The mechanical placement of orthopedic magnets within the human knee joint
by Deborah A Barber

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Engineering Mechanics
Montana State University
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Abstract:
A mechanical analysis of an orthopedic knee implant is presented. The analysis is performed on an
orthopedic knee implant that utilizes repelling magnets placed on the articulating surfaces of the tibia
and the femur. The repelling magnets theoretically serve to decrease the contact force and friction
within the knee joint. A three-dimensional mathematical model of the human knee joint is utilized to
analyze the mechanical effects of the implants within the knee. The geometry of the surface and the
effects of the ligaments are incorporated into the model. The model is evaluated at several flexion
angles.

The placement of the magnets within the knee joint is varied, and magnet strengths are proposed. The
model is then solved for the contact forces at the knee joint with and without the implanted magnets.
The decrease in contact force due to the presence of the magnets within the knee joint is evaluated. The
initial implant design consisted of a total of four magnets on the femoral surface, two medial and two
lateral, and two magnets on the tibial surface, one medial and one lateral. The initial design was
evaluated and the conclusion was made that a more effective design could be proposed. An implant that
utilized a series of three magnets on both the medial and lateral femoral surfaces repelling against a
single magnet placed on both the medial and lateral aspects of the tibial plateau was analyzed. The final
conclusion was made that the alternate design using six magnets on the femoral articulating surface and
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within the human knee joint. A summary of results for the initially proposed implant design and the
alternative design options are presented.
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WITHIN THE HUMAN KNEE JOINT

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MONTANA STATE UNIVERSITY- BOZEMAN
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Dr. Jim Dent

(Signature)

17 Apr 96

Date

Approved for the Department of Civil Engineering

Dr. Robert Oakberg

(Signature)

4/10/96

Date

Approved for College of Graduate Studies

Dr. Robert Brown

(Signature)

5/10/96

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<tr>
<td>$c$</td>
<td>position vector of a point on the tibial surface</td>
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<tr>
<td>$a$</td>
<td>vector connecting origins of coordinate systems</td>
</tr>
<tr>
<td>$a_{ij}$</td>
<td>coefficient of joint surface polynomial</td>
</tr>
<tr>
<td>$(e_x, e_y, e_z)$</td>
<td>unit vectors in $(x,y,z)$ system</td>
</tr>
<tr>
<td>$(e_\alpha, e_\beta, e_\gamma)$</td>
<td>unit vector in $(\alpha, \beta, \gamma)$ system</td>
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<tr>
<td>$f$</td>
<td>force in a spring</td>
</tr>
<tr>
<td>$F_e$</td>
<td>external force</td>
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<tr>
<td>$k$</td>
<td>spring stiffness constant</td>
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<tr>
<td>$l$</td>
<td>length between tibial and femoral spring insertion points</td>
</tr>
<tr>
<td>$l_0$</td>
<td>unstrained length of spring</td>
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<tr>
<td>$m$</td>
<td>number of springs</td>
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<tr>
<td>$M_e$</td>
<td>external moment</td>
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<td>$M_r$</td>
<td>flexion-extension moment</td>
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<td>$n$</td>
<td>degree of polynomial</td>
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<td>$n$</td>
<td>unit outward normal</td>
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<td>contact force</td>
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<tr>
<td>$r$</td>
<td>position vector of spring, tibial insertion</td>
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<tr>
<td>$T$</td>
<td>rotation matrix</td>
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<tr>
<td>$(x,y,z)$</td>
<td>tibial fixed coordinate system</td>
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<tr>
<td>$(\alpha, \beta, \gamma)$</td>
<td>femoral fixed coordinate system</td>
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\( \delta \) \hspace{1cm} \text{position vector of a point on the femoral surface}

\( \varepsilon_j \) \hspace{1cm} \text{strain in spring}

\( \varepsilon_{\eta j} \) \hspace{1cm} \text{strain in spring in extension}

\( \lambda \) \hspace{1cm} \text{unit vector for direction of flexion-extension moment}

\( \rho \) \hspace{1cm} \text{position vector of spring, femoral insertion}

\( \tau_a, \tau_\gamma \) \hspace{1cm} \text{tangent vectors to femoral joint surface}

\( \phi \) \hspace{1cm} \text{flexion-extension angle}

\( \phi^T \) \hspace{1cm} \text{vector describing rotations}

\( \psi, \omega \) \hspace{1cm} \text{rotation angles of the femur}
ABSTRACT

A mechanical analysis of an orthopedic knee implant is presented. The analysis is performed on an orthopedic knee implant that utilizes repelling magnets placed on the articulating surfaces of the tibia and the femur. The repelling magnets theoretically serve to decrease the contact force and friction within the knee joint. A three-dimensional mathematical model of the human knee joint is utilized to analyze the mechanical effects of the implants within the knee. The geometry of the surface and the effects of the ligaments are incorporated into the model. The model is evaluated at several flexion angles. The placement of the magnets within the knee joint is varied, and magnet strengths are proposed. The model is then solved for the contact forces at the knee joint with and without the implanted magnets. The decrease in contact force due to the presence of the magnets within the knee joint is evaluated. The initial implant design consisted of a total of four magnets on the femoral surface, two medial and two lateral, and two magnets on the tibial surface, one medial and one lateral. The initial design was evaluated and the conclusion was made that a more effective design could be proposed. An implant that utilized a series of three magnets on both the medial and lateral femoral surfaces repelling against a single magnet placed on both the medial and lateral aspects of the tibial plateau was analyzed. The final conclusion was made that the alternate design using six magnets on the femoral articulating surface and two on the tibial articulating surface is indicated to be the preferred mechanical placement for magnets within the human knee joint. A summary of results for the initially proposed implant design and the alternative design options are presented.
CHAPTER I
INTRODUCTION

The knee joint is the largest joint in the human body and one of the most complex. The joint consists of several structures including bones, ligaments, cartilage, muscles, and tendons. The complex mechanism that these structures make up allows the knee to perform its major functions: motion and the transmission of forces. When the structures of the knee begin to fail, however, the ability of the joint to perform these functions is greatly diminished. Damage to the knee joint can happen in many ways. One manner is through the effects of osteoarthritis. Osteoarthritis is a disease that attacks the articular cartilage that covers the ends of bones within a moving joint, such as the knee. In general, osteoarthritis affects the older population. This disease can cause great pain to those affected making weightbearing difficult or even impossible.

One method that has been utilized for many years to alleviate the pain and deformity associated with osteoarthritis is the replacement of the affected joint. As many as 150,000 total knee replacements are performed each year in the United States alone (Lancet, 1991). Long term results of total knee replacements are good with a success rate of 98% five years after the surgical implant. Although the results for the implants have been good, the surgery is extremely invasive, with a long rehabilitation time, and the procedure is quite costly. An
alternative to total knee replacement has been proposed by Jore Medical Corporation. Their alternative proposes the use of repelling magnets to create a relatively friction-free environment within a joint to restore motion and to alleviate pain by not letting the damaged joint ends grate upon each other. The use of magnets in medicine has been well documented for decades, however few applications have been found in orthopedics. At this point, only the concept has been proposed and a general design developed.

The intention of this research is to determine the precise location and strength of these magnets within a knee joint to provide for optimal mechanic behavior. This will be achieved through the use of a mathematical model of the human knee joint developed initially by Wismans, et al. (1980) and elaborated on by Huiskes, et al. (1985, 1988, 1990, 1991, 1995) and Blankevoort et al. (1991). The location of the magnets and a potential design of the implant will be proposed.

The design of an implant for the knee is a complex procedure. Knowledge of the anatomy, physiology, and biomechanics of the joint is necessary. Also, a full understanding of the modeling process and the specific model utilized is needed. In the following paper, a full background of the structures and mechanics of the knee is presented and the new knee model developed. The implications of success in this research project are staggering with the possible
result being a more effective implant that would be less invasive to implant with much lower projected costs than the knee replacements used today.
CHAPTER 2

THE KNEE JOINT

The knee is an extremely complex joint. It is made up of a number of components all of which play important roles in the functioning of the joint. In the following chapter, a general review of the anatomy and physiology of the knee as well as the motions of the joint will be presented. For those not familiar with the medical terminology used, brief descriptions are provided. A glossary of terms is also provided in the Appendix to utilize as a reference tool.

Anatomy is the study of the structures of the body and the relationships between them. Because it is the study of physical relations, a number of directional terms are necessary. The directional terms are all established from a reference axis known as anatomical position. In anatomical position, the person stands upright, facing forward, with feet on the ground, hands at the sides, and palms forward. Once in anatomical position, the body can be divided into a number of planes. A sagittal plane divides the body into left and right sides. A frontal plane divides the body into anterior and posterior (front and back) sides. Lastly, a horizontal plane divides the body into inferior and superior (bottom and top) sections. Now that the reference axis and the reference planes have been defined, a number of directional terms can be defined.
Directional terms are used to describe locations and functions of anatomical structures relative to each other. There are many directional terms used in the field of anatomy. However, only those that will be needed for understanding the language within this paper will presented. The directional terms discussed are: inferior, superior, superficial, deep, proximal, distal, medial, and lateral. Inferior means away from the head or toward the lower portion of a structure, while superior means towards the head or the upper portion of a structure. Superficial means toward or on the surface of the body, while deep is simply away from the surface of the body. Proximal is nearer to the attachment of an extremity and distal is farther away from the attachment (i.e. the wrist is distal to the elbow). As previously stated, anterior refers to the front of the body (or towards the front) and posterior refers to the back of the body. Lastly, medial structures are nearer to the midline of the body or a structure and lateral structures are further from the midline. Again all of these terms are reiterated in the attached glossary. While dealing with these terms, keep in mind that a structure, for example, can be both medial and lateral depending on what the structure is being referred to. All of these terms simply describe the relations between anatomical structure not their exact positions.

The knee is the largest joint in the human body. A joint is a junction between two or more bony components. A joint in the human body is comprised of several anatomical structures. The anatomical structures that play key roles in
the structure and function of the human knee joint are bone, cartilage, tendons, ligaments, and muscle. All, with the exception of muscle, are types of connective tissue.

Connective tissue is the most abundant tissue in the human body (Totora, Anagnostakos, 1990). Connective tissue, in general, serves to protect, support, and bind structures together. The structure of the connective tissue at the cellular level determines the properties and function of a tissue. At the cellular level, all types of connective tissue are made up of cells present within an intercellular matrix. The connective tissue qualities are largely determined by the intercellular substances. The intercellular substances are nonliving and can be fluid, gellike, or fibrous or combinations of these. Each type of connective tissue discussed here has this basic structure. Following, a brief description of the structure, properties, and functions of each of the pertinent connective tissues is presented.

Bone is the hardest of all the connective tissues. At a cellular level bone is a connective tissue whose solid structure accommodates for its protective and supportive role. At the microscopic level, bone consists of cells and an organic extracellular matrix. Bone differs from the other types of connective tissue, in that, within the extracellular matrix, it also contains a high volume of inorganic compounds, primarily mineral salts. The arrangement of the minerals within the organic matrix contributes greatly to the mechanical abilities of bone. The minerals are embedded within fibers of protein collagen. These collagen fibers
comprise the main portion of the extracellular matrix. These collagen fibers are extremely tough and pliable, yet they resist stretching. The layers of collagen fibers and mineral salts are “glued” together by a gelatinous substance consisting of protein polysaccharides. As a result, bone is a rigid material that is pliable enough to resist large stresses.

Bone has both physiological and mechanical functions. Physiologically, bone forms blood cells and stores calcium for use in the body. Mechanically, bone, in general, serves to provide support for the body, to act as a lever system to transfer forces and facilitate movement, and to protect internal organs. Bone is an anisotropic, viscoelastic material and is extremely dynamic with the ability to remodel in response to the forces and stresses applied to it. As a result, the structural and mechanical properties change with the forces acting on the bone.

There are four bones involved in the thigh and leg regions. They are the femur, tibia, fibula, and patella. These bones are shown in Figure 2.1 on the following page.

The femur (Figure 2.2) is the longest and heaviest bone in the body. Proximally, it articulates with the acetabulum of the pelvic girdle and distally, it articulates with the tibia. The distal end of the femur has several significant landmarks. The medial and lateral condyles of the femur are of particular importance. These condyles are the articulating surfaces on the femur relative to the knee joint. A condyle is defined as a large, rounded articular prominence on a
bone. On the femur, these prominences are convex and asymmetric. The medial condyle is longer and larger than the lateral condyle, however the width of the medial condyle is slightly smaller than the width of the lateral condyle.

Anteriorly, the condyles are separated by an asymmetric, shallow groove called the patellar surface where the patella itself articulates. Posteriorly, the condyles are separated by a depressed area called the intercondylar fossa.

Figure 2.1

Bones of the Lower Limb
The tibia (Figure 2.3) is the medial bone of the leg. This bone transmits the majority of weight of the body. The tibia articulates at the proximal end with the femur and the fibula and at its distal end with fibula and the talus bones of the ankle. The tibia expands at its proximal end into the medial and lateral condyles of the tibia which articulate with the femoral condyles. When comparing the femoral and tibial condyles, the circumference of the femoral condyles is twice as large as the length of the tibial condyles (Norkin, Levange, pp. 1983). The inferior surface of the lateral condyle articulates with the head of the fibula. The
The tibial condyles are also asymmetric, but are concave in shape. The articulating surface of the medial condyle is fifty percent larger than that of the lateral condyle (Norkin, Levange, 1983). The two tibial condyles are separated by a raised area called the intercondylar eminence. This proximal surface of the tibia is also referred to as the tibial plateau. Also of note is the tibial tuberosity. The tibial tuberosity is a roughened raised surface on the anterior side between the tibial condyles that serves as a muscle attachment site.

Figure 2.3
The Tibia and the Fibula
The fibula (Figure 2.3) and the patella are the two remaining bones. The fibula is lateral to the tibia and is much smaller. It is a non-weight-bearing bone that serves mainly for the attachment of muscles. It articulates in the ankle but not in the knee joint. Lastly, the patella, or the kneecap as it is commonly known, is a sesamoid bone that forms within the quadriceps tendon. The patella articulates only with the femur.

The next type of connective tissue is cartilage. Cartilage is a relatively solid type of connective tissue that can resist deformation well. It is, in fact, much more resilient than bone. Cartilage consists of a dense network of collagen fibers embedded in an intercellular matrix. The matrix in cartilage consists of chondroitin sulphate, a jellylike substance. As a result, the strength of cartilage is due mainly to the collagen fibers, while the ability of it to return to its original shape can primarily be attributed to the chondroitin sulphate. The primary functions of cartilage are transferring forces between articulating bones, distributing forces within joints, and allowing joint to move with minimal friction.

There are two types of cartilage involved in the construction of joints. The first type is fibrocartilage. The intercellular area of fibrocartilage consists primarily of bundles of collagen fibers although elastin fibers may also be present. The collagen fibers in this type of tissue are thick and heavy making fibrocartilage an extremely strong tissue. Fibrocartilage must be so strong and rigid because it
forms the intervertebral discs of the spine and the menisci of the knee joint and must withstand the weight of the body while allowing for joint movement. The menisci of the knee are two (medial and lateral) asymmetric, wedge-shaped fibrocartilaginous joint discs that are located on the tibial plateau. The menisci serve two major purposes. First of all, they deepen the articulating surfaces of the tibia enhancing the stability and improving the congruency of the articulating surfaces. And, secondly, they help to distribute the load across the joint during weight bearing.

The second type of cartilage involved in the construction of joints is articular, or hyaline, cartilage. Articular cartilage is the most common type of cartilage in the body. In articular cartilage, the cells are widely separated by the intercellular substance. The intercellular substance in articular cartilage consists of collagen, water, and protein-polysaccharides. As a result of its basic structure, articular cartilage has a glassy appearance and provides for a smooth surface. This type of cartilage can primarily be found covering the ends of bone at joints that are freely moveable. The main functions of articular cartilage are to absorb shock and to reduce friction within the joint. The large amount of water present in articular cartilage facilitates its ability to perform these functions. During joint motion when the joint is compressed, the fluid flows out of the cartilage and into the joint. After joint compression ceases, the fluid flows back into the cartilage
tissue. As a result of the presence of this fluid, healthy articular cartilage provides an articulating surface that is relatively friction-free.

Tendons and ligaments are the next types of connective tissue. They are both comprised of a special type of connective tissue called dense connective tissue. Dense connective tissue is characterized by close packing of collagen and elastin fibers. In ligaments, collagen fibers are mixed with elastin fibers, while in tendons primarily collagen fibers are present. These constitutive differences are a result of the functions that they each must perform. In the body, ligaments connect bone to bone and tendons connect muscles to bone. The arrangement of collagen fibers and the amount of elastin fibers present vary from joint to joint depending on the extent of stability and mobility that the tendons and ligaments must provide within a joint.

Although there are many ligaments and tendons present in the knee joint (Figure 2.4), only a few will be discussed here. The first are the collateral ligaments. There is a medial and a lateral collateral ligament (MCL and LCL). The medial collateral ligament is a broad band that is fused posteriorly with the capsule of the knee joint. It runs from the medial condyle of the femur to the medial condyle of the tibia. The lateral collateral ligament runs from the lateral condyle of the femur to the head of the fibula. The collateral ligaments serve to provide medial-lateral stability of the knee joint. The next ligament is the anterior cruciate ligament, or ACL. The ACL arises from the anterior surface of the tibia
and extends superiorly and posteriorly to attach to the posterior part of the medial surface of the lateral femoral condyle. The last ligament to be discussed is the posterior cruciate ligament, or PCL. The PCL arises from the posterior aspect of the tibia and extends to the anterior portion of the medial surface of the medial femoral condyle. The cruciate ligaments are responsible for providing anterior-posterior stability as well as providing rotary stability. Also of importance is the joint capsule. The joint capsule is a ligamentous sheath that encapsulates the joint. It is made up primarily of muscle tendons or expansions of them. Although the joint capsule does contain many gaps, the capsule does provide some overall stability within the knee joint.

Figure 2.4

The Ligaments of the Knee

(Totora and Anagnostakos, 1990, Page 222)
The patellar ligament is also of importance. The patellar ligament is an extension of the quadriceps femoris tendon. The quadriceps femoris tendon runs from the quadriceps muscle group on the anterior portion of the thigh to the patella. From there it becomes the patellar ligament and goes on to attach to the tibial tuberosity. This system attributes greatly to the mechanics of the knee's extension motion and will be discussed further.

The final type of tissue is muscle tissue. Muscle tissue contains cells, which appear as long fibers, that are specialized for contraction. Because of this capability muscle tissue provides motion, maintains static posture, and produces heat. There are several type of muscle tissue, however the only type we are concerned with is skeletal muscle. Skeletal muscle is called this because it is attached to bones. Other definitive characteristics are that skeletal muscle appears striated and can be voluntarily controlled unlike other types of muscle.

The muscular anatomy of the knee consists of three major group: the quadriceps, the hamstrings, and gastrocnemius. The quadriceps cover the anterior, medial, and lateral sides of the femur (Figure 2.5). There are four separate muscles in this group.

The rectus femoris appears as a separate muscle group at its origin. It is the only muscle of the quadriceps group that also passes across the hip where it serves to flex the hip. The other three heads are the vastus medialis, vastus lateralis, and the vastus intermedius. All three of these originate from landmarks
on the superior aspect of the femur. All four heads join together and insert on the patella which then attaches to the tibial tuberosity via the patellar ligament. This method of attachment serves to reduce friction within the knee joint and provides added leverage by holding the tendon away from the axis of motion (Jenkins, 1991). All four heads of the quadriceps act to extend the leg. They are the chief extensors of the knee joint.

Figure 2.5

The Musculature of the Thigh- Anterior

(Jenkins, 1991, Page 241)

The second muscle group is the hamstrings (Figure 2.6). There are three muscles in this group. They are the semitendinosus, semimembranosus, and
biceps femoris. All three of these muscles arise from the ischial tuberosity, a bony landmark on the pelvic girdle. They are all two joint muscles, or biarticular. At the hip they serve to extend the thigh. The semitendinosus inserts on the tibia inferior and medial to the tibial tuberosity. The location of this insertion is sometimes referred to as the pes anserinus. The semimembranosus inserts on the posterior, medial side of the tibial medial condyle. The biceps femoris inserts onto the head of the fibula. These three muscles comprise the posterior region of the thigh, and, at the knee, they serve to flex the leg. The last muscle is the gastrocnemius (Figure 2.6). It is a muscle of the leg that arises on the lateral and medial condyles of the femur and inserts on the calcaneus of the foot. It’s action at the knee is to flex the leg.

Figure 2.6
The Musculature of the Thigh and Leg- Posterior

(Jenkin, 1991, Page 263)
There are two articulations in the knee joint. They are the patellofemoral joint and the tibiofemoral joint, which are both synovial joints. A synovial joint is a joint in which there is a space between the articulating bones. The bones are in contact, but they are not connected. The space in between the bones is referred to as the synovial cavity. Due the synovial cavity and the arrangement of the ligament structures, synovial joints are freely moveable (diarthrotic). Synovial joints are surrounded by an articular capsule, also referred to as a joint capsule. The inner layer of the articular capsule is formed by the synovial membrane. A loose connective tissue with elastic fibers and some adipose (fat) tissue makes up this membrane. This membrane secretes a liquid called synovial fluid. The synovial fluid provides nourishment for the articular cartilage and also lubricates the joint. The amount of synovial fluid present varies from joint to joint. In the knee, a thin, viscous layer of approximately 3.5 ml of synovial fluid is present (Totora, Anagnostakos, 1990). The synovial fluid forms a thin film over the joint surfaces reducing friction and enhancing joint movement with minimal wear.

Two types of joints are represented in the knee complex (Figure 2.7). The patellofemoral joint is a gliding, or arthroidal, joint. It is the action between the patella and the patellar surface of the femur. The primary function of this articulation is to maximize the effectiveness of the quadriceps muscles when extending the leg. The tibiofemoral joint is a hinge joint, which is a type of diarthrosis. Hinge joints allow for movement mainly in a single plane. A hinge
joint is distinguished by the fact that in a joint of this type the convex articulating surface of one bone fits into the concave surface of another bone. The primary movements allowed by a hinge joint are flexion and extension.

Figure 2.7
The Knee Joint- Articulations

The motions at a joint are the result of one joint surface moving on the another. Terms like roll, slide, and spin are used to describe the movement of the articulating surface on one another. Generally, motion within joints is, in actuality, a combination of these motions. In describing the specific motions of any joint, certain terminology is needed. First of all, the range of motion of a joint is the entire range that a segment can move in a certain direction. Range of
motion is usually measured in degrees and is sometimes referred to as the anatomic range of motion because motion is limited to the anatomic bounds. There are four motions that take place at the knee. They are flexion, extension, lateral rotation, and medial rotation. Flexion is defined as a decrease in joint angle, while extension is an increase in this angle. Rotation in limbs occurs around the longitudinal axis. Internal rotation moves toward the midline, while external rotation moves away from the midline. Two other terms that may be of use are abduction, which is movement of a body segment away from the body, and adduction, which is the movement of a body segment towards the body. With this basic understanding of motions within the body, an overview of the specific motions of the knee can be presented.

In the knee, the vast majority of motion takes place in the sagittal plane. The primary motions are flexion and extension. For flexion/extension in the sagittal plane, the range of motion averages from 0 to 140 degrees (Nordin, Frankel, 1990). It has been noted that a range that goes from full extension (0 degrees) to 117 degrees of flexion is needed for an individual to be able to perform the activities of daily life (Nordin, Frankel, 1990). Rotation does take place at the knee, however, it can only take place when the knee is flexed. The maximum of rotation is reached when the leg is flexed to 90 degrees. Rotation increases until its maximum then, after 90 degrees of flexion, rotation begins to
decrease again. At 90 degrees, the maximum range for external rotation is about 30 degrees (Nordin, Frankel, 1990).

Describing the overall motion relative to the joint as the knee moves from extension to flexion, the following processes take place (Tria, Klein, 1992). The contact point on the femur begins to move posteriorly. This motion is a combination of rolling and sliding. Because of the larger size of the femoral condyle and medial tibial plateau surfaces when compared to the lateral surface, the traveled distance on the medial side is greater than that traveled on the lateral aspect. As a result, the tibia internally rotates on the femur as flexion occurs. The opposite process takes place as the knee extends. At terminal extension, the tibia externally rotates and "locks" into position under the femur. This locking process at full extension is called the screw-home mechanism.

This thorough review of the anatomy, physiology, and kinesiology of the knee will provide the basics to understand the mechanics of the knee. Mechanically, the knee has two major needs. First of all, the joint must be mobile enough to allow for the full range of motion needed for locomotion. Secondly, the knee must have enough stability to absorb the forces caused by both the weight of the body and the reaction force (Kreigbaum, Barthels, 1990). The anatomical structures and their individual mechanical properties provide both the strength and the flexibility to do this. In the following chapter, an overview of the biomechanical properties of the knee will be presented.
CHAPTER 3

BIOMECHANICS OF THE HUMAN KNEE

Biomechanics is the use of the fundamentals and methods of mechanics and engineering as they pertain to the human body. The same approaches that are used in engineering are utilized in biomechanics, only the system they are applied to is the human body. Terms such as lever, moment arm, torque, force, and mechanical advantage are as important in the field of biomechanics as they are in the field of structural engineering. In the following chapter, a general review of biomechanics, mechanics, and how they both apply to the knee joint will be presented. There are many practical applications for the field of biomechanics ranging from biology, to medicine, to human factors engineering. As a result, a general understanding of the field and how it applies to this research is necessary.

Biomechanics is a relatively new area of science. One definition that has been provided is: “Biomechanics is the science that examines forces acting upon and within a biological structure and effects produced by such forces (Nigg, Herzog, 1994).” Biomechanical research covers many areas. Studies performed about the functions of muscles, tendons, bones, the effects of loads on the body, as well as methods that can be applied to improve physical performance are all prominent areas of research within the field of biomechanics. One area of interest
relevant to this study is the use of biomechanical analysis for medical research and design.

The theoretical basis for biomechanics is the same as that for classical mechanics, Newton's laws of motion. The laws are as follows:

First law: A particle will remain in a state of rest or move in a straight line with constant velocity, if there are no forces acting upon the particle.

Second law: A particle acted upon by an external force moves such that the force is equal to the time rate of change of linear momentum ($F=ma$).

Third law: When two particles exert force upon one another, the forces act along the line joining the particles and the two force vectors are equal in magnitude and opposite in direction.

Their significance to biomechanics is as astounding as the effect they have in the approaches for classical mechanics. Overall, when approaching the field of biomechanics it is important to remember that the principles of mechanics as listed above remain the same.

As stated previously, mechanically the knee joint has two major needs. The joint must be mobile enough to allow for the needed range of motion, and it must be stable enough to withstand the large forces that the knee is required to
transmit. The structures in the knee make the joint capable to fulfill these requirements. The ligaments, bone, and musculature of the knee all contribute to the joint’s ability to perform these functions.

The ligamentous structure of the knee, in particular, contributes to the dual role that the knee must play. For example, when the knee is extended, the ligaments are arranged in such a manner that they provide added stability, and when the knee is flexed the ligaments become looser which allows the knee to adapt to changes in the direction of motion during locomotion. This example represents a key biomechanical characteristic of the knee. In short, the knee is more stable when extended and less stable when flexed (Kreigbaum, Barthels, 1990).

The musculature of the knee also contributes to the duality of the knee. Anterior support is provided by the quadriceps, medial by the sartorius and gracilis, lateral by the tensor fascia lata, and posterior by the hamstrings and gastrocnemius. As a result of these supporting structures, stability is provided by the muscles surrounding the knee joint. These muscular groups also, obviously, provide for the mobility in the joint as they are all principle movers of the knee.

The knee joint and its structures have many other biomechanical properties. Brief summaries of some of these characteristics are presented in the following paragraphs.
The knee is required to withstand immense joint reaction forces. In fact, the forces across the tibiofemoral joint can exceed four to six times total body weight (Tria, Klein, 1992). Smidt (1973) performed a biomechanical analysis of knee flexion and extension. In doing so, he calculated the magnitude of the forces present at the tibiofemoral joint. In his study, he determined that the magnitude of the compressive and shear forces at the joint were dependent on both the external load and on the degree of the joint angle. The study revealed that compressive force steadily declined during extension and the force gradually increased during flexion. The peak forces as determined by Smidt were 275 kg for extension and 269 kg for flexion. This analysis was done in a non-weightbearing situation. Therefore, the results show how much joint reaction force can be produced by only the muscles and other external forces.

As shown, the forces within the knee can become extremely large. However, the joint has its own system to distribute the load across the joint (McLeod, Hunter, 1980). At the patellofemoral joint, compressive forces from the femur are first absorbed by the femur. These forces are then transformed into tension forces that are found in the quadriceps tendon and the patellar ligament. This process enables the quadriceps muscle group to retain the femur. Finally, the viscoelastic quadriceps muscle and tendon group of the quadriceps act as a shock absorber. This action at the patellofemoral joint is significant because it protects
the body's other joints, especially the tibiofemoral joint, by distributing the shock loading in the knee.

The compressive loads that act across the tibiofemoral joint are still quite large. This is because this joint is responsible for transmitting the weightbearing loads. Although the force itself may not decrease, two structures help to decrease the stress over the knee joint. The first structures are the menisci. The menisci of the knee joint help to decrease some of the stress at the articulation. The menisci accomplish this by increasing the contact area over which the force acts. The arrangement of the collagen fibers in the menisci is such that the discs, essentially, expand under the compressive forces due to motion or weight-bearing causing the contact area to increase. Synovial fluid also assists in decreasing the stress within the joint. This is also accomplished by increasing the surface area of the joint thereby reducing the overall stress. The surface of the joint is increased by the flow of the fluid into the empty spaces within the joint during motion.

Besides assisting to decrease the stress within the joint, synovial fluid has another major mechanical function within the knee. As stated before, the friction in synovial joints is minimal. This effect is due primarily to the presence of synovial fluid within the joint. Although there is less than one cubic centimeter of synovial fluid in the knee joint, the effect the fluid has on the mechanics of the joint is remarkable. During flexion and extension, the synovial fluid moves within the joint. This allows the fluid to lubricate the entire articular surface. The
lubrication mechanism of the joint provides a near friction-free surface. In a healthy joint, the coefficient of friction is extremely low, ranging from 0.001-0.003. These values are extremely low, demonstrating the idea that when healthy the body functions, in some manners, very efficiently.

To gain a full understanding of the biomechanical processes that take place within the knee joint, a description of the mechanics of the bone ends moving upon each other is needed. In most studies about the knee joint, the knee is treated as a fictitious hinge. However, in reality the motion consists of a combination of rolling and sliding as the articular surfaces move on each other (Wongchaisuwat, Hemami, Buchner, 1984). In the study cited above the joint was not explored in three dimensions, however a realistic description of the motions is presented. In the study, “Control of Sliding and Rolling at Natural Joints,” by Wongchaisuwat, et al., the femoral surface is assumed to be an ellipse and the tibial surface a line. As in many studies, friction is neglected due to the presence of synovial fluid. The authors assumed that the tibia acted as a pendulum that swings about the femur without losing contact with the joint surface. In analyzing the motion of the joint surfaces, they divided the surface motion into three phases. First, with the knee in full extension, the femoral and tibial condyles are interlocked. Secondly, as the knee flexes, the condyles begin to roll like wheels. Then as flexion progresses, the ACL ligament becomes taut causing the tibia to not only roll, but also glide on the femoral condyles. Finally, as the knee moves
into full flexion, a pure gliding motion takes place as the tibia glides in order to maintain contact with the posterior portion of the femoral condyles. According to the study, the range of pure rolling is about 35 degrees, the range for combined rolling and gliding is about 50 degrees, and the range for pure gliding motion is approximately 55 degrees. In many studies, the motion is treated as either gliding or rolling, however, the more accurate description utilizes a combination of the two.

In conjunction with the motion of the bone ends, the biomechanics of the joint surfaces play a major role in the overall mechanics of the joint. The bone ends are covered with articular cartilage. Many studies also choose to neglect this fact and to treat the articulations as rigid. This approach is not accurate in that the joint surfaces are in actuality deformable. This is due primarily to the effects of the articular cartilage. However, bone also contributes to this effect. Bone is quite rigid, but it is a viscoelastic material that does experience some deformation when under loaded conditions. The effects of the bone deformation is minor, however, when compared to the effect of the articular cartilage. The manner in which articular cartilage withstands loads gives it the ability to sustain the large loads that cross joints without failing as well as allowing the cartilage to protect the subchondral bone from dynamic loads (Freeman, 1972).

In addition to lubricating the joint surfaces, articular cartilage improves the mechanical functioning of synovial joints in many ways. First of all, the bone
ends that meet in articulations are incongruous and there are small and large
collections within bones that causes them to not mate perfectly. Articular cartilage
improves this by compensating for the incongruities, leading to a increased
contact surface and, in turn, decreased stress at the joint. Secondly, most loads
that are applied to the body are large and applied rapidly. As a result, the bones
could be exposed to high impact loads that could lead to damage. Articular
cartilage acts as a shock absorber in this situation by damping the forces. Finally,
articular cartilage evenly distributes the loads across the joint to protect the bone
from the large loads that the joint experiences. Articular cartilage is able to adapt
and handle these mechanical challenges because of the manner in which it
deforms.

Articular cartilage displays the following characteristics when loaded in
compression (Freeman, 1972). When the load is initially applied, there is an
instantaneous deformation. This initial deformation constitutes a fourth of the
long term equilibrium deformation. After the instantaneous deformation, there is
a creep phase where the deformation increases continuously as a function of time
at a decreasing rate until an asymptotic value is reached. This creep phase in the
cartilage is due in part to the fluid flow within the matrix and to the time-
dependent characteristics of the extracellular matrix. This deformation pattern
demonstrates the viscoelastic properties of the material. The amount of
defformation that takes place is dependent on the magnitude of the applied load, as
well as the pressure within the matrix, and the elastic properties of the organic matrix.

Because articular cartilage is a "soft" tissue with a compressive modulus of less than 1.5 Mpa (Mow, Ateshian, Spiker, 1993), additional support must be provided to withstand and distribute the large joint loads. The additional support is provided by the fluid flow within the cartilage. The water content of articular cartilage ranges from 60 to 85 percent. When cartilage undergoes a deformation, the fluid within the tissue will flow. This is the result of a process called ploughing. Ploughing of cartilage is the alternate loading and unloading of the cartilage as the joint moves through a range of motion. This cyclic loading causes interstitial fluid flow and, in turn, fluid pressurization as well as energy dissipation. The pressurization of the fluid will, as a result, provide extra load support throughout the joint. As shown, articular cartilage plays a key role in increasing the mechanical abilities of synovial joints.

Other joint structures that have special mechanical properties are ligaments. Ligaments serve three major mechanical purposes. They transmit loads, maintain proper anatomical alignment of the joints, and guide joint motions. Ligaments display nonlinear behavior under deformation conditions. In general, they are usually modeled as a nonlinear, elastic material (they are actually a non-linear viscoelastic material). The reason for the nonlinear behavior has not been discovered. In looking for a structural explanation, some researchers have
concluded that the collagen fibers that make up ligaments are nonlinear themselves and behave nonlinearly as they are tensed. Ligaments, like articular cartilage, contain a large amount of water. They are made up of almost two-thirds water. The water in the ligaments is known to contribute to the viscoelastic behavior of the ligament. The manner in which water is involved in the mechanical behavior of ligaments is also an area that needs more research as it has not, to this point, been accurately explained. The reason for the vague information regarding the mechanical properties of ligaments can be attributed to wide variations between ligaments due to several conditions: ligament type, gender, age, activity, drugs, and diet (Nigg, Herzog, 1994). All of these factors affect the tissue sample and lead to wide variations in the test results. As a result, wide differences in the research regarding the biomechanical behavior of ligaments exists.

The final area of biomechanics that needs to be covered is that of the mechanics of locomotion. The mechanics of locomotion involve to a great extent the biomechanics of the musculature. The mechanical properties of a muscle have been simplified into a mechanical model developed by Roberts in 1967 (Morrison, 1970). The mechanical model of a muscle “unit” has three components: a contractile element which controls the level of tension within a muscle; an elastic component which acts in series with the contractile unit; an elastic component that acts in parallel with the contractile unit. In reality a muscle is much more
complex, however this model does provide general idea of how a muscle reacts mechanically. The contractile unit becomes ‘excited’ and contracts. The elastic element in series extends at a strength determined by the strength of the contraction, but adds active tension to the muscle. The elastic element that acts in parallel to the contractile component adds a passive component of tension that must be added to the active tension of the contractile unit to obtain the total tension imposed by the muscular unit.

A study performed by Morrison (1970) regarding the mechanics of muscle function in locomotion developed many conclusions in this area. As concluded in the study, the mechanism of muscle function during normal walking consisted of two primary phases. He determined that normal walking consisted first of a period of muscle lengthening as force increases, followed by a period of contraction under a diminishing load. He also concluded that during the first phase while the muscle was lengthening, the muscle absorbs energy which could play an important role in understanding the mechanical properties of muscle. Therefore, not only does muscle stabilize and move the knee, it also produces and absorbs energy.

During normal gait, the knee flexes to approximately 65 degrees. The overall function of the knee during walking is first impact absorption by increased flexion of the knee joint, followed by a period of increased extension. In general, as the speed of gait increases, the range of motion in the knee (and other lower
extremity joints) increases. In analyzing gait, there are two phases, a swing phase and a stance phase. In normal gait the swing phase comprises 35% of the gait cycle, and the stance phase comprises 65% of the gait cycle. The stance phase begins at the instant the heel of one extremity contacts the ground, a point referred to as heel strike, and ends when the toe of the same extremity is on the ground, a point known as toe off. The stance phase is characterized by the fact that some part of the foot is in contact with the ground at all times. The swing phase begins as soon as the toe of one extremity leaves the ground and ceases just prior to the heel strike of the same extremity. Data has shown that the knee is in extension at the point of initial contact, but in 15 degrees of flexion when the joint is loaded. The knee then begins to extend during the midstance phase. Next, the knee continues to extend until the terminal point of the stance phase, and then it flexes again during the preswing phase followed by continued flexion until the midswing phase until the knee prepares for initial contact again. Although there are many other components involved in true gait analysis, this brief description should assist in explaining the motions that take place within the knee during normal walking.

This chapter provided a general overview of the mechanics of the knee joint. The discussions in this chapter will assist in understanding the modeling methods applied and the components taken into consideration with the final implant design. In the following chapter, an overview about the magnetic implant
proposal will be presented followed by a discussion on the current research in prosthetic research and design.
In the field of orthopedic implants, the replacement of entire joints has become a common method utilized to relieve the pain and deformity caused by osteoarthritis and other illnesses or injuries. In the United States alone, about 150,000 total knee replacements are performed each year. Overall, the success rates of these replacements are quite good. However, the implantation of a total joint into the body is an extremely invasive procedure that is followed by a year of intensive rehabilitation. In the following chapter, an overview of a proposed implant design will be presented, as well as research about joint implant design, but first a review of osteoarthritis, the most common condition leading to the necessity for joint replacement, will be presented.

Osteoarthritis is defined as the degeneration of cartilage that allows the bone ends to touch. As a result, the friction on the joint surfaces is increased creating a bony reaction that, in turn, increases the friction even more. Osteoarthritis is a disease that primarily afflicts the elderly population. The disease is generally thought to be a result of a combination of aging, irritation of the joint, and wear and abrasion (Totora, Anagnostakos, 1990). This type of arthritis is often referred to as degenerative joint disease. Degenerative joint disease is a noninflammatory, progressive disorder that attacks the moveable
joints in the body. The disease tends to affect the weight bearing joints of the body, such as the knee.

As stated previously, osteoarthritis is characterized by a degeneration of the articular cartilage. As the cartilage deteriorates, new bone is formed in the subchondral areas and at the margins of the joint. This results in the exposure of the bone ends at the articulation. The exposure of the bone ends leads to the formation of small bumps, called spurs, that are the result of new osseous tissue deposits on the exposed bony areas of the articulation. The bone spurs decrease the area within the joint cavity, restrict joint movement, and lead to increased friction within the joint. In general, this type of arthritis affects only the articular cartilage. The synovial membrane of the joint usually remains undamaged, allowing some friction protection from the synovial fluid. However, increased joint surface friction remains a distinct symptom of this disease.

For the knee joint, Bennett, Waine, and Bauer (1942) demonstrated that cartilage changes that occur during the normal aging process can lead to osteoarthritis of the knee (Freeman, 1972). In some cases, a predisposing condition or injury leads to the onset of osteoarthritis, however in other cases a previously healthy joint is affected. The development of osteoarthritis is not guaranteed as a part of aging. The effects of this disease and the extent to which it affects the joint varies from individual to individual. It is not understood why
individuals demonstrating the same cartilage deterioration associated with aging may develop osteoarthritis while others may not.

In its advanced stages, osteoarthritis can become extremely painful and lead to joint deformities. One method to alleviate the pain and regain motion is the replacement of the joint. Although a high success rate is associated with the replacement of joints, an alternate method has been developed by Jore Medical Corporation. The present invention involves the use of implanted repelling magnetic material. The implant system has been designed to radically reduce the pain resulting from deterioration of the articulating surfaces of weightbearing joints.

The magnetic implant theory was first devised for use in prosthetic limbs. The general idea is to create a friction-free weight-bearing, shock-absorbing system. The need for such an invention is demonstrated in the example of a below-knee amputation with an artificial leg in place. In conventional prosthetics, the remaining portion of the leg is used to bear and distribute the forces that are transmitted through the leg. As demonstrated in the previous chapter, these forces can be extremely large. The weight-bearing surface of an amputated limb is created by the skin and muscle that covers the end of the remaining portion of the limb, called a stump. The skin and muscle at the end of the stump are not designed to transmit the weightbearing loads of the body. As a result, the soft tissue can become aggravated and sore, and even be prone to breakdown and
infection. In this case, two repelling magnets, one in the tibia and one in the prosthetic, could be put in place. Technically, this would result in reduced force and pressure on the stump end as well as the transferring of the weightbearing forces through the tibia as in an unaltered limb. The proposed theory for the prosthetic implant can be seen in Figure 4.1 and Figure 4.2 on the following pages (Diagrams provided by Jore Medical Corporation).

This same theory can be applied to the articulating surfaces in weightbearing joints. As stated above, damaged or diseased joints are characterized by an increase in the friction and wear upon the joint surfaces. In theory, implanting repelling magnets into the joint would relieve the friction within the joint, as well as increasing joint space, creating a relatively friction-free environment like that present in a healthy joint. The design proposed by Jore Medical Corporation for implantation into a knee joint is shown in Figure 4.3 and 4.4 on the following pages. The design originally proposed incorporates six magnets. Four on the femoral articulating surface and two on the tibial articulating surface. These magnets would be implanted in such a manner as the two articulating surfaces would repel each other creating a friction-free joint system. The magnets are composed of a material known as neodymium-iron-boron, $\text{Nd}_2\text{Fe}_{14}\text{B}$. The material properties of the magnets are summarized in the table following the implant schematics.
An implant containing magnetic material is fastened to existing bone. Magnetic material is embedded into the socket of the prosthesis. The materials are positioned to allow the magnetic field of the implant to repel the magnetic field of the prosthetic magnet. This MAGNETIC CUSHION invisibly transfers body weight to the prosthesis and dampens the shock of impact to the limb. The conventional below-the-knee prosthesis uses the skin and underlying soft tissue as the means of weight transfer and shock absorption. The MAGNETIC CUSHION returns this burden to the skeletal structure of the limb.

Figure 4.1

Magnetic Prosthetic Design
Figure 4.2

Magnetic Prosthetic Design- Detailed
MAGNETIC SUPPORT FOR DEFICIENT JOINTS

REPELLING MAGNETS ACROSS KNEE JOINT

JOINT SURFACES SHOWING MAGNETIC IMPLANTS

Figure 4.3
Implant Schematic
The magnetic material is encased within a specially-configured body comprised of an implantable material.

The specially-encased magnetic material is designed to be threaded into a housing which is implanted into the joint surface. Varying the depth of the magnetic material alters the distance between the opposing implants arranged on each joint surface and thereby allows the strength of the magnetic cushion to be adjusted for maximum benefit.

A locking pin is inserted to prevent disruptive changes to the distance between repelling magnetic materials.

Figure 4.4

Potential Knee Implant Design
Material Characteristics - Neodymium-Iron-Boron

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<td>Coercive Force (BHc)</td>
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<tr>
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<tr>
<td>Resistivity (r)</td>
<td>W·cm</td>
<td>1.6X10^4</td>
</tr>
<tr>
<td>Compressive Strength (s)</td>
<td>kg/cm²</td>
<td>7.4X10^3</td>
</tr>
<tr>
<td>Working Temperature</td>
<td>BC</td>
<td>&lt;80</td>
</tr>
</tbody>
</table>

Table 4.1

The above materials data was provided by Ogallala Electric, distributors of the magnetic material. Neodymium is a strong rare earth magnet. The neodymium-iron-boron magnets are produced by a powder metallurgy process. They provide the highest energy per unit volume of any commercially available magnetic material (Sandler, Meghji, Murray, Springate, Sandy, Crow, Reed,
These magnets are 70% more powerful than the other magnet commonly used in medical applications (samarium-cobalt magnets). The magnet's major downfall is that they are extremely brittle. As a result, they must be handled with care. The neodymium magnets are available in a variety of shapes and sizes, therefore they can be varied for use and strength needed.

According to magnetic engineers, the magnetic force can be isolated and controlled. Therefore, the magnets would be able to be implanted with the premise that the repelling force of the magnets will not actually increase joint loads by introducing excessive shearing forces.

Although this is a relatively new project, some testing has been performed on the effects on living tissue as a result of the magnets. Tests have been performed through the Central Washington Equine & Livestock Clinic on a lamb with magnetic implants in place on the femur. The post-mortem tests were run at the Washington Animal Disease Diagnostic Laboratory at Washington State University. The implanted magnets were encased in a FDA approved material (stainless steel). This experiment was a pilot experiment to prepare methods and reliability for future FDA tests. The magnets were kept in for a short time period (2 months). The results of the test were good. No adverse pathological changes were present in the tissue surrounding the magnet.

Magnets have been used in human medical devices for the last 42 years. Most of the early research on biocompatible magnets has shown no detrimental
effects. The majority of research that shows damaging effects of magnetic field are regarding time-varying magnetic fields. The magnets for this product, however, will be static repelling magnets. This is significant because to this point no detrimental effects to human tissue for short-term or long-term static magnetic field exposure have been observed.

The intention of this study is to determine the strength and location of these magnets with the human knee joint for optimal mechanical behavior. With the knowledge of the anatomy and biomechanics of the human knee and of the potential implant design, a mathematical model of the human knee can be applied to determine the placement and strength of the magnets within the knee. Many researchers have presented varying models that represent the knee joint. In the following chapter, a review of the many versions of knee models will be presented as well as a comparison between them.
CHAPTER 5

REVIEW OF BIOMECHANICAL MODELS OF THE KNEE

Modeling of systems attempts to represent reality by mathematical techniques. Models can be extremely useful in understanding the complex mechanisms of systems. The use of models is becoming more common in the field of biomechanics. Models are developed for two primary reasons. First, models are used to increase knowledge and provide insight into reality. Secondly, models are used to estimate or predict variables of interest. In this situation, both cases apply. Models of the knee provide insight into the actual motion of the knee and of the roles of the independent structures in the overall functions of the knee, and, for this study's particular application, models should have the ability to estimate the magnet placement and force magnitude and their effect on the mechanics of the joint. In the world of biomechanics, models exist for many tasks ranging from analyzing individual structures to entire joints and whole body segments. Several mathematical models are available in the literature for the human knee joint. In the following chapter, several of these will be reviewed and discussed.

In general, knee models can be divided into two categories. The two types are kinematic models and biomechanic models. Purely kinematic models are used to describe just the motion of the joint. In the case of a knee, a kinematic model could be used to describe the motions that take place between the tibia and the
femur. An example of model of this type would be a “four-bar mechanism.” This four bar model is used to represent the cruciate ligaments (ACL and PCL) and the articulating surfaces of the femur and the tibia. A “bar” represents each of these structures. The bars are all linked together. As the joint moves, the directions of the link connections move. The model represents how the linked structures move relative to each other during a range of motion. Biomechanic models, on the other hand, consider the forces and torques in the joint. Although the relative joint motions are likely taken into consideration when analyzing the joint forces, they are generally not the primary concern of this type of model. Biomechanic models are used to determine the forces in the structures, such as the ligaments or muscles, and the joint reaction forces. The models discussed here are all biomechanic models.

There are several steps involved in making a biomechanical model. First, a definition of the question to be answered must be determined. Next, the system must be defined. The next step is to review the existing knowledge, then the model procedure can be selected. The next step is very significant and challenging. The assumptions and simplifications must be decided upon. At this point, it must be decided what to neglect and what to include, as well as how much to simplify the remaining information without overlooking anything. The next step is to put the physical entities into mathematical terms and then solve for the appropriate variables. Finally, the model must be evaluated. All results of the
model must be checked for accurateness and validity. All good models follow this basic pattern. The models presented here all have basically followed this approach, however their questions, model type, and their assumptions vary greatly. All of the knee models presented have pertinent results, however their usefulness is dependent on the application. The knee is one of the most complex joints to model. The complexity of the knee and its functions will be demonstrated in the following discussion of available knee models.

Some researchers have attempted to quantify the response of the knee to external forces or displacements. This was the basis for a study done by Crowinshield, et al. (1976). The study presents a model which accounts for the geometry, characteristics of motion, and the material properties of the knee. The model is designed to permit the evaluation of the stability of the knee in several planes in response to external loading. The main factor for stability as presented in this model is the ligament structure. The ligaments were represented by 13 separate elements that represent portions of the cruciates and the collateral ligaments as well as portions of the joint capsule. The total stiffness of the joint was determined by the sum of the 13 ligament components. The assumptions in this model are: the tibia remains fixed and the femur is free to move; no bony deformation takes place; no viscoelastic properties are incorporated; and friction is neglected. Geometric relationships were determined through experimentation and then solved for six externally applied displacements. This model provides a
good base for understanding the ligament structure of the knee. The major
downfall of the model is that the ligaments are treated as straight line elements
and the actual material properties are neglected, rather than incorporated into the
model.

The next model, is a two-dimensional dynamic model of the human knee
joint (Moeizadeh, Engin, Akkas, 1983). This model is the first dynamic model
available in the literature. In this model, the knee joint is modeled as two rigid
bodies connected by the ligaments which are modeled as nonlinear elastic
elements. As in the prior model, friction is neglected. In this model, however, the
femur is fixed and the tibia is undergoing a general plane motion relative to the
femur. The geometry of the articulating surfaces was obtained experimentally.
The ligament and contact forces were then determined from which the equations
of motion were determined. The results of this model were shown to be
consistent with the data available for the knee joint. However, this model fails in
two ways. First of all, the data describing geometry of the surfaces is limited
because only profiles are analyzed which could lead to great variation within the
results. Secondly, the model only applies for the two-dimensional case.

Another model available for the two-dimensional case was developed by
Abdel-Rahman and Hefzy (1993). In this knee model, the joint is again modeled
as two rigid bodies. The femur is fixed with the tibia rotating about it, and the
bones are connected by ten nonlinear springs representing the ACL, PCL, MCL,
LCL, and the posterior portion of the articular capsule. This model is also a
dynamic model. The main difference between this model and Moeinzadeh’s
model is that the system of equations reduces to six nonlinear algebraic equations
while the previous model included three differential equations to describe the
motion and three algebraic equations to describe the surface geometry and
corresponding contact conditions. The work basically follows that presented by
Moeinzadeh, however the equations were simplified and the joint was analyzed
for its response to sudden impact. The major downfall of this model is that it is
again for the two-dimensional case.

The final model to be discussed was developed by Tumer and Engin
(1993). This model is a three-body segment dynamic model of the human knee.
This model is unique in that it analyzes two articulations, the patellofemoral joint
and the tibiofemoral joint. Distinguishing features of this model include: two
contact surfaces for each articulation, three muscle groups (quadriceps,
hamstrings, gastrocnemius), and the primary ligaments (ACL, PCL, LCL, MCL,
and the patellar ligament). This model is extremely complete in its description,
however it is also extremely complex. In a complete description of the model 59
parameters are specified: 15 coefficients describing the four surfaces; 20
coordinates for origins and insertions of the ligaments; 4 stiffness constants for the
4 ligaments (ACL, PCL, MCL, LCL); 4 strain lengths for ligaments in extension;
2 inertial properties of the tibia; 1 length of patellar ligament length; and a
parameter describing the fixed orientation of a \((x,y)\) coordinate system. As stated, this model is also a two-dimensional model which is again a downfall for this application. An eventual three-dimensional version of this model would be useful in many applications in that this model is the only one found to date in the literature that describes the interactions between the femur, tibia, and the patella.

Many other models are available in the literature. This is just a brief explanation of a few of the basic models available at this time. In the following chapter, an overview of the model chosen for this project is presented.
CHAPTER 6

THREE-DIMENSIONAL MATHEMATICAL MODEL OF THE HUMAN KNEE JOINT

After comparing several models of the knee joint, a three-dimensional mathematical model of the human knee joint was chosen. This model was developed by Wismans, et al, (1980) at Eindhoven University, The Netherlands. This model takes into consideration the geometry of the joint surface. This is necessary because the magnetic implants will be placed into the articulating surfaces of the tibiofemoral joint. The model also takes into consideration the material properties of the ligaments. This is also needed for this application, because the ligaments will remain intact when the magnets are placed into the joint. Of all the models researched this model has the particular features needed to analyze the type of implant discussed here.

This particular model describes the articulation between the tibia and the femur. The model consists of the proximal portion of the tibia, the distal portion of the femur, and the ligaments and articular capsule that connect them together. The articular cartilage is not taken directly into consideration. As a result, deformations of the cartilage and of the bones are neglected. The reasoning behind this assumption is that the deformations of these structures will be relatively small when compared to the displacements of the joint, therefore the
surface deformations are negligible. The menisci are also neglected in this model (as with the others) due to the complexity of introducing them into the model.

This model is limited to the quasi-static behavior of the tibiofemoral joint. The model describes the relative position of the femur to the tibia as a function of both the flexion/extension angle and the externally applied load. Also, friction between the joint surfaces is neglected due to the low coefficient of friction attributed to the cartilage and the synovial fluid.

Because of the above assumptions, the articular surface of the condylar surfaces are simulated as a curved, rigid surface. This greatly simplifies the model. As a result, the contact areas between the tibia and the femur are now reduced to contact points. It is assumed that both the lateral and medial articular surfaces are always in contact with each other. These assumptions, basically, result in a system of two rigid bodies in contact at two points.

In describing the position of the femur relative to the tibia, the tibia is assumed to be rigidly fixed. A coordinate system is applied to each of the tibia and the femur. The coordinate system attributed to the tibia is a fixed, orthogonal system \((x,y,z)\) with unit vectors \(\hat{e}_x, \hat{e}_y, \hat{e}_z\). The \(y\)-axis corresponds with the axis of the tibia, the \(x\)-axis is directed posteriorly, and \(z\)-axis is directed laterally (Figure 5.1). The coordinate system attributed to the femur is also a fixed orthogonal system \((\alpha, \beta, \gamma)\) with unit vectors \(\hat{e}_\alpha, \hat{e}_\beta, \hat{e}_\gamma\). The \(\beta\)-axis corresponds with the
axis of the femur, the $\alpha$-axis is directed posteriorly, and the $\gamma$-axis is directed laterally (Figure 6.1).

In describing the two coordinate systems relative to each other the following equation is applied (Figure 6.2):

$$\bar{c} = \bar{a} + \tilde{T} \cdot \tilde{\delta}$$  \hspace{1cm} (1)

where:

- $\tilde{\delta} =$ vector in $(\alpha, \beta, \gamma)$
- $\bar{c} =$ vector in $(x,y,z)$
- $\tilde{T} =$ orthogonal rotation matrix
- $\bar{a} =$ vector from origin of $(x,y,z)$ to origin of $(\alpha, \beta, \gamma)$
The rotation matrix discussed is the result of three rotations:

$$\bar{T} = \bar{T}(\psi, \omega, \phi)$$  \hspace{1cm} (2)

Where:
- \(\phi\) = flexion/extension angle; Between \(y\)-axis and projection of \(\beta\)-axis
- \(\psi, \omega\) = angles to describe rotation of the femur

The rotation angles are Euler angles that describe the angular rotation of the moving body through three successive rotations. The first rotation of the \((\alpha, \beta, \gamma)\) system is carried out through an angle \(\phi\) about the \(z\)-axis. The second rotation is through an angle \(\psi\) about the \(\alpha'\) axis that results from the first rotation. The third rotation is through the angle \(\omega\) about the \(\beta''\) that results from both of the prior rotations. This last rotation gives the final orientation of the \((\alpha, \beta, \gamma)\) system. The vector to describe these rotations is \(\phi^T = [\psi \omega]\). If a
and \( \phi^T \) are known, the position in the \((x,y,z)\) system of any point with a given position in the \((\alpha, \beta, \gamma)\) system can now be determined.

One characteristic that sets this model apart from the others is the description of the geometry of the joint surfaces. In a study previously done by Wismans and Struten (1977), the coordinates of points on the outside surfaces of the femur and the tibia were determined. The data obtained from this experiment was implemented into the formulation of the three-dimensional knee model. In the Wismans/Struben study, a device was constructed to measure the geometrical data on an anatomical specimen. The device was able to measure surface data coordinates for the tibia and the femur separately. From the coordinate measurements taken on the articulating surfaces, equations were generated for mathematical descriptions of the articular surfaces.

The equations for the joint surfaces were determined in the following manner. The position vector \( \mathbf{c}_i \) of a point on the outer surface of condyle \( i \) of the tibia \((i=1 \text{ for the lateral condyle and } i=2 \text{ for the medial one})\) was given by:

\[
\mathbf{c}_i = x\hat{e}_x + y_i(x,z)\hat{e}_y + z\hat{e}_z
\]

Where:
- \( x, y = y_i(x,z) \) and \( z \) are the coordinates of the point
- \( y \) is considered as a function of \( x \) and \( z \)

The function \( y_i(x,z) \) is approximated by a polynomial in \( x \) and \( z \) of degree \( n \):

\[
y_i(x,z) = \sum_{i=0}^{n} \sum_{j=0}^{n-i} a_{ij}x^i z^j
\]
The coefficients $a_j$ are calculated by minimizing the function as shown:

$$
\sum_{r=1}^{m} \left\{ y_r - y_i(x_r, z_r) \right\}^2 = \sum_{r=1}^{m} \left\{ y_r - \sum_{i=0}^{n} \sum_{j=0}^{m} a_j x_r^i z_r^j \right\}^2
$$

(5)

Where:
- $m =$ number of measured points
- $x_r, y_r, z_r =$ measured coordinates of point $r$

A mathematical approximation for the femoral articular surface was also achieved in the same manner. The position vector $\delta_i$ on the relevant part of condyle $i$ of the femur is approximated by:

$$
\delta_i = \alpha \hat{e}_\alpha + \beta_i(\alpha, \gamma) \hat{e}_\beta + \gamma \hat{e}_\gamma
$$

(6)

Where:
- $\alpha, \beta = \beta_i(\alpha, \gamma)$ and $\gamma$ are coordinates of the point
- $\beta$ is a function of $\alpha$ and $\gamma$

The accuracy of this approximation is dependent on the degree of the polynomial.

From the experimental data, the degree for the tibia is 3, and for the femur the degree of the polynomial is 4. The standard deviation for the results was smaller than 0.5 mm.

As previously stated, contact must always be present between the femur and tibia at both the medial and lateral articulating surfaces. Due to this, in order for the contact conditions to remain true the following equations must hold:

$$
\vec{c} = \vec{a} + \vec{T} \cdot \delta
$$

(7)
\[ (\vec{n}_i \cdot \vec{T} \cdot \vec{\tau}_{\alpha i}) = 0 \]  
\[ (\vec{n}_i \cdot \vec{T} \cdot \vec{\tau}_{\gamma i}) = 0 \]

Where:
\( \vec{n}_i \) is an outward normal of the tibial condyle \( i \)

\[ \vec{\tau}_{\alpha i} = \frac{\partial \vec{\delta}_i}{\partial \alpha} \]  
\[ \vec{\tau}_{\gamma i} = \frac{\partial \vec{\delta}_i}{\partial \gamma} \]

\( i = 1 \) for lateral condyle
\( i = 2 \) for medial condyle

Equations (10) and (11) are independent tangent vectors to the tibial condyle, \( i \).

Equation (7) determines that the contact point at \( i \) on the femoral condyle corresponds to the contact point at \( i \) on the tibial condyle. Equations (8) and (9) require that the cross product of the normal and the tangent must be zero (normal on tibial condyle and the tangent to the femoral condyle).

The above section describes the surface geometries of the articulating surfaces. The next section will cover the second significant factor in the model, the accurate modeling of the ligament structures. As stated in Chapter 3, ligaments have specific mechanical properties that affect the manner in which they react within a joint. Through use of mathematical descriptions of the
mechanical properties of these structures, the ligaments in this model are represented as nonlinear elastic springs (although they are actually a viscoelastic material). The majority of knee models in the literature choose to model the joint ligaments in this manner as it most accurately and easily describes the behavior at the macroscopic level of the ligaments when loaded. The main problem with this assumption is that the spring models do not (and cannot) take into consideration the contact of the ligaments or the joint capsule with the external bony surfaces or with other soft tissue structures.

The constitutive equation for the ligaments and the capsule was determined from data in other literature (Trent et al. (1976), Kennedy, et al. (1976), and Noyes and Grood (1976)). From the analysis of the data in this research, Wismans et al. determined the following mathematical description of the ligaments and the joint capsule.

The ligaments and the capsule are represented by $m$ springs. The springs are connected to insertion points on both the femur and the tibia. The location of the insertion point of spring $j$ on the femur relative to the $(\alpha, \beta, \gamma)$ system is denoted by the vector $\mathbf{p}_j$. The insertion point of the same spring $j$ on the tibia relative to the $(x, y, z)$ system is denoted by a vector $\mathbf{r}_j$ in the $(x, y, z)$ system (Figure 6.3).
The positions for these insertion points were measured on the same anatomical specimen used to analyze the joint surface geometry. The strained length of the spring must be determined. This length is calculated by the following equation:

$$l_j = \sqrt{\left(\tilde{r}_j - \tilde{a} - \tilde{T} \cdot \tilde{p}_j, (\tilde{r}_j - \tilde{a} - \tilde{T} \cdot \tilde{p}_j) \right)}$$

Where:

- $l_j$ = length of the strained spring
- $\tilde{a}$ = vector connecting the origins of the two coordinate systems

This equation is determined from the original equation that relates the two coordinate systems. The mechanical behavior for the ligaments was determined as a result of the assumption that they are an elastic material. The mechanical behavior is able to be approximated by a quadratic force-elongation function that is expressed by the following equations:

$$f_j = k_j (l_j - l_{oj})^2 \quad \text{if} \quad l_j > l_{oj}$$
\[ f_j = 0 \quad \text{if} \quad l_j \leq l_{oj} \quad (14) \]

Where:
- \( f_j \) = spring force
- \( k_j \) = spring constant
- \( l_{oj} \) = unstrained spring length
- \( j \) = spring

Next the initial, unstretched length of the spring must be determined to calculate the force within the spring. The strain in the spring is defined by:

\[ \varepsilon_j = \frac{l_j - l_{oj}}{l_{oj}} \quad (15) \]

The initial length of the spring is assumed to be the length of the ligament when the joint is in full extension. At the time when this model was initially developed, no accurate data was available on the strain in soft tissue structures as a result of the flexion-extension angle. This ligament strain was estimated based on rough data available in the literature. The data used to make the ligament strain assumptions was developed by Brantigan in 1941. Based on this rough data, assumptions were made for the strain when the springs were in extension. The extension strain is referred to as the initial strain and is represented by \( \varepsilon_{ej} \).

Knowing the initial strain, the unstrained length, \( l_{oj} \), can be determined from the above equation (Equation 15) and, in turn, the spring force can be calculated.
In this knee model, the ligaments and the joint capsule are represented by 7 nonlinear, elastic springs. The lateral collateral ligament (LCL), anterior cruciate (ACL), and the posterior cruciate (PCL) are all represented by a single spring. The medial collateral ligament is modeled as two springs, one posterior (PMC) and one anterior (AMC). The posterior capsule is also represented by two springs, one medial (MPC) and one lateral (LPC). Each of the modeled springs has a different spring constant and initial strain dependent on the ligaments location and mechanical function. The values for the spring constant \( k_j \) for each spring are based on the work of Trent (1976). A listing of the spring constant values and the initial spring strain values are listed below in Table 6.1.

<table>
<thead>
<tr>
<th>Structure</th>
<th>( k_j ) (N/mm²)</th>
<th>( e_{in} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral Collateral Ligament</td>
<td>LCL</td>
<td>15</td>
</tr>
<tr>
<td>Anterior Cruciate Ligament</td>
<td>ACL</td>
<td>30</td>
</tr>
<tr>
<td>Posterior Cruciate Ligament</td>
<td>PCL</td>
<td>35</td>
</tr>
<tr>
<td>Anterior Medial Collateral</td>
<td>AMC</td>
<td>15</td>
</tr>
<tr>
<td>Posterior Medial Collateral</td>
<td>PMC</td>
<td>15</td>
</tr>
<tr>
<td>Lateral Posterior Capsule</td>
<td>LC</td>
<td>10</td>
</tr>
<tr>
<td>Medial Posterior Capsule</td>
<td>MC</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 6.1

The next set of equations describe the equilibrium of the femur. These equations represent the forces and moments on the femur. The forces and
moments can be divided into two groups, internal loads and external loads (shown in Figure 6.4).

Figure 6.4

Force and Moment Representation

The internal loads consist of the forces in the springs and the contact forces present between the femur and the tibia. The internal loads due to the springs were given previously in Equations (13) and (14). The equation representing the contact forces is:

\[ \bar{p}_1 = p_1 \bar{n}_1 \]
\[ \bar{p}_2 = p_2 \bar{n}_2 \]

Where:
- \( \bar{n}_i \) = tibial unit outward normal (\( i=1 \), lateral, \( i=2 \), medial)
- \( \bar{p}_i \) = contact force
\( p_i = \text{magnitude of contact force} \)

In the current model, there is no friction present, therefore the contact forces will coincide with the normal forces as demonstrated in the above equation. The next group of forces is the external loads. The external loads include the following: muscle force, gravitational force, inertia, and patellar force if incorporated into the model, as well as any other external forces acting on the knee. The external forces are represented by a force \( F_e \) that acts on the femur in the \((\alpha, \beta, \gamma)\) system at the origin and by a moment \( M_e \). The following equations represent the general form for the external forces:

\[
\vec{F}_e = F_x \hat{e}_x + F_y \hat{e}_y + F_z \hat{e}_z \\
\vec{M}_e = M_x \hat{e}_x + M_y \hat{e}_y + M_z \hat{e}_z
\]  

(17)  

(18)

There is also a moment \( M_r \) present that acts to achieve the flexion-extension angle \( \phi \). In most cases this moment will be zero. The only time this moment will come into play is if the joint is in extreme positions such as hyperextension. This moment is described by:

\[
\vec{M}_r = M_r \vec{\lambda}
\]  

(19)

Where:

\( \vec{\lambda} = \text{a unit vector in direction of} M_r \) found using the principle of virtual work
The equations of the equilibrium for the femur are written using these general equations. These equilibrium equations are used to find the position of the femur relative to the tibia as a function of $\phi, F_e, M_e$. The equations are:

$$\bar{F}_e + p_1\bar{n}_1 + p_2\bar{n}_2 + \sum_{j=1}^{m} f_j\bar{v}_j = 0 \quad (20)$$

$$\tilde{M}_e + M_r\tilde{\lambda} + p_1 \cdot (\tilde{T} \cdot \tilde{\delta}_1) \times \bar{n}_1 + p_2 \cdot (\tilde{T} \cdot \tilde{\delta}_2) \times \bar{n}_2 + \sum_{j=1}^{m} \{ f_j (\tilde{T} \cdot \tilde{p}_j) \times \bar{v}_j \} = 0 \quad (21)$$

Where: $\bar{v}_j =$ unit vector from tibia to femur spring insertion points

$p_1\bar{n}_1 =$ contact force at lateral tibial condyle

$p_2\bar{n}_2 =$ contact force at medial tibial condyle

$f_j =$ spring force

$\tilde{T} =$ rotation matrix

$M_r\tilde{\lambda} =$ flexion-extension moment

$\tilde{M}_e =$ external moment

$\bar{F}_e =$ external force

$\bar{n}_1 =$ unit normal at lateral tibial condyle

$\bar{n}_2 =$ unit normal at medial tibial condyle

$\tilde{\delta}_1 =$ position vector at lateral contact point on femur

$\tilde{\delta}_2 =$ position vector at medial contact point on femur
The above mathematical model produces a system of 16 nonlinear equations with 16 unknowns. The equations are: 7, 8, 9, 10, 20, and 21. The unknowns are: the components of a, the femur rotations, the variables representing the contact points, and the magnitude of the flexion-extension angle. To numerically solve this system of equations, the set is first reduced then a Newton-Raphson iteration process is applied.

This model was not validated as part of the study, however many of the results closely agreed with those present in the literature. The only major differences between the model and reported data was in reference to internal-external rotation. These differences are likely due to the absence of the menisci in the model parameters.

This model is extremely complex, however all components of the model are directly applicable to the design of a joint implant. In the following chapter, the methods utilized in analyzing the magnetic implant using this model will be explained in detail.
CHAPTER 7
APPLICATION OF KNEE MODEL TO IMPLANT DESIGN

In the following chapter, the methods utilized in the mechanical analysis of the orthopedic, magnetic implant will be discussed. As presented earlier, the analysis is based directly on the work of J. Wismans (1980). Using the mathematical model, results were calculated to compare the effects of the magnetic implants on the mechanics of the knee. Several factors came into play in this analysis. A step by step presentation of the methods used to evaluate this analysis follows.

Several variables must be calculated individually to input into the mathematical model of the knee. The first set of variables necessary to analyze the knee model is the components of the \( \mathbf{a} \) vector. The \( \mathbf{a} \) vector is a vector that runs from the tibial coordinate system origin to the femoral coordinate system origin. The \( \mathbf{a} \) vector components are needed to calculate the ligament force vectors and the joint surface contact equations. The equation used to determine the \( \mathbf{a} \) vector is:

\[
\mathbf{c}_i = \mathbf{a} + \mathbf{P} \cdot \mathbf{d}_i
\]  

(1)

Where:

\( \mathbf{c}_i \) = vector describing an arbitrary point in \((x,y,z)\) system

\( \mathbf{a} \) = vector from origin of \((x,y,z)\) system to origin of the \((\alpha,\beta,\gamma)\) system.
\( \mathbf{T} = \text{Rotation matrix} \)

\( \mathbf{\delta}, = \text{vector describing an arbitrary point in } (\alpha, \beta, \gamma) \text{ system} \)

All of the individual components listed above must be determined to find the component values for the \( \mathbf{a} \) vector. The first variable to calculate is the rotation matrix. In evaluating the implant, the only motion analyzed was flexion. Although motion does occur in all three planes at the knee to some extent, to simplify the results pure flexion in the sagittal plane was chosen as the desired motion. The model was analyzed at four flexion angles: 0, 30, 60, and 90 degrees. There was assumed to be no hyperextension. The rotation matrix is determined through the use of Bryant, or Euler angles as described in the previous chapter. This results in the following matrix for the rotation matrix:

\[
\begin{bmatrix}
\cos \phi \cos \omega + \sin \phi \sin \psi \sin \omega & \sin \phi \cos \psi & \cos \phi \sin \omega - \sin \phi \sin \psi \cos \omega \\
-\sin \phi \cos \omega + \cos \phi \sin \psi \sin \omega & \cos \phi \cos \psi & -\sin \phi \sin \omega - \cos \phi \sin \psi \sin \omega \\
-\cos \psi \sin \omega & \sin \psi & \cos \psi \cos \omega
\end{bmatrix} \tag{2}
\]

Where:

\( \phi = \text{flexion/extension angle} \)

\( \omega = \text{varus/valgus motion} \)

\( \psi = \text{internal/external rotation} \)

Using this equation, the following rotation matrices were obtained for the corresponding flexion angles:
The next step in analyzing the model is to calculate the vectors that describe the articulating joint surfaces. These vectors are $c_i$ and $\delta_j$. To calculate these vectors, accurate geometrical descriptions of the articulating joint surfaces must be obtained. In the original study performed by Wismans, the joint surfaces were measured experimentally and input into the model. At that point, the data was used to approximate polynomials describing the articulating surfaces. In this application of the mathematical knee model, the data required to represent the joint surfaces was unavailable. Without proper equipment, precise measurements of the articulating surfaces was also unavailable. As a result, an approximation of the joint surfaces was utilized.

At $\phi = 0$  
\[ \tilde{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

At $\phi = 30$  
\[ \tilde{T} = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

At $\phi = 60$  
\[ \tilde{T} = \begin{bmatrix} 0.5 & 0.866 & 0 \\ -0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

At $\phi = 90$  
\[ \tilde{T} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
The geometry of the joint surfaces was approximated using the research of Kurosawa, et al. 1985 in their journal article entitled, "Geometry and Motion of the Knee for Implant and Orthotic Design." In their study, they concluded that the posterior femoral condyles could be accurately modeled as spherical surfaces. These spheres have an average radius of 20 mm and a medial-lateral spacing of 46 mm. This study was validated to show that when analyzing joint implants, for both motions and geometry, a surface description of a sphere for the posterior surface of the femur provided reliable results. As a result, this geometry for the articulating surface of the femur was chosen for use in analyzing the mathematical knee model. The condyles of the femur were chosen to be symmetric for this application. This varies from the true geometry of the joint because, in reality the medial condyle is slightly larger than the lateral condyle leading to slight amounts of rotation as the joint moves through its range of motion. The tibia was chosen to be a flat plane upon which the femoral surface moves. Again, this varies from the true geometry of the tibial plateau, however for modeling purposes this geometric description of the tibia proves to be reasonable.

To determine the locations of the vectors that describe the joint surfaces, the locations of the coordinate systems must be assigned. At full extension, the tibial coordinate system is centered on the tibial plateau with the initial contact points of the condyles a distance of 23 mm on either side. The femoral coordinate system is placed directly above that at a distance equal to the radius of the spheres.
modeling the condyles. With the coordinate systems in place, the equations for the joint surfaces can be modeled.

First of all, general equations to describe the surface geometry are derived. Determining the equation describing the surface of the femur requires the equation that describes the surface of a sphere. The general equation for a sphere is as follows:

\[ x^2 + y^2 + z^2 = r^2 \]  

(3)

This is the case when the coordinate origin is at the center. This, however, is not the case for the knee model. As a result, the proper equation to use is:

\[ (x - g)^2 + (y - h)^2 + (z - k)^2 = r^2 \]  

(4)

Where the location of the center of the sphere is at point \((g, h, k)\). From Kurosawa, et al., we know that the radius of the sphere is 20 mm and that the contact points of the condyles are separated by a distance of 46 mm. From this information we know that the coordinates for \((g, h, k)\) in the femoral system are:

\((0, 0, +/-23)\)

With this information, the equation for the surface of a sphere can be incorporated into the model to determine the equation for the femoral joint surface. The general equation for the femoral surface is:

\[ \bar{\delta}_i = \alpha \hat{e}_\alpha + \beta_i(\alpha, \gamma) \hat{e}_\beta + \gamma \hat{e}_\gamma \]  

(5)

The \(\beta_i\) portions of this equation are determined from the spherical surface equations for each femoral condyle which are:
Lateral (1) \[ \alpha^2 + \beta^2 + (\gamma - 23)^2 = 20^2 \] (6)

Medial(2) \[ \alpha^2 + \beta^2 + (\gamma + 23)^2 = 20^2 \] (7)

These equations are solved for \( \beta \) and substituted into the general surface equation. This yields the following for the lateral and medial femoral surface equations:

Lateral (1): \[ \delta = c \alpha \epsilon_x + \left( \sqrt{400 - \alpha^2 - (\gamma + 23)^2} \right) \epsilon_y + 23 \epsilon_z \] (8)

Medial (2): \[ \delta = c \alpha \epsilon_x + \left( \sqrt{400 - \alpha^2 - (\gamma - 23)^2} \right) \epsilon_y - 23 \epsilon_z \] (9)

The tibial surfaces are also described by a general equation: Which is:

\[ \bar{c} = x \epsilon_x + y \epsilon_y + z \epsilon_z \] (10)

The equations for the tibial surface are solved in the same manner as the equations for the femoral surface. The tibial surface, however, is described by a flat plane.

As a result, the equations for the tibial surfaces become:

Lateral (1): \[ \bar{c}_1 = x \epsilon_x + 23 \epsilon_z \] (11)

Medial (2): \[ \bar{c}_2 = x \epsilon_x - 23 \epsilon_z \] (12)

Using these equations, the contact conditions are now enforced. These equations must be enforced to support the assumption of the model that requires that contact is always maintained. This is done using the following equations:

\[ (\hat{n}, \vec{F} \cdot \vec{\tau}_w) = 0 \] (13)

\[ (\hat{n}, \vec{F} \cdot \vec{\tau}_y) = 0 \] (14)
These equations were evaluated to ensure that they held true throughout the range of motion.

From this point, the coordinates of the \( a \) vector were found using equation (1). The components of \( a \) at the corresponding joint flexion angles were determined to be:

\[
\begin{align*}
\phi = 0: & \quad \begin{bmatrix} x & 0 \end{bmatrix} &= \begin{bmatrix} -(-129 - \alpha^2 - \gamma^2 - 46\gamma)^{0.5} \\
\phi = 30: & \quad \begin{bmatrix} x & 0.5(-129 - \alpha^2 - \gamma^2 - 46\gamma)^{0.5} \\
\phi = 60: & \quad \begin{bmatrix} x & 0.866(-129 - \alpha^2 - \gamma^2 - 46\gamma)^{0.5} \\
\phi = 90: & \quad \begin{bmatrix} x & (-129 - \alpha^2 - \gamma^2 - 46\gamma)^{0.5} \end{bmatrix}
\end{align*}
\]

The variables \( x, \alpha, \beta \) still need to be determined for substitution into these equations. Using data from the motion of the knee obtained from Kurosawa, et al. 1985, and checking the values with the equations used in determining the contact conditions the following values were determined for the coordinate points \( x, \alpha, \beta \) (all distances in mm):

<table>
<thead>
<tr>
<th>Lateral:</th>
<th>Medial:</th>
</tr>
</thead>
<tbody>
<tr>
<td>At ( \phi = 0 ):</td>
<td>At ( \phi = 0 ):</td>
</tr>
<tr>
<td>( x = 0 )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>( \alpha = 0 )</td>
<td>( \alpha = 0 )</td>
</tr>
</tbody>
</table>
With numerical values for these variables, the remaining portions of the mathematical model can be evaluated.

The next component of the knee model to solve is the force effects of the ligaments on the system. The ligaments are represented by a number of non-linear elastic springs. The constitutive equations are based on data presented in the literature by Trent, et al 1976, Kennedy, et al 1976, and Noyes and Grood, 1976.

For calculating the needed information for the ligament forces, the following equations are used:
\[ \varepsilon_j = \frac{l_j - l_{oj}}{l_{oj}}, \text{ for the strain in the ligament} \]  
\[ l_{oj} = \frac{l_{ij}}{1 + \varepsilon_{ij}}, \text{ for the length of the ligament} \]
\[ f_j = k_j(l - l_{oj})^2, \text{ for the force of the ligament} \]  

Using these relationships as summarized in the previous chapter, the force vectors for the ligaments were determined. The data for the length and length changes of the ligaments was obtained from a study that also used Wismans' mathematical model. This study was performed by Blankevoort, et al 1991, and is entitled, "Passive Motion Characteristics of the Human Knee Joint." One objective of the study performed by Blankevoort, et al, was to analyze the force effects of the ligaments as well as other force/moment relations. Ligament data was collected from several subjects and input into the mathematical model as developed by Wismans. The ligament length data from this source is for pure flexion only and is summarized in the Table 7.1 on the following page:
LIGAMENT LENGTH SUMMARY

<table>
<thead>
<tr>
<th>LIGAMENT</th>
<th>K(N/MM²)</th>
<th>r</th>
<th>t</th>
<th>L₀</th>
<th>L₀ₗ</th>
<th>Lₗ</th>
<th>Lₗ</th>
<th>Lₗ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>φ =</td>
<td>φ =</td>
<td>φ =</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>ACL</td>
<td>30</td>
<td>0.05</td>
<td>0.03</td>
<td>28</td>
<td>28</td>
<td>27.9</td>
<td>27.8</td>
<td>27.7</td>
</tr>
<tr>
<td>PCL</td>
<td>35</td>
<td>-0.01</td>
<td>0.03</td>
<td>26.5</td>
<td>26.5</td>
<td>26.6</td>
<td>26.7</td>
<td>26.9</td>
</tr>
<tr>
<td>LCL</td>
<td>15</td>
<td>0.05</td>
<td>0.03</td>
<td>44.8</td>
<td>44.8</td>
<td>44.75</td>
<td>44.65</td>
<td>44.5</td>
</tr>
<tr>
<td>AMCL</td>
<td>15</td>
<td>-0.03</td>
<td>0.03</td>
<td>78.8</td>
<td>78.8</td>
<td>78.8</td>
<td>78.8</td>
<td>78.6</td>
</tr>
<tr>
<td>PMCL</td>
<td>15</td>
<td>0.05</td>
<td>0.03</td>
<td>87.6</td>
<td>87.6</td>
<td>87.55</td>
<td>87.5</td>
<td>87.35</td>
</tr>
<tr>
<td>LPC</td>
<td>10</td>
<td>0.05</td>
<td>0.03</td>
<td>48.3</td>
<td>48.3</td>
<td>48.25</td>
<td>48.1</td>
<td>47.9</td>
</tr>
<tr>
<td>MPC</td>
<td>10</td>
<td>0.05</td>
<td>0.03</td>
<td>48.3</td>
<td>48.3</td>
<td>48.25</td>
<td>48.1</td>
<td>47.9</td>
</tr>
</tbody>
</table>

Table 7.1

The strain in each ligament is then be calculated from the above data. If the strain in the ligament is negative, the spring is relaxed and the force in the spring is set equal to zero. If the strain is positive, there is a force present. First, the ligament strain is calculated. Those with negative strain are represented by a zero, because they will have an effective force of zero. The majority of the ligaments had a negative strain as the joint moved into flexion. This is because most of the ligaments have a primary function of providing added stability while the knee is in extension and relaxing during flexion to allow the knee to move freely as the joint flexes. A summary of the calculated results are presented in the following table:
LIGAMENT STRAIN SUMMARY

<table>
<thead>
<tr>
<th>LIGAMENT</th>
<th>STRAIN</th>
<th>STRAIN</th>
<th>STRAIN</th>
<th>STRAIN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi = 0$</td>
<td>$\phi = 30$</td>
<td>$\phi = 60$</td>
<td>$\phi = 90$</td>
</tr>
<tr>
<td>ACL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PCL</td>
<td>0</td>
<td>3.77x10^-3</td>
<td>7.55x10^-3</td>
<td>0.015</td>
</tr>
<tr>
<td>LCL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AMCL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PMCL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LPC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MPC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7.2

From this table the force in the ligaments as a function of joint angle can be calculated. The only ligament that has a force is the PCL. As a result, the force magnitudes at the corresponding joint angles for the PCL are:

At $\phi = 0$: $f_j = 0N$

At $\phi = 30$: $f_j = 0.35N$

At $\phi = 60$: $f_j = 1.4N$

At $\phi = 90$: $f_j = 5.6N$

The next task is to determine the insertion points of the affected ligament, the PCL. The insertion points of the ligament are needed to determine the line of action of the ligament force. The general ligament force vector is given by:

$$\vec{f} = f_j \vec{v}_j$$  \hspace{1cm} (20)
Where:

\[ v_j = \frac{1}{l_j} \left( \vec{r}_j - \vec{a} - \vec{T} \cdot \vec{\rho}_j \right) \]  

(21)

Where:

\( r_j \) = the vector from the origin \( x,y,z \) coordinate system to the tibial insertion
\( \rho_j \) = the vector from the origin of the \( \alpha,\beta,\gamma \) coordinate system to the femoral insertion

The data for the insertion points of the ligaments were obtained from "Control of Sliding and Rolling as Natural Joints," by Wongchaisuwat, et al. 1984. From data presented in Wongchaisuwat, et al. 1984, the insertion points for the PCL were given, yielding the following results for the vectors from the coinciding coordinate system to the insertion point of the PCL ligament:

\( \vec{r}_j = -29\hat{e}_x \)
\( \vec{\rho}_j = 16\hat{e}_x - 16\hat{e}_y \)

Using this information and the \( \vec{a} \) vector components, the force vector calculated for the PCL at the specified joint angles are:

At \( \phi = 0 \): \( \vec{f}_j = 0 \)

At \( \phi = 30 \): \( \vec{f}_j = -9.03\hat{e}_\alpha + 7.5\hat{e}_\beta \)

At \( \phi = 60 \): \( \vec{f}_j = -40.5\hat{e}_\alpha + 11.36\hat{e}_\beta \)

At \( \phi = 90 \): \( \vec{f}_j = -192.87\hat{e}_\alpha \)
With the above information, the mathematical knee model can be evaluated using the force and moment equations. The only missing variable is the external force. The external force was chosen to be 150N in compression. This value was used for two reasons. First of all, the same external force was applied in the study that was used to determine the ligament data. Secondly, the force is reasonable load to cross the knee joint. This external force was chosen as it was large enough to have an effect on the joint mechanics without causing an excessively large magnetic force to be calculated. This force is smaller than the contact forces that typically cross the knee joint, but large enough to indicate the effects that the magnets have to the mechanics of the knee joint.

The model was initially solved for the contact force without the orthopedic magnets in place to provide baseline results to compare to. From this point the magnets were theoretically implanted into the knee and the contact forces were again solved for. In theory, the contact forces should decrease due to the effects of the magnets. The extent to which the magnets decrease the contact forces is what is to be determined.

The placement of the magnets into the mathematical model result in the addition of magnetic forces into the system of equations. The number of additional forces is dependent on the number of magnets placed in the joint for the analysis. The magnetic force was determined to be a point load, placed at the center of the magnet, directed along the line between the repelling magnets on the
tibia and the femur. The magnetic force was simplified to this extent due to the fact that the magnets can be isolated by the design of the implant system containing the magnet. The force/distance relationship of the magnets whereby the force magnitude between magnets is directly proportional to the square of the distance between them was used to determine the force. This relationship is represented by the equation:

\[ f_m = \frac{k}{d^2} \]  

(22)

Where \( k \) is the "strength" of the magnet. The true force generated by a magnet is the result of several components including the magnetic induction and the magnetic field intensity. However, in this example the forces from the magnets are being treated as point loads. As a result, the properties of the individual magnets are not used to determine force. Rather, the force will determine the field strength once an acceptable design is chosen. The magnetic design itself is an area that will require future research and is not discussed here.

Once the force relationship for the magnets was determined, the placement of the magnets within the joint was chosen. In all cases, the magnets were placed along the line of the contact points, i.e. at +/-23 mm in the z-direction of the tibial coordinate system and +/- 23 mm in the \( \gamma \) direction of the femoral system. Also, in all cases the two magnets were placed on the tibia at +/- 23 mm in the z-direction. The magnet location on the tibia was not varied within this study.
However, the placement and number of the magnets on the femur was altered, and the magnets were placed at several locations. The model was evaluated with the magnets at certain angles along the spherical surface of the femur. The angles chosen represent the location angle along the sphere not the flexion angle of the joint. The model was evaluated with one, two, or three magnets on each femoral condyle placed at several angles to obtain varied results that would represent changes in the contact force due to the implantation of the magnets.

The magnetic implant was analyzed using the mathematical model and various results were found. The following chapter summarizes the results found with this mechanical evaluation of the magnetic orthopedic knee implant.
CHAPTER 8
RESULTS

The mechanical calculations of the knee model with the magnetic implants in place yielded results that varied greatly from one implant configuration to another. As stated in the previous chapter, the model was run both with various magnet placements and without any magnets in place at all. In the following chapter a summary of the results of this study will be presented.

First of all, the model was solved for the contact forces without the magnets in place. For this analysis, the data as summarized in the previous chapter was utilized and an external compressive force was applied to the femoral surface. When solving the model equations for this example, the following results were obtained (Table 8.1):

<table>
<thead>
<tr>
<th>Flexion Angle</th>
<th>Contact Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>30</td>
<td>82.27</td>
</tr>
<tr>
<td>60</td>
<td>138.7</td>
</tr>
<tr>
<td>90</td>
<td>96.435</td>
</tr>
</tbody>
</table>

Table 8.1
These contact forces are the force on each femoral condyle, lateral and medial. The contacts forces on each femoral condyle are taken to be the same due to the symmetry of the model. Table 8.1 represents the baseline contact forces to which all future results are compared to.

From this point, a number of magnetic implant configurations were analyzed. In the first run of equations. The magnetic “strength” $k$ was chosen to make the contact force 0N when the joint was in full extension. The $k$ value determined in this instance was 296.5 N. The original design proposed by Jore Medical was utilized and two magnets were placed on each femoral condyle. Two placement configurations were attempted. In the first magnetic configuration, one magnet was placed at 0 degrees along the sphere and one was placed at 45 degrees on each femoral condyle (Case 1). In the second configuration, one magnet was placed at 30 degrees and the other at 60 degrees (Case 2). The results from these calculations are presented in the table below:

<table>
<thead>
<tr>
<th>Flexion Angle (°)</th>
<th>Contact Force for Control Group (N)</th>
<th>Contact Force for Magnets: 0 and 45 (N)</th>
<th>% Decrease</th>
<th>Contact Force for Magnets: 0 and 45 (N)</th>
<th>% Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75</td>
<td>0</td>
<td>100%</td>
<td>72.25</td>
<td>3.6%</td>
</tr>
<tr>
<td>30</td>
<td>82.27</td>
<td>71.99</td>
<td>12.5%</td>
<td>63.3</td>
<td>23.1%</td>
</tr>
<tr>
<td>60</td>
<td>138.7</td>
<td>102.35</td>
<td>26.2%</td>
<td>-147.4</td>
<td>206.3%</td>
</tr>
<tr>
<td>90</td>
<td>96.435</td>
<td>23.89</td>
<td>75.2%</td>
<td>64.6</td>
<td>33.0%</td>
</tr>
</tbody>
</table>

Table 8.2
From the summary in Table 8.2, it can be seen that the contact force does decrease when the magnets are in place. In fact, as represented by the negative value at 60 degrees of flexion for Case 2, the joint actually loses contact which violates the assumptions placed on the model making the results invalid. In an attempt to decrease the contact force, this scenario was re-evaluated using a $k$ value of 500 N. The results of this run are presented in the table below (Table 8.3):

<table>
<thead>
<tr>
<th>Flexion Angle</th>
<th>Contact Force (N) Control Group Magnets: 0 and 45</th>
<th>Contact Force (N) Magnets: 0 and 45</th>
<th>% Decrease Magnets: 30 and 60</th>
<th>Contact Force (N) Magnets: 30 and 60</th>
<th>% Decrease Magnets: 30 and 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75</td>
<td>-51.5</td>
<td>168.7%</td>
<td>70.335</td>
<td>6.22%</td>
</tr>
<tr>
<td>30</td>
<td>82.27</td>
<td>62.7</td>
<td>24.3%</td>
<td>50.25</td>
<td>38.9%</td>
</tr>
<tr>
<td>60</td>
<td>138.7</td>
<td>76.84</td>
<td>44.6%</td>
<td>-339.7</td>
<td>344.9%</td>
</tr>
<tr>
<td>90</td>
<td>96.435</td>
<td>-25.9</td>
<td>126.8%</td>
<td>47.667</td>
<td>50.1%</td>
</tr>
</tbody>
</table>

Table 8.3

As shown, the results for this example are first of all, very erratic and secondly, are invalid at certain flexion angles due to the presence of negative numbers. As a result of the widely varied values for the contact forces, several other runs were made using magnet configurations that were different than that proposed by Jore Medical.
The first alternative attempted was the use of only one magnet per femoral condyle. For this analysis, one magnet was placed at 45 degrees along the surface of the sphere and a magnet of strength $k=150$ was used. The results for the contact force in this situation is presented in Table 8.4.

### Contact Force for One Implanted Magnet: $k=150$

<table>
<thead>
<tr>
<th>Flexion Angle</th>
<th>Contact Force (N) Control Group</th>
<th>Contact Force (N) Magnet at 45</th>
<th>% Decrease Magnet at 45</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75</td>
<td>74.09</td>
<td>1.21%</td>
</tr>
<tr>
<td>30</td>
<td>82.27</td>
<td>79.25</td>
<td>3.67%</td>
</tr>
<tr>
<td>60</td>
<td>138.7</td>
<td>105.9</td>
<td>23.6%</td>
</tr>
<tr>
<td>90</td>
<td>96.435</td>
<td>23.03</td>
<td>76.11%</td>
</tr>
</tbody>
</table>

Table 8.4

As can be seen, this does decrease the contact force at the higher joint flexion angles, but does little at the lower joint angles. Due to this, another configuration using three magnets per femoral condyle was analyzed.

In evaluating the contact force with three magnets in place on the femoral condyle, four separate magnet placements were analyzed. The first placed magnets at 0, 25, and 50 degrees. The second placed magnets at 0, 22.5, and 45 degrees. The third placed magnets at 0, 30, and 60 degrees. Finally, the fourth placed magnets at 22.5, 45 and 67.5 degrees. All of these provided reasonable
results, although some were distinctly better than others. The results for these implant configurations are presented in Table 8.5.

Contact Force for Three Implanted Magnets: $k=150$

<table>
<thead>
<tr>
<th>Flexion Angle</th>
<th>Contact Force (N) Control Group</th>
<th>Contact Force (N) Magnets: 0,25,50</th>
<th>Contact Force (N) Magnets: 0,22.5,45</th>
<th>Contact Force (N) Magnets: 0,30,60</th>
<th>Contact Force (N) Magnets: 22.5,45,67.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75</td>
<td>6.17</td>
<td>5.7</td>
<td>6.915</td>
<td>72.16</td>
</tr>
<tr>
<td>30</td>
<td>82.27</td>
<td>47.25</td>
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Table 8.5

Again in this case, some of the calculated values vary immensely. However, in general, better results were found with this placement design than with the previous configurations. The best results were found when the magnets were placed at 0, 22.5, and 45 and $k$ was set to 150.

The effects of the magnets on the contact force can also be compared graphically. In the following chart, the best set of results from each magnet
configuration (one magnet, two magnets, and three magnets) is compared to the initial values found for the contact force prior to the inclusion of the magnets into the model. This graphical description demonstrates the wide differences in the contact force that the placement of the magnets within the knee joint contributes to. Figure 8.1 shows both the effectiveness and ineffectiveness that design of the implant has on decreasing the contact forces at the knee.

Magnetic Force Comparisons

![Magnetic Force Comparisons](image)

Figure 8.1
All of the data summarized in this chapter demonstrates the wide variety of results that were obtained for this study. The variances displayed among magnet placement options reflect several design questions that are essential to the mechanical effectiveness of the joint implant. The next chapter discusses the reasons for the discrepancies as well as potential solutions to alleviate the problems with the implant.
CHAPTER 9
DISCUSSION AND SUMMARY

The results of the mechanical analysis for the magnetic orthopedic knee implant were presented in the preceding chapter. The following chapter shall focus on a discussion of those results. Reasons for the results will be presented and conclusions on the mechanical effectiveness of the initial implant design will be summarized. Also, a proposal for areas of future research will be offered.

The mathematical model itself is an area that has room for discussion. In my review of literature, I have not seen a case where any mathematical model of the knee has directly been used to solve a real life problem. In validation of the models, some areas, such as ligament forces, that have a large amount of published data available have been analyzed to prove the effectiveness of the models. However, no studies were found that use a mathematical approach to approximate the effectiveness of a joint implant. As a result, there is little data which to compare. The results for the model without magnets in place demonstrate that the contact forces increase through a range of motion. These results are valid and the trend for the contact force to increase through a range of motion has been documented in many studies. However, there is discrepancy when the flexion angle is at 90 degrees. The decrease in the contact force at this point could be attributed to many factors. The approximation of the joint surface is likely the primary reason for error at this point in the evaluation. Although a
sphere adequately represents the joint surface in many instances, the effect of the change in internal motion (from rolling at lower joint angles to gliding at higher flexion angles) may affect the results at the higher joint flexion angle. Also, the absence of the menisci, the patella, muscles, and other vascular structures may contribute to the lower force values at this point. In general, the results for the initial model analysis are reasonable. Because the goal of this study was to observe the amount of decrease in the contact force attributed to the implantation of repelling magnets, the actual values are not highly significant; the amount that they decrease due to the placement of the magnets is.

Once the magnets were theoretically placed into the knee joint within the confines of the mathematical model, changes in the contact forces were observed. The extent of the change in the contact forces varied a great deal from case to case. The calculations done using the initial model design with two magnets in place on each femoral condyle yielded results that demonstrated that this model design is not as effective as planned. The contact forces did decrease throughout the range of motion, but the amount of the decrease ranged from approximately 10 N to 75 N when the magnetic strength was assumed to be 296.5 N. If a higher value was chosen for the strength of the magnet, negative values were calculated for the contact force which violates the model assumptions. Due to the wide variances in the results for the two magnet system other alternative magnet placements were proposed.
The first alternative incorporated the use of only one magnet. The magnet was placed at 45 degrees along the surface of the sphere. This alternative decreased the contact force at 60 and 90 degrees, but was nearly ineffective at 0 and 30 degrees. The large separation distance between the two repelling magnets led to this ineffectiveness. Finally, an analysis with three magnets in place on each femoral condyle was performed.

The evaluation done with three magnets yielded the best results. The magnets were placed at various locations for this approach to the model. The optimal results were obtained when the magnets were placed at 0, 22.5, and 45 degrees. In this model analysis the contact force decreased an average of 57.7 N. The amount of decrease did vary from approximately 40 N to 69 N, however the results were, in general, more consistent. The decreases were higher at lower flexion angles and lower at higher flexion angles. This observation can likely be attributed to the alternate joint surface utilized in the model calculations. Designs that used more magnets than three per femoral condyle were not attempted. Placing too many magnets would only defeat the purpose of this implant’s goal to simplify the repair of damaged joints. The results found in the model analysis for three magnets demonstrate that the magnets would be effective in decreasing the contact forces within a joint.

The best set of results were obtained when the initial design for the knee implant system by Jore Medical was altered to include one more set of magnets.
Although the results were not perfect with this design either, they were fairly consistent and contact force decreases were recorded at all flexion angles. As a result of this, I conclude, first of all that the implant design itself must be altered to represent the optimal mechanical behavior as presented here and to provide a consistent "cushion" within the joint as it moves through a range of motion. Although the alternative suggested in this study requires improvements, it will give future researchers the tools as well as guidelines upon which to base the additional designs.

The results of this study provide a baseline from which to start design and testing. Actual force testing of the magnets both in a controlled setting and implanted into living bone needs to be performed to determine the true effectiveness of the implant design. The model should not be used as the primary design tool due to the many assumptions made within the mathematical model itself and due to the approximation of the joint surface. Both of these items contribute to error within the analysis. The simplification of the magnetic force also contributes to the amount of error. However, the error due to the simplification of the magnetic force would likely lead to conservative results regarding the effectiveness of the magnets. This is because in reality the magnet force is produced by a field that affects a larger area than that produced by the point loads utilized in this analysis.
Although there was potential for error within the model analysis, the results found demonstrated the effects of the magnets as their placement and strength were altered. This reflects the fact that implanted repelling magnets do have the desired effect of decreasing the contact force and, in turn, the friction within the knee joint. The amount was dependent primarily on the placement of the magnets, but the general result was the same. The placement of repelling magnets within the knee joint changed the mechanics of the joint in that the magnitude of the contact force decreased.

This conclusion of the analysis demonstrates that the magnetic knee implant is a viable option for the alleviation of human joint damage. A great deal of future research must be done. Continued tissue and pathological testing must be performed, and mechanical testing of implanted magnets must be initiated. The mechanics of the joints with implanted magnets must be observed and additional force and moment testing must be performed to ensure that the magnets won't increase joint forces due to shearing effects. Also, monitoring must be done to ensure that the magnets do not alter the inherent mechanical properties of the tissues in which they are placed. Of particular concern is bone. Bone has the ability to remodel in response to forces placed on it. The magnetic implants must be designed so as to limit this effect to prevent detrimental remodeling of the bone surrounding the implant. Although the effects on bone are particularly important, the long term effects that the continuous repelling magnetic force will have on the
ligaments and the surrounding muscles are also of significance. Both of these tissues will eventually change in some manner due to these forces. The extent and benefits or downfalls of the adaptations should be monitored. Finally, additional theoretical evaluations should be performed. The use of finite element analysis may be one approach that would provide additional insight into the mechanics of the implants and their effects on the surrounding tissues. To summarize the suggestions for future studies, the following are possible areas for continued research:

1. Cellular and pathological testing on the long term effects of biological tissue in the presence of a static magnetic field.

2. A study on the mechanics of a joint with an actual magnetic implant in place.

3. A study on the long term mechanical effects on the individual issues themselves such as bone, ligaments, and cartilage.

4. A study to observe the changes in the muscular strength and recruitment due to the implantation of the magnets into a joint.

5. A finite element analysis of the knee joint with and without the magnets theoretically placed into the joint.

As this brief list demonstrates, the magnetic implant theory presented in this study offers many future areas of research and presents many questions that have yet to be answered.
In conclusion, although this analysis does not offer exact answers in regards to the mechanical effectiveness of magnetic knee implants, it does provide the base from which to redesign the implant for optimal mechanical performance. This study also indicates that the implants, in theory, accomplish their goal of decreasing contact force and friction. The decrease in the contact forces when the magnets are in place demonstrates this. In the future, the use of repelling magnetic implants within human joints is a viable option that has the potential to revolutionize the field of orthopedics, primarily the area of joint replacements. With proper continued research and improved design, an implant for the knee utilizing the theories of magnets may eventually lead to an alternative to full joint replacements.


Nigg, B.M., and Herzog, W., Biomechanics, Great Britain, John Wiley & Sons Ltd., 1994.


APPENDIX

GLOSSARY OF TERMS
**Abduction** Movement away from the axis or midline of the body or one of its parts.

**Acetabulum** The rounded cavity on the external surface of the coxal (hip) bone that receives the head of the femur.

**Adduction** Movement toward the axis or midline of the body or one of its parts.

**Agonist** Muscle that causes motion.

**Anatomical Position** A position of the body universally used in the anatomical descriptions in which the body is erect, facing the observer, the upper extremities are at the sides, the palms of the hands are facing forward, and the feet are on the floor.

**Anthropometrics** The area of study that is concerned with the body’s physical composition.

**Antagonist** A muscle that has an action opposite that of the prime mover (agonist) and yields to the prime mover.

**Anterior** Nearer to the front of the body. Also called ventral.

**Aponeurosis** A sheetlike tendon joining one muscle with another or with bone.

**Arthritis** Inflammation of a joint.

**Arthroscopy** A procedure for examining the interior of a joint, usually the knee, by inserting an arthroscope into a small incision.

**Articular Capsule** Sleeve-like structure around a synovial joint composed of a fibrous capsule and a synovial membrane.

**Articular Cartilage** Hyaline cartilage attached to the articular bone surface.

**Articular Disc** Fibrocartilage pad between articular surfaces of bones of some synovial joints. Also called a meniscus.

**Articulate** To join together as a joint to permit motion between parts.

**Atrophy** Wasting away or decrease in size of a part due to a failure, abnormality of nutrition, or lack of use.
Biarticulate Muscle  A muscle that crosses two joints.

Bursa  A sac or pouch of synovial fluid located at friction points, especially about joints.

Calcaneal Tendon  The tendon of the soleous, gastrocnemius, and plantaris muscles at the back of the heel. Also called the Achilles tendon.

Cartilage  A type of connective tissue consisting of chondrocytes in lucanae embedded in a dense network of collagenous and elastic fibers and a matrix of chondroitin sulphate.

Chondrocyte  A cell of mature cartilage

Chondroitin Sulphate  An amorphous matrix material found outside of a chondrocyte.

Collagen  A protein that is the main organic constituent of connective tissue.

Concentric Tension  The contraction of a muscle during which the muscle shortens and causes movement of one or more attached segments.

Connective Tissue  The most abundant of the four basic tissue types in the body, performing the functions of binding and supporting.

Deep  Away from the surface of the body.

Deep Fascia  A sheet of connective tissue wrapped around a muscle to hold it in place.

Diathrosis  Articulation in which opposing bone move freely, as a hinge joint.

Distal  Farther from the attachment of an extremity to the trunk or a structure; farther from the point of origin.

Dorsiflexion  Bending the foot in the direction of the dorsum (upper surface).

Dynamic (Isotonic) Tension  The contraction of a muscle where the muscle changes length.
**Eccentric Tension** The contraction of a muscle during which muscle lengthens and resists motion of a segment.

**Edema** An abnormal accumulation of interstitial fluid.

**Extension** An increase in the angle between two bones.

**Fascia** Connective tissue sheet or membrane that binds other structures.

**Flexion** A folding movement in which there is a decrease in the angle between the bones.

**Frontal Plane** A plane that divides the body into anterior and posterior sections.

**Hinge Joint** A synovial joint in which a convex surface of one bone fits into the concave surface of another bone, such as the elbow, knee, or ankle.

**Hyperextension** Extension beyond anatomical position.

**Inferior** Away from the head or toward the lower part of a structure.

**Insertion** The manner or place of attachment of a muscle to the bone that it moves.

**Interstitial Fluid** The portion of extracellular fluid that fills the spaces between the cells.

**In Vitro** Outside the living body and in an artificial environment.

**In Vivo** Inside of the body.

**Isokinetic** Refers to a method of strength training in which the speed of segmental rotation is kept constant throughout the range of motion.

**Isometric** A muscle contraction which the tension on the muscle increases, but there is minimal muscle shortening so that no movement is produced.

**Kinesiology** The study of the movement of body parts.

**Kinesthesia** Ability to perceive extent, direction, or weight of movement.

**Lateral** Farther away from the midline of a structure.
Ligament  Dense, regularly arranged connective tissue that connects bone to bone.

Leg  Part of the lower extremity between the knee and the ankle.

Medial  Closer to the midsagittal plane of the body or the midline of the structure.

Origin  The place of attachment of a muscle to the more stationary bone.

Osseous  Bony

Ossification  Formation of bone.

Plantar Flexion  Bending of the foot in the direction of the plantar surface (sole).

Posterior  Nearer to the back of the body. Also called dorsal.

Proximal  Nearer to the attachment of an extremity to the trunk or a structure; nearer to the point of origin.

Range of Motion  The total amount of angular displacement through which two segments may move.

Rotation  Moving a bone around its own axis, with no other movement.

Sagittal Plane  A vertical plane that divides the body into left and right portions.

Skeletal Muscle  An organ specialized for contraction, composed of striated muscle fibers, supported by connective tissue, attached to a bone by tendon or aponeurosis.

Superficial  Closer to the surface of the body.

Superior  Closer to the head or upper part of a structure.

Synergist  A muscle that assists the prime mover by reducing undesired action or unnecessary movement.

Synovial Cavity  The space between the articulating bones of a synovial joint.
**Synovial Fluid** Secrets of synovial membranes that lubricates joints and articular cartilage.

**Synovial Joint** A fully moveable diarthrotic joint in which a synovial cavity is present between the two bones.

**Tendon** A white, fibrous cord of dense regularly arranged connective tissue that connect muscle to bone.

**Thigh** The portion of the lower extremity between the hip and the knee.

**Valgus** The condition in which the segment tends outward from its proximal end to its distal end.

**Varus** The condition in which the segment tends inward from its proximal end to its distal end.