Comparing the cognitive differences resulting from modeling instruction: using computer microworld and physical object instruction to model real world problems
by Mark David Oursland

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University
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Abstract:
This study compared the modeling achievement of students receiving mathematical modeling instruction using the computer microworld, Interactive Physics, and students receiving instruction using physical objects. Modeling instruction included activities where students applied the (a) linear model to a variety of situations, (b) linear model to two-rate situations with a constant rate, (c) quadratic model to familiar geometric figures.

Both quantitative and qualitative methods were used to analyze achievement differences between students (a) receiving different methods of modeling instruction, (b) with different levels of beginning modeling ability, or (c) with different levels of computer literacy. Student achievement was analyzed quantitatively through a three-factor analysis of variance where modeling instruction, beginning modeling ability, and computer literacy were used as the three independent factors. The SOLO (Structure of the Observed Learning Outcome) assessment framework was used to design written modeling assessment instruments to measure the students’ modeling achievement. The same three independent factors were used to collect and analyze the interviews and observations of student behaviors.

Both methods of modeling instruction used the data analysis approach to mathematical modeling. The instructional lessons presented problem situations where students were asked to collect data, analyze the data, write a symbolic mathematical equation, and use equation to solve the problem.

The researcher recommends the following practice for modeling instruction based on the conclusions of this study. A variety of activities with a common structure are needed to make explicit the modeling process of applying a standard mathematical model. The modeling process is influenced strongly by prior knowledge of the problem context and previous modeling experiences. The conclusions of this study imply that knowledge of the properties about squares improved the students’ ability to model a geometric problem more than instruction in data analysis modeling. The uses of computer microworlds such as Interactive Physics in conjunction with cooperative groups are a viable method of modeling instruction.
COMPARING THE COGNITIVE DIFFERENCES RESULTING FROM MODELING INSTRUCTION: USING COMPUTER MICROWORLD AND PHYSICAL OBJECT INSTRUCTION TO MODEL REAL WORLD PROBLEMS

by

Mark David Oursland

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education

Montana State University-Bozeman
Bozeman, Montana

January, 1996
APPROVAL

of a thesis submitted by

Mark David Oursland

This thesis has been read by each member of thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

\[\text{Date}\] 4/1/97
Co-chairperson, Graduate Committee

\[\text{Date}\] 4/Fe\text{b} 97
Co-chairperson, Graduate Committee

Approved for the Major Department

\[\text{Date}\] 3/13/97
Head, Major Department

Approval for the College of Graduate Studies

\[\text{Date}\] 3/7/97
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Signature  Mark D. Ousland
Date  3/3/97
To my wife, Sheryl; daughters, Beth, Kirsten, and Michel; and sons, Paul, Austin, and Grant
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ABSTRACT

This study compared the modeling achievement of students receiving mathematical modeling instruction using the computer microworld, *Interactive Physics*, and students receiving instruction using physical objects. Modeling instruction included activities where students applied the (a) linear model to a variety of situations, (b) linear model to two-rate situations with a constant rate, (c) quadratic model to familiar geometric figures.

Both quantitative and qualitative methods were used to analyze achievement differences between students (a) receiving different methods of modeling instruction, (b) with different levels of beginning modeling ability, or (c) with different levels of computer literacy. Student achievement was analyzed quantitatively through a three-factor analysis of variance where modeling instruction, beginning modeling ability, and computer literacy were used as the three independent factors. The SOLO (Structure of the Observed Learning Outcome) assessment framework was used to design written modeling assessment instruments to measure the students’ modeling achievement. The same three independent factors were used to collect and analyze the interviews and observations of student behaviors.

Both methods of modeling instruction used the data analysis approach to mathematical modeling. The instructional lessons presented problem situations where students were asked to collect data, analyze the data, write a symbolic mathematical equation, and use equation to solve the problem.

The researcher recommends the following practice for modeling instruction based on the conclusions of this study. A variety of activities with a common structure are needed to make explicit the modeling process of applying a standard mathematical model. The modeling process is influenced strongly by prior knowledge of the problem context and previous modeling experiences. The conclusions of this study imply that knowledge of the properties about squares improved the students’ ability to model a geometric problem more than instruction in data analysis modeling. The uses of computer microworlds such as *Interactive Physics* in conjunction with cooperative groups are a viable method of modeling instruction.
CHAPTER 1

INTRODUCTION AND REVIEW OF THE LITERATURE

Introduction

In the mid-1980s, documents such as *A Nation at Risk* (National Commission of Excellence in Education, 1983) and *Educating Americans for the 21st Century* (National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1983) persuaded mathematics educators to debate educational reform as a matter of national concern. According to Davis (1992), the mathematics education community’s formal response was *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989).

Historically, the reasons for formal adoption of standards are: to ensure quality, indicate goals, and promote change (NCTM, 1989). The NCTM (1989) used three criteria to decide what mathematical content should be included in the curriculum standards: (a) What is needed to think mathematically?, (b) What mathematical processes are needed by society?, and (c) What applications mathematics has in our technological society? First, the NCTM (1989) viewed learning mathematics as an active process rather than a collection of mastered concepts and procedures. The NCTM (1989) stated, “‘knowing’ mathematics is ‘doing’ mathematics” (p. 7). Second, the utility of mathematics as a tool is constantly changing as computers change the way we process information (NCTM, 1989). Finally, the NCTM (1989) stated, “changes in technology and the broadening of the area in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself” (p. 7).
Student activities which enhance with each curriculum standard have two main characteristics: (a) Every activity grows out of a problem situation, and (b) activities are designed to actively involve students in mathematics.

Traditional teaching emphasis on practice in manipulating expressions and practicing algorithms as a precursor to solving problems ignore the fact that knowledge often emerges from the problems. This suggests that instead of the expectation that skill in computation should precede work problems, experience with problems helps develop the ability to compute. Thus, present strategies for teaching may need to be reversed; knowledge often should emerge from experience with problems. (NCTM, 1989, p. 9-10)

"Mathematical problem solving, in its broadest sense, is nearly synonymous with doing mathematics" (NCTM, 1989, p. 137). The NCTM (1989) stated:

The mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can—(a) use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content; (b) apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics; (c) recognize and formulate problems from situations within and outside mathematics; and (d) apply the process of mathematical modeling to real-world problem situations.” (p. 137)

An important component of mathematical problem solving is the process of mathematical modeling. Modeling is a subset of problem solving in which the task presented to the solver is nonroutine and the finding or designing of an appropriate model is part of the task (Burkhardt, 1989). Students may perform modeling by either applying models they know to new situations or building a new model (Niss, 1989). Reinterpreting a problem from a mathematical perspective gives a person mathematical power to simplify, structure, and generalize the problems. As the emphasis in the mathematics curriculum
moves towards real-life problems, mathematical modeling will naturally gain more attention (Burkhardt, 1989). Blum and Niss (1989) contended that the process of modeling develops the competencies and attitudes needed for fostering creative and explorative problem solving.

Davis (1992) stated that modeling instruction that uses new technologies to represent mathematical concepts offers better instruction than traditional methods. Facilitating a change to alternative methods of mathematics instruction requires time and energy (Davis, 1992). The rationale behind using alternative representations in mathematics is to provide students with a less foreign view of abstract concepts or processes (Fennema & Franke, 1992). In the instruction of mathematical modeling, three instructional methods are predominant: (a) two dimensional drawings, (b) physical objects (manipulative), and (c) computer mediums (Hiebert & Carpenter, 1992). Drawing has been the most traditional instructional method, but the use of physical objects has gained great popularity, especially in foreign countries (Robitailli & Travers, 1992). The newest method utilizes computer modeling, which is the focus of this study.

In the past, mathematical notation systems have been static; however, the new computer mediums offer a whole new class of dynamic and interactive notations (Kaput, 1992). One aspect of studying the educational benefits of computer media is coming to terms with the rapid and revolutionary possibilities they provide. To better understand the new environment created by computer media, a study of the interactions between mental processes and physical actions is needed (Kaput, 1992). The National Research Council (1987) recommended a systematic program for the development of pilot education systems using computer-centered microworlds that engage learners in science- and mathematics-linked tasks.

A computer microworld is a computer program that generates an interactive environment where students create and test hypotheses. Computer visualizations of
mathematical concepts that model real world situations will grab and hold a student's attention. These microworlds that model situations from outside the classroom become more than just mathematical activities—they become part of the students' lives (diSessa, 1984). For example, when a moving vehicle is simulated, students can precisely measure and study relationships between distance and time. Microworlds are used in this way to facilitate active learning. The dynamic nature of computer microworlds seems to naturally facilitate an environment where students mathematically model problem situations (Hoyles & Noss, 1992). One such microworld is Interactive Physics, which models the physical laws of nature so that students can construct and run lifelike experiments and demonstrations making the process of mathematical modeling more interesting and understandable. Interactive Physics and similar computer microworlds can act as a transitional bridge between the physical and mathematical worlds.

Research Problem

The problem of this study was to compare the modeling achievement of students receiving mathematical modeling instruction using a computer microworld and students receiving instruction using physical objects. This experiment used the computer microworld Interactive Physics and physical objects in selected sections of Math 150, Finite Mathematics, taught during the 1995 Summer Semester at Montana State University.

Definition of Terms

1. **Alternative representations** refers to familiar objects that represent abstract mathematical objects to provide students with a less foreign view of abstract concepts or processes (Hiebert & Carpenter, 1992).

2. **Affect** refers to the feeling that a student attaches to a practice or concepts (Schoenfeld, 1992).
3. **Cognitive** refers to mental concepts and processes constructed in a mental architecture that coordinates interaction between memory and recall (Schoenfeld, 1992).

4. **Computer literacy** is a person's procedural and cognitive proficiency on general computer usage. This is heightened by his/her exposure to the use of a computer as a tool.

5. **Computer Microworld** is a computer generated environment with a coherent system of metaphor designed to organize the users' thinking as they build and explore specific tools and simulations (Kaput, 1992).

6. **Extended abstract** is the use of an abstract general principle or hypothesis that is derived from the given information (Wilson, 1990).

7. **Interactive Physics** (Knowledge Revolution, 1993) is a computer microworld that simulates the physical laws of nature. It is an interactive coherent system of recognizable metaphors that forms a programming system intended to facilitate the building of physical simulations. These animated constructions help organize the user's thinking and visualization (Kaput, 1992).

8. **Mathematical modeling** is a subset of problem solving in which the task presented to the solver is nonroutine and the finding or designing of an appropriate mathematical model is part of the task (Burkhardt, 1989). Students perform mathematical modeling by applying their knowledge of known mathematical models to new situations (Niss, 1989).

9. **Mathematical modeling achievement** is cognitive behavior that a student exhibits and is identified as belonging to one of the five levels in the SOLO taxonomy.

10. **Multistructural** students use two or more discrete related pieces of information in the given information (Wilson, 1990).

11. **Physical objects** (manipulative) are concrete representations of abstract concepts or processes used for instructional purposes (Kaput, 1992).
12. **Plasticity** is a term used to explain the feature of linking actions of notation systems within microworlds (Kaput, 1992).

13. **Prestructural** students use no obvious pieces of information coming directly from the given information (Wilson, 1990).

14. **Unistructural** students use one obvious piece of information coming directly from the given information (Wilson, 1990).

15. **Relational** students use two or more pieces of information from the stem to integrate understanding of the information of the stem (Wilson, 1990).

16. **SOLO (Structure of the Observed Learning Outcome)** defines levels of attainment in the cycle of learning which measures the student's level of attaining modeling abstraction within a certain context (Wilson, 1990).

17. **SOLO superitem** is the instrument that measures student behaviors and identifies the student's position on the SOLO Taxonomy (Wilson, 1990).

18. **SOLO Taxonomy** is constructed from this theoretical basis by identifying five levels of student cognitive responses on a continuum specific to a particular modeling context (Wilson, 1990).

19. **Transfer distance** is a measure of the student's ability to apply a previous model to a new situation (Hiebert & Carpenter, 1992).

20. **Transparency** is the degree to which a notation system or medium reveals the key mathematical concepts embedded in a problem situation (Kaput, 1992).

**Need for the Study**

The recent increase in accessibility of computers in the classroom and development of interactive software is having an impact on mathematical instruction methods. Specifically, computer microworlds have unique possibilities to offer in the area of mathematical modeling instruction. To uncover the instructional possibilities, computer
microworlds are compared to familiar instruction, such as manipulatives (Kaput, 1992). A controlled comparison can define the relationship that exists between technology, mathematical modeling, and pedagogy (Heibert & Carpenter, 1992). This study may help determine which alternative representation best reveals the mathematical concepts or procedures in the problem situation.

Questions to Be Answered

1. Is there a modeling achievement difference between students receiving instruction using the computer microworld *Interactive Physics* or those receiving instruction using physical objects?

2. Does pretreatment modeling ability effect the overall modeling achievement of the students?

3. Does the computer literacy of students effect their overall modeling achievement?

4. Does method of instruction and pretreatment modeling ability interact on modeling achievement?

5. Does method of instruction and computer literacy interact on modeling achievement?

6. Does computer literacy and pretreatment modeling ability interact on modeling achievement?

7. Does method of instruction, pretreatment modeling ability, and computer literacy interact on modeling achievement?
Framework for Comparison

To narrow the modeling scope of this study, four epistemological factors are used to describe and identify each modeling situation: (a) purpose, (b) method of modeling instruction, (c) nature of model produced, and (d) method of verification to narrow the modeling scope of this study (Secretary's Commission on Achieving Necessary Skills, 1992). The purpose of the modeling activities is to teach students how to apply linear and quadratic functions to new situations. The study compares the modeling achievement of students who received two different methods of modeling instruction. Both methods of instruction have identical learning outcomes, problem solving activities, and mathematical content. The instructional manipulatives used are the only differences between the two methods of instruction. The modeling activities are designed to help students write a symbolic mathematical model to solve the problem presented. The students verified their newly created symbolic models by comparing them to instructional manipulative context.

The modeling achievement of the students is measured by comparing the cognitive behaviors of students to predetermined cognitive levels of modeling behaviors. These predetermined cognitive levels are derived from the SOLO taxonomy that are described in chapter two (Romberg, Zarinnia, & Collis, 1990). The assessment instrument used to identify cognitive modeling behaviors is SOLO superitems. Three of these tests are used to assess two linear and one quadratic problem situation. Since modeling is such a complex process, the effects of two other factors are measured: the students' beginning modeling ability and computer literacy. Other extraneous variables, such as students' interests and beliefs about mathematics, are monitored through constant comparison using class observations, student interviews, students' daily learning logs, and teacher interviews (Webb, 1993).
Review of the Literature

Introduction

Mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics (Schoenfeld, 1992). Teachers with a deep knowledge of mathematics usually tailor their teaching to a more conceptual view of mathematics, conversely teachers with a weaker knowledge of mathematics tend to use a rote structure when teaching mathematics (Jones, 1995). Many teachers have trouble teaching for both conceptual knowledge and procedural knowledge. The root of this problem is the perception that conceptual knowledge is important but takes too much time away from developing basic computational skills (Jones, 1995). Primarily, most teachers focus on procedural knowledge, yet their profession stresses the equal importance of teaching both conceptual knowledge and procedural knowledge (Schoenfeld, 1992). The mathematics profession has identified a gap between how it views mathematics and how it is taught. To reduce this gap, mathematics educators need new resources to assist in developing new perspectives on mathematical curriculum and pedagogy (NCTM, 1991).

The most influential document in promoting instructional changes in mathematics education has been the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). In the *Standards*, mathematical problem solving is stressed as the focus to teaching mathematical understanding. The authors of the *Standards* (NCTM, 1989) stated, “Problem-solving is the essence by which mathematics is both constructed and reinforced” (p. 37). The NCTM (1989) called for students to learn mathematics by (a) using problem solving approaches to investigate and understand mathematical content, (b) applying integrated mathematical problem solving strategies in order to solve problems from within and outside mathematics, (c) recognizing and formulating problems from...
situations within and outside of mathematics, and (d) applying the process of mathematical modeling to real-world problem situations.

The ability to invent a symbolic system that models a real world situation is central to mathematics (Krumholtz, 1989). This supports the notion that problem solving is at the very core of mathematics education (Schoenfeld, 1992). In the past, the mathematics that students have learned in school has had little to do with the way they have applied it outside of the classroom (Cuoco, 1995). Curricula need to be centered on mathematical habits of mind rather than on specific mathematical content (Cuoco, 1995). The problem-solving approach can be used to teach all the important mathematical concepts (NCTM, 1989). Instructors must learn the features and ramifications of pedagogy that encourages problem solving habits.

This researcher considers whether computer microworlds enhance the use of mathematical modeling in the mathematics curriculum. This investigation is divided into three main parts. The first section develops the theoretical framework for exploring pedagogical issues of mathematics education. The second section identifies the fundamental aspects of mathematical modeling. The final section defines and explores the possible roles of computer microworlds in the instruction of mathematical modeling.

**Theoretical Framework of Mathematics Education**

One of the most strongly held beliefs in mathematics education today is that mathematics should be taught for understanding. In this section old and new psychological theories of learning are used to build a framework for examining the various aspects of understanding mathematics. Then, this framework is used to discuss the key issues of learning and pedagogical theory related to understanding mathematics.
Internal and External Representations of Understanding

The framework for considering the understanding of mathematics is built on the assumption that knowledge is represented internally and that these internal representations are structured. Mathematical ideas must be represented in ways that our minds process them. Communication requires that these representations be made external in the form of spoken language, written symbols, or physical objects (Dossey, 1992). Because mental representations are not observable, discussion of the exact structure of such representations is based on inference. During the early part of this century, such discussion was viewed as nonscientific by psychologists because the representations were not observable (Hiebert & Carpenter, 1992). Presently, psychological research on mental representations is central to cognitive science (Hiebert & Carpenter, 1992).

In laying down the framework for investigating the learning and teaching of mathematics, two assumptions regarding internal and external representations are observed. First, it is assumed that there exists some relationship between external and internal representations. In a mathematical setting, it can be said that the nature of the external mathematical representations which a person uses to model an idea expresses the way a person thinks about that mathematical idea (Kaput, 1992). This view of understanding an idea is important for our discussion on instruction because the external representations (words, pictures, symbols, or objects) that students use in communicating their ideas are the only information that instructors have to assess the students' understanding of those ideas. This is not a precise observation, but an assumption that students who can represent an idea in multiple external presentations understand the idea more completely (Hiebert & Carpenter, 1992). The second assumption is that internal representations are connected to each other. Again, these connections can only be inferred, but it makes sense to view connections between ideas forming internally when they are being demonstrated externally (Hiebert & Carpenter, 1992). External connections can be constructed between
mathematical ideas in the same or differing representations. Connections between ideas are often based on similarities or differences observed when students find patterns and predictable regularities between the ideas (Hiebert & Carpenter, 1992).

To discuss connections within an internal representation, it is useful to build a visual representation of the relationship between the different ideas. These visual representations are models and are specifically referred to as networks in the literature. The two prominent types of networks found in the literature are vertical hierarchies and webs (Hiebert & Carpenter, 1992). A network of connections can be thought of as a hierarchy. Some representations fit as details underneath or within other representations. Another type of network is a web. Nodes or ideas have threads between them which represent student made connections or relationships (Hiebert & Carpenter, 1992).

This framework of external and internal representations for understanding mathematics is used to address specific implications of current research and for discussing issues of mathematics instruction, both past and present.

Learning Viewed Through External and Internal Representations

When a student has connected an idea to his/her internal network of ideas, it is considered to be understood (Hiebert & Carpenter, 1992). The degree of understanding depends on the number and strength of the connections. This definition of understanding is not new but has been a constant theme throughout the mathematical education literature of Brownell, Dewey, Polya, and Wertheimer (cited in Hiebert & Carpenter, 1992). Even though constructivism does not directly imply that facilitating the creation of relationships between mathematical ideas is vital to teaching for understanding, it certainly promotes such educational practices.

When assimilating new information, there are two kinds of connections or relationships that learners construct: (a) relationships based on similarities and differences,
and (b) relationships based on inclusion. Students usually create relationships of the first type by working on the same mathematical concept in different external representations. A problem that frequently occurs in mathematics education is that students tend to focus on similarities and differences presented by the external representation rather than those of the mathematical idea. This is why symbolic representations are so important to mathematics education. Although written symbols are very abstract, they also represent mathematical ideas very distinctly. The processes of building relationships through similarities and differences reappears throughout a student’s career in mathematics (Schoenfeld, 1992).

Connections built on inclusion relationships assume that one mathematical idea or procedure is a special case connected to another. A common way of applying problem-solving strategies is to view new problems as special cases of a general type of previously solved problem (Schoenfeld, 1992).

When the process of learning mathematics is viewed from a framework of internal and external representations, mathematical understanding is exhibited when students reveal connections or relationships between existing and new information (Polya, 1973). Learning is initiated through situations that challenge the existing network’s relationship and organization of ideas. Students change their existing network through the formation and realignment of relationships between major ideas. Aligning the student’s understanding of a mathematical idea with its true abstract meaning is the task of the instructor. The accommodation of new information only insures the formation and realignment of relationships. Making sure the learning process leads to greater mathematical understanding is the next issue to be addressed.

Teaching Viewed Through External and Internal Representations

Written symbols are keys to understanding and using mathematics because they are the most exact external representations in mathematics. Written mathematical symbols are
also an international language allowing for easy communication of ideas within the field of mathematics. Also, the abstract characteristics of mathematical symbols can be an educational drawback. When mathematics is taught using only symbols, it becomes a tool for manipulating those symbols rather than representing complex ideas. It is important to remember that these symbols are not created arbitrarily, but arise from needs of mathematics activities, science, and daily life (Dossey, 1992). The meaning of written symbols can develop from outside the symbol system by creating connections to other representations or by developing connections within the symbol representation itself. Basically, there are two types of mathematical symbols: symbols that stand for quantities and symbols that stand for relationships between quantities. Once meanings are established for individual symbols it is possible to think about creating meanings for rules and processes that govern actions on these symbols (Hiebert & Carpenter, 1992).

The argument that meaning can be derived through building relationships within a symbol system was advocated by Brownell in 1938. When building meaning for symbols by connections within a symbol system, the symbol must first represent a mathematical object rather than a mark on paper. Then, patterns about how that symbol is related to other symbols in the system can be established (Hiebert & Carpenter, 1992). Students must realize that symbols carry identities of their own and that they may be related to each other and operated on in well-defined manners.

Written symbols are an intellectual tool used to communicate what is already known to others and to personally organize and manipulate mathematical ideas. Both functions of symbols require connections: The public function requires connections with real-life representations, while the personal function requires connections within the symbol system (Hiebert & Carpenter, 1992). In either case the role of research should be toward describing and understanding how to facilitate the construction of these connections. Research must compare the strengths and weaknesses of different instructional methods to
understand what effect they have on the relationships students construct between mathematical symbols (Hiebert & Carpenter, 1992).

Understanding the meaning of both quantifier and action symbols can be facilitated by introducing alternative representations. Through using different representations, students are encouraged to construct relationships between written symbols and alternative representations. There are three predominant types of alternative representations used in the instruction of mathematical modeling: (a) physical three-dimensional objects, (b) two-dimensional drawings, and (c) computer media (Hiebert & Carpenter, 1992). The rationale behind using alternative representations in mathematics is to provide students with a less foreign view of abstract concepts and processes (Fennema & Franke, 1992). Drawings have been the most traditional instructional method, with physical objects gaining great popularity especially in foreign countries (Robitailli & Travers, 1992). Intuitively, the concrete nature of physical three-dimensional objects provides students with the most familiar alternate representation of an abstract situation or mathematical idea. Most research has shown that the instructional use of physical objects yields mixed results. This research reveals that students do not automatically transfer relationships understood, while using physical objects, to the mathematical ideas they represent (Hiebert & Carpenter, 1992). The transfer of understanding from concrete representations to abstract ideas depends on how similar the student perceives the representations (Kaput, 1992). Two-dimensional drawings are less concrete than physical objects but more flexible in adapting to abstract objects (Hiebert & Carpenter, 1992). The traditional two-dimensional drawing is considered the standard alternative representation (Kaput, 1992). Computer media are the third alternative representation and are the focus of this study. In the past, mathematical notation systems have been static; however, the new electronic media have opened up a whole new class of dynamic and interactive representations (Kaput, 1992). The key to
studying the educational benefits afforded by computer media is coming to terms with the rapid and revolutionary possibilities they provide.

It is important to note that just because one external representation is more distant from the problem context than another does not mean it is less useful (Hiebert & Carpenter, 1992). In most cases, written symbols are the preferred representation. Even though contextual, they are the most distant from the quantities they represent. It is usually helpful for students to fill this gap between quantities and written symbols with alternative representations, such as manipulatives (Hiebert & Carpenter, 1992). From this perspective, the effectiveness of an alternative representation is measured by its ability to fill the gap between quantities and written symbols. Another consideration, when measuring the effectiveness of alternative representations, is the social situation in which the representation is used. Features of a representation are personal because they are influenced by the students' previous experiences and knowledge. On the other hand, the alternative representation offers a powerful way of focusing students' attention on a shared experience for communicating ideas. Even though it is unclear what features students attach to alternative representations, the fact that they are public representations is a powerful instructional tool (Hiebert & Carpenter, 1992).

The purpose of this study is to assess the instructional benefits of a new computer microworld environment. To do this, the interactions between mental processes and physical actions of students is studied (Kaput, 1992). A benefit resulting from classroom computer usage is a change in the patterns of instruction. When a classroom is technology-rich, the learning environment becomes more dynamic because the students and the teacher become natural partners in developing mathematical ideas and solving mathematical problems. This relationship between technology, problems solving, and the teacher's role as a facilitator needs a better definition. One way to determine the instructional value of
computer media is to compare it to other alternative representations and decide which representation best reveals hidden mathematical ideas (Hiebert & Carpenter, 1992).

Mathematical Modeling

Introduction

A philosophy of mathematics instruction calls for activities that help mathematicians, teachers, and students experience the invention of mathematics (Burkhardt, 1989). To apply this philosophy to current mathematics instruction, a much broader range of instructional strategies than the standard explanation-example-exercise approach is needed (Burkhardt, 1989). In the previous sections, a framework has been developed for examining the issues of understanding mathematics through building connections. Within this framework, students do not understand mathematical ideas from a single representation but from multiple representations and from their daily lives (NCTM, 1991). The creation of a mathematical representation in a problem-solving setting makes modeling a candidate for increased emphasis in the mathematics curriculum. The modeling process naturally creates an environment where students can make connections between mathematical ideas and processes (Niss, 1989). To effectively use modeling, mathematics instructors need to know which methods of modeling instruction promote the best environment for students to create connections between mathematical ideas.

According to Dossey (1992), students can use mathematics in two ways, as a tool or as a language of communication. Students can use mathematics as a tool to distance themselves and focus objectively on observed differences in problem situations. In doing so, mathematics becomes a mirror whereby a problem situation can be separated from its surroundings. Written symbols are abstract representations that encourage analysis through manipulation to discover hidden relationships in the problem situation. Also,
students can view mathematics as a process of communication between people and groups of people. The language of mathematics is useful in defining and discussing problem situations—mathematics is a means of problem-solving communication. This perspective of mathematics is directed more toward a synthesis of mathematical concepts. Mathematical relations are constructed using mathematical language in order to provide a universal representation for communicating problem situations.

Mathematics is the use of symbols as tools to make the observable more focused. The symbols, in turn become the medium for communicating mathematical perspectives of reality. The invention of such systems in a problem setting is mathematical modeling. If the modeling process is made more creative through increased freedom and flexibility within planned activities, mathematics becomes more meaningful and personal to students (Niss, 1989). Modeling naturally uses mathematics as a tool and language for solving problems. These two perspectives create the framework for discussing mathematical modeling. Mathematical modeling instruction is discussed in three parts: theoretical, pedagogical, and assessment.

**Theoretical Basis for Mathematical Modeling Instruction**

Mathematical modeling is a subset of problem solving in which the task presented to the solver is nonroutine and the finding or designing of an appropriate mathematical model is part of the task (Burkhardt, 1989). Many examples of modeling have been reviewed and they fall into one of two categories. Representations of the modeling process either focused on the student’s knowledge or the student’s processes and products. An explanation of these two modeling types is reviewed and becomes the foundation for discussing modeling instruction.
Models of Mathematical Modeling. The first modeling type focuses on the knowledge that the student brings to the modeling situation. Skovsmose (1989) broke mathematical modeling (Figure 1) into three different cognitive domains: (a) mathematical knowledge, (b) modeling knowledge, and (c) reflective knowledge used to monitor the process of modeling. Reflective knowledge focuses on a conceptual or metaknowledge used by the student to synthesize their knowledge of mathematical concepts and modeling theories.

Figure 1. Skovsmose (1989) Model of Mathematical Modeling

Many mathematics education reform programs that emphasis mathematical modeling built their curriculum using this perspective of modeling. An example of such a program is the Computer-Intensive Algebra Program (Heid & Zbiek, 1995). CIA focused on mathematical modeling instead of classic word problems; it centers on algebraic concepts, modeling strategies, and use of multiple representations (Heid & Zbiek, 1995). The three main curriculum objectives correspond directly to the three areas of knowledge identified by Skovsmose.

The second modeling type focuses on the processes and products of the student during the modeling process. In the Standards (NCTM, 1989), the process of mathematical modeling is illustrated by the diagram shown in Figure 2. First, students must formulate the problem and decide what is important from the given information.
Next, they invent the mathematical model, which is sometimes referred to as mathematization. In this step, the students apply their knowledge of the situation and known mathematical models to identify quantifiable aspects of the situation and relationships existing between them for the purpose of creating a useful mathematical model. Once the model is created, the students use their mathematical knowledge to find a solution to the problem within the model. Since the answer only makes sense in the problem situation, the mathematical solution is applied to the problem situation. Finally, the solution needs to be assessed and analyzed through reflective thought (Warzel, 1989).

Figure 2. *Standards* Model of Mathematical Modeling (NCTM, 1989)

This perspective on the modeling process was used by mathematics educators interested in what the students need to do rather than what they need to know. A process of successive approximations is an example of this perspective of modeling (Edwards, 1995). This process is started by creating many models, each imitating more properties of the problem situation. The process is completed by searching for the best model through
comparing the different models to the problem situation (Edwards, 1995). Edwards had his students create many models and searching for the best model through transformation, interpretation, and validation.

In 1992, through the Secretary's Commission on Achieving Necessary Skills, the U.S. Department of Labor published a report about the needs of the 21st century workplace. Part of this report includes the design for a unit in mathematical modeling. The design includes four stages of mathematical modeling which correspond to the NCTM's model type: (a) evaluate and interpret data, (b) build models, (c) solve models, and (d) communicate findings (SCAN, 1992). The report (SCAN, 1992) indicated that less time needs to be spent solving theoretical models and more time needs to be spent collecting and analyzing data.

In this study, the modeling knowledge and mathematics knowledge are limited to specific contexts. The modeling strategies are limited to a single process of collecting data and fitting a function rule to the data. The mathematical concepts are limited to linear and quadratic relationships. The use of only three representations is the only limiting factor on the modeling processes. The three representations used are algebraic symbols, graphical visualization, and real-world problem situation.

**Pedagogical Basis for Mathematical Modeling Instruction**

So far, the discussion of mathematical modeling has been general and theoretical in nature. Now, the specific issues of teaching mathematical modeling are addressed. This discussion includes three pedagogical perspectives of mathematical modeling and their implications for this study.

**Pedagogical issues of mathematical modeling.** Burkhardt (1989) separated mathematical modeling into two distinct types: A mathematical model can be (a) a form of a
standard model used to illustrate the power of a mathematical technique or idea, giving students a model to explore a particular mathematical idea or process; or (b) a situation in which the students may be asked to focus on an interesting problem that can be better understood through a mathematical model. In the latter type of modeling, the emphasis is on the process of modeling itself: Power over the situation is emphasized, not the mathematical content.

When instructors are focusing on the student's ability to apply a standard model to new problem situations, the main educational consideration is to what degree the mathematical model and problem are isomorphic (Usiskin, 1989). This refers to the transfer distance of a problem which is not easily measured. The difficulty in measuring the transfer distance arises because students begin modeling a situation with different experiences, knowledge of models, modeling processes, and problem contexts. Another consideration is the students' interest in creating a mathematical model to solve the problem. Student must be motivated to creatively invent a unique mathematical model or the modeling process falls short of a solution and becomes just a simple application of the standard model (Burkhardt, 1989). Ideally, instructors must create a learning environment were the modeling process is both explicit and personally motivating (Usiskin, 1989).

Students learn about the modeling process by analyzing and assessing existing models. When instructors are focusing on the students' ability to create a model, the main educational consideration is creating activities where students are motivated to personalize the presented problem. Unless students view the problem as personally curious and relevant to their own lives, motivation, commitment, and understanding of their modeling processes are rote rather than personal insight (Burkhardt, 1989). In this type of activity, students are not just asked to build a model but to analyze and assess it as well (Niss, 1989).
Burkhardt's perspective on modeling instruction is evident in many of the mathematics education reform programs. An example of such a program was the Interactive Mathematics Program (Alper, Fendel, Fraser, & Resek, 1995). IMP begins with a motivating problem too difficult for the students to solve initially (Alper et al., 1995). Then students are given examples of easier problems to model designed to lead them to discover a standard model that can be used to solve the initial problem.

Niss (1989) divided the process of mathematical modeling into three modes: (a) Students acquire knowledge of an existing model or modeling process, (b) students perform modeling by applying their knowledge of existing models to new situations, and (c) students critically analyze and assess models.

Students acquire knowledge about existing models and modeling practices through modeling experiences (Niss, 1989). For students to successfully use the modeling process to solve problems, they must draw on a knowledge base that consists of a large number of models and modeling experiences (Burkhardt, 1989). In the process of modeling, students learn how to perceive what is invariant by observing what remains the same under variance (Kaput, 1992). Hoyles and Noss (1995) created this type of activity with the dynamic geometry environment Cabri (Texas Instruments, 1992), to teach understanding about geometric models. A picture of a car, snowman, or faces was mixed up and the students used the Cabri tools (Texas Instruments, 1992) and their knowledge of geometry to discover insights and understanding about particular geometric concepts (Hoyles & Noss, 1995).

The process of model building has its roots in communication: describing the original situation, searching for existing models, agreeing on the important attributes, choosing a suitable mathematical system, operating with the model, clarifying the aims, and accepting the mapping of the model (NCTM, 1989). The instructor must facilitate all of these actions for the students to model successfully. An educational metaphor for this type
of communication is interaction games. "Through interaction games, students can learn to naturally operate within a framework that answers the main metacognitive questions: 'What shall I do? and What can I hope for?'" (Warzel, 1989, p. 125).

A current review of mathematical research shows positive results when modeling activities treat students as engineers or builders rather than scaled-down scientists (diSessa, Hammer, Sherin, & Kopankowiski, 1991). Usiskin (1989) reasoned that students need structure to make the modeling process explicit. Dynamic games and simulations are an example of activities where students work within bases of modeling tools (diSessa et al., 1991). For example, in diSessa et al.'s (1991) research dealing with the Boxer microworld, students felt they had invented graphing of real world situations from designed microworld simulations. From these activities, they gained ownership of this type of modeling and extended this modeling process to many real-life situations independent of any lesson guidelines. Activities of this type often lead to student-driven activities which are more interesting for the students.

In this study, the modeling processes are acquiring knowledge of an existing model and applying their knowledge of existing models to new situations. The treatment activities are used to give the students experience in using one of two standard models in the context of a computer microworld or physical objects. The modeling assessment instruments are used to test the student's ability to create and use the standard models for the purpose of solving a unique problem situation.

The Standards (NCTM, 1989) have identified mathematical modeling's place in the mathematical curriculum through connections between mathematical modeling, problem solving, and mathematical content. The acceptance of a problematic approach to mathematics education has given rise to the prominence of modeling instruction (Skovsmose, 1989). Through modeling instruction, students can engage in problem
solving tasks that involve interrelationships (Figure 3) among these three curriculum components (NCTM, 1989).

Figure 3. Modeling and Mathematical Connections, *Standards* (NCTM, 1989)

Mathematics educators are not advocating new mathematical content, just new ways of presenting and viewing mathematical concepts. "We need to develop curricula centered on mathematical habits of mind rather than on specific content" (Cuoco, 1995, p. 187). Modeling can be integrated into the mathematics curriculum through using activities that include the following processes (Skovsmose, 1989):

1. A model can be used to develop knowledge by making connections with students' existing knowledge of a known model.
2. A model can lead to greater clearness and simplicity of structure to connect with existing knowledge.
3. A model can combine different theories.
4. A model can lead to new possibilities for confirmation of theoretical hypotheses.
5. A model can show examples of technical processes.
6. A model can be used to analyze the structure of a situation.
7. A model can be used to visualize difficult abstract patterns.
8. A model can be used to solve non-mathematical problems by mathematical means.
9. A model can encourage a creative atmosphere in the mathematics classroom.
10. A model can motivate students to engage in mathematics activities.

Modeling is the focus of this study, but models are only designed to represent something and should not be confused with being an exclusive part of mathematics education. All 10 of these modeling processes are used in designing the activities for this study.

All three perspectives express views of mathematical modeling that agree and support the others. Transfer distance and problem interest are two dynamic educational parameters of mathematical modeling that interact with different modes of modeling. The students' knowledge of mathematical models becomes their library for writing the abstractions of invariance. When students are motivated to solve a problem, their existing knowledge of mathematical models and the modeling processes is used to create a model for the purpose of solving a problem.

**Assessment Basis for Mathematical Modeling Instruction**

“Although assessment is done for a variety of reasons, its main goal is to advance students’ learning and inform teachers as they make instructional decisions” (NCTM, 1995, p. 13). Inferences about cognitive processes cannot be observed directly, they must be based on students' performances (NCTM, 1995). Present assessment reform practices are shifting toward judging the development of students' mathematical power and away from assessing the students' knowledge of specific facts and isolated skills (NCTM,
1995). When students build mathematical models, they integrate many different aspects of their knowledge (Marshall, 1990). This is the reason an assessment framework must assess more than students' recall of conceptual and procedural knowledge. An assessment framework must also assess students' ability to integrate different pieces of knowledge to create a model (Marshall, 1990). A coherent assessment framework must be developed that assesses students' ability to recognize and retrieve needed information and then synthesize these pieces of knowledge into usable mathematical models (Kulm, 1990). Most items on standardized instruments measure only isolated pieces of information (Marshall, 1990). The key to developing a comprehensive mathematical modeling assessment framework is specifying the mathematical thinking processes and abilities expected of the students at key points in the modeling process (Kulm, 1990).

Most of our traditional instructional practices are based on guesswork rather than on research about students' capabilities (Kulm, 1990). To make instructional decisions based on research, an assessment instrument must be created that can gauge a student's growth in modeling ability. When discussing modeling achievement, a developmental continuum becomes a convenient framework for viewing levels of modeling (Wilson, 1990). Attaching student performances to portions of a modeling continuum for practical purposes becomes an effective method of quantifying modeling achievement (Kulm, 1990). At the other extreme, using qualitative assessment allows for the full impact of individual modeling differences to be taken into account and standardization is not as important (Wilson, 1990). The primary focus of this study is to compare the effect of two methods of modeling instruction on students' modeling achievement. Individual differences will be minimized so that overall modeling achievement differences can be studied. To best serve this research agenda the SOLO (Structure of the Observed Learning Outcome) Taxonomy will be used as the assessment framework (Romberg et al., 1990).
The SOLO Taxonomy approach is based on explicit modeling developmental levels. Piaget's Theory of Learning Cycles is used as a theoretical basis of investigation to define the level of abstraction utilized by students in handling the elements of modeling (Romberg et al., 1990). The cycles of learning progress from concrete actions to abstract concepts. The SOLO Taxonomy is constructed from this theoretical basis by identifying five levels of cognitive responses of a mathematical modeling continuum. The modeling continuum ranges from level 0 where students respond with only irrelevant information, to level 4 where students respond by extending the use of the created model to information beyond the stated problem. The SOLO superitem is the instrument that measures these student behaviors by identifying the student's position on the SOLO Taxonomy (Wilson, 1990).

A general survey of how the SOLO taxonomy is constructed begins with a content-by-behavior matrix where student responses are classified according to a structure of predetermined learner outcomes. One dimension of the matrix is the cognitive levels of mathematical modeling. The other dimension of the matrix is related to the context of the situation being modeled. When the researcher writes the stem or stimulus of a SOLO superitem, a situation is stated followed by four questions devised in such a way that each question requires the student to have the ability to respond to a particular level on the SOLO Taxonomy: (a) Prestructural--no obvious piece of information coming directly from the stem, (b) Unistructural--one obvious pieces of information coming directly from the stem, (c) Multistructural--two or more related discrete pieces of information in the stem, (d) Relational--two or more pieces of information from the stem to integrate understanding of the information in the stem, and (e) Extended Abstract--an abstract general principle or hypothesis that is derived from the stem (Wilson, 1990).

A study conducted at the National Center for Research in Mathematical Science Education used the SOLO scheme of assessing mathematical modeling. Mark Wilson wrote an article addressing the use of SOLO taxonomy in the book, Assessing Higher
Order Thinking in Mathematics, edited by Gerald Kulm and published in 1990. The primary purpose of the article was to analyze the creation and use of seven SOLO superitems in measuring the modeling achievement of middle school and high school students. Seven SOLO superitems were written and analyzed as a part of a larger study (Wilson, 1990). The study consisted of two experimental groups (25 classes) and one control group (10 classes) from middle school and high school students in the states of Wisconsin and Connecticut (Wilson, 1990). The SOLO superitems were used as part of a pre-test of the 1238 students in the study (Wilson, 1990). The seven SOLO superitems were analyzed individually using both quantitative and qualitative data. The qualitative data was obtained from classroom observations and student interviews (Wilson, 1990). The quantitative data resulted from scoring the test items according to the SOLO scores ranging from 0 to 4. Analysis of each question in the SOLO superitems was done using the partial credit model developed by Masters (1982). The partial credit model used the scores of the 1238 students to estimate step difficulty between each of the four questions in the SOLO superitem (Wilson, 1990). The partial credit analysis was described for each of the seven SOLO superitems. The qualitative data was used to discuss conclusion about each test item's effectiveness in identifying students' modeling ability (Wilson, 1990).

Conclusions from the analysis of the seven SOLO superitems pointed out that there were some specific problem with particular items (Wilson, 1990). Two types of problems discussed were: (a) patterns of interdependence between questions and (b) inconsistent response pattern for scoring (Wilson, 1990). Methods for identifying and reworked question were discussed. Questions with confusing wording were stated as the major reason for inconsistent response patterns (Wilson, 1990). Collecting interview data at the time the tests are being scored was recommended as the best method for assessing test validity (Wilson, 1990). Test validity can be assessed by comparing the modeling levels
determined from the interviews with those determined from the quantitative scores (Wilson, 1990).

There are three attributes of the SOLO superitem that make it a suitable instrument for this study: (a) SOLO can be used to detect cognitive development at any level of abstraction, (b) SOLO can be used to detect cognitive development within any cognitive context, and (c) SOLO can be made increasingly complex in its structural organization (Romberg et al., 1990). The SOLO scheme of translating students’ open responses into quantitative modeling scores is an assessment method aimed at answering many of the pedagogical questions of mathematical modeling (Wilson, 1990). This researcher uses this method of assessment to measure the effectiveness of traditional manipulative instruction to new computer microworld instruction.

Computer Microworlds

Introduction

The use of computer simulations to illustrate and validate mathematical models is becoming more prominent in mathematics education as interactive and dynamic software becomes available (Hoyles & Noss, 1995). The most common type of computer-generated environment that is both interactive and dynamic is the computer microworld (Kaput, 1992). Microworlds are computer software programs that simulate some phenomena of reality so that students can interact with them to build models to solve problems (Kaput, 1992). For example, a microworld that simulates the physical laws of nature uses students’ knowledge of nature to organize a system of model-building tools where students construct and run experiments. *Interactive Physics* (Knowledge Revolution, 1990) allows users to create or run a wide variety of science laboratory explorations. These explorations require that the user be knowledgeable in physics and mathematics (Kaput, 1992).
The key to studying the instructional benefits of computer microworlds is to find a perspective that reaches beyond the particulars of microworlds to create a framework where distinction between microworlds and traditional representations can be made (Kaput, 1992). To make a distinction between mathematical ideas and particular aspects of the physical world, very specific language is needed. The terms "medium" and "notation system" are used to explain external representations more specifically. A medium is the physical aspect of reality in which the notation system is initiated (Kaput, 1992). Examples of mediums include paper-pencil, physical objects, spoken language, and computer displays. A notation system is the set of rules that define the objects and allow actions on them. This is essentially abstract until the decision is made to initiate it using the physical world (Kaput, 1992). The key interest in this study is comparing the special nature of microworld media to traditional media. Specifically, this study compares the computer microworld Interactive Physics to manipulatives.

A medium which is dynamic and interactive poses many pedagogical problems and possibilities (Hoyles & Noss, 1995). Computer microworlds are also capable of supporting a variety of notational forms, which is called representational plasticity (Kaput, 1992). Microworlds are not only dynamic and interactive, but also represent mathematical objects in multiple notations. The discussion of how the unique abilities of computer microworlds affect mathematical modeling instruction has been divided into four categories: the student's (a) ability to translate between microworld notation and the mathematical model, (b) ability to accentuate selected details while suppressing other details during modeling, (c) ability to vary notations to uncover complex concepts, and (d) ability to reflect on actions performed (Kaput, 1992).
Translations Between Microworld Notations and Mathematical Models

Although too much present-day mathematics education deals with transformations within notation systems or transformations between notation systems guided by specific rules, most true mathematical activities involve transformations between multiple representations (Kaput, 1992). How easily students can translate the actions and characteristics of a representation to the mathematical objects being modeled depends on the medium and notation system (Kaput, 1992). Ideally, a mathematical representation should have notations and media that transparently reveal the hidden mathematical objects and relationships. Media that link different notations allow students to see an abstract mathematical idea in many different representations. When modeling real world situations understanding equivalent mathematical equations is influenced greatly by understanding the connections between multiple representations (Heid & Zbiek, 1995). Microworlds enable students to make these connections through using the dynamic notations of the microworlds which are linked so that consequences in one notation are observable in other notations. This overlay of different notations reveals the more abstract symbolic notation that is standard in mathematical communication (Kaput, 1992).

An interesting approach to modeling involves programs like ANIMATE where students build dynamic diagrams that are linked through algebraic notations (Kaput, 1992). Another interesting linkage of notations is the way many algebra microworlds dynamically link graphs and character string notations. In these programs, changes in mathematical equations result in instantaneous graphical changes. This shows how multiple linked notations can be used to dynamically reveal different aspects of a complex idea.

Accenting Selected Structures and Suppressing Details

The interactive nature of the microworld media distinguishes it from traditional static media and traditional video media (Kaput, 1992). When you write something on
paper, it does not interact with the paper or a notation structure. The paper does not provide support or constraints in using a mathematical notation system. Conversely, all notation systems are naturally interactive since a user must interact with them to use them (Kaput, 1992). The interactive feature of microworlds makes them a powerful tool in motivating students to create mathematical models because the media respond to students' input. Traditional media only record ideas and notational representations already produced within the students' minds. There is no motivation from the media to create or invent a mathematical model to solve a problem. Microworlds also have built-in constraints and supports that give the students structure in their search for a solution to a problem (Kaput, 1992). It is the nature of simulations of the physical world to suppress details and give control over the chosen characteristics being modeled. Computer microworlds provide such capabilities in science and technology by making apparent things that are not usually noticed (diSessa, 1984).

The interactive and structural features of computer microworlds create environments in which doing mathematics makes sense to students, making learning intrinsically motivating (National Research Council, 1987). For example, in “Mess up” students play with geometrical ideas in an intuitive and dynamic way to change a computer drawing using geometrical concepts (Hoyles & Noss, 1995). Students are forced to think about specific geometric concepts because specific geometric constructions are used to draw the figures. Geometric understanding comes as students manipulate the figures in order to make and test hypotheses (Hoyles & Noss, 1995).

**Vary Notations to Uncover Complex Concepts**

Complex ideas are almost never adequately represented by one notational system. Each notational system reveals a different characteristic or aspect of the mathematical object and its relationship to other mathematical objects being modeled. The ability to link
different mathematical notation systems helps make complex ideas explicit for students (Kaput, 1992). Computer microworlds are a medium where students can understand and construct unique mathematical ideas (Heid & Zbiek, 1995). Computers allow students to build notations in the process of modeling which reinforces mathematical habits stressed by the Standards.

Microworlds that simulate some aspect of reality help students transform reality into a simpler notation system and, in many cases, the possibility of multiple notation systems simultaneously. For this reason microworlds are a great method for teaching students how to create, use, and see the significance of using mathematical models to solve problems.

**Microworlds as a Procedure-Capturing System**

Complex structures and processes are perceived through reflective thought and notation interaction (Hiebert & Carpenter, 1992). Procedure-capturing systems, like those in many microworlds, provide for reflective thought through stored procedures and actions within the media (Kaput, 1992). Many times, students need to see an action repeated many times before a pattern becomes apparent. Microworlds such as *Interactive Physics* (1990) allow an experiment or action to be saved in a form that has control tools like a VCR. Students can stop, play, slow down, speed up, or review any action. Basically, students can recall the history of their actions within the media at any time and review them as many times as needed.

**Using Technology to Address Issues of Mathematical Modeling**

Plasticity and linking representations are valuable features of computer microworlds, but what specific possibilities of mathematical modeling might these new dynamic notation systems address? In the mathematical modeling section it is noted that the process of modeling engages students in the abstraction of invariance. The use of
computer microworlds to observe variance introduces four important instructional issues with respect to mathematical modeling: (a) How is variance represented?, (b) How is variance between notation systems represented?, (c) How naturally can the real situation be translated to the microworld notation systems?, and (d) Do the features facilitate reflection and crystallization of the concept observed? (Kaput, 1992). These issues are addressed specifically to *Interactive Physics*, the microworld used in this study.

Transformations within a particular notation system are the key to uncovering the hidden concepts revealed by invariance. Students must understand why and what actions result in changes in microworld representations. The microworld *Interactive Physics* models the physical universe, obeying most of the laws taught in high school and college physics. The leading authority in this field is diSessa (1984) of the University of California - Berkeley who developed and tested a microworld called Dynaturtle, in which children try to control objects in an environment that obeys the physical laws of the universe. Microworlds, such as *Interactive Physics*, draw on the students’ own intuition for understanding the cause and effect within the microworld. What is not understood is what role the students’ beliefs about computer usage have on their use of computer microworlds (Kaput, 1992).

The phrase, “translations between notation systems,” refers to actions in one notation system affecting another notation system, giving the students many views of the same concept. In *Interactive Physics*, animation, numerical values, and graphical coordinates can be seen simultaneously. This is the feature of plasticity that microworlds do so well by linking multiple representations and displaying them side by side. Then, students can connect meaning between the different notation systems naturally. Research has shown (Kaput, 1992) that microworlds link representations well. What remains to be shown is what effect this linkage has on the internalization of the key mathematical ideas important to the problem being solved (Kaput, 1992).
Constructing and testing mathematical models through the investigation of personal hypotheses needs to be understood within the context of both the physical situation being modeled and the microworld. The NCTM (1989) emphasized that the instructional issues surrounding the investigation approach to mathematical problem solving is key to teaching mathematics for understanding. Microworlds naturally facilitate such active learning and construct meaning to the mathematics they are studying (Hoyle & Noss, 1992). There is much remaining to be studied about the instructor's role in facilitating an interactive computer environment. In addition, the instructional material to support these microworld environments needs to be written and tested (Anderson, Boyle, Franklin, & Reiser, 1985).

One of the research committees that has recommended the use of microworlds is The Committee on Research in Mathematics, Science, and Technology Education (National Research Council, 1987). Its recommendation is a systematic program for the development of pilot educational systems using computer-created microworlds that engage students in science and mathematics-linked tasks, thereby advancing both the acquisition of knowledge and the learning of reasoning and problem-solving skills.

To successfully model a situation, the student must consolidate the relationships and concepts of the problem into conceptual objects and processes (Skovsmose, 1989). The feature of built-in constraints and interactive input are basic to facilitating the modeling process (Kaput, 1992). The feature of stored and captured processes aids the student's memory in recalling past action. The personal construction of the model within the microworld notation has been found to be the key feature of microworlds that enables reflection on the modeling process (Kaput, 1992). Interactive Physics obeys the physical laws of nature so that students can construct and run real-world experiments and demonstrations through metaphoric tools, allowing a concise study of the important relationships. Research also indicates that computer microworlds interactively and intuitively visualize the process of mathematical modeling (Kaput, 1992). This makes them
ideal for exposing students to mathematical modeling. It is to be hoped that computer microworlds, such as *Interactive Physics*, will act as a transitional bridge between problem situations and mathematical models, making the process of modeling more interesting and understandable. Because of the dynamic nature of computer microworlds, they seem to naturally facilitate mathematical modeling (Hoyle & Noss, 1992).
CHAPTER 2

PROCEDURES

Introduction

The primary purpose of this study is to determine the modeling achievement differences of students who receive one of two different methods of instruction in mathematical modeling. Both methods incorporate small cooperative groups, but the problem situations are presented to one group by means of physical objects and to the other with the computer microworld \textit{Interactive Physics}. Both methods of instruction used the data analysis approach to mathematical modeling. Instructional method is not the only independent variable; the students’ computer literacy and beginning modeling ability are used as independent variables also. The students are classified by modeling abilities into four groups using a pretreatment SOLO superitem, Tom’s Sweet Shop (Appendix A). These four classifications are related to the five levels of the SOLO Taxonomy by making the first three beginning modeling levels correspond to the first three SOLO levels and the final two SOLO levels correspond to the final SOLO class. The students’ computer literacy is determined by the Computer Literacy Mastery Test (Appendix B), which was designed to identify proficiency in the Macintosh operating system. The four beginning modeling levels and the two computer literacy levels are used to assign each of the students from the two Finite Mathematics classes to one of eight sampling blocks. The students in each of these blocks are randomly assigned to receive one of the two instructional methods. The treatment groups meet in different rooms with one of the two Finite Mathematics instructors who are chosen at random. The instructors are to facilitate investigative problem solving in
cooperative groups and the students are to work in groups to complete and hand in modeling worksheets.

The framework for comparing the students’ modeling achievement (the dependent variable) focuses on measuring the modeling transfer distance students experience when applying known mathematical models to new problem situations. The three modeling contexts for instruction and assessment of students are to (a) apply a linear model to given data, (b) apply a linear model to a two-rate situation with a constant distance, and (c) apply a quadratic model to a familiar geometric object. The students’ modeling achievement is assessed using SOLO superitems. SOLO superitems use a series of 4 questions to identify the students’ modeling level. Each of the three posttreatment SOLO superitems (Appendix A), are set in one of these modeling contexts. These three situations do not cover all modeling situations but they represent three situations with varying degrees of transfer distance from the known linear and quadratic models.

Since modeling is such a complex process, other extraneous variables are monitored through constant comparison using class observations, student interviews, student learning logs, and teacher interviews (Webb, 1993).

**Pilot Study**

**Introduction**

During the 1993 spring semester, the researcher received permission for a section of Math 150 to participate in a mathematical modeling activity using the computer microworld *Interactive Physics*. The focus of this study was to determine the relationship between mathematical modeling pedagogy and student behaviors when using the microworld *Interactive Physics*.
Understanding the issues of studying the process of mathematical modeling when technology is used was the purpose of this study. The goal was to find pedagogical questions that could be answered through further research. Hoyles and Noss conducted a study in 1992 where traditional methods of instruction were used in conjunction with a LOGO computer microworld. They found that the dynamic nature of computer microworlds appears to naturally facilitate mathematical modeling. Two natural questions arise from their study: (a) Would similar results appear when using other computer microworlds? and (b) Would the use of instructional grouping such as cooperative groups enhance the modeling process?

The purpose of this pilot study was to formulate research questions associated with the use of computer microworlds in modeling instruction. The researcher developed hypotheses which were used in formulating pedagogical guidelines for implementing computer microworlds into mathematics instruction.

The instructor facilitated problem solving by asking questions which encouraged students to investigate and develop strategies for understanding and solving the given problems. The instructor intervened when students were having problems with using the microworld, since this can lead to frustration and an unproductive sequence of work (Anderson et al., 1985). Aspects of the learning environment created under these situations have given some indication as to whether this learning environment is manageable and accessible.

The traditional style of instruction is designed to teach facts. Since many of the goals stated in the Standards (NCTM, 1989) are not facts but processes, like reasoning and problem solving, the pilot study did not compare microworld instruction to traditional direct instruction. The pilot study instead focused on students' behaviors within the described educational environment. Analyzing the interviews was done through comparing and contrasting the recorded behaviors.
Method

Qualitative analysis was used to study the pedagogical effects of using a computer microworld, an instructor acting as a facilitator, and students organized into cooperative groups on the learning behaviors of students. Prior to this study a hypothesis on students' behaviors, within the context of computer microworlds, was not found in a current review of the literature. Through qualitative study of the patterns in the students' behaviors as they interacted with the instructor, the microworld program, and other students, the researcher developed hypotheses for further study. A qualitative method allowed the researcher to ground this study in behaviors of the teacher-student-subject-matter interactions (Patton, 1990).

The sample was a section of Finite Mathematics (Math 150) during spring 1993 semester. The study included 23 of the 39 students enrolled in the section. Finite Mathematics is one of the core requirements for those students receiving degrees that do not require a specific mathematics course. The prerequisites for this course are high school algebra II or its equivalent. Two of the students studied were taking the course without the prerequisites. Nineteen of the students were between the ages of 19 and 23, and 4 of the students were over 27 years of age. The study used Macintosh computers and no computer training was given before or during the activity.

At the beginning of the class period the students were told to report to Room 1-144 Willson which is a computer lab with 9 Macintosh ci's and ten DEC 486/SX's. When the students arrived, they were given a worksheet called Experts on Gravity (Appendix E) and told to arrange themselves in groups of 2 or 3 at the nine Macintosh Computers scattered throughout the room. The instructor then passed out a blank sheet of paper to each group to be used as a log of the group’s work during the activity.
The activity instructions were presented both verbally from the instructor and in writing on worksheets. The instructor’s oral instructions were these:

This is a break from the algebra lessons in the book so that we can work on problem solving. You are to answer the questions from the work sheet as completely as possible with words or mathematical expressions. Put these answers on the Log Sheet I gave you along with all work that you do in chronological order. The information should include data, conjectures, checking conjectures, graphs, conclusions. Each group will hand in one of these log sheets at the end of the class period. This is a picture of what I was referring to on the worksheet. (The instructor was pointing to a picture drawn earlier on the board of a person dropping a rock off a cliff.) The computer is a tool to help you run experiments on falling objects and in turn answer the questions on the work sheet Experts on Gravity. Here are the four steps to running the experiment referred to in the work sheet. (The instructor pointed to four steps that had been previously written on the board and to a picture of the computer screen.) I will help you with the program called Interactive Physics and answer questions that will stimulate a problem solving approach but do not ask if your answer is correct or what the answer is. Raise your hand if you have a question and I will come to where you are working. You have about 30 minutes to work on these problems. At the end of the lesson if any of you could stay and sign up for a ten minute interview, I would appreciate it. Are there any questions? Good, then you may begin.

The students read and answered the three questions on the worksheet Experts on Gravity. They kept logs of their work which included conjectures, data, tables, graphs, calculations, definitions, and conclusions. These logs were used as data on student behavior. A video camera was used to record student behaviors during the activity and during the postactivity interviews. The students worked for 29 minutes while the instructor
answered student questions, recording some. A VHS camera was started and focused on three groups during the entire lesson. The instructor had sufficient time to monitor each group’s progress at least once.

The instructor lead the following discussion with about 7 minutes remaining in the period. These were the questions asked in the closing activity:

1. What makes the rocks fall?
2. How did you answer question number one?
3. How can we measure how fast the rocks are falling?
4. What is this measurement called?
5. What is acceleration?
6. How did you answer question number two?
7. Did anyone use any other method to find velocity?
8. How did you answer question number three?
9. Did anyone use any other method to find the height of the cliffs?

During question 9 the bell rang and the discussion was finished quickly. The students were told to place their log sheets on the front table.

The VHS camera was refocused on one of the desks in the room where interviews began approximately 10 minute later. Eleven of the 23 students were interviewed. Many of the questions were the same but the instructor found that interactive questioning caused the students to reveal more insights into their thoughts during the activity.

As a follow-up to a pattern found in abstraction to mathematical equations, the students were given the following exercise on a quiz during the following class period: Solve the equation \( F = \frac{9}{5}C + 32 \) for \( C \). The follow-up experiment was compared to data from the interviews.
Analysis of Data

In the analysis of the data, common patterns were observed around already existing areas of interest. Data on the students’ experiences were collected from many different sources. The bulk of the information in the pilot study came from the postactivity interviews. These interviews were reflective in nature, the main purpose was to discover the students’ affective and cognitive processes during the activity. Quotes, research observations, and students’ written work were used to document, illustrate and clarify the patterns found in the areas of interest. The five areas of interest analyzed were: student-teacher interaction, reasoning, problem solving, mathematical interaction, and student-student interaction.

Conclusions

The researcher made the assumption that microworld instruction is a viable instructional method when analyzing the student behaviors. With this assumption, cooperative groups, and microworld instruction created a good environment for testing and creating conjectures. The computer microworld provided a common language for discussing and researching such possibilities.

Because the computer microworld is only acting as a mathematical tool, the instructor, acting as a facilitator, can help the students stay on task without inhibiting the exploration of mathematical concepts. This has been a problem in lessons that do not guide the students to a set behavior because exploration boundaries are usually too artificial. The microworld program established natural boundaries by limiting the input and output of the data. This feature of Interactive Physics, coupled with the students’ creative use of conjectures, made this activity highly successful.
Pedagogical Questions to Study

Present mathematics education research indicates that the dynamic notation systems inherent in computer microworlds should be integrated into the mathematics curriculum (Kaput, 1992). Conclusions from student behaviors have supported this claim. Identifying the relationships between pedagogical and student behaviors is much more difficult. Before computer microworlds can be used as an instructional tool pedagogical guidelines needed to be identified.

The goal of this pilot study was to identify relationships between pedagogical and student behaviors for the purpose of creating a knowledge base to use in further research. The pilot study showed that:

1. Students need to be more aware of the reasoning and problem solving dynamics within their group. Personality conflicts within groups and a poor understanding of problem solving strategies were the major reasons many groups stopped working on the problem solving activity.

2. The problem situations represented by the computer microworld should simulate real-life situations as closely as possible. Over half of the students could not relate their solutions to the real world problem, simulated by the computer microworld.

3. Familiarity with Macintosh operations was very beneficial to operating the microworld. Most students who stated that they did not like the problem solving activity had trouble operating the computer microworld.

4. The researcher’s assessment of the students’ mathematical understanding from the interviews aligned with the written responses from the worksheet. The researcher found that the written solutions on the worksheet revealed the student’s mathematical understanding of the problem in 9 out of the 11 interviews.
5. Confidence in suggesting and testing conjectures was the most important factor in moving toward a correct solution. Groups that used the computer microworld to test 2 or more conjectures reached a successful solution every time.

**Setting of the Study**

The study, to identify the pedagogical differences between two methods of mathematical modeling instruction using cooperative groups, was conducted at Montana State University during May and June of 1995. Montana State University is in the city of Bozeman, Montana. Approximately 11,000 students (56 % men and 44 % women) attend MSU. Approximately 80 % of the students at Montana State University are Montana residents. MSU, the only land-grant university in the state, is the major research institution in the state.

**Population Description**

The population of this study was students enrolled in Finite Mathematics, Math 150, during the 1995 first six week summer session. The course covered reading, comprehension and writing skills in the language of mathematics, which included vocabulary, grammar, syntax, and logic, with emphasis on understanding, expressing, proving, and thinking mathematically.

The 1995 first interim summer session is a 6 week session starting May 22 and ending June 30. The enrollment for the 1995 summer semester was projected to be similar or above this number. Students enrolling in Finite Mathematics have completed Algebra I, Geometry, and Algebra II in high school and have either passed a placement exam to check proficiency at the college algebra level (Math 103), passed the placement exam and a high school equivalency, or passed College Algebra (Math 103). Finite Mathematics is a core course for students seeking degrees that do not require a specific mathematics course.
The Null Hypotheses of the Study

The three modeling contexts of this study are the three dependent variables. The three modeling context of the study are: (a) students applying a linear model to data, (b) students applying a linear model to a two-rate situation with a constant distance, and (c) students applying a quadratic model to a familiar geometric figure. The same set of seven hypotheses are tested in each of the three modeling context.

Null Hypotheses

Main Effects

1. There are no significant modeling achievement differences between students receiving modeling instruction using the computer microworld Interactive Physics and those using physical objects.

2. There are no significant modeling achievement differences between students at different beginning levels of modeling ability.

3. There are no significant modeling achievement differences between students who differ with respect to computer literacy.

Two-Factor Interactions

4. Levels of beginning modeling ability and methods of modeling instruction do not significantly interact on modeling achievement.

5. Levels of beginning modeling ability and computer literacy do not significantly interact on modeling achievement.

6. Methods of modeling instruction and computer literacy do not significantly interact on modeling achievement.
Three-Factor Interactions

7. Methods of modeling instruction, computer literacy, and levels of beginning modeling ability do not significantly interact on modeling achievement.

Experimental Treatment

Each mathematics lesson presented during the study is an application of either a linear or a quadratic model. Each of the eight lessons was evaluated by two or three mathematics instructors for content and construct validity. After a series of revisions, the lessons were used in instruction during the 1994 first summer interim with Finite Mathematics students. At this time student feedback was used to check for clarity, construct validity, and assessment alignment with the four SOLO superitems. Some revisions were made in the lessons and they were again tested with Finite Mathematics students during the 1994 fall semester. Copies of the four SOLO superitems can be found in Appendix A.

The instructional treatment was administered in 1995 from June 20 to June 29 in eight sessions. During the summer, Finite Mathematics met twice daily for two 70-minute sessions. The researcher limited the treatment to the later session. Since both sections of Finite Mathematics were taught at the same time, students from both sections were randomly assigned one of the two treatments. One of the instructors met with one of the newly formed treatment groups in the Willson Hall computer lab while the other instructor met in Lindfield Hall. Although two weeks is a short instructional duration, the researcher feels it is an appropriate length of time because it reflects the current time spent in most high school units dealing with the application of linear and quadratic models.

The lessons were introduced by a traditional "whole class" lecture-discussion method of instruction. The teachers also used instructional time to clarifying and assist students with individual questions during class. Students were arranged in small
cooperative learning groups for both methods of instruction. No instruction similar to either of these two instructional methods was used prior to this 2-week treatment period.

The patterns of instruction for both treatments was similar. The small groups for both treatments were given activities to complete by answering worksheet questions about mathematical modeling. The instructor was available always to visit with the groups, answering questions, encouraging discussion, and praising the individual groups for their efforts.

There were four stages of treatment during the class periods.

1. After the cooperative groups were assigned, but before the worksheets had been distributed, the instructor described the objectives of the activity, reviewed prerequisites for the activity, and made sure each group was properly equipped.

2. After the worksheets were distributed, the students proceeded to work cooperatively to complete the worksheets. The worksheets required the students to investigate the given problem by using the manipulatives to make and test conjectures with the goal of formalizing a mathematical model. Specifically, the students were asked to analyze the data by checking the numeric pattern characteristics of linear and quadratic models. Students first performed the numeric analysis for the linear model by studying the difference ratios of the independent and dependent variables. If no linear pattern was found, the students checked for a quadratic pattern by squaring the data of the independent variable and studying difference ratios of the squared independent variable and the dependent variable. To confirm their symbolic model, the students plotted the data and their model on the same graph to perform a visual reliability test.

3. The second part of the worksheet was designed to motivate students to apply and abstract their mathematical model to other situations.
4. The class period ended with the instructor leading a class discussion of group discoveries and difficulties.

Stages 1 and 4 lasted about 5 minutes and stages 2 and 3, combined, lasted about 40 minutes. The first two sessions were used to orient students on the features and investigative methods of the different instructional methods. The remaining six sessions investigated linear and quadratic models in different problem situations. Table 1 gives a description of the six worksheets used for the treatment. The content and models used in each activity are outlined.

Both instructors were given similar lessons and instructional guidelines to minimize the effect of the instructor variable. To assure that the treatments were well-defined, specific pretreatment training was given to both instructors. Observations and interviews with the students and instructors were conducted during the treatment to monitor any differences in treatments due to instructional practices.
Table 1. Description of Treatment Activities

<table>
<thead>
<tr>
<th>Work Sheet Title</th>
<th>Days</th>
<th>Mathematical Model</th>
<th>Major Learning Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Looking for Relationships</td>
<td>Two</td>
<td>Linear, Quadratic</td>
<td>Students will be able to identify the connections between the relation, computer microworld, graphical, and symbol models.</td>
</tr>
<tr>
<td>How Far Will it Roll</td>
<td>One</td>
<td>Linear</td>
<td>Students will be able to write and use a model of the horizontal movement of a rolling ball with mathematical symbols.</td>
</tr>
<tr>
<td>Can Modeling</td>
<td>One</td>
<td>Quadratic</td>
<td>Students will be able to write and use a model of the relationship between the radius and volume of a cylinder with mathematical symbols.</td>
</tr>
<tr>
<td>Swing that Pendulum</td>
<td>One</td>
<td>Quadratic</td>
<td>Students will be able to write and use a model of the movement of a pendulum with mathematical symbols.</td>
</tr>
<tr>
<td>Swim and Row</td>
<td>One</td>
<td>Linear</td>
<td>Students will be able to write and use a model of the relationship between time and rates when distance is a constant with mathematical symbols.</td>
</tr>
<tr>
<td>Watch for Those Flying Objects</td>
<td>Two</td>
<td>Linear, Quadratic</td>
<td>Students will be able to write and use a model of the horizontal and vertical movement of projectiles with mathematical symbols.</td>
</tr>
</tbody>
</table>
To check that randomization controlled for the contaminating variables of selection of the population, a detailed pretreatment investigation of the two treatment groups was conducted on: beliefs about computers, beliefs about mathematics, and attitudes toward mathematics. This investigation was conducted through pretreatment interviews. One student was interviewed from each of the 16 treatment blocks.

The contaminating variable of communication between the two treatment groups was controlled by assigning the two treatment groups to different instructors in different locations. The students would contaminate the study if they had any motivation to teach either of the two different treatment methods to each other. The main motivation for such communication would be in receiving a higher grade evaluation on achievement. The motivation and availability of students to teach each other the two methods of instruction was controlled by limiting the instructional treatment to the separate treatment locations. This was done by having the treatment groups meet in different buildings, and by requiring the students to complete and hand in the worksheets during each instructional session. No homework, practice problems, or help sessions were conducted outside the treatment area. Student evaluations came from the completed worksheets and the posttreatment SOLO superitems. This minimized their incentive for comparing methods of instruction.

The contaminating variables of testing and maturation were not problems in this study because the same instruments and treatment time was used for both groups. Since students can drop out of Finite Mathematics during the treatment, mortality could have been a problem in this study, but no students dropped out of the class during the treatment period. Contamination from regression was not a problem in this study because modeling achievement was used to form sampling blocks.

Contamination from the statistical instruments was the final problem of the experimental design to be addressed. The SOLO superitems' construct and content validity and reliability were tested, and adjusted as a result of that testing. The validity, reliability,
and floor and ceiling effects of the SOLO superitems are discussed in the SOLO superitem section.

**Instructor's Special Training**

The instructors met with the researcher prior to May 22, 1995, and reviewed the purposes and methods of this study. At that time, the instructors were randomly selected to use either *Interactive Physics* or physical apparatus to teach the modeling unit. The instructors were given the guidelines and training for their instructional manipulatives. Each instructor received and practiced each of the eight lessons and received all instructional materials (activity sheets, daily logs, permission forms, test questions, and copy of the interview questions). Instructor training was conducted 3 weeks prior to the start of the treatment unit to give the instructors time to balance their lesson plans in order to accommodate for these lab days.

**The Instruments Used in the Study**

The following measuring instruments were used in this study.

1. SOLO Superitem--Appendix A
2. Class Observation
3. Interviews of Students--Appendix C
4. Interviews of Instructor
5. Students’ Daily Logs
6. Computer Literacy Mastery Test--Appendix B

**SOLO Superitem**

The SOLO (Structure of the Observed Learning Outcome) taxonomy will be the assessment framework used to measure the modeling achievement of the students. The
SOLO taxonomy uses five stages to describe the modeling process: (a) Prestructural--students use no obvious piece of information coming directly from the stem, (b) Unistructural--students use one obvious pieces of information coming directly from the stem, (c) Multistructural--students use two or more related discrete pieces of information in the stem, (d) Relational--students use two or more pieces of information from the stem to integrate understanding of the information in the stem, and (e) Extended Abstract--students use an abstract general principle or hypothesis that is derived from the stem (Wilson, 1990). The student's position on the SOLO taxonomy is measured using an instrument called a SOLO superitem (Wilson, 1990). The SOLO superitem begins with information about a problem situation, followed by four open-ended questions designed to correspond to specific levels of the modeling process (Wilson, 1990). An example of one of the seven superitems is given in Figure 4. Due to the age of some of the students, only the first four levels were assessed.

Figure 4. SOLO Superitem 7 (Wilson, 1990)

7. A teacher tries to guess the seasons and month when any child in her class was born. If the teacher was to guess the season, she would most likely get 1 correct for every 4 guesses. If the teacher was to guess which month any child was born, she would be likely to get 1 correct for every 12 guesses.

A. If the teacher used the season to make her guesses, how many times do you think she would have been correct with four children? Answer ________

B. The teacher has 12 girls and 16 boys in her class. She guessed the month in which each girl was born and the season in which each boy was born. In how many of her 28 guesses was she likely to have been correct? Answer ________

C. If the teacher guessed 7 right out of 16 for the seasons and 6 right out of 12 for the months, how many more correct guesses altogether has she made than you would expect by chance? Answer ________

(p. 203)
The same process used in a 1982 study and described by Wilson (1990) was used to create the four SOLO superitems for this study. In the 1982 study, Wilson embedded seven SOLO superitems in a mathematics pre-test for 1238 high school and middle school students. Wilson’s research focused on the evaluation of SOLO superitems as an instrument for assessing mathematical modeling. He concluded that the SOLO framework was workable and possessed potential as a source for identifying procedural abilities in many different problem solving contexts (1990). A recommendation was made that qualitative data from the answer sheets and interviews were valuable in assessing the integrity of the test questions to link student responses to SOLO taxonomy levels (Wilson, 1990).

The modeling process used during instruction followed a data analysis approach. The researcher adapted the behavior matrix in the SOLO taxonomy to align with the modeling instruction. The SOLO modeling levels were modified as follows: (a) Level 0--students could not identify information found the stem to answer a question about the independent variable; (b) Level 1--students identified information found the stem to answer a question about the independent variable; (c) Level 2--students answered a question about the relationship between the dependent variable and the independent variable; (d) Level 3--students could write a symbolic mathematical equation from the information in the stem; and (e) Level 4--students applied the information in the stem to answer a problem similar the stem situation. Each SOLO superitem started with a stem consisting of a table, diagram, or problem explanation. The stem was followed by four open-ended questions following specifically corresponded to the student’s ability to achieve the first through the fourth modeling level. No questions answered correctly identified the student’s modeling ability as level 0. The four SOLO superitems consisted of one pre-treatment test of a linear situation and three post-treatment tests corresponding to the three modeling contexts.
addressed in the instructional treatment. Each SOLO superitem went through extensive reliability and construct validity testing which is discussed in the following sections.

Reliability Testing

The four SOLO superitems' reliability depends primarily on how consistently students respond to the questions over time. This consistency in students' responses to SOLO superitem questions was measured by the researcher in a test-retest method after the superitems where revised for content and construct validity. The reliability of the SOLO superitems were tested twice because some of the questions were changed after the first testing to adjust the construct validity. The final reliability testing was conducted on students from a course of Mathematics for Elementary Teachers during the 1995 fall semester. The students were given one of the four superitems as an in-class modeling assessment and then given the same superitem 3 weeks later. The instruction during these 3 weeks was transformational geometry, which was not related to any of the four SOLO Superitems. The test-retest measurements were correlated using a Pearson-r (See Table 2 for results).

Table 2. Pearson-r Correlations for Test-Retest Measurement of SOLO superitems

<table>
<thead>
<tr>
<th>SOLO superitems</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom's Sweet Shop</td>
<td>.891</td>
</tr>
<tr>
<td>Biathlon</td>
<td>.855</td>
</tr>
<tr>
<td>Land Lord Dilemma</td>
<td>.866</td>
</tr>
<tr>
<td>Square</td>
<td>.874</td>
</tr>
</tbody>
</table>
Construct Validity

Construct validity was checked through interviews with Finite Mathematics students during June of 1994 and again on students from a course of Mathematics for Elementary Teachers during September of 1995. Both times the students were given one of the four superitems as an in-class modeling assessment. Participants were interviewed after taking the superitems to evaluate the accuracy of the SOLO superitem ability to identify the modeling levels of the students. Wilson explained this method of assessing students' modeling levels from interviews in an article about a study conducted in 1982 at the National Center for Research in Mathematical Sciences Education (1990). One of his recommendations was that qualitative data from the answer sheets and interviews were valuable in assessing the integrity of the test questions to link the student responses to levels of mathematical modeling. In the 1982 study, the SOLO scores were validated through student interviews corresponding to the written test questions. To model this approach to test evaluation a sample of one fourth of the students were interviewed after completing one of the four SOLO superitems. The stem or original problem was put before the students and similar questions to those of the SOLO superitem are asked the student. If student responded confidently the researcher asked the next question but if the student seemed confused the researcher reworded the question until he was sure that the student understood the question and he understood the student's response. The researcher scored each student's interview in the same manner the written responses were scored. The researcher did not look at the student's written responses until after the interviews were completed. The SOLO superitems were restructured until 80% of the students were identified at the proper modeling levels. This process was completed twice and the second time the correspondence between the interviews and the written responses of all four SOLO superitems were above 80%.
Student Interview Procedures

This study used student interviews for three purposes: (a) to describe the population; (b) monitor the affect and effect of the treatment on students' problem solving behaviors; and (c) monitor the treatment's effect on the students' beliefs about mathematical modeling, computer usage, and physical manipulatives. Interviews were conducted at three different times during the study: (a) before treatment, (b) halfway through treatment, and (c) after treatment.

Interviews Before Treatment

The purpose of the first interview was to determine their (a) attitudes toward mathematics; (b) beliefs about modeling, computer usage, and physical manipulatives; and (c) practices and reflections about the SOLO superitem pretest.

This interview was conducted on the same day students completed the SOLO superitem pretest. One student was randomly selected from each of the 16 blocks of the population. Students were asked to reflect on their strategies, beliefs, and practices on each question of the pretest. Then the students were asked to reflect on their attitude toward the different questions on the pretest. Finally, the students were asked questions about their ability and attitude toward mathematics, computer usage, and the physical manipulatives. Notes were taken of each interview and the SOLO superitem pretests were kept. (See Appendix C for student interview questions.)

Interviews During Treatment

The during-treatment interview occurred one week after treatment began. The interviews were similar to the pretreatment interview with one exception: Students also were asked about their knowledge of the other treatment group's activities.
Interviews After Treatment

This interview was conducted in the same manner as the during-treatment interview and occurred on the day that the three posttreatment SOLO superitems were given.

Instructor Interview Procedures

The purpose of this interview was to describe the treatment from the instructors' perspectives and describe extraneous effects that influenced the instructors' use of these two methods of instruction. The instructors were interviewed before, during, and after the treatment period. The purpose of these interviews was to monitor the differences in the practices of the instructors.

Classroom Observations

Twice per week the researcher observed and recorded the practices of the students and instructors during treatment. The purpose of these observations was to describe the practices of the instructor and students. This allowed the researcher to better describe the actual treatment.

Student's Daily Logs

The students kept daily logs of their perceptions and attitudes toward the lessons. This consisted of written responses on the activity sheets by the students. The students responded to the activity questions and made comments about the activity.

Computer Literacy Mastery Test

The purpose of the Computer Literacy Mastery Test was to identify those students who could successfully operate a mathematics-oriented program for the first time on a Macintosh computer (Appendix B). To create this instrument a domain matrix for computer literacy using mathematics software on a Macintosh computer was developed.
This was done through interviewing mathematics instructors, MSU mathematics students, and Macintosh educational representatives. Three content domains emerged, (a) system use, (b) program menu use, and (c) translating computer representations to mathematics. The matrix identifying the performance indicators for each of these domains is shown in Table 3. From this matrix, a four page test was constructed to indicate proficiency in Macintosh program computer operation. An instructor at Montana State University whose area of expertise was Macintosh instruction reviewed the instrument for content validity. After changes were made, it was administered to 9 students to check for construct validity and question clarity. This was done by attaching a cover letter and evaluation sheet to the instrument. The instrument was adjusted and tested again with 8 students for construct validity by interviewing students after they had answered all the questions. The researcher used the verbal and written responses to check for alignment between actual performance and instrument scores. The goal was to identify the students that were not computer literate and identify areas where they needed instruction. After the instrument had been adjusted again, 8 students were test-retested at a 3-week interval. Nine out of the 10 students were assessed at the same level of computer literacy on both the test and the retest which was an acceptable level of reliability for the researcher.
Table 3. Domain Matrix for Computer Literacy of Mathematics Software on Macintosh Computers

<table>
<thead>
<tr>
<th>System Use</th>
<th>Program Menu Use</th>
<th>Translation of Computer Visualization to Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operate a Macintosh computer within the last month</td>
<td>Operate more than one Macintosh program within the last month</td>
<td>Recognize a computer representation of a math object</td>
</tr>
<tr>
<td>Open a file or program</td>
<td>Open, save, and print a file from within a program</td>
<td>Recognize a numerical computer object</td>
</tr>
<tr>
<td>Find a file or program on a diskette and hard drive</td>
<td>Find a program command when given its menu location</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quit a program</td>
<td></td>
</tr>
</tbody>
</table>

This instrument was given to the students at the same time as the pre-treatment SOLO Superitem. The score from this instrument was used to evaluate whether students were computer literate or not computer literate.

Limitations and Delimitation

The main limitation of this study is its short duration. Eight lessons over 2 weeks is not enough time for students to make substantial changes in modeling achievement. The researcher used this instructional duration because it reflects the current time spent in most high school units dealing with the application of linear and quadratic models. The argument for this decision is, if a difference in modeling achievement is not achievable in two weeks then it is not enough of a change in student achievement for making instructional decisions.

A delimitation of this study is the fact that the process of mathematical modeling is complex. This makes controlling for extraneous variables such as multiple instructors difficult. Therefore, what was not controlled by the random sampling process will be monitored descriptively.
CHAPTER 3

ANALYSIS OF DATA

Introduction

This chapter presents the analysis of the data collected for this study. The analysis of the modeling achievement scores from the three modeling contexts is the primary focus of this study. Since modeling is such a complex process, other extraneous variables are monitored through comparing and contrasting the student and instructor interviews with class observations, student learning logs, and student achievement scores. This chapter is organized into six sections: (a) description of the sample, (b) methods of analysis, (c) tests of hypotheses, (d) summary of results, (e) analysis of student interviews, and (f) analysis of instructor interviews.

Sampling

Background

This study was limited to two sections of Finite Mathematics meeting simultaneously in different buildings. These two sections were taught during the first 6-week summer session of 1995, starting May 22 and ending June 30, with enrollments of 24 and 21. The sample was randomly chosen from a pool of the two sections.

The demographics of the students' backgrounds which is noted in the before treatment interviews show that the sample of this study covers a wide range of ages, humanity majors, and mathematical backgrounds.
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Sampling Results

The researcher used a randomized block design (Neter, Wasserman, & Kutner, 1985) to assign students to receive one of the two methods of modeling instruction.

Students were assigned to one of four beginning modeling levels and one of two computer literacy levels according to their scores on the pretreatment SOLO superitem, Tom’s Sweet Shop, and the Computer Literacy Mastery Test. These instruments were administered on May 26, 1995. The students were arbitrarily given identification numbers to identify them during the sampling procedure. The sorting of students into the sampling blocks is given in Tables 4 and 5.

Table 4. Identified Students in the Sampling Blocks

<table>
<thead>
<tr>
<th>SOLO 0</th>
<th>SOLO 1</th>
<th>SOLO 2</th>
<th>SOLO3&amp;4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Literate</td>
<td>1,6,10,17,19,20, 22,25,38,42</td>
<td>34,40</td>
<td>14,27,29, 31,37,43</td>
</tr>
<tr>
<td>Computer Illiterate</td>
<td>3,16,23,28, 32,35,36,39</td>
<td>21,45</td>
<td>2,4,9,41</td>
</tr>
</tbody>
</table>

Table 5. Block Size of the Sampling Blocks

<table>
<thead>
<tr>
<th>SOLO 0</th>
<th>SOLO 1</th>
<th>SOLO 2</th>
<th>SOLO 3&amp;4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Literate</td>
<td>10</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Computer Illiterate</td>
<td>8</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

After placing the students into the sampling blocks they were randomly selected to receive one of the two methods of instruction. Half the students were selected to receive modeling instruction using Interactive Physics by a process using randomly generated numbers. The results are revealed in Table 6. The remaining students received modeling instruction using physical objects (see Table 7).
Table 6. Sample Using Interactive Physics

<table>
<thead>
<tr>
<th></th>
<th>SOLO 0</th>
<th>SOLO 1</th>
<th>SOLO 2</th>
<th>SOLO 3 &amp; 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Literate</td>
<td>1,10,19,20, 22</td>
<td>34</td>
<td>14,27,37,43</td>
<td>7,13, 44</td>
</tr>
<tr>
<td>Computer Illiterate</td>
<td>3,23,32</td>
<td>21</td>
<td>4,2</td>
<td>5,8, 11,18</td>
</tr>
</tbody>
</table>

Table 7. Sample Using Physical Objects

<table>
<thead>
<tr>
<th></th>
<th>SOLO 0</th>
<th>SOLO 1</th>
<th>SOLO 2</th>
<th>SOLO 3 &amp; 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer Literate</td>
<td>6,17,25,38, 42</td>
<td>40</td>
<td>29,31</td>
<td>24,26,30,33</td>
</tr>
<tr>
<td>Computer Illiterate</td>
<td>16,28,35,36, 39</td>
<td>45</td>
<td>9,41</td>
<td>12,15</td>
</tr>
</tbody>
</table>

Analysis of SOLO Test Items

The purpose of the SOLO Superitem analysis is to measure what effects instructional method, beginning modeling ability, and computer literacy have on the modeling achievement of the students. This analysis is done for three posttreatment SOLO superitems (Appendix B) addressing three modeling contexts: (a) students applying a linear model to data, (b) students applying a linear model to a two-rate situation with a constant rate, and (c) students applying a quadratic model to a familiar geometric figure.

The modeling achievement is measured by identifying five SOLO modeling achievement levels. These levels are at least ordinal and can be considered interval since the difficulty of the levels can be quantified using Masters Partial Credit Model (cited in Wilson, 1990). Each step or item is scored as either a failure or success (Figure 5). If the correct pattern of wrong and correct responses is broken, then the score is eliminated from the study.
The modeling achievement of the students is analyzed for the three SOLO superitems. Modeling achievement is measured using the SOLO modeling levels. The three independent factors are beginning modeling ability (Factor A), method of instruction (Factor B), and computer literacy (Factor C). Figure 6 presents a schematic of the study’s design. Note that SOLO level 5 is pooled with level 4 due to its ceiling effect.

A three-factor ANOVA was used to study the three-factor interaction, the three two-factor interaction effects, and the three main effects of the three independent factors.

Using the computer statistic package of Stat 512+, in a Power Macintosh 7100/80AV, all seven null hypotheses were tested for each of the three posttreatment SOLO superitems.

**Figure 5. Performance Levels of a SOLO Superitem Example**

<table>
<thead>
<tr>
<th>Questions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student A</td>
<td>Wrong</td>
<td>Wrong</td>
<td>Wrong</td>
<td>Wrong</td>
<td>0</td>
</tr>
<tr>
<td>Student B</td>
<td>Correct</td>
<td>Wrong</td>
<td>Wrong</td>
<td>Wrong</td>
<td>1</td>
</tr>
<tr>
<td>Student C</td>
<td>Correct</td>
<td>Correct</td>
<td>Wrong</td>
<td>Wrong</td>
<td>2</td>
</tr>
<tr>
<td>Student D</td>
<td>Correct</td>
<td>Correct</td>
<td>Correct</td>
<td>Wrong</td>
<td>3</td>
</tr>
<tr>
<td>Student E</td>
<td>Correct</td>
<td>Correct</td>
<td>Correct</td>
<td>Correct</td>
<td>4</td>
</tr>
</tbody>
</table>
Choosing of an Alpha for the SOLO Superitem

In testing the null hypotheses, the researcher runs risk of rejecting them when they in fact are true (a Type I error). There is also the risk of failing to reject null hypotheses when they are false (a Type II error). Since the primary focus of this study was to determine which instructional method tested is more effective for teaching mathematical modeling, the consequence of a Type I error could be that a mathematics instructor may invest time and resources to implement the instructional method that apparently has proven superior when, in fact, there is no difference in student behaviors. The consequence of a Type II error would be to implement no change in the instructional method and deny the students a better learning environment. The researcher sees neither error as more serious and believes that an alpha = .05 provides a reasonable balance between the two.

Analysis of SOLO Superitem—Land Lord Dilemma

The context of the SOLO superitem, Land Lord Dilemma is to measure the student’s achievement in applying a linear model to data.

Table 8. Three Factor Analysis of Variance on Land Lord Dilemma

### Anova table for a 3-factor Analysis of Variance on $Y_1$: TEST1

<table>
<thead>
<tr>
<th>Source:</th>
<th>df:</th>
<th>Sum of Squares:</th>
<th>Mean Square:</th>
<th>F-test:</th>
<th>P value:</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLO (A)</td>
<td>3</td>
<td>4.011</td>
<td>1.337</td>
<td>1.417</td>
<td>.2612</td>
</tr>
<tr>
<td>TREAT (B)</td>
<td>1</td>
<td>4.8</td>
<td>4.8</td>
<td>5.089</td>
<td>.0331</td>
</tr>
<tr>
<td>AB</td>
<td>3</td>
<td>.587</td>
<td>.196</td>
<td>.207</td>
<td>.8904</td>
</tr>
<tr>
<td>COMPUTER LIT. (C)</td>
<td>1</td>
<td>.079</td>
<td>.079</td>
<td>.084</td>
<td>.7741</td>
</tr>
<tr>
<td>AC</td>
<td>3</td>
<td>3.662</td>
<td>1.221</td>
<td>1.294</td>
<td>.2984</td>
</tr>
<tr>
<td>BC</td>
<td>1</td>
<td>.131</td>
<td>.131</td>
<td>.139</td>
<td>.7123</td>
</tr>
<tr>
<td>ABC</td>
<td>3</td>
<td>4.917</td>
<td>1.639</td>
<td>1.738</td>
<td>.185</td>
</tr>
<tr>
<td>Error</td>
<td>25</td>
<td>23.583</td>
<td>.943</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There were no missing cells found.
Test for Three-factor Interactions

The first test conducted is for three-factor interactions. The $p$-value of ABC interaction is .185 (Table 8). The researcher concludes that there are no significant three-factor interactions.

Test for Two-factor Interactions

The second test conducted was for the two-factor interactions of AB, AC, and BC. The $p$-value for AB interaction, the interaction between beginning modeling level and instruction treatment, is .8904 (Table 8). The researcher concludes that no AB interactions are present.

The $p$-value for AC interaction, the interaction between beginning modeling level and computer literacy, is .2984 (Table 8). The researcher concludes that no AC interactions are present.

The $p$-value for BC interaction, the interaction between instructional treatment and computer literacy, is .7123 (Table 8). The researcher concludes that no BC interactions are present.

Test for Main Effects

The third test conducted is for main effects of each factor, beginning modeling ability (Factor A), method of instruction (Factor B), and computer literacy (Factor C). Since no significant three-factor or two-factor interactions are found, attention is focused on testing for the main effects of each factor on modeling achievement.

With a $p$-value for Factor A of .2612 (Table 8), the researcher concludes that no Factor A main effects are present.

With a $p$-value for Factor B of .0331 (Table 8), the researcher concludes that Factor B main effects are present. Modeling achievement scores for students who received
instruction using the computer microworld, *Interactive Physics* are significantly higher than for students who received instruction using physical objects.

With a P-value for factor C of .7741 (Table 8), the researcher concluded that no factor C main effects were present.

After examining the previous series of seven separate $F$-tests at a .05 level of significance, the researcher concludes that no three-factor or two-factor interactions are present and that Factor B, instructional treatment, is the only statistically significant main effect on modeling achievement. The researcher concludes that the method of modeling instruction has a significant effect on the modeling achievement of students applying a linear model to a data set. The modeling achievement scores show that those students who received computer instruction using *Interactive Physics* score higher than those students who received instruction using physical objects.

**Analysis of SOLO Superitem—Biathlon**

The context of the SOLO superitem, Biathlon is to measure the student’s achievement in applying a linear model to a two-rate situation with a constant distance.

Table 9. Three Factor Analysis of Variance of Biathlon

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F-test</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLO (A)</td>
<td>3</td>
<td>2.183</td>
<td>.728</td>
<td>.54</td>
<td>.6591</td>
</tr>
<tr>
<td>TREAT (B)</td>
<td>1</td>
<td>.978</td>
<td>.978</td>
<td>.726</td>
<td>.4021</td>
</tr>
<tr>
<td>AB</td>
<td>3</td>
<td>1.672</td>
<td>.557</td>
<td>.414</td>
<td>.7445</td>
</tr>
<tr>
<td>COMPUTER LIT. (C)</td>
<td>1</td>
<td>.28</td>
<td>.28</td>
<td>.208</td>
<td>.6524</td>
</tr>
<tr>
<td>AC</td>
<td>3</td>
<td>.387</td>
<td>.129</td>
<td>.096</td>
<td>.9618</td>
</tr>
<tr>
<td>BC</td>
<td>1</td>
<td>.001</td>
<td>.001</td>
<td>3.929E-4</td>
<td>.9843</td>
</tr>
<tr>
<td>ABC</td>
<td>3</td>
<td>1.233</td>
<td>.411</td>
<td>.305</td>
<td>.8214</td>
</tr>
<tr>
<td>Error</td>
<td>25</td>
<td>33.687</td>
<td>1.347</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There were no missing cells found.
Test for Three-factor Interactions

The first test conducted is for three-factor interaction. The $p$-value of ABC interaction is .8214 (Table 9). The researcher concludes that there are no significant three-factor interactions.

Test for Two-factor Interactions

The second test conducted is for the two-factor interactions of AB, AC, and BC. The $p$-value for AB interaction, the interaction between beginning modeling level and instruction treatment, is .7445 (Table 9). The researcher concludes that no AB interactions are present.

The $p$-value for AC interaction, the interaction between beginning modeling level and computer literacy, is .9616 (Table 9). The researcher concludes that no AC interactions are present.

The $p$-value for BC interaction, instructional treatment and computer literacy, is .9843 (Table 9). The researcher concludes that no BC interactions are present.

Test for Main Effects

The third test conducted is for main effects of each factor, beginning modeling ability (Factor A), method of instruction (Factor B), and computer literacy (Factor C). Since no significant three-factor or two-factor interactions are found, attention is focused on testing the main effects of each factor on modeling achievement.

With a $p$-value for Factor A of .6591, Factor B of .4021, and Factor C of .6524 (Table 9), the researcher concludes that no main effects are present.

After examining the previous series of seven separate $F$-tests at a .05 level of significance the researcher concludes that no three-factor or two-factor interactions or main effects are present. Therefore, the researcher concludes that the method of modeling
instruction has no significant effect on the modeling achievement of students when applying a linear model to a two-rate situation with a constant distance.

Analysis of SOLO Superitem—Squares

The context of SOLO superitem, Squares is to measure the student’s achievement in applying a quadratic model to a familiar geometric figure.

Table 10. Three Factor Analysis of Variance of Squares

<table>
<thead>
<tr>
<th>Source:</th>
<th>df:</th>
<th>Sum of Squares:</th>
<th>Mean Square:</th>
<th>F-test:</th>
<th>P value:</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLO (A)</td>
<td>3</td>
<td>15.412</td>
<td>5.137</td>
<td>5.759</td>
<td>.0039</td>
</tr>
<tr>
<td>TREAT (B)</td>
<td>1</td>
<td>.034</td>
<td>.034</td>
<td>.038</td>
<td>.8471</td>
</tr>
<tr>
<td>AB</td>
<td>3</td>
<td>4.113</td>
<td>1.371</td>
<td>1.537</td>
<td>.2295</td>
</tr>
<tr>
<td>COMPUTER LIT. (C)</td>
<td>1</td>
<td>2.034</td>
<td>2.034</td>
<td>2.28</td>
<td>.1436</td>
</tr>
<tr>
<td>AC</td>
<td>3</td>
<td>2.097</td>
<td>.699</td>
<td>.783</td>
<td>.5144</td>
</tr>
<tr>
<td>BC</td>
<td>1</td>
<td>2.447</td>
<td>2.447</td>
<td>2.743</td>
<td>.1102</td>
</tr>
<tr>
<td>ABC</td>
<td>3</td>
<td>2.558</td>
<td>.853</td>
<td>.956</td>
<td>.4288</td>
</tr>
<tr>
<td>Error</td>
<td>25</td>
<td>22.3</td>
<td>.892</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

There were no missing cells found.

Test for Three-factor Interactions

The first test conducted is for three-factor interaction. The $p$-value of ABC interaction is .4288 (Table 10). The researcher concludes that there are no significant three-factor interactions.

Test for Two-factor Interactions

The second test conducted is for the two-factor interactions of AB, AC, and BC. The $p$-value for AB interaction, the interaction between beginning modeling level and
instruction treatment, is .2295 (Table 10). The researcher concludes that no AB interactions are present.

The \( p \)-value for AC interaction, the interaction between beginning modeling level and computer literacy, is .5144 (Table 10). The researcher concludes that no AC interactions are present.

The \( p \)-value for BC interaction, the interaction between instructional treatment and computer literacy, is .1102 (Table 10). The researcher concludes that no BC interactions are present.

**Test for Main Effects**

The third test conducted is for main effects of each factor, beginning modeling ability (Factor A), method of instruction (Factor B), and computer literacy (Factor C). Since no significant three-factor or two-factor interactions are found, attention is focused on testing the main effects of each factor on modeling achievement.

With a \( p \)-value for Factor A of .0039 (Table 10), the researcher concludes that Factor A main effects are present. To identify what aspect of the beginning modeling levels significantly effected modeling achievement, the researcher later used the Scheffe’ method of post hoc comparisons. Since the \( p \)-value for Factor B is .8471 and for Factor C is .1436 (Table 10), the researcher concludes that no Factor B or C main effects are present.

The Scheffe’ method of post hoc comparisons is used for two reasons: (a) It allows for flexibility in comparing different combinations of groups and (b) it is a conservative method of comparison (Neter et al., 1985).

The comparisons that were of interest to the researcher are:

1. Simple comparison between consecutive levels.
2. Complex comparison between grouped lower and upper levels.
In Table 11, the group means and sizes are listed. In Table 12, five post hoc comparisons conducted by the researcher are recorded. Only one comparison is found to be statistically significant at the $\alpha = .05$ level, the comparison between the two lower levels and the two upper levels. The achievement scores of the students in the two lower beginning modeling levels are significantly lower than students in the two upper beginning modeling levels.

Table 11. Treatment Means of Beginning Modeling Levels

<table>
<thead>
<tr>
<th>Beginning Modeling Levels</th>
<th>Beginning Modeling Level Means</th>
<th>Group Sizes of the Beginning Modeling Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>2.133</td>
<td>15</td>
</tr>
<tr>
<td>Level 2</td>
<td>0.75</td>
<td>4</td>
</tr>
<tr>
<td>Level 3</td>
<td>2.111</td>
<td>9</td>
</tr>
<tr>
<td>Level 4</td>
<td>2.923</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 12. Comparisons of Treatment Means of Beginning Modeling Levels

<table>
<thead>
<tr>
<th>Comparisons</th>
<th>Schieffe' F-tests</th>
<th>P-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1 vs. Level 2</td>
<td>0.3919</td>
<td>0.7595</td>
</tr>
<tr>
<td>Level 2 vs. Level 3</td>
<td>0.3328</td>
<td>0.8016</td>
</tr>
<tr>
<td>Level 3 vs. Level 4</td>
<td>0.2275</td>
<td>0.8767</td>
</tr>
<tr>
<td>Levels1&amp;2 vs. 3&amp;4</td>
<td>3.0609</td>
<td>0.0390</td>
</tr>
<tr>
<td>Levels 1-3 vs. 4</td>
<td>2.4709</td>
<td>0.0757</td>
</tr>
</tbody>
</table>

The researcher concludes after the previous series of seven separate $F$-tests at a .05 level of significance, that no three-factor or two-factor interactions are present and only the statistically significant main effect is Factor A, beginning modeling level. The researcher concludes that beginning modeling level has a significant effect on the modeling achievement of students who are applying a quadratic model to a familiar geometric figure.
The post hoc comparisons show that students in the lower two beginning modeling levels have achievement scores that are significantly lower than students in the upper two beginning modeling levels.

**Summary of SOLO Superitem Analysis Results**

In the three modeling context of this study, statistically significant main effects in modeling achievement are found in SOLO superitem Landlord Dilemma (Test 1) and SOLO superitem, Squares (Test 3). In the SOLO superitem Landlord Dilemma, the main effect of treatment is found to be significant with students receiving computer instruction achieving significantly higher scores. In the SOLO superitem Squares, the main effect of beginning modeling level is found to be significant. Through post hoc comparisons it is found that students in the lower two beginning modeling levels achievement scores are significantly lower than students in the upper two beginning modeling levels.

**Analysis of Student Interviews**

**Introduction**

The analysis of the data from student interviews focuses on three concerns: (a) comparing SOLO written responses with the students’ verbal explanations of practices, (b) comparing changes in students’ attitudes toward modeling, and (c) comparing the SOLO written responses with students’ verbal beliefs about modeling and the manipulative.

Interviews were conducted at three different times during the study: (a) before treatment, (b) halfway through the treatment period, and (c) after treatment. One student was randomly selected from the 16 blocks of the sample.
Demographic Description of Sample

The sample consisted of 21 men and 24 women. The majority of the students were liberal arts and education majors. Over three fourths of the students had not used a Macintosh computer recently and only two of the students had used a Macintosh computer for any other use than word processing. Over half of the students viewed themselves as poor mathematical problem solvers, yet most spoke highly of the mathematics course they were presently taking.

Before Treatment Interviews

Introduction

The analysis of the data of before treatment interviews focuses on three concerns related to differences between the two treatment groups: (a) comparing SOLO written responses with the students' verbal explanations of practices, (b) comparing differences in students' attitudes toward modeling, and (c) comparing the SOLO written responses with students' verbal beliefs about modeling and the manipulative.

Comparing SOLO Written Responses with the students'

Verbal Explanations of Practices

The before treatment interview is compared with the SOLO superitem pretest responses to identify patterns between SOLO levels and the modeling practices of students. Two specific patterns that the researcher looked for are conflicting verbal and written responses and differences between sample blocks.

The before treatment written responses are weak in reflecting the student's true modeling ability in many cases because the SOLO pretest was given to them at the end of class and many students did not spend enough time to completely answer all questions
thoroughly. This is a universal pattern among those interviewed, students verbally revealed more modeling information than they wrote on the SOLO pretest. Six out of the 16 interviewed show that their modeling level is at least one level higher than their written responses reveal. For example, Student #16 did not write anything on the SOLO superitem; but when verbally interviewed, the student was able to work through a rough explanation of the first question. This identifies this student’s SOLO modeling level as a two instead of a one. When asked why no response at all was written on the pretest, the student explained that the questions on the test seemed too difficult. The verbal and written responses of all three of the SOLO posttests identify the same modeling levels in 13 out of 16 interviews. This difference may be explained by the SOLO superitem instrument. When asked how much time students spent on each of the three SOLO superitems, their responses ranged from 2 to 15 minutes. When asked how seriously they felt their achievement of these tests was, the students most often responded that it was important for them to do well. Many of them went on to explain that since these test questions were given on the same day as the final exam, they were unsure what grade significance the tests carried. Some students responded that they felt good about their answers on test questions because they felt confident in modeling since the most recent class time had been spent on this topic. The researcher did not find a pattern difference between any of the factors that the sample of students were blocked on.

The only other comparison pattern between verbal and written responses is students on both the SOLO pre and posttests usually attempted to graph the data on question four even if they were unable to answer the corresponding question. Students who did this almost unanimously responded they were confident in graphing and felt they might get the question correct. The majority of these students did not graph the data correctly and when interviewed about their graph were unsure of why they drew what they did.
Comparing SOLO Written Responses with the Students’ Verbal
Explanations Attitudes Toward Modeling

Only one attitude pattern between the students’ written responses and verbal attitudes became evident: Achievement and attitude toward modeling are positively related. With the exception of one student, all the students that spoke positively about the SOLO problems could answer two or more of the questions correctly. The students that answered only the first question unanimously responded with negative responses about any type of mathematical problem solving.

Comparing SOLO Written Responses with the Students’ Verbal Explanations
of Beliefs About Modeling

All the students interviewed revealed that they tried to answer the questions in the SOLO problem by relating them to mathematical problems that they had previously solved or had been taught to solve. Students that answered less than two of the questions correctly had very similar responses. Here is an example of a typical response, "I had seen problems like this one before but just could not remember the formula." Students that answered two or more of the questions usually referred to algebra. No students interviewed referred to any problem solving strategies such as guess and check or finding a similar simpler problem.

During Treatment Interviews

Introduction

These interviews are used primarily to monitor extraneous factors affecting modeling achievement such as modeling practices and attitude toward modeling activities.
Another purpose of these interviews is to monitor the students’ knowledge of the other treatment group’s activities to check for treatment contamination. During the interviews the students were asked to explain their written work (Daily Logs), as well as their cooperative and individual modeling practices (Observation Records).

Comparing Written and Verbal Responses with Attitudes Toward Modeling

Only one attitude pattern between the students’ written responses and verbal attitudes became evident: Most students are indifferent about using the computer to solve mathematical problems but students using physical objects are usually very opinionated about the worth of the lessons. Comments from students receiving computer instruction included, “it’s OK” and “I like it.” Comments from students receiving physical object instruction included, “I don’t like all this stupid science stuff,” contrasted with, “This is really fun, math should be taught like this all the time.”

Comparing the Written and Verbal Practices of the Student from the Treatment Groups

There are four differences in modeling behavior between the two treatment groups, (a) collection of data, (b) proficiency in handling the modeling medium, (c) revelation of problem situation, and (d) time spent in different aspects of modeling.

In both treatment groups the students collected data from the problem situation, analyzed the data, and answered questions about both the mathematical modeling and the real life situation. The students in both treatment groups were asked to collect specific measurement within the different mediums. Students using the computer medium struggled during the first lesson with questions such as, “Where am I suppose to get the time the ball is in the air?” In the remaining lessons the students responded within their cooperative groups with responses such as this when collecting data, “Just use the box
with that name on it and write down the numbers.” Students using the physical objects used stop watches and tape measures to collect their data. The majority of the students said that in every problem situation it was hard to figure out what measurement was wanted on the lesson. One student explained, “It is hard to know what time is wanted and then everyone reads the stop watches differently so we never know if we are right.”

The majority of students in the computer medium group became proficient in analyzing collected data into either a linear or quadratic mathematical model by the second lesson. This is not true of those students in the physical objects group, many of these students still struggled with how to analyze the data to find mathematical relationships. A student interviewed explained the problem this way, “How do you know which numbers to use to see if the slope is constant?” Once these same students had a mathematical equation relating two or more measurements, they had no trouble answering questions about the real life situation using their model. Students who used the computer to collect their data oftentimes did not know how to use their mathematical equation to answer the real life situation.

A typical student from the physical object group struggled in collecting the data and analyzing it into an equation. This same student very successfully could relate a solution back to the original problem situation. In contrast a typical student from the computer instructed group collected and analyzed data easily, but frequently did not see any connection to a real life solution. Observations of the two treatment groups show that many students in the physical object group related all modeling steps back to the original real life situation. For example, one student from the physical object group was quoted as saying, “But if we multiply the radius times four, does that mean it would be a larger or smaller circle than this one?”

The students in the physical object group usually spent 15 to 20 minutes collecting data while the students in the computer instructed group usually spent 5 to 10 minutes
collecting data. Due to errors in the method of measuring, one group using physical objects spent 40 minutes collecting data in lesson two.

**Verbal and Written Evidence of Treatment Contamination Due to Student Communication**

Some students did acknowledge that they talked to students in the other treatment group. The student interviews do not reveal any student attitude statements that suggest a preference of treatment. Most often, if a student commented about the other treatment group, it is because they knew a student in the other treatment group.

**After Treatment Interviews**

**Introduction**

The analysis of the data of the after treatment interviews focuses on three concerns related to differences between the two treatment groups: (a) comparing SOLO written responses with the students' verbal explanations of practices, (b) comparing changes in students' attitudes toward modeling, and (c) comparing the SOLO written responses with students' verbal beliefs about modeling and the manipulative.

**Comparing SOLO Written Responses with Students’ Verbal Explanation of Modeling Practices**

Only one pattern is found and it is on question three of SOLO test Land Lord Dilemma. The students that received computer instruction frequently explained the process of applying the linear model to the data. This pattern directly corresponds with the student’s SOLO responses. Three out of 8 students interviewed from the computer instruction treatment group scored three or four on the SOLO test Land Lord Dilemma and each of these 3 students explained the process of applying a linear model to data very well.
None of the students that were interviewed from the physical object instruction treatment group scored a three or higher on this SOLO test. One of the students interviewed from this treatment group did adequately explain how to apply a linear model to the data even though that student only scored a two on the test.

Comparing SOLO Written Responses with Explanation of Students’ Attitudes Toward Modeling

There are two patterns that emerged dealing with attitudes toward modeling. The first pattern is on the SOLO test Land Lord Dilemma. Nine out of the 16 students interviewed responded no to the question, “Would the computer or physical objects that you used in class help you solve this problem?” Six of those 9 students who responded no had received physical object instruction. These six students also received a score of one or less on the SOLO test. The interviews from the computer treatment group do not reveal the same pattern. Only 1 out of the 3 students who responded no received a low score on the SOLO test. Five of the students from the computer treatment group who responded yes received a score of three or higher.

The second pattern identified is students frequently responded that SOLO test Biathlon was not like any test they had seen before. Most students could clearly talk about the problem stated in the SOLO test, but they also did not think that the computer or physical objects would help them solve this problem. This pattern is evident across treatment groups and SOLO test scores.

Comparing SOLO Written Responses with Explanation of Students’ Beliefs About Modeling and Manipulatives

Two patterns emerged from the student responses in reference to beliefs about modeling and one pattern is identified in reference to manipulatives. The patterns are general and not specific to any of the three individual SOLO tests.
Most students felt that the SOLO tests did not reveal how much they knew about mathematical modeling. A typical response was, “I know more about this problem than getting one out of four questions right.” Students frequently responded negatively to the question, ”Are these problems what you expected to be doing in this course?” The majority of the students from both treatment groups responded that the SOLO tests and the treatment lessons were very different from other math courses they had taken.

Finally, the majority of the students who received computer instruction responded positively about the benefits of the computer as a tool for learning mathematical modeling. Conversely students receiving physical instruction frequently spoke negatively about the characteristics of this method of instruction. The students from the physical object treatment group were mixed in their attitude about whether using physical objects enhanced the modeling process. Their responses were not extremely negative, only noncommittal. For instance a typical student response was, “I suppose.”

Analysis of Instructor Interviews

Introduction

The information from the instructors’ interviews is analyzed together with the SOLO superitem scores, student’s daily logs, and classroom observations to monitor and explain the characteristics of the instructional treatments. The analysis of the data from the instructors’ interviews focuses on three concerns: (a) comparing SOLO superitem scores with the instructors’ explanation of instructional treatments’ successfulness. (b) differences between treatment practices of the instructors, and (c) comparing and contrasting the treatments’ effects caused by differing beliefs held by the instructors about mathematical modeling. The instructors were interviewed before the treatment, during the treatment, and after the treatment.
Comparing Students' SOLO Superitem Scores with Instructors' Views of Instructional Treatments' Successfulness

When the instructors were asked to assess the instructional methods effectiveness, the assessment information not only revealed the strengths and weaknesses of the two instructional methods but also revealed attitude changes toward their instructional methods.

The instructor using computer instruction noted that a positive characteristic of this method of instruction was the active interest in learning the new material that the majority of students demonstrated during the treatment period. This instructor indicated that a frustrating aspect of using *Interactive Physics* for instructions was that he was unable to answer many of the questions the students had about the lessons. Even though the instructor using computer instruction admitted to being frustrated at times; in all three interviews, the computer lessons were mentioned in positive terms.

The instructor teaching with the physical objects found the experiments very interesting, but felt that over half of the students were unable to see past the experiment settings to learn how to apply linear and quadratics equations to solve problems. He had strong feelings on why this happened: "Students spent too much time collecting data and goofing around." He explained further that he felt this method would work better if only one linear and quadratic experiment were used and the remaining instructional time was spent in lecture.

The instructor's change in attitude toward the instructional method he used is important because it attached personal worth to the methods of instruction. Instructors make pedagogical decision based on what they believe (Hiebert & Carpenter, 1992). The researcher identified a change in attitude by comparing the before treatment and after treatment interviews. Both instructors indicate that the unit on modeling would be a good change for their class. The researcher takes this type of response as neither positive or negative. The instructional practices of both instructors indicate that the initial expectations
toward their assigned instructional method is at least equivalent to their common instructional practices. The interview and observational patterns indicate that the instructor using the computer instruction remained positive throughout the study while the instructor using physical objects became less positive toward the method of instruction he used.

**Differences Between Treatment Practices of Instructors**

The class observations and instructor interviews reveal only two differences in instructor practices. The first difference in instructional practices is that the instructor using the computers talked less and expected the students to work on their own most of the time. If a student did not ask a question of the instructor, little personal communication occurred between the student and the instructor. In contrast the instructor using the physical objects interacted with the students as a group, as well as individually. When the students using physical objects were doing their experiments he often gave out helpful hints as he monitored their work. The second difference in instructional practices is the perceived difference in importance of completed assignments during treatment. Curiously, instructors explained that the completed worksheets from the treatment lessons would count as homework assignments. In both classes that meant that a completed assignment was worth approximately 10 points out of 600. But the students being given computer instruction were constantly asking the instructor if they got credit for a particular lesson.

**Comparing Instructors' Beliefs About Mathematical Modeling**

The instructors had differences in beliefs about mathematical modeling from the start. To begin with, the instructor using computer instruction viewed mathematical modeling as a tool to solve real life problems, while the instructor using physical object instruction viewed mathematical modeling as a process that developed a mathematical perspective. The after treatment interviews reveal that both instructors used their beliefs
about mathematical modeling to assess the successfulness of the instructional method they used. The instructor using the computer instruction was interested in the students actively modeling. In contrast the instructor teaching with the physical objects felt again that too much of the student’s time was spent collecting data rather than studying different examples of applying linear and quadratic equations to problem situations.
CHAPTER 4

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

Introduction

This chapter includes a summary of the findings of the study, conclusions based on analysis of the data, implications of the conclusions, and recommendations for future research.

Summary of the Study

The primary purpose of this study is to compare the modeling achievement of students receiving mathematical modeling instruction using the computer microworld Interactive Physics and the achievement of students receiving instruction using physical objects. Both methods of instruction incorporated small cooperative groups and identical learning objectives. The methods differed only in the medium with which the students worked.

This study was conducted in two sections of Finite Mathematics, during the 1995 summer semester at Montana State University. These two sections were taught the first 6-week summer session of 1995, starting May 22 and ending June 30, with enrollments of 24 and 21 respectively.

To minimize the differences which could effect modeling achievement, the independent factors of computer literacy and beginning modeling ability were used along with the instructional method to group students. The students were classified by modeling abilities into four groups using a pretreatment SOLO superitem. These four classifications were related to the five levels of the SOLO taxonomy. The students' computer literacy was
determined by a mastery test designed to identify proficiency in the Macintosh operating system. Four beginning modeling levels and two computer literacy levels were used to assign each of the students from the two Finite Mathematics classes into one of eight blocks of students. The students in each of these blocks were then randomly assigned to receive one of the two instructional methods. The two instructional groups simultaneously met in different buildings to minimize the contamination of the instructional methods.

After the students completed the modeling unit, three SOLO Superitem tests were given to the students. The SOLO superitem instrument measured the students' modeling achievement by matching written cognitive behaviors to five predetermined cognitive levels of the SOLO Taxonomy (Romberg et al., 1990). The three modeling contexts for instruction and assessment of students were: (a) applying a linear model to data, (b) applying a linear model to a two-rate situation with a constant distance, and (c) applying a quadratic model to a familiar geometric object. Each of the nine treatment activities and three posttreatment SOLO superitems were set in one of these modeling contexts.

A three-factor ANOVA is used to study the three-factor interaction, the three two-factor interaction effects, and the three main effects. Using the computer statistic package Stat 512+ and in a Power Macintosh 7100/80AV, all seven hypotheses are tested for each SOLO superitem at the alpha level of .05. Since modeling is such a complex process, other extraneous variables are monitored through comparing and contrasting the student and instructor interviews with class observations, student learning logs, and student achievement scores. The analysis of the data from observations and interviews focuses on three areas: (a) comparing SOLO written responses with the students’ verbal explanations of practices, (b) comparing changes in students’ attitudes toward modeling, and (c) comparing the SOLO written responses with students’ verbal beliefs about modeling and the manipulatives.
Conclusions Based on Analysis of the SOLO Superitems

**Land Lord Dilemma**

This instrument is designed to measure student’s achievement in interpreting, building, and solving a linear problem situation. The researcher concludes from analysis of the three-factor ANOVA at a .05 level of significance that no three-factor or two-factor interactions are present. Only the main effect of factor B, instructional treatment, is statistically significant on modeling achievement. The mean modeling achievement scores of students who received computer instruction using *Interactive Physics* are significantly higher for the modeling task than those of students that received instruction using physical objects. Modeling achievement scores reflect modeling knowledge independent of instructional medium because modeling achievement is an assessment using paper and pencil. The researcher concludes that students were able to use the process of data analysis to model linear situation more efficiently after instruction using the computer microworld *Interactive Physics* than instruction using physical objects. This conclusion supports Hoyles and Noss’s (1992) findings: In their research with junior high students physical manipulatives were compared to a computer microworld similar to the manipulatives. They concluded that the dynamic nature of computer microworlds seem to naturally facilitate an environment where students mathematically model problem situations. The conclusion that a significant number of students revealed proficiency at modeling linear situations reveals that the computer microworld *Interactive Physics* is an effective medium for modeling instruction in at least linear situations.
Biathlon

This instrument is designed to measure student’s ability to apply a linear model to a two-rate situation with a constant distance. With regards to the three-factor ANOVA at a .05 level of significance, no three-factor interactions, two-factor interactions, or main effects are present. The statistical measurement reveals that the differences in modeling achievement scores can not be attributed to any of the three factors for this instrument. The researcher designed the modeling unit activities with the assumption that a two-rate situation with a constant distance is so similar to other linear situations that only one activity would be needed to enable students to assimilate this model from their knowledge of the standard linear model. In further study of the literature, the researcher found evidence which stated that students need a variety of experiences to make any new problem situation understandable (Kysh, 1995). On the bases of this evidence the researcher concludes that there were not enough treatment activities to adequately attribute achievement score differences to the instructional methods.

Squares

This instrument is designed to measure student’s ability to apply a quadratic model to a familiar geometric figure. The researcher concludes from analysis of the three-factor ANOVA at a .05 level of significance that no three-factor or two-factor interactions are present. Only the main effect of factor A, beginning modeling ability, is statistically significant on modeling achievement. The researcher concludes that beginning modeling level has a statistical effect on the modeling achievement of students who are applying a quadratic model to a familiar geometric figure. Interview data reveal that the beginning modeling scores are suspect: The before treatment interviews show the scores from the
SOLO superitem Tom's Sweet Shop are universally lower than the student’s true ability. This evidence makes this conclusion suspect but still valid in the opinion of this researcher.

To identify where the significant differences between the modeling levels occur, the researcher used the Scheffe’ method of post hoc comparisons at a .05 level of significance. The achievement scores in beginning modeling levels 1 and 2 are significantly lower than the scores in beginning modeling levels 3 and 4. This is the only comparison found to be statistically significant at the .05 level.

The student behaviors associated with the student’s beginning modeling ability and modeling achievement are used to discuss the conclusions of the SOLO superitem Squares. A pretreatment SOLO Superitem Tom’s Sweet Shop is used to identify the beginning modeling ability of students and a posttreatment SOLO superitem Squares is used to identify the student’s ability to model a quadratic problem situation. The two lower modeling levels correspond to written student responses indicating they are unable to use any or only one piece of important information from the problem situation. The upper two modeling levels correspond to written responses indicating students use two or more pieces of information from the problem situation to write a symbolic mathematical model.

Evidence from the student’s written responses on the SOLO superitem and posttreatment interviews reveal that the majority of the students who successfully modeled this quadratic situation did so from their understanding of the geometric properties of squares. The researcher concludes that most of the students found it easier to write a symbolic mathematical model in this particular problem situation from their knowledge about squares rather than using a data analysis approach. Students who showed ability in modeling before the study were able to use their modeling intuition to successfully model a situation not made clear by recent instruction. The literature supports the idea that there are many ways to model a situation; students are searching the problem situation for any information
that will fit this situation into a similar successful modeling experience that they recall (Burkhardt, 1989).

The statistical differences of the achievement scores reported in this study support and reasonably extend the present literature, but the strength of the treatment is suspect. This study registered 21 hypotheses, each are tested at a .05 level. By chance alone, 1 hypothesis would be expected to be significant, this study found 2.

**Conclusions Based on Analysis of the Student and Instructor Interviews**

There is a dual purpose of the analysis of the student interviews: (a) monitor deviations from the experimental ideal and (b) identify differences between the instructional groups that emerged during the treatment period.

**Deviations from the Experimental Ideal**

One of the researcher’s concerns is that the blocked random sampling process would fail to produce similar groups. From the before treatment interviews, the researcher concludes that no response patterns would indicate any differences between the two instructional groups.

Communication between students receiving different instructional treatments is addressed in the design of this study. To monitor the influence of this threat on the study, students were asked questions specifically addressing this concern. No students responded in a way that would indicate that they had searched for or obtained information regarding the other group’s instruction.

Two instructor differences may have affected the study. One difference deals with attitude changes of the instructors toward the method of instruction that they were using. The other difference detected is in their instructional practices.
The during treatment and posttreatment interviews reveal attitude differences toward the instructional methods. Both instructors began teaching with positive attitudes toward their prescribed method of instruction; however, during the instruction period, the instructor using physical objects indicated that the method of instruction that he was using needed changes. He indicated that this method of instruction was not working as well as a traditional lecture method of instruction for learning objectives. This is not true of the instructor using computer instruction. He remained positive toward the method of instruction throughout the instruction period.

The instructors were given guidelines to follow in their role as facilitators of the cooperative groups. A difference in instructional practices emerged as the instructors showed their individuality and personality in classroom practices. The instructors differed in the amount of verbal instructions they gave and the way they collected the activity sheets. The study is designed to compare two instructional mediums and minimized all other instructional differences. For this reason the researcher can not make any conclusions regarding these differences in instructional practices and discusses them further in the future research section.

Differences in the Students’ Modeling that Emerged Within and Between Instructional Groups

Interviews and other observable data were collected before, during, and after the instructional treatment. The researcher is able to detect differences in the modeling practices, achievement, and attitude of students. Conclusions regarding these differences are addressed with respect to differences within and between instructional groups.
Differences observed within the instructional groups

The students receiving computer instruction learned a sense of empowerment and confidence in writing and using of linear models. This pattern appears midway through the study and is quite evident in the posttreatment interviews. The number of responses demonstrating modeling confidence became prominent and are accompanied by comments indicating an enjoyment of the activity. This pattern of success is prominent in the group receiving computer instruction. The group using physical objects does not show this pattern of change in modeling success and attitude.

One distinguishable pattern develops and remains to the end of the treatment period. Regardless of the instructional group, students who had a high beginning modeling ability usually enjoyed and successfully completed the modeling activities. Likewise, students who had a low beginning modeling ability complained about and struggled with the modeling activities. This pattern supports the main effects analysis where beginning modeling level is found to be significant when students were modeling a quadratic situation and contradicts the main effects analysis where instructional treatment is found to be significant when students were modeling a linear situation.

Differences Observed Between the Instructional Groups

Four observable differences emerge between the instructional groups: (a) difference in attitude toward the modeling activities, (b) difference in modeling practices, (c) difference in knowledge and success of connecting numeric patterns with symbolic expressions, and (d) difference in knowledge and success of writing symbolic equations.

From the during treatment and posttreatment interviews it is evident that the students receiving computer instruction enjoyed the modeling activities. Students using
physical objects responded differently; they frequently responded about how hard the activities were to complete.

It is evident from class observations and during treatment interviews that the instructional method affects how long it takes to collect the data. It took the students using physical objects anywhere from 10 to 20 minutes to collect their data, while students using the computer took 2 to 10 minutes to collect their data.

The group receiving instruction using physical objects revealed frustration in identifying numerical patterns and the group receiving computer instruction showed increased confidence and success in finding and identifying numeric patterns. Many of the students using physical objects expressed an inability to accommodate for data with measurement errors. Although the instructors addressed strategies for dealing with imperfect data, the students expressed an inability to use these concepts to find numeric patterns that best fit the data.

Writing and using the symbolic mathematical models is a struggle for the students receiving instruction using physical objects. Many students commented that they could not write the symbolic equations because the numerical analysis did not make sense to them. When students using the physical objects were successful in writing and using an equation to model the problem situation, they were able to use their model to explain relationships to the original problem situation. Students receiving computer instruction understood the process of writing the equations. It is evident from the observations and interviews that they knew the form of their desired model. Even though more of the students receiving computer instruction were able to write equations to model the problems situations, many times they were unable to use their model to explain relationships in the original problem situation.
Implications

Introduction

The conclusions of this study indicate that in a Finite Mathematics course at Montana State University, the method of instruction of a unit on mathematical modeling has some effect on modeling achievement. The problems addressed involved linear and quadratic relationships. The method of instruction differed in the medium that students used to model problems situations: One group worked with the computer microworld *Interactive Physics* and the other physical objects to model and solve problem situations.

In both instructional groups, students were asked to collect, evaluate, and interpret data so that, through seeing patterns or relationships, the students could write a symbolic model.

There are three implications from the conclusions of the analysis: (a) implications of differences in what factors were significant in the three SOLO superitems, (b) implications of the significance of instructional method on modeling achievement on the SOLO superitem Land Lord Dilemma, and (c) implications of the significance of beginning modeling ability on student’s modeling achievement on the SOLO superitem Squares.

Implications of Differences in What Factors Showed to be Significant in the Three SOLO Superitems

The three posttreatment SOLO superitems produce different statistical results. The researcher views these differences as key to explaining some of the connections between modeling pedagogy and modeling achievement of students. To understand the results of this study, the connections between the three independent factors and three modeling contexts are discussed. A three-way ANOVA is used to measure differences in student modeling achievement scores according to their beginning modeling ability, the method of modeling instruction they received, and their computer literacy. The purpose of this
analysis is to identify and explain the effect that the two different methods of modeling instruction have on the student’s achievement. The modeling achievement of students is measured in three different modeling contexts corresponding to the three posttreatment SOLO superitems. These are the three contexts addressed in the instructional modeling unit.

The statistical significant results of the three SOLO superitems are: (a) Land Lord Dilemma shows significant difference between instruction groups, (b) Squares shows significant difference based on the beginning modeling ability of students, and (c) Biathlon shows no significant difference between any of the three independent factors or their interactions. Due to the difference in achievement scores two statements from the literature on mathematical modeling are relevant. First, why is it that no independent factors or their interaction are significant for the SOLO superitem Biathlon. Second, why is it that the SOLO superitems Land Lord Dilemma and Squares result in different independent factors showing a significant amount of variation in the modeling achievement scores of the students.

Burkhardt (1989) stated that students must be able to apply knowledge about known models to new modeling contexts in order to successfully create and use a model. Students successfully model a situation when their modeling experiences are sufficient that they perceive them as similar enough to apply to proposed problem situations (Niss, 1989). Recent evidence shows that students need a variety of experiences to make any new problem situation understandable (Kysh, 1995). The researcher used a treatment of only one activity assuming that a two-rate situation with a constant distance is almost identical to linear situations. The researcher concludes that there were not enough treatment activities to adequately attribute achievement score differences to the instructional methods. This implies that any modeling situation requiring students to vary their present understanding of how to apply a model must have a variety of experiences. Students need structure to make
the modeling process explicit if they are to gain ownership of that modeling process or type of model (diSessa at al., 1991).

In addressing the two modeling contexts that produced significant differences in modeling achievement scores, the researcher draws your attention to the variation in the students’ modeling behaviors. First, it is important to focus on the analysis of the observational data. The main difference between the two instructional methods is the time and success with which students were able to search for numerical patterns. Two different methods students can use to write symbolic relationships are from their knowledge of the relationships in the situation or from the numerical patterns found from collected data. In this case most students who successfully modeled the Land Lord Dilemma did it through finding numerical patterns and most students who successfully modeled the Square did it through their knowledge of geometry. The unit activities taught modeling from a numerical analysis approach. Thus, a significant number of student achieved better modeling scores in finding numerical relationships through computer instruction than physical object instruction. The students who successfully wrote a symbolic model for the Square Superitem did so using a different method of modeling than was taught. The students who successfully wrote a symbolic model used their knowledge about squares rather than data analysis. This idea of different methods of modeling is not new. Skovsmose (1989) pointed out that knowledge about the modeling context is also a factor when students are searching for connections between previous modeling experiences and the proposed problem to be modeled. The metacognitive process by which students monitor the model building process searches for all relevant connections with past experiences (Niss, 1989). It is this understanding of the modeling processes that reveals the pedagogical complexity of mathematical modeling. From this perspective there are many ways students search for information to create a mathematical model of the problem situation (Adams & diSessa,
1991). When the modeling instruction and assessment focus on a particular modeling process, factors such as knowledge of the problem context can influence the results.

Implications of the Significance of Instructional Method on Modeling Achievement on the SOLO Superitem, Land Lord Dilemma

On the posttreatment SOLO superitem Land Lord Dilemma, significant differences in modeling achievement are found between the instructional groups at the .05 level in modeling achievement. The context of this SOLO superitem was applying a linear model to a given set of data. Within this modeling context, the modeling achievement scores of students receiving computer instruction using *Interactive Physics* were higher than students receiving instruction using physical objects. Observation and interviews reveal that the instructional groups differed in (a) attitude toward the modeling activities, (b) modeling practices, (c) knowledge and success in connecting numeric patterns with symbolic expressions, and (d) knowledge and success in writing symbolic equations.

To understand the implications of instructional method on modeling achievement, a discussion of the differences in instructional treatment is needed. Both instructional groups worked on activities that differed on one point only: the medium used by students to collect the data. The medium in which the students performed the modeling process was the only difference in the activities of the two groups. After the students collected the data, the process of analyzing the data, creating a symbolic model, and using their model to find solutions in the problem situation was identical. The observational data show that the students receiving computer instruction spent less time collecting data and more time analyzing the data than the students who received instruction using physical objects. The students receiving computer instruction were more successful and enjoyed the process of analyzing data more than the students using physical objects. It is unclear to the researcher whether deciphering measurement error or the process of taking physical measurements
accounts for these differences. This uncertainty is left for future research, but the researcher is confident that students receiving computer instruction show greater success and knowledge in the process of discovering numerical patterns and writing symbolic equations from them.

The fact that most of the students who received computer instruction received significantly higher modeling scores on a linear problem situation through the use of numerical analysis is an important pedagogical discovery. The current mathematical research from the U.S. Department of Labor Commissions in 1992 proposed that the analyzing and interpreting of numeric data to find patterns and relationships is one of the five skills of competence needed the in the United States work place. Computer microworlds such as Interactive Physics can be used successfully as a pedagogical medium for the instruction of the numerical analysis approach of mathematical modeling.

Implications of the Significance of Beginning Modeling Ability on Student's Modeling Achievement on the SOLO Superitem, Squares

A significant difference in modeling achievement is detected among beginning modeling levels at an alpha of .05 on the posttreatment SOLO superitem Squares. Applying a quadratic model to a familiar geometric object was the context of this SOLO superitem. Within the context of this superitem, the modeling achievement scores of students who had high beginning modeling abilities is higher than those students with low beginning modeling ability. The significant differences between combined levels 1 and 2 and combined levels 3 and 4 are identified through post hoc comparisons at the .05 level of significance.

Most students did not successfully model the problem situation the SOLO Superitem Squares. Only a couple of the students who did successfully find a symbolic quadratic model of this situation modeled the situation through numerical analysis. One
possible explanation is that numeric analysis of data best described by a quadratic model is more difficult than fitting data to a linear model. If this were the case, then more numerical analysis instruction on quadratic situation is needed by students regardless of their instructional medium or method. In the modeling activities that students performed during treatment, they were asked to analyze the data by checking for the numeric patterns of both the linear and quadratic models. Students first performed the numeric analysis for the linear model by studying the difference ratios of the independent and dependent variables. If no linear pattern were found, the students checked for a quadratic pattern by squaring the data of the independent variable and studying difference ratios of the independent variable squared and the dependent variable. To confirm their symbolic model, the students plotted the data and their model on the same graph to perform a visual reliability test. It is the extra step of squaring the independent variable data when checking for a quadratic model that influences the researcher to view data analysis of quadratic situations as more difficult than linear ones.

The researcher concludes that most of the students found it easier to write a symbolic mathematical model for this particular problem situation from their knowledge of squares and previous modeling experiences. Usiskin (1989) stated that the difficulty of a modeling situation is dependent on how different the problem situation is from known mathematical models. When creating a model any information about the problem situation or a previously used model can be utilized to successfully create a workable model (Skovsmose, 1989). The research and conclusions imply to the researcher that instruction in modeling using data analysis does not improve the students ability to model this situation, although previous modeling experiences and knowledge of the properties of squares did. Further implications of this discussion are left for further research.
Implications for Practice

This section summarizes the implications for mathematical modeling instruction reached by the researcher based on the conclusions of the study. These three implications for practice of modeling instruction are the researcher's recommendations resulting from this study.

1. A variety of modeling experiences with a common instructional structure are needed to make explicit the modeling process of applying standard models to new situations.

2. In the literature review it was stated that students monitor the model building process by searching for all relevant information from past experiences (Niss, 1989). From this perspective students are motivated to search for all relevant information in the process of creating a mathematical model (Adams & diSessa, 1991). The conclusions of this study imply that knowledge about squares improved the students' abilities to model the problem situation more than modeling instruction using data analysis. When modeling instruction and assessment practices focus on a particular modeling process, factors such as knowledge of the problem context and other modeling experiences influence the students' modeling practices. Instructors must be aware that each student initiates the modeling process with different past experiences and mathematical understanding.

3. A significant number of the students receiving computer instruction received higher modeling scores using numerical analysis modeling of linear problem situation. The current mathematical research from the U.S. Department of Labor Commissions in 1992 proposed that the analyzing and interpreting of numeric data to find patterns and relationships are one of the five skills of competence needed in the United States work place. Modeling instruction using computer microworlds such as Interactive Physics in conjunction with cooperative groups are a viable method for teaching the numerical analysis approach to mathematical modeling.
Recommendation for Further Study

This study was conducted to determine the effects of using a computer microworld or physical objects in instruction of mathematical modeling on modeling achievement. For the unit designed, some significant differences are found. It is not certain if similar results would be obtained if different modeling activities or assessment problems were used. Further study needs to be conducted with similar activities where the focus of the modeling instruction is on a different method of modeling. Also, the same study should be conducted with two improvements: (a) more instructional time given to the linear situation of two rates with a constant distance and (b) a different quadratic problem situation for the posttreatment SOLO superitem.

Results from this study indicate students working in groups receiving modeling instruction using the computer microworld Interactive Physics rather than physical objects are more successful at using the data analysis method of modeling. It is unclear what features of the computer microworld medium make the process of data analysis more understandable. Students who collected data from physical objects responded that they were often frustrated. Many times the collection of data took a group 20 minutes. These students also became frustrated with how to handle outliers and pick the best fit of two mathematical models when analyzing the data. Further research needs to be conducted to identify whether it is the process of measuring or measurement error that produces the largest difference between the two mediums. Also, further research should be conducted to determine if including instruction on how to collect data using physical objects produces similar results.

The study also shows that the instructional medium did not influence the achievement scores of the quadratic problem situation. Students from both instructional groups show success at modeling a quadratic situation by using an intuitive method of
modeling. Further research should be conducted to determine if more data analysis instruction would produce a difference in modeling ability using the data analysis approach. Other research is needed to explore the results of using activities that use different methods of modeling problem situations.

The instructors were given guidelines to follow in their role as facilitators of the cooperative groups. A difference in instructional practices emerged as the instructors showed their individuality and personality in classroom practices. The instructor using computer instruction was soft spoken and interacted very little with the students unless they directly asked a question. In contrast, the instructor using the physical objects was constantly moving from group to group interjecting comments and asking questions of group members. Another instructional difference surfaced on the grading of the modeling unit worksheets. The completed worksheets were worth 10 points in both classes; however, the instructor using computer instruction kept more noticeable account of those points which caused the students in his group to take more care in completing and returning the worksheets. Further research should be conducted to determine which are the best instructional practices when using computer microworlds for modeling instruction.

Results from this study indicate that students were not given enough instruction in how to model a linear situation that included two rates with a constant distance. One other possibility is that the SOLO superitem Biathlon is more difficult than the other two superitems. The construction of the SOLO superitem instruments does not support this hypothesis. The scores on SOLO superitem Biathlon and Land Lord Dilemma are within a half SOLO level. Using this study, further research is needed that includes more instructional time where a linear situation includes two rates with a constant distance.

The SOLO system of assessing modeling achievement had a major influence on the conclusions of the study. The SOLO taxonomy is relatively unknown as an assessment framework and needs more research dedicated to apply it to a variety of modeling contexts.
The recommendations of this study would be made stronger if the study were repeated using a different assessment instrument. Then results of the two studies could be compared. This would reveal information about the SOLO system of assessing mathematical modeling and validate or discredit the conclusions of this study.
REFERENCES


Appendix A

SOLO Superitem
Tom’s Sweet Shop

Tom has a company that makes candy bars and sells them in boxes of 10. Tom calculates his income, cost of production, and net gain/loss for a week from the following table.

<table>
<thead>
<tr>
<th>Number of Boxes</th>
<th>Gross Income</th>
<th>Tom’s Cost</th>
<th>Net Gain/Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.00</td>
<td>$2002.00</td>
<td>-$1999.00</td>
</tr>
<tr>
<td>2</td>
<td>$6.00</td>
<td>$2004.00</td>
<td>-$1998.00</td>
</tr>
</tbody>
</table>

1. If Tom did not sell any boxes of candy, what would be his net loss?

2. If Tom sold 10 boxes of candy, what would be his net loss or gain?

3. Write a mathematical expression that gives the net gain/loss in terms of N when given c, the number of boxes of candy.

4. Graph this equation below and indicate what is the minimum number of candy bars that must be sold for Tom to make a profit.


**Land Lord Dilemma**

A large apartment rental company has 2500 units available, and 1200 are currently rented at $450 per month. The landlord has set monthly expenses and expenses per tenant. The below table shows these figures.

<table>
<thead>
<tr>
<th>Number of tenants</th>
<th>Gross Income</th>
<th>Landlord's Cost</th>
<th>Net Gain/Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>$540,000</td>
<td>$40,000</td>
<td>$500,000</td>
</tr>
<tr>
<td>1210</td>
<td>$544,500</td>
<td>$40,200</td>
<td>$504,300</td>
</tr>
</tbody>
</table>

1. According to the above table if the number of tenants is 1220 what is the net gain by the landlord?

2. If there were no tenants in the building would the landlord lose or gain money and by how much?

3. Write a mathematical equation that gives the net gain/ loss N when given t, the number of tenants.

4. Graph this equation below and show what the graph would look like if the rent where raised $10 per month.
Biathlon

Joe intends to travel from Bozeman to Three Forks by both running and biking. The total distance is 33 miles. Joe can consistently cover a mile in 10 minutes by running and 4 minutes by biking.

1. If Joe ran 5 miles then how many miles did he bike on his trip from Bozeman to Three Forks?

2. If Joe ran 10 miles and biked the rest. What will be the total time it will take him to travel from Bozeman to Three Forks?

3. Write an equation that gives $T$, the total time spent running and biking the 33 miles when given $d$, the distance ran.

4. If Joe made the trip in two hours and thirty six minutes how many miles did he run?
Squares

How are the perimeters and areas of squares related? For instance if the perimeter of a square is 12 then the area is 9. The length of the sides of the squares are used in the following formulas to find these measurements.

Area of a square = (length of the sides)^2  Perimeter of a square = (length of the sides) x 4

1. Find the perimeter and area of three different sized squares.

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
</table>

2. If the perimeter increase from 12 to 16 how much did the length of each side of the square change?

3. Given the measurement for the perimeter P of a square, find an equation that will give the area A.

4. Let us say that we were working with equilateral triangles instead of squares. Using the equation below find the equation of the area A, of the triangle, if the perimeter P is given.

Area of an equilateral triangle = .433 (length of the sides)^2
Appendix B

Computer Literacy Mastery Test
These questions will be used to check your familiarity with the Macintosh computer operations.

How many times did you use a Macintosh computer in the last two months. (Circle number)

1 None
2 1-2
3 3-4
4 5-6
5 more than 7 times

How many different Macintosh computer programs have you use in the last two months. (Circle number)

1 None
2 1-2
3 3-4
4 5-6
5 more than 7 times

In the following scenario, you down at a Macintosh computer, displaying the screen displayed in figure #1.

1. You are instructed to open a file called new, on the hard drive. Which of the following best describes the first step that you would perform in opening the file new?

1 Type "Open new" (Go to question #3)
2 Release on "Open" in the Edit menu (Go to question #3)
3 Double click the mouse on the hard drive icon (Go to question #2)
2. Once the hard drive has been opened the computer screen will look like figure #2. Notice that the file new is visibly represented by an icon. Which of the following procedures best describes the first step you would perform in opening this file?

1. Type "Open new" (Go to question #3)
2. Double click the mouse on the new icon (Go to question #3)
3. Use the mouse to drag the new icon to the open menu (Go to question #3)

In a new scenario you have opened a file called disk to Bur in a word processing program. The screen that looks like figure #3.

3. Which of the following best describes how you would print the file Disk to Bur?

1. Release on "Print" under the file menu (Go to question #4)
2. Use the keys <command>, <open apple>, and <P> (Go to question #5)
3. Use the keys <open apple> and <P> (Go to question #4)
4. Type "Print" (Go to question #5)
When you are printing the file **Disk to Bur** the screen that looks like figure #4. This screen indicates that an imagewriter will be used to print your file. Since you would like to print your file on a Laserwriter, you select cancel.

4. Which of the following procedures best describes the next step that you would perform in changing to a different printer now that you have clicked on cancel in Figure #4.

1. Release on "**Printer**" from under the file menu (Go to question #5)
2. Release on "**Chooser**" from under the apple menu (Go to question #5)
3. Release on "**Printer Set-up**" from under the file menu (Go to question #5)

5. While still working with the file Disk to Bur you would like to open the file **projectile**. Which of the following procedures best describes the next step you would perform to complete this task?

1. Release on "**Open**" from the file menu (Go to question #6)
2. Double click on the **projectile** icon in the upper right hand window corner of the screen (Go to question #8)
3. Use the keys <open apple> and <O> (Go to question #6)
6. In your search of the file *projectile* it is not visible in the list of files shown in Figure #5. Which of the following procedures best describes your next step in locate the file *projectile*.

![Figure #5](image)

1. Click on the "Open" button (Go to question #8)
2. Click on "Read Only" Box (Go to question #8)
3. Select the down arrow to scroll down the list (Go to question #7)

7. The file *projectile* is still not visible; which of the following procedures would describe how you would locate *projectile* in the old backups folder.

1. Select the "Open" button (Go to question #8)
2. Double click on the icon old backup (Go to question #8)
3. Click on "Desktop" and then click on the "Open" button (Go to question #8)

8. Which of the following best describes the first procedure, you would perform in saving a file that you have been working on, called Deborah-9/5, with the new name of *ball*?

1. Release on "Save" under the file menu (Go to question #10)
2. Release on "Save As" under the file menu (Go to question #9)
3. Use the keys <open apple> and <S> (Go to question #10)
4. Type "Save ball" (Go to question #10)
9. Once you have a screen such as figure #6, what would be the best way to save the file with the new name, **ball**.

![Save File Dialogue Box](image)

*Figure #6*

1. Click on the "Save" button (Go to question #10)
2. Highlight "Deborah-9/5" with the mouse and type "ball" then click on the "Save" button (Go to question #10)
3. Click on "ball" and then click on the "Save" button (Go to question #10)

10. What best describes how you would end the program.

1. Shut off the computer
2. Type "Quit"
3. Release on "Quit" from the file menu
4. Release on "Quit" from the apple menu
Appendix C

Student Interview Questions
BEFORE TREATMENT INTERVIEWS
1. What do you expect to be doing in this course?
2. When you are asked to write a mathematical equation to explain or solve a problem situation; how do you do it?
3. Do you think a computer could help you solve a problem situation mathematically?
4. Do you think physical objects, like a science experiment, could help you solve a problem situation mathematically?
5. Did the four questions you answered on the worksheet revealed your ability to solve the problem presented?

DURING TREATMENT INTERVIEWS
1. Are you doing what you expected to in this course?
2. When you are asked to write a mathematical equation to explain or solve a problem situation; how do you do it?
3. Do you think a computer or physical objects could help you solve a problem situation mathematically?
4. Does this problem seem like any that you have done before?
5. Do you have any idea what the other groups are doing?

AFTER TREATMENT INTERVIEWS
1. Are you doing what you expected to in this course?
2. When you are asked to write a mathematical equation to explain or solve a problem situation; how do you do it?
3. Do you think a computer or physical objects could help you solve a problem situation mathematically?
4. Does this problem seem like any that you have done before?
5. Did the test questions reveal what you know about mathematical modeling?
Appendix D

SOLO Superitem Scores
<table>
<thead>
<tr>
<th>SOLO</th>
<th>TREATMENT</th>
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Treatment of 1 is computer microworld instruction.
Treatment of 2 is physical object instruction.
Computer Literacy of 1 means the student was computer literate.
Computer Literacy of 2 means the student was not computer literate.
Appendix E

Pilot Study Worksheet—Gravity Experts
Experts on Gravity

We may not be experts on gravity but we should it effects most of what we do every day.

When I was climbing a mountain this past summer in Glacier Park I had a lot of fun dropping rocks off of cliffs. I noticed a lot of similarities about gravity's effect on these rocks.

I would like you to answer some questions about how gravity effects rocks dropped off a cliff by using the program Interactive Physics. Your teacher will help you operate Interactive Physics if you need help with the program. Design perform experiments by collecting data and drawing graphs. Then try to make conclusion about how the questions below could be answered with a mathematical equation, graph, or well worded explanation. On the second sheet of paper given to you please keep all your work as a log of your work on these problems.

Terms to know:
Gravity -- Accretion of objects to the earth.
Accretion -- the change in rate of the velocity of an object.
Velocity -- the rate of an objects movement measured by distance per time
Distance -- Measurement of displacement between two points in space

I dropped rocks off of many different cliffs. So your answers need to work for any height cliff.

1. Does the mass of the rock matter when determining the velocity at which the rock falls?

2. How could we find out what velocity the rock is hitting the ground hogs at the bottom of the cliff?

3. How could we figure out how tall a cliff is if we dropping a rock from the top and watch it hit at the base of the cliff?