Standards from the beginning of the study. Her initial interpretation of a mathematically empowered student was one who could use reasoning, critical thinking, and manipulatives to figure things out, and her observations and actions in this classroom setting helped her to confirm and strengthen these notions about mathematical power.

Mrs. Alder interpreted mathematical power as confidence -- students
having the confidence to use their thinking to solve the problems of today's, not yesterday's, society. She associated empowering instruction with making mathematics real and meaningful, posing real life problems, and utilizing good assessment practices (i.e., "knowing a lot about what each child knows and what needs to be learned.") She had previously experienced innovative curricula as a student and during professional inservice programs. An active leadership role on the school district's Math Curriculum Committee helped Mrs. Alder to be well informed and tuned-in regarding the spirit as well as the specific content of the NCTM Standards, and she routinely modeled instruction which fostered the development of mathematical power. The researcher's data supports the finding that this strong modeling helped her student teacher, Miss Aragon, learn ways to enact her beliefs in a classroom setting.

Mrs. Birch struggled with mathematics as a student. Perhaps because of this, she exhibited a strong sensitivity to her students' mathematical dispositions, and a belief that students needed a variety of tools and a variety of strategies in order to become more mathematically powerful. Early in her career, Mrs. Birch taught as she was taught, by the book, with a few games and activities thrown in to increase motivation. As her career developed, she came to understand mathematics teaching and learning in new ways. Mrs. Birch noted that she "relearned" mathematics with more understanding as she learned to teach for understanding, and she continually refined and adjusted her teaching to accommodate her continually growing insights. This pattern was evident in the
research setting, as Mrs. Birch identified areas of transition which she herself had targeted for improvement, such as conducting stronger discourse and looking at algorithmic instruction differently. Some residual lack of confidence from her mathematics background seemed to contribute to some apprehension about her ability to conduct discourse and “stay ahead” of her students. She said, “I’m just not yet where I want to be with that.” Mrs. Birch’s attitude of openness to change, and her success in reconstructing her own mathematical and pedagogical experience, appear to have enabled her to teach far more effectively than she was taught.

Miss Barnaby, Mrs. Birch’s student teacher, was a successful and conscientious student in traditional mathematics classes, but she also remembered some negative experiences and frustrations. (See the section: Profiles in Chapter 3). 1) One negative experience involved work with advanced math, which she didn’t understand. 2) Another was associated with an innovative math class where she had to do some creative thinking on her own, which she found far more frustrating than the predictable textbook problems to which she was accustomed. 3) A third memory came from a fraction problem on a test; she didn’t know the answer and was frustrated because she didn’t know how to figure it out. 4) And finally, she mentioned that in high school she had trouble conveying her ideas. The belief system that Miss Barnaby described in her interviews seemed to correspond closely to the above experiences: for instance, 1) Teachers need to offer a variety of approaches to
help students understand. 2) Teachers should foster creativity. 3) Teachers should give students strategies that will help them figure out problems. 4) Students should interact and communicate. Miss Barnaby’s initial interpretation of a mathematically empowered student was one who could analyze, convey, and communicate mathematical ideas, and relate mathematics to their everyday lives. This espoused belief system is very reflective of the Standards, and Miss Barnaby experimented with ways to enact these beliefs during her student teaching. But in the end, the data supports the finding that Miss Barnaby’s instructional actions reflected her old experiences far more strongly than her newly espoused beliefs. She tended to teach as she was taught; her beliefs and actions are not yet congruent.

Research Question 2

What knowledge, beliefs, and criteria do the cooperating teachers use as they enact a mathematics curriculum based on the NCTM Standards, and how are these conveyed to the student teachers? To answer this question, observations regarding the tasks, discourse, environment, and analysis in each classroom were recorded each day.

Research Question 2A: What is the nature of the tasks in which the students are engaged each day, and how are the tasks selected? In both classrooms, learner outcomes which were aligned with the NCTM Curriculum and Evaluation Standards were used to guide task selection. In both classes,
problem solving and real world applications (Standards 1 and 4 in the NCTM Curriculum and Evaluation Standards) were at the center of the learning tasks selected by the cooperating teachers. Both cooperating teachers used the textbook as a skeleton framework for math topics, but relied for the most part on resources such as Marilyn Burns materials, Lane County guides, TOPS decks, and Mathland for selecting the activities and problems that were used each day. These resources were all oriented toward activity based learning. The textbooks themselves were not used in either class. Even though multiplication and division happened to be units of study in both classes, conceptual development and problem solving were the instructional focus and rote practice of computation skills was minimal. In both classrooms, the daily tasks chosen by the cooperating teachers were ones which enabled students to construct and extend their knowledge of important mathematics topics and processes.

The student teachers used the same pool of resource materials as the cooperating teachers when planning their lessons. It can be inferred from observations and exit interview data that Miss Aragon learned how to recognize, select, and create worthwhile tasks, and learned how to present these tasks in empowering ways, while Miss Barnaby’s understanding of the connection between the choice of task and the empowerment of students was less clear. At midterm she said, “As far as presenting and just going through the motions of the day, I think I can do that, but I need help planning the day.” She continued to need help with task selection and design through the semester. At the end of
the term, Miss Barnaby said, "If I've learned anything, it's that students still need concrete objects, and even if something seems simple to me, they still need concrete objects to manipulate." The data collected showed that she did incorporate the use of manipulatives in her planning, but in ways that fostered the rote manipulation of materials rather than the construction of mathematical ideas.

**Research Question 2B:** What is the nature of the discourse among students and teachers that occurs during the mathematics lesson? The routine of Classroom A was purposefully structured by Mrs. Alder to provide daily opportunities for students to communicate and reason. (Standards 2 and 3 in the NCTM Curriculum and Evaluation Standards). The "problem solver" which began each math lesson called on students to explain, question, justify, analyze, listen, and respond as they shared and compared their solution strategies as a community of learners. Although orchestrated by the teacher, this discourse was student-initiated and the focus was on helping each person make sense of the problem. Informal discourse between students often occurred during the math activity which followed the problem solver, and formal discourse again took place during the closure, although this discussion was usually less open-ended and more directed by the teacher. This non-didactic, empowering pattern of discourse which is supported by the literature (e.g., Lampert 1991; Wheatley 1992) was adopted by Miss Aragon through an induction schedule which allowed her to observe Mrs. Alder's routines and techniques, then try it herself,
then observe, then try it, and so on throughout the semester until by semester's end she was quite skillful in facilitating student discussion.

In Classroom B, patterns of discourse varied for each participant. Opportunities for whole class discourse, emphasizing mathematical communication and reasoning as described in Standards 2 and 3, were sporadic and less frequent, and depended on the type of lesson and who was conducting it. Mrs. Birch always engaged student participation during lessons in a way that ensured attention and involvement. If the lesson was a procedural one, the discussion was “guided” toward shaping the desired responses. If the lesson was more problem solving in nature, Mrs. Birch was interested in eliciting a variety of thinking processes and strategies. But in either case, there was constant interaction and interchange between Mrs. Birch and the students. Cooperative problem solving groups were used more frequently in Classroom B than in Classroom A, and student to student discourse in these small groups tended to be lively and mathematically stimulating. However, during Miss Barnaby's lessons, most of the discourse centered around Miss Barnaby's directions, and whether students knew what to do. She did most of the talking, and student responses were primarily short answer. Empowering student discourse was not present during Miss Barnaby's lessons.

The induction schedule in this classroom (that is, the rate and method in which teaching responsibility was transferred to Miss Barnaby), though quite typical as will be discussed in the next section, may have contributed to Miss
Barnaby's limited ability to conduct empowering discourse. During the first part of the semester while Mrs. Birch was leading discussions in which student reasoning and creative thinking was emphasized, Miss Barnaby attended to individuals who seemed to need extra help. During the middle part of the semester, when Miss Barnaby was able to devote more time to observing the class as a whole, Mrs. Birch was modeling procedural lessons which primarily required student recall. Miss Barnaby was then asked to plan and deliver the problem solving lessons without having observed her cooperating teacher model effective student discourse during problem solving. During the final part of the semester, Miss Barnaby taught a fraction unit. Mrs. Birch intended this unit to be taught in a way that would allow students to construct a personally meaningful understanding of fraction concepts, but Miss Barnaby taught it in a procedural manner, and conducted discourse which primarily required student recall. Mrs. Birch expressed regret that this unit had not been "team taught" earlier in the semester as a way to model empowering instruction.

Research Question 2C: What are the features of the classroom environment associated with the mathematical empowerment of students? Both classrooms provided a physical environment which generated and reinforced mathematical ideas. Interactive and / or student created bulletin boards, hands on materials, multiplication tables, charts displaying problem solving steps and strategies, and students' work were all prominently displayed. Both classrooms presented a social and emotional climate which was orderly, relaxed,
challenging, motivating, and which conveyed the expectation of respect and cooperation. In Classroom A, some of the messages about mathematics that were communicated by Mrs. Alder and reinforced by Miss Aragon were: we value your thinking process even more than your answer; we expect everyone to contribute their ideas; we are interested in your language; and we will help you connect your language to mathematical language. Some of Classroom B's messages communicated by Mrs. Birch through her activities and embedded management were: it's important to keep trying; working with others can help you solve problems when everyone cooperates and contributes; and mathematics is useful and applicable to your life. These messages were affirmed by Miss Barnaby as desirable, but she sometimes communicated different ones (for example, math is trying to figure out what the teacher wants you to do and say; math is boring; math doesn't make too much sense sometimes) while she was trying out several teaching styles.

**Research Question 2D:** How do the cooperating teacher and student teacher reflect on (analyze) the effectiveness of the program components -- the tasks, the discourse, the environment, the pedagogy -- in the empowerment of students?

**Student data:** In both classrooms, informal assessment was integral to instruction. Mrs. Alder and Mrs. Birch relied on verbal and written feedback from the daily activities to monitor how students were doing, and they used this information to make and adjust further plans. Both consistently prompted their
student teachers to do the same. Miss Aragon effectively incorporated both formal and informal assessment practices. Miss Barnaby was just beginning to be able to utilize techniques of informal assessment – observation, interaction, questioning – toward the end of student teaching. Both cooperating teachers experimented with new alternative assessments (a district expectation during this year) such as journal entries and performance tasks, and also used familiar, more traditional chapter tests. Students in both classrooms performed well on the chapter tests. The alternative assessment data was useful to the teachers, but difficult to classify in terms of comparing student achievement.

Reflective Practice: Both cooperating teachers and both student teachers actively self-monitored the effects of their instruction, as evidenced by journal entries, evaluation sessions which were scripted, and interviews. However, this is somewhat tautological in that those same processes of the research setting requested that participants be reflective. Reflection was also enhanced by the natural processes of appropriate student teaching supervision. Nevertheless, the ability to regularly monitor the effects of the tasks, discourse, and environment on the mathematical empowerment of students is considered an important component of effective implementation of the Standards, and this habit of reflection was actively modeled by both cooperating teachers and adopted by both student teachers.

Mentoring: Mrs. Alder’s mentoring style in this instance was remarkably low profile, and primarily took the form of modeling. The well established
classroom routine, and Mrs. Alder's firm sense of the goals of her units of study, provided most of the guidance and direction. Mrs. Alder rarely offered direct feedback or evaluation of Miss Aragon's lessons; rather, they operated as a team of cohorts, with Mrs. Alder decidedly in the lead role. They tended to discuss students' responses and where to go next mathematically, rather than specifically analyzing the lesson components. But then, the lessons tended to go well. Miss Aragon was a keen observer, exercised a great deal of initiative and energy, and was visibly seen to grow more confident and skillful in her pedagogy through the course of the semester. During the exit interview, the researcher probed at some depth to determine if this degree of growth and competence was the norm for Mrs. Alder's proteges. It was not; Miss Aragon was "top of the line." Mrs. Alder indicated during the exit interview that she had been, and might be, more specifically directive and analytical with a different student teacher. That is, if a student teacher was not showing evidence of a growing understanding of mathematical power, then

we would do more together, take more time for assessment . . . I would want to know where they're coming from . . . we can remediate, or drop back and start over. . . . I might be more directive too, say I know you struggled with this one, now you're going to do this one, let's plan it out together, and this is how I would do it; try this -- much more specific. . . . I would want to see less textbook, more problem solving and hands on, but you also want to hear them talk about what they see children learning. You can have hands on and problem solving and not have learning.

Mrs. Birch's mentoring style was more visible and more active. She and Miss Barnaby sat down after math several times a week to reflect on the lesson
and make further plans. Mrs. Birch gave Miss Barnaby short assignments. Sometimes they were fairly directive (e.g., look over this alternative assessment task and present it next week), and sometimes fairly open-ended (e.g., look through these materials and find several extension activities we can use for division). When they talked together, Mrs. Birch always elicited Miss Barnaby's perspective first, and was very skilled at engaging Miss Barnaby in a dialogue where the strengths and weaknesses of the lesson were brought out in clear and helpful ways. Miss Barnaby likewise was amazingly alert and aware of aspects of her lessons which did not go well. She recognized when students were bored or didn't understand. As the semester evolved, Mrs. Birch expressed to the researcher some concern and frustration that even though Miss Barnaby was consistently responsive to the evaluations of her lessons, and would adjust the next lesson accordingly, Mrs. Birch was still not seeing overall progress in key areas. These areas were classroom management, what Mrs. Birch called "taking the temperature of the class", and planning lessons that were motivating and allowed students to do their own thinking, what Mrs. Birch called "making it relevant" and "letting them discover." Interview and observational data collected throughout the semester documented Mrs. Birch's ongoing effort to provide the appropriate balance of direction and autonomy, as she supervised a student teacher who was having more difficulty learning to teach for mathematical power.
Research Question 3

What was the influence of placement in a Standards-based classroom on these student teachers' understanding of mathematical power and related pedagogy?

Miss Aragon began the student teaching experience with an intuitive notion of mathematical power that was closely aligned with a cognitive, constructivist belief system. This placement enabled her to enact those beliefs, to see what they meant when applied to real children -- twenty two of them at once! She began with firm beliefs, but tentative behaviors. Although somewhat shy and quiet in her personality and demeanor, she was not at all shy about becoming involved, and wanted to take on as much responsibility as possible. She had a personal goal to become confident as a teacher, and so she took on responsibility and challenging assignments with the idea that it would help her achieve this goal of greater confidence. She appreciated Mrs. Alder's willingness to let her struggle with and resolve management issues independently, and felt this was important to her sense of self confidence. And she was seen to grow more confident each week -- this student teacher became empowered even as the students in the classroom were becoming mathematically empowered. The influence of being placed in this Standards-based classroom on Miss Aragon's understanding of mathematical power and related pedagogy can be summarized:
1) The most influential aspect of her placement was the opportunity to witness and practice the presentation of real problem solving and the orchestration of empowering discourse on a regular schedule.

2) The resources to which Miss Aragon was directed by Mrs. Alder also empowered her to plan and deliver lessons which centered on developing mathematical power. The Marilyn Burns book *About Teaching Mathematics* became a strong guide and ally.

3) Miss Aragon's awareness of assessment practices increased dramatically as well. She learned about ways to assess as an integral part of the instruction process, as well as observing and reflecting on alternative assessment practices, such as journal entries and performance tasks and student interviews, which were used during her tenure. She created her own assessment plans and rubrics for her Oceans Unit and for a number of other activities.

4) Although Miss Aragon began student teaching with a great deal of intuitive and practical understanding of some of the elements of mathematical reform, the evidence indicates that through her placement with Mrs. Alder, Miss Aragon got a jump start on teaching for mathematical power.

Miss Barnaby experienced more cognitive and affective dissonance during her placement. She seemed to bring to the classroom foundational notions about how students learn and what constitutes understanding that were quite behavioral, which clashed with her more recently acquired cognitive ideas.
Miss Barnaby seemed to recognize her weak areas at the beginning of the student teaching experience, and conveyed some initial apprehension, particularly about whether she would be able to manage or control the class effectively, and whether she would be able to plan lessons that were appropriate to the students' cognitive level. And in fact, these were two areas in which she experienced difficulty throughout the semester. Miss Barnaby was consistently curious and often puzzled about students' learning processes. The influence of placement in this Standards-based classroom on Miss Barnaby's understanding of mathematical power can be summarized:

1) Miss Barnaby was able to use the language of mathematical empowerment as expressed in the Standards quite fluently to express her beliefs about the best ways to teach and learn mathematics. Her instructional focus was clearly on teaching for understanding; however, she was in the process of reconstructing for herself a meaningful understanding of what it means for students to understand, and so her instructional actions often did not match her own definition of empowering mathematics. Miss Barnaby's interpretations of mathematical power, recorded at intervals, illustrate an awareness of mathematical power as an abstract concept which she was not able to make concrete by the end of student teaching:

(Beginning of semester - mathematical power is:) . . . to be able to analyze, convey, communicate mathematical ideas, to use problem solving in everyday life, not just see it as a subject but how they can use it

(Midterm - mathematical power is:) . . . when they work in cooperative groups, discussing problems, figuring out a strategy. I think that's what it's all about is trying to solve problems, working together . . .
(End of semester - mathematical power is:) ... to be able to use what they learn in class, and incorporate it in their lives. Not necessarily doing computations, but different ways to solve problems, and that they don't have to rely on other people all the time. Trying to figure it out on their own. (You’ve got to) help kids relate what they’re doing in class to their experiences.

2) Miss Barnaby was able to notice some aspects of a mathematically empowering environment; but was sometimes unable to relate these things to the key idea of helping students make sense of things for themselves when she taught. For example, Miss Barnaby earlier cited the math warmup activities, the use of visual references and manipulatives, the positive encouraging atmosphere, and cooperative groups as empowering elements of Mrs. Birch’s mathematics program. She used some of these ideas to model empowering pedagogy when she led the averaging project (see Chapter 3); but these components were not used in empowering ways during her instruction of fractions during a later series of lessons.

3) Miss Barnaby was able to recognize many of her own deficit areas; but was not able to translate this awareness into more empowering teaching patterns. For example, she knew when a lesson she had planned was too hard or too easy, but assumed she would just have to try to “guess better” about students’ cognitive levels, rather than using student cues to adjust her approach or activity. And she sensed when students were disinterested, but could not maintain a pedagogy which drew from students’ own interests and background experiences, even though Mrs. Birch pointed her in this direction several times.
4) It appeared that Miss Barnaby did not benefit from Mrs. Birch's modeling of Standards-based instruction to the same extent that Miss Aragon benefited from her placement. Yet the data confirms that through her placement with Mrs. Birch, Miss Barnaby was exposed to many components of Standards-based instruction: less reliance on a textbook and greater use of outside resources to select worthwhile tasks; use of manipulatives to develop a conceptual understanding of procedures; and the use of cooperative groups and pairs, to name a few. We cannot currently measure or know what influence this placement will have as Miss Barnaby continues to grow through experience; it's even possible that in the long term she will have benefited as much or more than Miss Aragon. Therefore, the most influential and possibly the most needed aspect of Miss Barnaby's placement relative to an awareness of mathematical power, may prove to be Mrs. Birch's active modeling of self-reflection and analysis and her commitment to personal and professional growth. This capacity of a teacher to be open to change and new ideas is fundamental to "implementing the Standards" in a way that fosters students' development of mathematical power.

**Implications**

Studies of the classroom, whether qualitative or quantitative, are not conducive to controlling all variables. For instance, we cannot know precisely
the extent to which differing backgrounds, beliefs, placements, or mentoring practices influenced each of these student teacher's understanding of Standards-based instruction and related pedagogical skill. Further, because a case study cannot be replicated, we can only speculate what might have happened if, say, these student teachers' placements had been reversed, or if they had been assigned to traditional classrooms.

This study was qualitative in design, its purpose to observe, describe, and interpret, rather than to impose a treatment. Yet the student teaching component of teacher preparation programs is inherently a sort of treatment, whose intent is to strengthen the pedagogical knowledge and skill of the preservice teacher. This section suggests adjustments or refinements to the traditional student teaching format, implied by the findings of this study, which might increase a preservice elementary teacher's awareness and understanding of mathematical power and related pedagogy. There are many effective "pre/Standards" traditional and nontraditional elementary teachers, from whom student teachers can learn a great deal. But that is not enough. If the mathematical reforms called for by the Standards are to move from the innovative fringe to center stage in our classrooms, we must become proactive in structuring our preservice courses and field placements to emphasize and develop an understanding of mathematical power. There is "a shift in the importance that the world outside the schools increasingly places on thinking and problem solving. Procedural skills alone do not prepare students for that
world. Therefore, students deserve a curriculum that develops their mathematical power." (NCTM 1995, 11). Mathematical power includes:

- the ability to explore, conjecture, and reason logically; to use a variety of mathematical methods effectively to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity.
- Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions.
- Students' flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power. (NCTM 1989, 5; NCTM 1991, 1)

The central concern of this study — how a student teacher comes to understand the notion of mathematical power and its related pedagogy as described in the NCTM Standards documents — can be seen as an interplay between the effects of the cooperating teachers' modeling and mentoring practices on the student teachers' abilities to observe, practice, and reflect; set against the backdrop of beliefs and prior knowledge which participants bring to the setting. In this discussion, modeling will connote showing by example, demonstrating, serving as an exemplar or ideal, a practice worthy of imitation. Mentoring comes from the Greek word for a wise and trusted counselor, and will be construed as showing the way by guiding, advising, leading, or influencing.

Implications from this study fall into five main categories. The results of this study confirm that 1) prior mathematical experiences and 2) beliefs about how learning takes place are two important sources of influence on a student teacher's ability to understand mathematical power.
The results of this study also indicate that placement in a Standards-based classroom is an important, but not necessarily sufficient, condition for developing a student teacher's understanding of mathematical power. Several additional conditions emerged from this study as potential contributors to a student teacher's developing understanding of mathematical power. They are: 3) the critical role of student discourse; 4) the pattern and schedule by which a student teacher is inducted into (takes over) the teaching responsibility; and 5) the degree of prescriptiveness and support which the cooperating teacher provides during supervision. The implications of these five categories are discussed in the remainder of this section.

Background Experiences

Miss Aragon’s and Miss Barnaby's prior experiences as students of mathematics contributed to differences in their “readiness” to understand mathematical power. It would seem that the more opportunities preservice teachers have to be constructive learners, the more likely they will be to “teach as they have been taught” with an emphasis on developing mathematical power. In Everybody Counts (1989), a report documenting the need for changes in the teaching and learning of mathematics, the authors note:

Too often, elementary teachers take only one course in mathematics, approaching it with trepidation and leaving it with relief. Such experiences leave many elementary teachers totally unprepared to inspire children with confidence in their own mathematical abilities. . . . Those who would teach mathematics need to learn mathematics in a style consistent with the way in which they will be expected to teach . . . Since teachers teach much as they were taught, university courses for prospective teachers
must exemplify the highest standards for instruction... prospective teachers should learn mathematics as a process of constructing and interpreting patterns, of discovering strategies for solving problems, and of exploring the beauty and applications of mathematics. Above all, courses taken by prospective teachers must create in these teachers confidence in their own abilities to help students discover richness and excitement in mathematics. (NRC 1989, 64-66)

The NCTM Professional Development Standards (1991) call for preservice and inservice training and preparation that will increase teachers’ awareness of the notion of mathematical power by offering university level mathematics and mathematics education courses where instruction which “implements the Standards” is modeled, and preservice teachers’ own personal mathematical power is nurtured and developed.

Beliefs About How Learning Takes Place

These two student teachers’ entering belief systems regarding how mathematics is learned affected their ability to understand and implement mathematics lessons which developed students’ mathematical power. Specifically, the belief that knowledge is constructed rather than transmitted is fundamental to understanding mathematical power as envisioned in the Standards documents (NCTM 1991, 2; NRC 1990, 28-29; NRC 1989, 58-59).

The experiences of Miss Barnaby indicate that this belief is easier to articulate than to act upon. While Mrs. Birch modeled and encouraged a constructivist philosophy, Miss Barnaby continued to fall back on methodologies that relied on telling and directing. However, this research data indicates that
placement in a Standards-based classroom can provide a realistic context for a student teacher to confront and reflect on his or her own beliefs as they relate to the mathematical empowerment of students. Other studies have corroborated that cognitive growth can take place in the long term when preservice teachers' existing beliefs are challenged, and they are encouraged to be reflective while in a state of "disequilibrium" (e.g., Thompson 1992; Lundeberg and Fawver 1994; Raymond and Santos 1995).

Miss Aragon's belief that mathematical learning occurs through the active construction and sharing of ideas was developed and reinforced during this study when she saw, first hand and every day, students exploring, conjecturing, and reasoning, as they communicated and discussed their own various solution strategies to challenging problems. Research from the University of Wisconsin's Cognitively Guided Instruction Project (Carpenter, Fennema, and Peterson 1984) confirms that when teachers are provided evidence about the constructive nature of student learning, they are more likely to use this knowledge to effect changes in their practice which are more empowering to students (Carpenter et al. 1989).

The Critical Role of Discourse

The cooperating teachers in this study were both exemplary, master teachers who evidenced in their daily planning and instruction their thorough understanding of mathematical empowerment. These teachers differed in many
ways (e.g., Mrs. Birch was more apt to use technology and cooperative groups during problem solving; Mrs. Alder was more likely to embed procedural instruction within the context of problem solving); yet students in both classrooms were becoming mathematically empowered. However, the discriminating feature in these two classrooms relating to the respective student teacher's ability to discern the meaning of mathematical power, was the saliency of the discourse.

In Classroom A, student discourse was a regular part of the daily routine, allowing Miss Alder to actually witness the process of how students become better mathematical thinkers. Each day began with a problem solving time which always contained a period of empowering discourse; that is, students shared their various strategies; all students were expected to explain and justify their thinking; and all were expected to listen and learn from one another, not just from the teacher. From the perspective of the student teacher, this daily 20 minute segment provided a conspicuous, almost tangible, display of how learning takes place in the minds and through the actions of students, and how a teacher can facilitate this process. Student discourse is an important vehicle through which the processes of mathematical empowerment (explaining, reasoning, conjecturing, sense-making) become visible.

In Classroom B, Miss Barnaby saw students engaged in worthwhile tasks in a stimulating and collaborative environment, and she engaged in the processes of analysis regularly. These are all important components of teaching
for mathematical power. But she did not have as many opportunities to witness student discourse -- the process whereby students make a problem make sense. Perhaps more opportunities to see this process would have helped Miss Barnaby understand the difference between the quality of learning that occurs when a student is helped to figure things out for himself and make his own connections, and the pseudo-learning that occurs when the teacher tells and the student follows directions.

This study indicates that placement in a Standards-based classroom where discourse is a prominent feature, and the orchestration of discourse is strongly modeled, enhances a student teacher's understanding of mathematical power.

It follows that efforts must be made to increase classroom teachers' awareness of the critical role of discourse in implementing the Standards. As Wheatley points out, it is not enough to simply complete worthwhile tasks. Students could conceivably be kept so busy with active learning projects that they are not given opportunities to actively reflect on and discuss their thinking -- and it is through this process of reflection and discussion that sense-making occurs (Wheatley 1992). Many teachers who come from a traditional background may be delivering a Standards-based curriculum in traditional, relatively unempowering, ways. Others who, like Mrs. Birch, are aware of the importance of discourse, need encouragement to trust the process. From Everybody Counts:
Teachers themselves need experience in doing mathematics — in exploring, guessing, testing, estimating, arguing, and proving — in order to develop confidence that they can respond constructively to unexpected conjectures that emerge as students follow their own paths in approaching mathematical problems. Too often, mathematics teachers are afraid that someone will ask a question that they cannot answer. Insecurity breeds rigidity, the antithesis of mathematical power. (NRC 1989, 65)

Preservice and inservice experiences are needed which highlight the central role of discourse in the mathematical empowerment of students, and which provide concrete information about ways teachers can, in Vygotskian terms, “help students bridge their zone of proximal development.” (Jones and Thornton 1993, 22).

Patterns of Induction

The induction patterns of Classrooms A (3rd grade) and B (4th grade) (that is, the methods by which the student teachers were gradually brought into a teaching role, and the schedules and rates at which responsibility for teaching mathematics was transferred to the student teachers) were subtly but powerfully different. The issue of how much observation and how much actual teaching constitutes a successful student teaching experience is an ongoing concern, both for student teachers and for cooperating teachers (e.g., Johnston 1994). The opportunity to see empowering and effective methodology modeled is an important component of learning-to-teach which differentiates field experience (practical knowledge) from coursework (theoretical knowledge), as is the opportunity to learn from direct, supervised teaching experience. The schedules
of induction for each student teacher presented earlier (Table 2, page 76 and Table 4, page 106) enumerated the student teachers' days of observation, shared teaching, and full responsibility as these unfolded through the semester. Data in Table 7, which follows, directly compares the induction patterns of these student teachers with each other and with the average induction pattern from a national survey of student teaching programs conducted in 1982 (Guyton and McIntyre 1990).

**TABLE 7: Schedules of Induction by Percent of Total Mathematics Lessons During Student Teaching**

<table>
<thead>
<tr>
<th>Observed only</th>
<th>Shared a lesson</th>
<th>Full responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miss Aragon</td>
<td>25%</td>
<td>49%</td>
</tr>
<tr>
<td>Classroom A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miss Barnaby</td>
<td>34%</td>
<td>15%</td>
</tr>
<tr>
<td>Classroom B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>National average</td>
<td>14%</td>
<td>26%</td>
</tr>
</tbody>
</table>

It should be noted that the data from the 1982 survey included secondary as well as elementary student teaching experience, and was not limited to mathematics teaching. The pattern in Classroom B, which was encouraged in the student teaching handbook used by these participants, was fairly typical for an elementary placement: Miss Barnaby did most of her observing at the beginning; Mrs. Birch and Miss Barnaby alternated days during the middle
period; and Miss Barnaby had full responsibility for planning and teaching mathematics during the final three weeks. A few lessons were shared (meaning Mrs. Birch taught during part of the period and Miss Barnaby taught during part of the period), but for the most part either one or the other taught for the whole period. Planning and evaluation were collaborative, Miss Barnaby was supported, and mentoring was active. In other words, Miss Barnaby was not thrown to the wolves; she was well supervised. Yet this traditional pattern of induction did not enable Miss Barnaby to take advantage of Mrs. Birch's ability to model empowering instruction.

By contrast, the particular structure of the mathematics period in Classroom A maximized Miss Aragon's opportunities for both observing and teaching. Statistically, Miss Aragon was able to observe effective, Standards-based teaching during 75% of the mathematics periods, and she practiced teaching during 75% of the mathematics periods as well; while Miss Barnaby observed effective, Standards-based instruction during 50% of the mathematics periods and practiced teaching during 65% of the mathematics periods. In Classroom A there were two distinct segments of the lesson, the problem solving opener and the content activity, and Miss Aragon frequently taught one segment while observing the other. Because this pattern also allowed Mrs. Alder to continue to teach problem solving and content activities regularly throughout the semester, Miss Aragon was provided with a sort of "constant comparison" from which to refine her teaching.
This data implies that Miss Aragon's heavy dose of shared teaching (50% of her lessons were shared as compared to 15% of Miss Barnaby's lessons), where opportunities for observation and autonomy were closely integrated throughout the semester, helped her to realize and incorporate aspects of mathematical power in her teaching at a steady rate of growth. Although Miss Aragon brought considerable skill to the situation, it seems that this pattern of induction would be likely to promote steady growth in pedagogy no matter what the entry point of the student teacher. Miss Barnaby's ongoing struggle with pedagogical issues on the other hand, may point to a need for more frequent, more ongoing modeling. Although modeling (direct demonstration of the instructional practices which develop mathematical power) and mentoring (coaching, guiding, providing feedback, influencing, and leading a student teacher toward an understanding of mathematical power) are both important components of student teacher supervision and were both utilized by both cooperating teachers; the results of this study suggest that modeling may be even more important than mentoring in helping a student teacher construct an initial understanding of mathematical power and related pedagogy.

This implication is supported by previous research concluding that methods modeled by cooperating teachers have a strong influence on student teachers (Copeland 1975; 1977). Copeland's 1975 study tried to determine if preservice training in effective questioning techniques via microteaching (practicing mini-lessons with peers) would lead to more effective questioning
during student teaching. He concluded that such training did not result in significant instances of that behavior during student teaching when compared with control student teachers. However, in the 1977 followup study, Copeland found that when cooperating teachers were trained in the teaching method, there was a definite correlation with the use of that method by student teachers; whereas student teachers who were trained, but paired with cooperating teachers who were not trained, again showed no training effects. Therefore, to increase preservice teachers' awareness and understanding of mathematical power and instructional practices which foster the development of mathematical power, preservice courses and field placements are needed where empowering discourse, and other recommended strategies, are strongly modeled throughout the semester, rather than just at the beginning.

Further, the nature of each student teacher's activity when the cooperating teacher was responsible for the lesson differed in this way: When Mrs. Alder taught, Miss Aragon was seen to keenly observe, attending directly to Mrs. Alder's behavior and observing the behavior and responses of students from a holistic viewpoint. However, when Mrs. Birch taught, Miss Barnaby sometimes observed closely, but often floated around the room, becoming involved with individual students whom she felt needed extra help or clarification. Thus her observational focus, and her ability to benefit from modeling, was often narrower. This suggests that the quality of the observation is as important as the opportunity to observe. An observational instrument or
tool could be helpful in directing a student teacher's, and cooperating teacher's, attention to mathematically empowering aspects of the teaching and learning process. In addition to this sensitizing function, such a tool might also serve as a catalyst for communication and conferencing.

**Degree of Prescription and Support**

How prescriptive should a cooperating teacher be in guiding his or her student teacher toward an understanding of mathematical power?

In Classroom B, Miss Barnaby had considerable latitude for planning and delivering lessons. Mrs. Birch conveyed a strong belief that although she was there to share her practical knowledge and experience, Miss Barnaby, like her fourth grade students, needed to be allowed to figure things out for herself in order for her learning to be meaningful. When Miss Barnaby found fractions frustrating to teach, Mrs. Birch empathized:

> A lot of her frustration, I as a teacher have felt. There's a lot to learn to become a proficient teacher. You come into student teaching with a degree of confidence and you come into math itself with a degree of confidence and they're not always the same... she's still of the generation that was brought up on book instruction and I think they say you tend to teach the way you were taught a lot of times; she still has to relearn a lot... you have to do some of what you feel comfortable with and then venturing out...

Thus Mrs. Birch was presented with a dilemma regarding if and when she should directly intervene (as opposed to coaching from the sidelines), especially during Miss Barnaby's fraction unit, when things were not going as Mrs. Birch would
have wished. Students receiving instruction from a student teacher often “suffer,” if only in the sense that although the instruction is usually not “bad,” it’s also unlikely to be as good as it would be with their regular teacher. Feiman-Nemser and Buchmann (1985) note that the legitimate purpose of a public school classroom is to maximize the learning of its students, which often acts at cross-purposes to using the classroom as a laboratory for learning to teach.

Mrs. Birch’s dilemma, translated into the terms of this study, is this: Our goal is for student teachers to understand the constructivist nature of mathematical power. We do not want our student teachers to show and tell mathematics to their students. We want them to help their students actively construct mathematical knowledge. How do we accomplish this task? Preferably by helping our student teachers actively construct their own understanding of mathematical power and its related pedagogy, rather than by showing and telling.

This does not imply that cooperating teachers should maintain a hands off policy. Just as teachers in a constructivist classroom must play an active role in helping their students “bridge their zones of proximal development” (see Chapter 1), so must a cooperating teacher be proactive in guiding student teachers toward teaching for mathematical power. To this end, a reasonably prescriptive and predictable routine (which would take various forms among Standards-based classrooms) seems helpful in shaping the student teacher’s pedagogical skill.
Classroom A provided Miss Aragon with a fairly prescriptive and predictable routine. The structure for the lessons was firm and consistent, and Miss Aragon was expected to conform to the established pattern; that is, begin with a problem solver, get students to talk about it mathematically, introduce an activity, provide worktime, provide closure. This structure did not vary, but within the structure Miss Aragon could be as creative as she wished, and indeed her lessons evidenced a great deal of original thinking and preparation time. The problem solving opener, very constructivist in its flavor, offered Miss Aragon a valuable window into third graders' thinking, and established a firm foundation of pedagogical success from which she was able to extend her teaching into the longer activity portion of the lesson.

Two recent studies address this issue.

Vare (1994) contrasts two styles of supervision -- one termed applied science, the other parental reflective practice. Her study took place in a university microteaching laboratory, where preservice teachers prepared and delivered lessons to their peers. These lessons were critiqued by one of two instructors. The supervision of the instructor who used a clinical, applied science style was characterized by:

- detached observation (an attitude of distance);
- normative standards (research-based knowledge about effective teaching practices);
- an unsupported, solo performance (no interference); and
- private feedback (written comments from peers, a private conference with the supervisor).
In contrast, the supervision of the instructor who used a parental, reflective practice style was characterized by:

- parental connection (a caring relationship);
- personalized, practice-based knowledge (suggestions which derived from real teaching and which included knowledge of the particular prospective teacher's characteristics);
- supported performance (cues and prompts while teaching); and
- public feedback (discussions afterward in which peers gave positive feedback followed by suggestions, and teaching was considered a shared experience rather than a solo performance) (Vare 1994, 211-213).

Vare's study focused on practice teaching prior to a student teaching placement; however, the implication from the study was that a supported practicum (a parental, reflective practice orientation), which includes appropriate intervention, is more helpful than a more clinically oriented or applied science practicum, because it results in a greater amount of practical, useful knowledge with minimal disruption to the classroom routine. The results of Vare's study would seem to encourage Mrs. Birch to go ahead and intervene when lessons fall apart; because the student teacher will gain useful knowledge and the students will suffer less. However, another research study by Knapp and Peterson sympathizes with Mrs. Birch's dilemma and supports her strong constructivist belief that a student teacher -- or any teacher -- must be allowed to explore and develop new instructional practices at her own pace and in her own way:

Knapp and Peterson (1995) report on a follow up study to the original Cognitively Guided Instruction (CGI) Project of the 1980s (Carpenter et al. 1989). In the original study, 40 first grade teachers in a summer inservice were
provided with direct evidence in the form of research reports, videotapes, and interviews, about how children learn to add and subtract when left to their own devices. The teachers debated and discussed the meaning of this constructivist philosophy for their own instruction. The next year, these CGI teachers exhibited stronger constructivist beliefs and practices in their classrooms, and student achievement was higher in both problem solving and number fact knowledge than in non CGI classrooms.

The longitudinal study involved 20 of the original 40 participants, and was conducted to determine “how these changes had weathered the test of time.” Three distinct groups emerged:

1) ten teachers had “conceptualized” or internalized CGI ideas, and their constructivist beliefs drove their curriculum in a variety of ways (like our Miss Aragon);

2) four teachers had “proceduralized” the CGI ideas; that is, they used them occasionally as a supplement or add-on to their existing curriculum. These teachers still saw mathematics as a body of knowledge and skills to be transmitted rather than a constructive activity.

3) six teachers had conceptualized CGI ideas but were still teaching in a primarily procedural manner (like our Miss Barnaby). Some of these teachers did not realize that their espoused beliefs and practice did not match; others were very aware of the discrepancies and felt conflicted, but enumerated many constraints as to why they could not enact their constructivist beliefs.
Researchers were most concerned with Group 3 teachers, wondering if:

a greater degree of prescriptiveness or specificity in curriculum would have made them initially more secure . . . However, we are concerned that such prescriptiveness or specificity could increase teachers' tendencies to 'proceduralize' CGI or might compromise the sense of flexibility and personal ownership many teachers in this study gained through being encouraged to create and develop their own CGI-based practices. (Knapp and Peterson 1995, 62)

This is Mrs. Birch's dilemma in a nutshell.

Data from the current study shows that modeling can be a powerful medium for helping preservice teachers develop a conceptual understanding of mathematical power. However, modeling could become showing-and-telling if the "presentation" of empowering mathematics is proceduralized, and the result could be teacher clones who go through certain empowering motions in certain prescribed ways. Journal, interview, and observational data from this study indicate that in Classroom A, even though Mrs. Alder regularly modeled practices which develop mathematical power, it was the 3rd grade students who displayed their mathematical power during discourse that most strongly affected Miss Aragon's understanding of mathematical power. Similar data indicate that in Classroom B, even though Mrs. Birch modeled practices which develop mathematical power, Miss Barnaby did not see evidence of 4th graders' mathematical power as frequently and consistently, because opportunities for student discourse were fewer. Therefore, the modeling of greatest influence is the modeling of powerful mathematical processes by students — classrooms where conjecturing, reasoning, and communicating are the norm — for this allows
the student teacher to have direct experience with the nature and quality of
empowering mathematics, so that beliefs are affected. Past research indicates
that these new beliefs and understandings about mathematical power are then
likely to affect practice in any number of empowering ways (Knapp and Peterson
1995).

Recommendations for Practice

If they are to effectively implement Standards-based instruction which
centers on helping elementary students develop mathematical power, beginning
teachers must be familiar with the three NCTM Standards documents, and the
notion of mathematical power which comprises the foundation of these
documents. The implications of this study support recommendations called for
in the Professional Standards and in other reform documents (NRC 1989; NRC
1990), which will help preservice teachers develop such an understanding of
mathematical power. These recommendations are:

1) Teacher education institutions that have not already done so must
revise preservice mathematics and mathematics education courses so that
prospective teachers receive the kind of instruction they are expected to deliver.
Prospective teachers' understanding of mathematical power will increase when
preservice courses:
1a) Present models of instruction and assessment that are aligned with the goal of mathematical empowerment, and especially ways to promote empowering discourse. It is important that methods instructors and supervisors help a preservice teacher develop a "feel" for what empowering discourse looks and sounds like, by learning to observe not just the teacher, but also the student discourse patterns. Videotapes published by NCTM, Marilyn Burns, and others could be helpful at the preservice level.

1b) Incorporate research based findings regarding the way learners construct their own meaningful mathematical knowledge; and

1c) Focus on a strong content knowledge base which emphasizes doing as a means to knowing mathematics.

2) Field experiences must be structured to help student teachers develop pedagogical skill which nurtures the development of mathematical power in students. Especially, field service programs must seek placements with teachers who are implementing the Standards, in classrooms and schools where:

2a) Empowering discourse is modeled, as was the case in Classroom A.

Jones and Thornton summarize principles related to the orchestration of discourse which can serve as a knowledge base for preservice and inservice experiences:
Be sensitive to the knowledge that children bring to a learning setting. Let interactive problem solving guide learning. Present some problems that challenge and stretch. Use on-going assessment to monitor instruction. Provide opportunities for modeling of higher-order thinking. Encourage, explore and accept different solutions and strategies. Enrich communication: have children explain and justify their thinking. Use a variety of teaching and learning strategies. Be willing to travel unknown pathways to nurture children's potential. (Jones and Thornton 1993, 27)

2b) Mentoring practices are used which are collaborative and supportive, as was the case in Classrooms A and B, and reasonably prescriptive regarding the orchestration of discourse, as was the case in Classroom A.

2c) Mentoring tools are used to highlight an awareness of mathematical power. The cooperating teacher in Classroom B expressed a need for a concrete means to help a student teacher to better analyze which lessons nurture mathematical power and which do not, and why. An observation instrument, journaling, peer coaching, and videotaping are examples of tools which could focus the student teacher’s attention on aspects of mathematical power. (See Appendices B, D, and G).

2d) An induction pattern is utilized in which both observation and autonomous teaching are integrated throughout the term, so as to provide the student teacher with an ongoing model for empowering instruction, as was the case in Classroom A.
3) The issue of finding or developing field placement sites such as those described in Recommendation #2 leads to a recommendation for collaboration between the teacher preparation institution and the public schools. Professional Development Schools or other collaborative arrangements can play an important role in placing student teachers in quality classrooms where master teachers are implementing mathematics programs that reflect sound and desirable practices. Consideration should be given to the idea of training cooperative teachers to enhance the possibility that such practices will be modeled and encouraged.

Recommendations for Further Research

This study focused on how a student teacher’s understanding of mathematical power can develop when paired with a teacher who is implementing ideas from the NCTM Standards documents. Such placements are not yet the norm. Research is needed investigating teachers’ and student teachers’ understanding of mathematical power among other pairings. For example, a study of student teachers who understand mathematical power, placed with cooperating teachers who do not, could generate theories about how to help prospective teachers become proactive in furthering their own ability to teach for mathematical power, whatever their student teaching and eventual career placements.

Because each cooperating teacher / student teacher pairing is always
unique, we cannot expect to generate a magic formula regarding the perfect combination of mentoring practices vs. modeling practices by cooperating teachers. However, research which investigates the amount of time spent on mentoring vs. modeling, or which explores various blends or combinations of modeling and mentoring practices (see page 165 for definitions), could extend or further illuminate the patterns which emerged in this study.

An experimental study which compares a larger number of student teachers in nontraditional induction schedules (i.e., a schedule which allows ongoing observation of master teaching throughout the placement, or one in which a team teaching approach is used) with student teachers in typical placements (i.e., observe for several weeks, then teach for the rest of the placement) could shed more light on whether the nontraditional induction pattern better facilitates an awareness and understanding of mathematical power.

Because student discourse was highlighted in this study as a critical element in the mathematical empowerment of students, research which focuses on discovering instructional strategies that promote lively discourse would promote the goals of mathematical reform. Related to this is the question of how a student teacher can be empowered to help students construct their own meaningful understandings of mathematics concepts, even within a more traditionally structured lesson.

Finally, long term studies of student teachers could provide information about ways in which student teachers implement, extend, or disregard their
knowledge of instructional strategies which nurture students' development of mathematical power.

Concluding Statement

This study highlighted the reform goal of "mathematical power for all students" as described in the NCTM Standards documents (NCTM 1989; NCTM 1991; NCTM 1995). The study recounted the experiences of two student teachers placed with cooperating teachers who were implementing the NCTM Curriculum Standards, for the purpose of determining how such a placement can strengthen a student teacher's understanding of mathematical power and help the student teacher develop empowering instructional strategies. The NCTM Professional Teaching Standards were used to guide and interpret observations related to the tasks, discourse, environment, and analysis of teaching and learning in each classroom. Multiple data collection methods, including observation and field notes, journals, and interviews, provided data regarding participants' understanding of mathematical power and related pedagogy. Although both cooperating teachers were effectively implementing the Standards, and both student teachers verbally indicated an understanding of the meaning of mathematical power, one student teacher, Miss Aragon, was able to demonstrate a deeper understanding of the instructional practices which help students develop mathematical power. The instructional variable which seemed
to have the strongest potential for making the processes of mathematical power visible and concrete to a student teacher, was the routine presence of empowering student discourse about challenging mathematical problems which occurred daily in Classroom A. Another important feature of Miss Aragon's student teaching experience was a schedule of responsibility which allowed her to observe empowering pedagogy being modeled by her cooperating teacher throughout the term, rather than just at the beginning. Finally, Mrs. Birch's mentoring style, which encouraged Miss Barnaby, the less effective student teacher, to actively reflect on her teaching, was also noted to be an important influence on Miss Barnaby's growing understanding of mathematical power.

Miss Aragon, the 3rd grade student teacher, and Miss Barnaby, the 4th grade student teacher, each brought to the research setting a unique set of circumstances: prior educational and personal experiences, attitudes and beliefs, personality and character traits, strengths and weaknesses, goals and ambitions. Yet they were alike in their enthusiasm and commitment to teaching, and in their generous participation in and support for this research project.

The two cooperating teachers in this study were exemplars of two ideals from the recent literature: Mrs. Alder was a master "constructivist teacher" and Mrs. Birch was the "reflective practioner" par extraordinaire. They each paid a great service to their student teachers and to the mathematics education profession by sharing their special gifts.
REFERENCES
REFERENCES CITED


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APPENDIX A

NCTM STANDARDS K-4
NCTM Curriculum Standards for K-4

1. Mathematics as Problem Solving
2. Mathematics as Communication
3. Mathematics as Reasoning
4. Mathematical Connections
5. Estimation
6. Number Sense and Numeration
7. Concepts of Whole Number Operations
8. Whole Number Computation
9. Geometry and Spatial Sense
10. Measurement
11. Statistics and Probability
12. Fractions and Decimals
13. Patterns and Relationships
APPENDIX B

OBSERVATION TEMPLATE
Date: Lesson Objective:

Task:

Discourse:

Analysis:

Mrs. Alder Miss Aragon Environment
Lesson Objective:

Use duration to find unit price

Task:

1. Warm up 3.50 - 3.50 round to nearest cent (Whole in lunch count, beginning)
2. Directions for worksheet activity - finding unit price
3. Small group work time
4. Whole class for #2 - Discussion (not)

Discourse:

Video taping

(2) As if end papers 4/6 comment you over directions - So if 3 pm 50 how do you find price 5 1) 3/50 2) Sequential modelling of dir. algorithm w. need answer.

Discourse: directions for end paper

(3) Signs of new elements - round down, but directions contact.

(4) Signs of new elements - round up. No reason.

(5) Signs of new elements - round up. No reason.

Finally get stuck at 3. I could help. I came by to explain and I.

Analysis:

Hand lesson. I didn't explain it very well. I could see their frustration, in hall. So was frustrated too. No motivation. They didn't care.

Hand planned to do whole with closure. No - were just going.

Shrink have done something different as task - Scott Foresman

Mrs. B. | Miss B. | Environment
---|---|---
1. Lead | Observe | Next. Colorful.
2. Do video cam | E. Incomplete | Spanish - sixth.
3. Floyd w help | Lead. Incomplete | 5th
4. Video cam | Lead. Incomplete | 5th
5. Assessment | E. Incomplete | 5th
6. Guide w/ help | E. Incomplete | Music

Reflection


decide next time to assign work that is more relevant to their lives.


decide to spend more time on the concept of finding unit price.


decide to provide additional examples to clarify the process of dividing with decimals.


decide to give more feedback on the students' understanding of the concept.


decide to incorporate more visual aids to enhance comprehension.


decide to differentiate instruction to cater to students' varying levels of understanding.
ENTRY INTERVIEW GUIDE:
Background, Beliefs, Philosophy, Goals
(*indicates question to cooperating teacher only)

I really appreciate your willingness to be a primary subject in my doctoral research. As you know, I will be observing (the mathematics teaching and learning in your classroom for the next 12 weeks; as well as your process of mentoring your student teacher) or (the mathematics teaching and learning in this classroom, as well as your student teaching experience as it relates to mathematics teaching and learning). The purpose of this interview is to help me acquire some information about your background, beliefs, philosophy, and goals as they relate to mathematics education. This information will help provide a perspective for my observations and my "findings". It is not my purpose to evaluate you as a teacher / student teacher. There are no right or wrong answers; you will not be held "accountable" for your ideas; I am just interested in learning more about you as we begin this research together, and feel this information will be helpful to me as I describe the events that happen during the next 13 weeks. I would like to tape record our interview so that we can maintain a natural pace and don't have to slow down for me to write; also so that I can pay complete attention to you. Some of the questions will be easy; others will require more thought; you're welcome to take as much time as you like.
Tell me about your math background

Start with elementary school if you can and tell me what you remember about math.
What mathematics classes have you taken? What were they like?
What did you learn?
Do you remember taking math tests? (SAT; ACT; achievement tests, etc.) What was that like?
How did you feel about yourself as a math student?

How did you decide to become a teacher?

How would you describe your philosophy of education?
What is the purpose of school?
What makes a good teacher?
Where do you think your ideas about these things came from?
* Has your philosophy changed at all since you started teaching? How?
* What are your strengths as a teacher? Weaknesses?

I’d like to find out more about your philosophy of mathematics education.
What is math?
How would you define or describe it?
* Has your understanding of what mathematics is changed at all since you started teaching? In what way?
How important is math in the curriculum?
* How much time do you spend planning for mathematics compared to your other responsibilities?
What are the most important things for students to learn in mathematics?
What’s the best way for students to learn mathematics?
What’s the best way to teach math?
What are the components of a high quality mathematics program?
* What do you do about individual differences?
* Has your mathematics instruction changed since you began teaching? In what ways?
Where have your ideas about teaching math come from?
* How do you (will you) know if your students are learning what you want them to learn? What are some ways to know if a student understands?
What does it mean to be a good student in mathematics?
* How do you feel about yourself as a mathematics teacher?
I'm looking at how a student teacher learns to teach for mathematical power.
What does the term mathematical power mean to you?
What does it mean to be empowered mathematically?

Let's talk about this student teaching experience, in general and as it relates to the mathematics portion of your day.

(Cooperating teachers only): What are your long range plans in mathematics for the next 12 weeks?
What are the main resources you use to plan your math program?
What are your mathematical goals for your students during this period of time?
(generically)
How will you know if they have met these goals?
What do you believe is the purpose of student teaching?
How would you describe your role and responsibilities as cooperating teacher?
What are your goals for your student teacher? What are your particular goals for your student teacher in mathematics?

(Student teachers only): What do you believe is the purpose of student teaching?
How has your education program prepared you for this experience? Do you feel well prepared?
What strengths so you bring to this experience?
What are your weaknesses?
How are you feeling about starting your student teaching experience?
What are your expectations?
What are your concerns?
Do you think that you could be hired as a teacher right now and do OK?
What do you hope to learn from student teaching?
When you become a new teacher with your own class, what will you use to guide you as you plan your math program? How will you know what to teach?

I'd like to find out how familiar you are with the NCTM Standards.
Have you heard of them? Where did you hear about them?
Have you read them?
Do you have a copy?
Can you describe the process standards vs. the content standards?
What is the main idea behind the Standards?
* What is your opinion about the Standards and their usefulness to teachers?
* How do you use or not use the Standards to guide your mathematics program?
1. How is your student teacher doing? What are her responsibilities to date? (grading, planning, observing, working with kids, etc.) Strengths? Weaknesses? (lack of experience doesn’t count as a weakness)

2. What is your daily/weekly routine in working with your student teacher? (Plans, assignments, use of class time, evaluation, etc.) What is your long range plan — assignments from student teaching handbook, from you -- for transfer of responsibility?

3. Focusing on your mathematics lessons and program -- what do you hope she notices as she observes your lessons?

4. Review goals of research: How do student teachers learn to teach for mathematical empowerment?
   1. Placed with a teacher who does so
   2. Develop an understanding for what math empowerment is.
   3. Develop the skills and pedagogy associated with math empowerment.

I would like your input in reflecting on some of the key components of teaching for math power. Not the ideology by itself, but what are the things you do specifically and purposefully in planning and teaching your mathematics lessons to develop strong math ability in your students?

Also reflect on things you do, could do, have done, to convey these ideas about math empowerment and pedagogical skills to your student teacher(s).
1. What are your responsibilities so far?

2. How are you feeling about the level of responsibility you have right now?

3. What are your feelings about teaching at this point in time? Have you picked the right profession? Are you having second thoughts?

4. I'd like to review the goals of my research with you. I'm studying how student teachers learn to teach for math empowerment. One influence that might help a student teacher teach for math power is to be placed with a cooperating who does so, so I made sure that would happen by choosing these cooperating teachers. Another influence would be for student teachers to develop an understanding of what mathematical power is. You shared some ideas about that in our first interview. What have you added to your ideas about what constitutes mathematical power? Do you feel the students in this class are being empowered mathematically, and what does that mean?

5. What are some of the routine components of your teacher's math lessons that you feel lead to math empowerment?

6. What do you notice about the tasks your teacher plans, the content of the lessons? Which ones are empowering?

7. What can you tell me about student dialogue and discussions in this class?

8. How does the environment -- the emotional climate as well as the physical environment -- contribute to empowering students?

9. What about analysis, or the process of evaluating mathematics lessons?

10. How are things going with your university supervisor? How are things going in general?
1. You know that I'm looking at how student teachers increase their knowledge of Standards-based instruction and their awareness of mathematical power, and that you were invited to participate in this research because of your knowledge of the Standards and your commitment to improving mathematics instruction in your classroom.

   In January you described mathematical power as ______. Is there anything more you have learned this semester about what it means to empower students mathematically, or anything you would like to add to these ideas?

2. I'd like to review some of the activities and lessons that took place during this semester. Thinking back over this time period, which activities, lessons, routines, components of lessons, etc. would you categorize as mathematically empowering?

3. What is your impression of your student teacher's beliefs about teaching and learning mathematics?

4. How did you perceive your student teacher's understanding of mathematical empowerment at the beginning of the semester? How has she grown in her understanding of Standards-based, empowering instruction, as opposed to more traditional instruction? Does she have a sense of it and value it? (Do you think she's philosophically aligned, if not pedagogically aligned, with the vision of teaching and learning contained in the Standards?) What obstacles or hurdles has she struggled with as she moves in that direction?

5. What role does classroom management play in learning to teach for mathematical empowerment?

6. How would you compare this student teacher's progress with other student teachers you have supervised?

7. What are reasonable expectations for student teachers regarding their ability to understand and implement Standards-based instruction? We can't expect any student teacher to start out as a master teacher, but do you think it's possible to identify certain practices, attitudes, skills, behaviors that would indicate growth in that direction?
8. Two avenues cooperating teachers have for influencing a student teacher’s teaching effectiveness include modeling and mentoring. What teaching practices did you try to or hope to model for your student teacher? As a supervisor or mentor, what are some things you did or do to try to increase a student teacher’s understanding of Standards-based instruction and empowerment?

9. What kinds of experiences have been or would be valuable in helping a student teacher move in the direction of the Standards? (What math teaching experiences should elementary student teachers have?)

10. What are your thoughts regarding how much the educational community should try to influence our beginning elementary math teachers in the direction of the Standards, vs. how much autonomy they should have to develop their own teaching style? Can the two be blended? What if a student teacher expresses strong beliefs in the value of drill and practice and a strong textbook orientation? How do you feel about intervention at different points in the student teaching experience? interrupting, redirecting, co-teaching, etc.

11. How important is the solo? What are some positive effects or advantages? What are some negative effects or disadvantages?

12. How do you feel about the effectiveness of the traditional structure of the elementary student teaching experience? (observe, gradually add responsibility, solo at the end) Do you have thoughts on how it could be improved or changed?

13. Do you have any recommendations for what teacher education programs and public schools could do to help beginning teachers learn more about how to teach for mathematical power?
1. Let's start with your final two weeks. How would you evaluate your “solo” weeks?

2. What age group would you like to work with and why?

3. What have you seen or learned from your cooperating teacher’s way of teaching math that you will use in your classroom?

4. Can you picture ways in which your math instruction might differ from your cooperating teacher’s when you start teaching?
   How often did you use or refer to the textbook or manual? Will you use it more or less often when you teach?

5. What were the best things about this student teaching experience?

6. What was disappointing about this student teaching experience?

7. Do you feel there was anything missing from this experience? Anything that could have been more helpful to you?

8. Management is an important component of teaching and an area that many student teachers are concerned about. What have you learned about management?

9. Has this student teaching experience helped increase your awareness and understanding of what mathematical power is and how to teach for it? Explain.
   Looking at your own teaching for math power, what do you feel good about? What areas will you continue to improve in?

10. What are some things you will do as a teacher to be sure your students are empowered mathematically?

11. What have you learned about assessment?

12. You mentioned these influences on the development of your teaching beliefs and your ideas about mathematics at the beginning of the semester: (list). Which ones have been the most influential in shaping your ideas about the best ways to teach and learn mathematics? Can you rank them?
13. Has your student teaching experience met the expectations you had at the beginning of the semester?

14. Do you have any recommendations for what teacher ed programs and public schools could do to help beginning teachers learn more about how to teach for mathematical power?

15. What does the future hold for you?
APPENDIX D

SAMPLE JOURNAL PAGES
Lesson Objective: Lesson Date: 2/10/95

What stands out in your mind as you think back on today's lesson?

(Miss B) and I talked about the validity of the assessment task and will look to see if we can still use it. (Miss B) will also work on her awareness of the overall "temperature" of the room — looking for level of engagement in the task, understanding and the time factor spent on the task for lesson.

Lesson Objective: Lesson Date: Feb. 1

What stands out in your mind as you think back on today's lesson?

This lesson at first seemed easy. They've done area before and they seemed to have grasped two-digit multiplication the day before. Today they didn't seem to want to use the area formula on the board. We were all frustrated.
Lesson Objective: Chap. Test

Problem solving

Lesson Date: Wednesday 2/1

What stands out in your mind as you think back on today’s lesson?

When I figure out a problem solver, I

don’t realize the many many ways there are to

come up with the same answer! Leave it to

teachers to come up with every possible way to

figure out an answer! They are great! I

learn a lot from them.

Lesson Objective: 3-22

What stands out in your mind as you think back on today’s lesson?

* Liked problem-solving – allowed
  for practicing various multiplication
  families

* Everyone seemed to do well on
  standardized test (even those who
  didn’t demonstrate solid understanding
  on yest. alternative assessment) –
  they can do computations not truly
  understand concept
APPENDIX E

DATA COLLECTION SCHEDULE
DATA COLLECTION SCHEDULE
(171 Hours)

January 3, 1995  DAY 1 of Observation and Fieldnotes
8:30 - 10:15 Classroom B
2:00 - 3:45 Classroom A

January 4  Entry Interview -- Miss Barnaby
January 4  Entry Interview -- Miss Aragon
January 9  Entry Interview -- Mrs. Alder
January 10 Entry Interview -- Mrs. Birch

January 26  Midterm Interview -- Mrs. Birch
February 10 Midterm Interview -- Mrs. Alder
February 13 Midterm Interview -- Miss Barnaby
February 13 Midterm Interview -- Miss Aragon

January 27  Videotape -- Miss Aragon
February 21 Videotape -- Miss Barnaby
February 22 Tape -- Miss Aragon
February 24 Tape -- Miss Barnaby
February 27 Tape -- Mrs. Alder
March 9  Tape -- Miss Barnaby
March 9  Tape -- Mrs. Birch
March 26  Tape -- Miss Barnaby
April 4  Tape -- Miss Barnaby
April 5  Videotape -- Miss Barnaby
April 5  Videotape -- Miss Aragon

April 6  DAY 44 of Observation and Fieldnotes -- Classroom A
April 7  DAY 47 of Observation and Fieldnotes -- Classroom B

April 20  Exit Interview -- Miss Aragon
April 21  Exit Interview -- Miss Barnaby
April 21  Exit Interview -- Mrs. Birch
April 27  Exit Interview -- Mrs. Alder
APPENDIX F

ALTERNATIVE ASSESSMENT

TIME AND MONEY
Show With Models

1. Draw hands on the clock face to show the time at which open swim begins.

Swimmers must prepare to get out of the pool 2 minutes before the open swim ends.
Draw hands on the next clock face to show the time swimmers prepare to get out of the pool.

What time would this be on a digital clock?

If you stayed swimming for as long as the pool is open, how long would you swim? Explain your answer.

Use play money to show two ways you could pay the exact amount for open swim. Draw what you show.
RUBRIC FOR TIME AND MONEY ASSESSMENT

Give one point unless otherwise indicated.

_______ Can tell time to the 5 minute interval

_______ Can tell time to the minute

_______ Can write time from a clock

_______ Shows understanding that money amounts can be represented in more than one way (one point for each representation - possibility of 2 points).

_______ Can calculate elapsed time

_______ Uses all available data (takes 2 minutes off of total)

_______ Is able to communicate thinking clearly (1 point - minimal; 2 points - adequate; 3 points - complete)

_______ TOTAL
APPENDIX G

UNIT PRICING WORKSHEET AND EVALUATION
Explore as a Team

Suppose you want to buy one of an item that is priced by the group, such as 2 for 99¢ or 3 for 25¢. You can divide to find the unit price, or price for one. If there is a remainder, round the price up to the nearest cent.

1. Look at the price signs. Guess which items have the same unit price.

2. Discuss your guesses with other students on your team. Then compute unit prices. Were your guesses correct?

3. Think of other ways to write the prices so that the unit prices are the same. Write the prices for groups of 8 or fewer items. Discuss your answers with other team members.

4. Make lists of prices that go with each unit price. Find a rule for writing prices that will have a certain unit price.

TIPS FOR WORKING TOGETHER

Be a good tutor. Make up similar problems or easier ones to help someone understand.
Video tape

Lesson # 1 - Find Unit Prices

Teacher position: in front of the room near the chalk board

This lesson mainly consisted of following a worksheet. The directions were read by me as the teacher. Then I went directly to the first problem.

The first example that I did was to find the unit price for 1 apple. I read from the worksheet "3 apples cost 50¢." I asked how can I find out how much one apple is? (then reread the directions and called on a student who stated "3 ÷ 50. I asked "3 ÷ 50 ?" She then reworded her answer 50 ÷ 3.

Next I proceed to work through the steps of division with the class. Throughout this time I would call on the students to work the different steps of the problem. The dividend had a remainder so I asked the student "Do we round the remainder up or down? A few students called out answers but I again asked the question and again a few students called out an answer. I had heard the answer twice. A good idea here would have been to question the students as to why I should round up or down and relate it to the policies of grocery stores, but I did not do that. I said we round down.

After this, I told the students that we'll be working in groups in the hall. I handed out blank paper and the work sheets then called out the names of people in a group. The students got up to go in the hall. I had paused and then decided that I should give some more directions to the students. I told them to keep the noise level down because other classes were working. I also told them to choose a recorder, a leader, and an organizer. I required everyone to record the information on their own paper. Then I let the students leave.

During the first part of the lesson I was able to determine by watching the video that there needed to be more of an introduction. The students would have understood the lesson better if it had been related to experiences outside of school. Watching this video I had the feeling of me being chained to the worksheet and unable to branch outside of it. It had the
potential of being a better lesson and incorporating students' knowledge and experience of group prices and unit prices.

A few other problems that I saw in the lesson were the lack of instruction with the problems and the meaning of the remainders. It seemed that I had basically read the information from the worksheet and did not develop it to real experiences. I should also have defined group prices and unit prices. Another factor that could have helped the students during their work time is to read through all of the questions and discussed them with the class before breaking up into groups. This would have taken care of a lot of problems that I remember arising during the group time. The remainder posed problems for the students and I think that a better discuss of the policy of stores towards prices would have helped to take care of that.

The last problem I felt that this lesson had was the use of my language. I felt that I used the words "OK" and "so then" too many times. I also gave the students the signal to break into groups by my body language which then created the problem of refocusing their attention quickly for more instruction.

The next part of the lesson which was tape was the closure. The students had found the unit price for all items on the worksheet. I immediately started with the last problem. The last problem was vague. The students seemed to have problems on using the unit price to find group prices. The group prices had to be different from the original group prices. I needed to clarify the question in order to work the problem. Watching this video, it was not clear to me what I as the teacher wanted. I had asked the question, "How many cards do we want? We are going to change the number, what number are we going to change it to?" After thinking about this question, it seemed first of all unmotivating and ambiguous. It implies that there is a specific number to choose but in actuality there isn't. During this time the students were not responsible for any of the information and this detracted from keeping their attention.

I had ended the lesson with "Does anyone have any questions which I feel is a very weak ending and portrays insecurity. This could have been avoided.
Fraction Equivalency Chart

<table>
<thead>
<tr>
<th>1/2</th>
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<th>1/2</th>
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</thead>
<tbody>
<tr>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
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<tr>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
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<td>1/5</td>
<td>1/5</td>
<td>1/5</td>
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<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
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<td>1/8</td>
<td>1/8</td>
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<tr>
<td>1/10</td>
<td>1/10</td>
<td>1/10</td>
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<tr>
<td>1/12</td>
<td>1/12</td>
<td>1/12</td>
</tr>
</tbody>
</table>

(Continued diagram with fractions equivalent to 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/10, and 1/12.)
APPENDIX J

ALTERNATIVE ASSESSMENT: TAXMAN
Play several games of Taxman with these ten numbers:

1 2 3 4 5 6 7 8 9 10

Make a record of your best game. Be sure to show which numbers you took and the order in which you took them, not just the final score.

Then answer the following questions:

1. Did you beat the Taxman? ________

2. What number did you choose first? _____ Why?

3. Do you think anyone could ever play a better game than your best game? ______ Explain why or why not.

4. Suppose you were going to play Taxman with the whole numbers from 1 to 95. What number would you choose first? _____ Why?
The Taxman

Develop strategies based on systematic analysis

Present convincing arguments

Use uniform introduction for all classes

Generalize from specific cases

Suggested time allotment
One class period

Student social organization
Working alone or in pairs, following a videotaped introduction

Task

Assumed background:
This task assumes that children are familiar with the concept of factors of whole numbers and, in particular, with prime numbers. It also assumes that they have had some experience in developing multi-step strategies and in articulating those strategies coherently.

Presenting the task:
The problem is to analyze a game that we assume is unfamiliar to the children. Hence the first task for the teacher is to introduce the rules of the game. One way of doing this is to show a videotape in which a teacher shows a small group of
The scene opens in a classroom setting, with a teacher and a group of six students. There is a chalkboard on one wall.

Teacher: Today I'm going to show you how to play a new number game, called Taxman. The game is played with a list of numbers starting with the number one. For example, the Taxman game with six numbers would start with this list: [Teacher writes list on chalkboard]

1  2  3  4  5  6

There are two players, You and the Taxman. Every time it is your turn, you can take any number in the list, as long as at least some factors of that number are also in the list. You get your number, and the Taxman gets all of the factors of that number that are in the list. For example, if you take 4, . . .

Student A: . . . then the Taxman would get 2!

Teacher: Why?

Student A: Because 2 is a factor of 4.

Student B: The Taxman'd get 1, too.

Student C: Oh, yeah, because 1's a factor of 4.

Teacher: So if you took 4, then the list would look like this: [writes]

1  2  3  4  5  6

Now you have 4 points . . .

Student D: . . . and the Taxman has 3 points. [Teacher writes]

You          Taxman
4            2
             1
Teacher: So, the first rule of the Taxman game is:

1. The Taxman must get something every time.

That means you can't choose if there aren't any factors of that number still in the list. The only other rule is this:

2. When none of the numbers in the list has any factors left in the list, then the game is over, and the Taxman gets all the numbers that are left in the list.

Student E: I don't get the second rule.

Teacher: Let's play a game together to see what this second rule means. We'll use the same list:

1 2 3 4 5 6

What would you like to start with?

Student F: 6. [Teacher gives chalk to Student F, who crosses off the “6” and records 6 points for You.]

1 2 3 4 5 6

You 6

Student A: So the Taxman gets 3 and 2 [crosses them off the list] ...

Student B: ... and 1 [crosses off 1 and updates the score].

1 2 3 4 5 6

You 6

Taxman 3

2

1

Student F: So now you could take 5.

Student E: No you can't! 5 doesn't have any factors that are left on the list.

Student F: Oh, OK. The 1's not there any more. How about 4?
Student D: That won't work either. The factors of 4 are 1 and 2, and they're both gone already.

Student F: Oh, I see. So the game is over?

Teacher: Right. At this point, the only the numbers left are 4 and 5. Neither of these has any factors that are still in the list. So the game is over, and the Taxman gets both the 4 and the 5: [writes]

<table>
<thead>
<tr>
<th>You</th>
<th>Taxman</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

Student C: So You lost — 6 to 15.

Student B: I bet we can do better than that.

Teacher: Well, let's see. We'll start with our list again: [writes]

| 1   | 2   | 3   | 4   | 5   | 6   |

Student F: How about if we start with 5 this time? [Student F crosses off the 5]

Student B: Hah! The Taxman only gets 1. [Crosses off 1 and records scores]

<table>
<thead>
<tr>
<th>You</th>
<th>Taxman</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Student A: Now take 6.

Student D: No, wait a minute. If you do that, the Taxman'll get 3 and 2.

Student E: And then you won't be able to take the 4 . . .

Student D: . . . 'cause the 2 and 1'll be gone.

Student A: So take the 4 now.
Student C: Yeah, that'd be better. Then the Taxman'll get only the 2.

Student F: And you'll still be able to take the 6.

Teacher: Wait a second. I don't follow you. So what do you want to do now?

Student B: Take the 4, so the Taxman gets the 2. [Crosses off and records scores.]

1 2 3 4 5 6

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

You Taxman

Student E: And now you can choose the 6 because the 3 is still left for the Taxman. [Crosses off and records scores.]

1 2 3 4 5 6

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

15 Taxman

Student D: So this time, You won, 15 to 6!

Teacher: Do you think you could ever do better, starting with the list from 1 to 6?

Student A: I don't think so...

Student C: I'm positive there's no way to do any better than that.

Teacher: How do you know?

Student C: Well, look. Every time you play, the Taxman has to get something, right? So that means...
Following the viewing of the videotape, let students play a game of Taxman using more than six numbers. In small groups they can play Taxman-8:

1 2 3 4 5 6 7 8

Together the group should choose the numbers to claim for You; then one student removes that number and updates the score, while another member of the group removes the factors and updates the Taxman's score. Remind the students that the Taxman must always get something.

Student assessment activity: See the following pages. A Spanish translation is provided after the English version.

Protorubric

Characteristics of the high response:

<table>
<thead>
<tr>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nobody could, because I got the 5 highest numbers 7, 9, 6, 8 + 10 = 90 to get the lowest 5 numbers.</td>
</tr>
</tbody>
</table>

Question 3

Do 39 first because the only thing that goes into 39 is 1 and 39 is the largest factor that's like that. Anything you do the Taxman will get 1.

Question 4

The high-level response is one that demonstrates an optimal game, communicates it effectively, and generally shows understanding of choosing the largest available prime as the best first move.

In response to the questions about Taxman-10, the pair of students present a winning game with
the optimal score, in a form that can be followed clearly. For example, this table is given:

<table>
<thead>
<tr>
<th>You</th>
<th>Taxman</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>40</td>
<td>15</td>
</tr>
</tbody>
</table>

The first number chosen, 7, is justified on the basis that it is the largest prime on the list, or on a basis that amounts to a complete analysis of the game (e.g., “I chose the 7 first because if I chose the 10, then . . .; and if I chose the 9, then . . .; and if I chose the 8, then . . .”).

No better game is possible because “You” has captured the five largest numbers on the list, while giving the Taxman the five smallest numbers.

For the game of Taxman-95, 89 is chosen as the first move. The justification asserts that if any prime is to be chosen at any point in the game, it must be on the first turn. Since 89 is the largest prime number in the list, it’s the best first move.

A somewhat less advanced response is simply that 89 is the largest number on the list that has only one factor on the list.

An even less satisfactory response is to declare that the largest prime is the optimal opening move, but then to misidentify 91, 93, or 95 as a prime.

*Characteristics of the medium response:*

A winning game is described, although it need not be an optimal one.

The first number chosen is justified simply on the basis that it works out to be a winning first move.

A correct answer, with some justification, is given to Question 3. (Of course, this response will have
to depend on the best game that the students can find. If a less-than-optimal game is described, then an answer of "Yes" is correct here.)

Some number other than 89 is suggested as the first move, with a rationale that includes some reference to the number's factors. For example, "95, because it's big, and the Taxman would get only 5, 19, and 1."

*Characteristics of the low response:*

Some game is described, but sketchily and perhaps ambiguously (that is, it may not be possible to tell in what order the numbers were selected).

No justification is provided for the first number chosen, or the justification does not take into account the factors of the number.

In question 4, no reference is made to factors of the number selected as the first move. For example: "95, because it is the biggest number you can get."