Comparing the cognitive differences resulting from modeling instruction : using computer microworld and physical object instruction to model real world problems
by Mark David Oursland

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University

Abstract:
This study compared the modeling achievement of students receiving mathematical modeling instruction using the computer microworld, Interactive Physics, and students receiving instruction using physical objects. Modeling instruction included activities where students applied the (a) linear model to a variety of situations, (b) linear model to two-rate situations with a constant rate, (c) quadratic model to familiar geometric figures.

Both quantitative and qualitative methods were used to analyze achievement differences between students (a) receiving different methods of modeling instruction, (b) with different levels of beginning modeling ability, or (c) with different levels of computer literacy. Student achievement was analyzed quantitatively through a three-factor analysis of variance where modeling instruction, beginning modeling ability, and computer literacy were used as the three independent factors. The SOLO (Structure of the Observed Learning Outcome) assessment framework was used to design written modeling assessment instruments to measure the students' modeling achievement. The same three independent factors were used to collect and analyze the interviews and observations of student behaviors.

Both methods of modeling instruction used the data analysis approach to mathematical modeling. The instructional lessons presented problem situations where students were asked to collect data, analyze the data, write a symbolic mathematical equation, and use equation to solve the problem.

The researcher recommends the following practice for modeling instruction based on the conclusions of this study. A variety of activities with a common structure are needed to make explicit the modeling process of applying a standard mathematical model. The modeling process is influenced strongly by prior knowledge of the problem context and previous modeling experiences. The conclusions of this study imply that knowledge of the properties about squares improved the students' ability to model a geometric problem more than instruction in data analysis modeling. The uses of computer microworlds such as Interactive Physics in conjunction with cooperative groups are a viable method of modeling instruction.

COMPARING THE COGNITIVE DIFFERENCES RESULTING FROM MODELING INSTRUCTION: USING COMPUTER MICROWORLD AND PHYSICAL OBJECT INSTRUCTION TO MODEL REAL WORLD PROBLEMS

by

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Signature Mark D. Ouwland

Date 3/3/97

To my wife, Sheryl; daughters, Beth, Kirsten, and Michel; and sons, Paul, Austin, and Grant

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ABSTRACT

This study compared the modeling achievement of students receiving mathematical modeling instruction using the computer microworld, *Interactive Physics*, and students receiving instruction using physical objects. Modeling instruction included activities where students applied the (a) linear model to a variety of situations, (b) linear model to two-rate situations with a constant rate, (c) quadratic model to familiar geometric figures.

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The researcher recommends the following practice for modeling instruction based on the conclusions of this study. A variety of activities with a common structure are needed to make explicit the modeling process of applying a standard mathematical model. The modeling process is influenced strongly by prior knowledge of the problem context and previous modeling experiences. The conclusions of this study imply that knowledge of the properties about squares improved the students' ability to model a geometric problem more than instruction in data analysis modeling. The uses of computer microworlds such as *Interactive Physics* in conjunction with cooperative groups are a viable method of modeling instruction.

CHAPTER 1

INTRODUCTION AND REVIEW OF THE LITERATURE

Introduction

In the mid-1980s, documents such as *A Nation at Risk* (National Commission of Excellence in Education, 1983) and *Educating Americans for the 21st Century* (National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1983) persuaded mathematics educators to debate educational reform as a matter of national concern. According to Davis (1992), the mathematics education community's formal response was *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989).

Historically, the reasons for formal adoption of standards are: to ensure quality, indicate goals, and promote change (NCTM, 1989). The NCTM (1989) used three criteria to decide what mathematical content should be included in the curriculum standards: (a) What is needed to think mathematically?, (b) What mathematical processes are needed by society?, and (c) What applications mathematics has in our technological society? First, the NCTM (1989) viewed learning mathematics as an active process rather than a collection of mastered concepts and procedures. The NCTM (1989) stated, "'knowing' mathematics is 'doing' mathematics" (p. 7). Second, the utility of mathematics as a tool is constantly changing as computers change the way we process information (NCTM, 1989). Finally, the NCTM (1989) stated, "changes in technology and the broadening of the area in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself" (p. 7).

Student activities which enhance with each curriculum standard have two main characteristics: (a) Every activity grows out of a problem situation, and (b) activities are designed to actively involve students in mathematics.

> Traditional teaching emphasis on practice in manipulating expressions and practicing algorithms as a precursor to solving problems ignore the fact that knowledge often emerges from the problems. This suggests that instead of the expectation that skill in computation should precede work problems, experience with problems helps develop the ability to compute. Thus, present strategies for teaching may need to be reversed; knowledge often should emerge from experience with problems. (NCTM, 1989, p. 9-10)

"Mathematical problem solving, in its broadest sense, is nearly synonymous with doing mathematics" (NCTM, 1989, p. 137). The NCTM (1989) stated:

> The mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can--(a) use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content; (b) apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics; (c) recognize and formulate problems from situations within and outside mathematics; and (d) apply the process of mathematical modeling to real-world problem situations." (p. 137)

An important component of mathematical problem solving is the process of mathematical modeling. Modeling is a subset of problem solving in which the task presented to the solver is nonroutine and the finding or designing of an appropriate model is part of the task (Burkhardt, 1989). Students may perform modeling by either applying models they know to new situations or building a new model (Niss, 1989). Reinterpreting a problem from a mathematical perspective gives a person mathematical power to simplify, structure, and generalize the problems. As the emphasis in the mathematics curriculum moves towards real-life problems, mathematical modeling will naturally gain more attention (Burkhardt, 1989). Blum and Niss (1989) contended that the process of modeling develops the competencies and attitudes needed for fostering creative and explorative problem solving.

Davis (1992) stated that modeling instruction that uses new technologies to represent mathematical concepts offers better instruction than traditional methods. Facilitating a change to alternative methods of mathematics instruction requires time and energy (Davis, 1992). The rationale behind using alternative representations in mathematics is to provide students with a less foreign view of abstract concepts or processes (Fennema & Franke, 1992). In the instruction of mathematical modeling, three instructional methods are predominant: (a) two dimensional drawings, (b) physical objects (manipulative), and (c) computer mediums (Hiebert & Carpenter, 1992). Drawing has been the most traditional instructional method, but the use of physical objects has gained great popularity, especially in foreign countries (Robitailli & Travers, 1992). The newest method utilizes computer modeling, which is the focus of this study.

In the past, mathematical notation systems have been static; however, the new computer mediums offer a whole new class of dynamic and interactive notations (Kaput, 1992). One aspect of studying the educational benefits of computer media is coming to terms with the rapid and revolutionary possibilities they provide. To better understand the new environment created by computer media, a study of the interactions between mental processes and physical actions is needed (Kaput, 1992). The National Research Council (1987) recommended a systematic program for the development of pilot education systems using computer-centered microworlds that engage learners in science- and mathematics-linked tasks.

A computer microworld is a computer program that generates an interactive environment where students create and test hypotheses. Computer visualizations of mathematical concepts that model real world situations will grab and hold a student's attention. These microworlds that model situations from outside the classroom become more than just mathematical activities--they become part of the students' lives (diSessa, 1984). For example, when a moving vehicle is simulated, students can precisely measure and study relationships between distance and time. Microworlds are used in this way to facilitate active learning. The dynamic nature of computer microworlds seems to naturally facilitate an environment where students mathematically model problem situations (Hoyles & Noss, 1992). One such microworld is *Interactive Physics,* which models the physical laws of nature so that students can construct and run lifelike experiments and demonstrations making the process of mathematical modeling more interesting and understandable. *Interactive Physics* and similar computer microworlds can act as a transitional bridge between the physical and mathematical worlds.

Research Problem

The problem of this study was to compare the modeling achievement of students receiving mathematical modeling instruction using a computer microworld and students receiving instruction using physical objects. This experiment used the computer microworld *Interactive Physics* and physical objects in selected sections of Math 150, Finite Mathematics, taught during the 1995 Summer Semester at Montana State University.

Definition of Terms

1. **Alternative representations** refers to familiar objects that represent abstract mathematical objects to provide students with a less foreign view of abstract concepts or processes (Hiebert & Carpenter, 1992).

2. **Affect** refers to the feeling that a student attaches to a practice or concepts (Schoenfeld, 1992).

3. **Cognitive** refers to mental concepts and processes constructed in a mental architecture that coordinates interaction between memory and recall (Schoenfeld, 1992).

4. **Computer literacy** is a person's procedural and cognitive proficiency on general computer usage. This is heightened by his/her exposure to the use of a computer as a tool.

5. **Computer Microworld** is a computer generated environment with a coherent system of metaphor designed to organize the users' thinking as they build and explore specific tools and simulations (Kaput, 1992).

6. **Extended abstract** is the use of an abstract general principle or hypothesis that is derived from the given information (Wilson, 1990).

7. ***Interactive Physics*** (Knowledge Revolution, 1993) is a computer microworld that simulates the physical laws of nature. It is an interactive coherent system of recognizable metaphors that forms a programming system intended to facilitate the building of physical simulations. These animated constructions help organize the user's thinking and visualization (Kaput, 1992).

8. **Mathematical modeling** is a subset of problem solving in which the task presented to the solver is nonroutine and the finding or designing of an appropriate mathematical model is part of the task (Burkhardt, 1989). Students perform mathematical modeling by applying their knowledge of known mathematical models to new situations (Niss, 1989).

9. **Mathematical modeling achievement** is cognitive behavior that a student exhibits and is identified as belonging to one of the five levels in the SOLO taxonomy.

10. **Multistructural** students use two or more discrete related pieces of information in the given information (Wilson, 1990).

11. **Physical objects** (manipulative) are concrete representations of abstract concepts or processes used for instructional purposes (Kaput, 1992).

12. **Plasticity** is a term used to explain the feature of linking actions of notation systems within microworlds (Kaput, 1992).

13. **Prestructural** students use no obvious pieces of information coming directly from the given information (Wilson, 1990).

14. **Unistructural** students use one obvious piece of information coming directly from the given information (Wilson, 1990).

15. **Relational** students use two or more pieces of information from the stem to integrate understanding of the information of the stem (Wilson, 1990).

16. **SOLO (Structure of the Observed Learning Outcome**) defines levels of attainment in the cycle of learning which measures the student's level of attaining modeling abstraction within a certain context (Wilson, 1990).

17. **SOLO superitem** is the instrument that measures student behaviors and identifies the student's position on the SOLO Taxonomy (Wilson, 1990).

18. **SOLO Taxonomy** is constructed from this theoretical basis by identifying five levels of student cognitive responses on a continuum specific to a particular modeling context (Wilson, 1990).

19. **Transfer distance** is a measure of the student's ability to apply a previous model to a new situation (Hiebert & Carpenter, 1992).

20. **Transparency** is the degree to which a notation system or medium reveals the key mathematical concepts embedded in a problem situation (Kaput, 1992).

Need for the Study

The recent increase in accessibility of computers in the classroom and development of interactive software is having an impact on mathematical instruction methods. Specifically, computer microworlds have unique possibilities to offer in the area of mathematical modeling instruction. To uncover the instructional possibilities, computer microworlds are compared to familiar instruction, such as manipulatives (Kaput, 1992). A controlled comparison can define the relationship that exists between technology, mathematical modeling, and pedagogy (Heibert & Carpenter, 1992). This study may help determine which alternative representation best reveals the mathematical concepts or procedures in the problem situation.

Questions to Be Answered

1. Is there a modeling achievement difference between students receiving instruction using the computer microworld *Interactive Physics* or those receiving instruction using physical objects?

2. Does pretreatment modeling ability effect the overall modeling achievement of the students?

3. Does the computer literacy of students effect their overall modeling achievement?

4. Does method of instruction and pretreatment modeling ability interact on modeling achievement?

5. Does method of instruction and computer literacy interact on modeling achievement?

6. Does computer literacy and pretreatment modeling ability interact on modeling achievement?

7. Does method of instruction, pretreatment modeling ability, and computer literacy interact on modeling achievement?

Framework for Comparison

To narrow the modeling scope of this study, four epistemological factors are used to describe and identify each modeling situation: (a) purpose, (b) method of modeling instruction, (c) nature of model produced, and (d) method of verification to narrow the modeling scope of this study (Secretary's Commission on Achieving Necessary Skills, 1992). The purpose of the modeling activities is to teach students how to apply linear and quadratic functions to new situations. The study compares the modeling achievement of students who received two different methods of modeling instruction. Both methods of instruction have identical learning outcomes, problem solving activities, and mathematical content. The instructional manipulatives used are the only differences between the two methods of instruction. The modeling activities are designed to help students write a symbolic mathematical model to solve the problem presented. The students verified their newly created symbolic models by comparing them to instructional manipulative context.

The modeling achievement of the students is measured by comparing the cognitive behaviors of students to predetermined cognitive levels of modeling behaviors. These predetermined cognitive levels are derived from the SOLO taxonomy that are described in chapter two (Romberg, Zarinnia, & Collis, 1990). The assessment instrument used to identify cognitive modeling behaviors is SOLO superitems. Three of these tests are used to assess two linear and one quadratic problem situation. Since modeling is such a complex process, the effects of two other factors are measured: the students' beginning modeling ability and computer literacy. Other extraneous variables, such as students' interests and beliefs about mathematics, are monitored through constant comparison using class observations, student interviews, students' daily learning logs, and teacher interviews (Webb, 1993).

Review of the Literature

Introduction

Mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics (Schoenfeld, 1992). Teachers with a deep knowledge of mathematics usually tailor their teaching to a more conceptual view of mathematics, conversely teachers with a weaker knowledge of mathematics tend to use a rote structure when teaching mathematics (Jones, 1995). Many teachers have trouble teaching for both conceptual knowledge and procedural knowledge. The root of this problem is the perception that conceptual knowledge is important but takes too much time away from developing basic computational skills (Jones, 1995). Primarily, most teachers focus on procedural knowledge, yet their profession stresses the equal importance of teaching both conceptual knowledge and procedural knowledge (Schoenfeld, 1992). The mathematics profession has identified a gap between how it views mathematics and how it is taught. To reduce this gap, mathematics educators need new resources to assist in developing new perspectives on mathematical curriculum and pedagogy (NCTM, 1991).

The most influential document in promoting instructional changes in mathematics education has been the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). In the *Standards,* mathematical problem solving is stressed as the focus to teaching mathematical understanding. The authors of the *Standards* (NCTM, 1989) stated, "Problem-solving is the essence by which mathematics is both constructed and reinforced" (p. 37). The NCTM (1989) called for students to learn mathematics by (a) using problem solving approaches to investigate and understand mathematical content, (b) applying integrated mathematical problem solving strategies in order to solve problems from within and outside mathematics, (c) recognizing and formulating problems from situations within and outside of mathematics, and (d) applying the process of mathematical modeling to real-world problem situations.

The ability to invent a symbolic system that models a real world situation is central to mathematics (Krumholtz, 1989). This supports the notion that problem solving is at the very core of mathematics education (Schoenfeld, 1992). In the past, the mathematics that students have learned in school has had little to do with the way they have applied it outside of the classroom (Cuoco, 1995). Curricula need to be centered on mathematical habits of mind rather than on specific mathematical content (Cuoco, 1995). The problem-solving approach can be used to teach all the important mathematical concepts (NCTM, 1989). Instructors must learn the features and ramifications of pedagogy that encourages problem solving habits.

This researcher considers whether computer microworlds enhance the use of mathematical modeling in the mathematics curriculum. This investigation is divided into three main parts. The first section develops the theoretical framework for exploring pedagogical issues of mathematics education. The second section identifies the fundamental aspects of mathematical modeling. The final section defines and explores the possible roles of computer microworlds in the instruction of mathematical modeling.

Theoretical Framework of Mathematics Education

One of the most strongly held beliefs in mathematics education today is that mathematics should be taught for understanding. In this section old and new psychological theories of learning are used to build a framework for examining the various aspects of understanding mathematics. Then, this framework is used to discuss the key issues of learning and pedagogical theory related to understanding mathematics.

Internal and External Representations of Understanding

The framework for considering the understanding of mathematics is built on the assumption that knowledge is represented internally and that these internal representations are structured. Mathematical ideas, must be represented in ways that our minds process them. Communication requires that these representations be made external in the form of spoken language, written symbols, or physical objects (Dossey, 1992). Because mental representations are not observable, discussion of the exact structure of such representations is based on inference. During the early part of this century, such discussion was viewed as nonscientific by psychologists because the representations were not observable (Hiebert & Carpenter, 1992). Presently, psychological research on mental representations is central to cognitive science (Hiebert & Carpenter, 1992).

In laying down the framework for investigating the learning and teaching of mathematics, two assumptions regarding internal and external representations are observed. First, it is assumed that there exists some relationship between external and internal representations. In a mathematical setting, it can be said that the nature of the external mathematical representations which a person uses to model an idea expresses the way a person thinks about that mathematical idea (Kaput, 1992). This view of understanding an idea is important for our discussion on instruction because the external representations (words, pictures, symbols, or objects) that students use in communicating their ideas are the only information that instructors have to assess the students' understanding of those ideas. This is not a precise observation, but an assumption that students who can represent an idea in multiple external presentations understand the idea more completely (Hiebert & Carpenter, 1992). The second assumption is that internal representations are connected to each other. Again, these connections can only be inferred, but it makes sense to view connections between ideas forming internally when they are being demonstrated externally (Hiebert & Carpenter, 1992). External connections can be constructed between mathematical ideas in the same or differing representations. Connections between ideas are often based on similarities or differences observed when students find patterns and predictable regularities between the ideas (Hiebert & Carpenter, 1992).

To discuss connections within an internal representation, it is useful to build a visual representation of the relationship between the different ideas. These visual representations are models and are specifically referred to as networks in the literature. The two prominant types of networks found in the literature are vertical hierarchies and webs (Hiebert & Carpenter, 1992). A network of connections can be thought of as a hierarchy. Some representations fit as details underneath or within other representations. Another type of network is a web. Nodes or ideas have threads between them which represent student made connections or relationships (Hiebert & Carpenter, 1992).

This framework of external and internal representations for understanding mathematics is used to address specific implications of current research and for discussing issues of mathematics instruction, both past and present.

Learning Viewed Through External and Internal Representations

When a student has connected an idea to his/her internal network of ideas, it is considered to be understood (Hiebert & Carpenter, 1992). The degree of understanding depends on the number and strength of the connections. This definition of understanding is not new but has been a constant theme throughout the mathematical education literature of Brownell, Dewey, Polya, and Wertheirmer (cited in Hiebert & Carpenter, 1992). Even though constructivism does not directly imply that facilitating the creation of relationships between mathematical ideas is vital to teaching for understanding, it certainly promotes such educational practices.

When assimilating new information, there are two kinds of connections or relationships that learners construct: (a) relationships based on similarities and differences, and (b) relationships based on inclusion. Students usually create relationships of the first type by working on the same mathematical concept in different external representations. A problem that frequently occurs in mathematics education is that students tend to focus on similarities and differences presented by the external representation rather than those of the mathematical idea. This is why symbolic representations are so important to mathematics education. Although written symbols are very abstract, they also represent mathematical ideas very distinctly. The processes of building relationships through similarities and differences reappears throughout a student's career in mathematics (Schoenfeld, 1992). Connections built on inclusion relationships assume that one mathematical idea or procedure is a special case connected to another. A common way of applying problem-solving strategies is to view new problems as special cases of a general type of previously solved problem (Schoenfeld, 1992).

When the process of learning mathematics is viewed from a framework of internal and external representations, mathematical understanding is exhibited when students reveal connections or relationships between existing and new information (Polya, 1973). Learning is initiated through situations that challenge the existing network's relationship and organization of ideas. Students change their existing network through the formation and realignment of relationships between major ideas. Aligning the student's understanding of a mathematical idea with its true abstract meaning is the task of the instructor. The accommodation of new information only insures the formation and realignment of relationships. Making sure the learning process leads to greater mathematical understanding is the next issue to be addressed.

Teaching Viewed Through External and Internal Representations

Written symbols are keys to understanding and using mathematics because they are the most exact external representations in mathematics. Written mathematical symbols are also an international language allowing for easy communication of ideas within the field of mathematics. Also, the abstract characteristics of mathematical symbols can be an educational drawback. When mathematics is taught using only symbols, it becomes a tool for manipulating those symbols rather than representing complex ideas. It is important to remember that these symbols are not created arbitrarily, but arise from needs of mathematics activities, science, and daily life (Dossey, 1992). The meaning of written symbols can develop from outside the symbol system by creating connections to other representations or by developing connections within the symbol representation itself. Basically, there are two types of mathematical symbols: symbols that stand for quantities and symbols that stand for relationships between quantities. Once meanings are established for individual symbols it is possible to think about creating meanings for rules and processes that govern actions on these symbols (Hiebert & Carpenter, 1992).

The argument that meaning can be derived through building relationships within a symbol system was advocated by Brownell in 1938. When building meaning for symbols by connections within a symbol system, the symbol must first represent a mathematical object rather than a mark on paper. Then, patterns about how that symbol is related to other symbols in the system can be established (Hiebert & Carpenter, 1992). Students must realize that symbols carry identities of their own and that they may be related to each other and operated on in well-defined manners.

Written symbols are an intellectual tool used to communicate what is already known to others and to personally organize and manipulate mathematical ideas. Both functions of symbols require connections: The public function requires connections with real-life representations, while the personal function requires connections within the symbol system (Hiebert & Carpenter, 1992). In either case the role of research should be toward describing and understanding how to facilitate the construction of these connections. Research must compare the strengths and weaknesses of different instructional methods to understand what effect they have on the relationships students construct between mathematical symbols (Hiebert & Carpenter, 1992).

Understanding the meaning of both quantifier and action symbols can be facilitated by introducing alternative representations. Through using different representations, students are encouraged to construct relationships between written symbols and alternative representations. There are three predominant types of alternative representations used in the instruction of mathematical modeling: (a) physical three-dimensional objects, (b) two-dimensional drawings, and (c) computer media (Hiebert & Carpenter, 1992). The rationale behind using alternative representations in mathematics is to provide students with a less foreign view of abstract concepts and processes (Fennema & Franke, 1992). Drawings have been the most traditional instructional method, with physical objects gaining great popularity especially in foreign countries (Robitailli & Travers, 1992). Intuitively, the concrete nature of physical three-dimensional objects provides students with the most familiar alternate representation of an abstract situation or mathematical idea. Most research has shown that the instructional use of physical objects yields mixed results. This research reveals that students do not automatically transfer relationships understood, while using physical objects, to the mathematical ideas they represent (Hiebert & Carpenter, 1992). The transfer of understanding from concrete representations to abstract ideas depends on how similar the student perceives the representations (Kaput, 1992). Two-dimensional drawings are less concrete than physical objects but more flexible in adapting to abstract objects (Hiebert & Carpenter, 1992). The traditional two-dimensional drawing is considered the standard alternative representation (Kaput, 1992). Computer media are the third alternative representation and are the focus of this study. In the past, mathematical notation systems have been static; however, the new electronic media have opened up a whole new class of dynamic and interactive representations (Kaput, 1992). The key to studying the educational benefits afforded by computer media is coming to terms with the rapid and revolutionary possibilities they provide.

It is important to note that just because one external representation is more distant from the problem context than another does not mean it is less useful (Hiebert & Carpenter, 1992). In most cases, written symbols are the preferred representation. Even though contextual, they are the most distant from the quantities they represent. It is usually helpful for students to fill this gap between quantities and written symbols with alternative representations, such as manipulatives (Hiebert & Carpenter, 1992). From this perspective, the effectiveness of an alternative representation is measured by its ability to fill the gap between quantities and written symbols. Another consideration, when measuring the effectiveness of alternative representations, is the social situation in which the representation is used. Features of a representation are personal because they are influenced by the students' previous experiences and knowledge. On the other hand, the alternative representation offers a powerful way of focusing students' attention on a shared experience for communicating ideas. Even though it is unclear what features students attach to alternative representations, the fact that they are public representations is a powerful instructional tool (Hiebert & Carpenter, 1992).

The purpose of this study is to assess the instructional benefits of a new computer microworld environment. To do this, the interactions between mental processes and physical actions of students is studied (Kaput, 1992). A benefit resulting from classroom computer usage is a change in the patterns of instruction. When a classroom is technology-rich, the learning environment becomes more dynamic because the students and the teacher become natural partners in developing mathematical ideas and solving mathematical problems. This relationship between technology, problems solving, and the teacher's role as a facilitator needs a better definition. One way to determine the instructional value of computer media is to compare it to other alternative representations and decide which representation best reveals hidden mathematical ideas (Hiebert & Carpenter, 1992).

Mathematical Modeling

Introduction

A philosophy of mathematics instruction calls for activities that help mathematicians, teachers, and students experience the invention of mathematics (Burkhardt, 1989). To apply this philosophy to current mathematics instruction, a much broader range of instructional strategies than the standard explanation-example-exercise approach is needed (Burkhardt, 1989). In the previous sections, a framework has been developed for examining the issues of understanding mathematics through building connections. Within this framework, students do not understand mathematical ideas from a single representation but from multiple representations and from their daily lives (NCTM, 1991). The creation of a mathematical representation in a problem-solving setting makes modeling a candidate for increased emphasis in the mathematics curriculum. The modeling process naturally creates an environment where students can make connections between mathematical ideas and processes (Niss, 1989). To effectively use modeling, mathematics instructors need to know which methods of modeling instruction promote the best environment for students to create connections between mathematical ideas.

According to Dossey (1992), students can use mathematics in two ways, as a tool or as a language of communication. Students can use mathematics as a tool to distance themselves and focus objectively on observed differences in problem situations. In doing so, mathematics becomes a mirror whereby a problem situation can be separated from its surroundings. Written symbols are abstract representations that encourage analysis through manipulation to discover hidden relationships in the problem situation. Also, students can view mathematics as a process of communication between people and groups of people. The language of mathematics is useful in defining and discussing problem situations--mathematics is a means of problem-solving communication. This perspective of mathematics is directed more toward a synthesis of mathematical concepts. Mathematical relations are constructed using mathematical language in order to provide a universal representation for communicating problem situations.

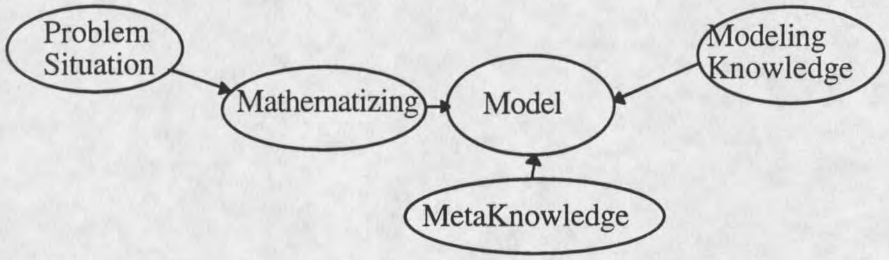
Mathematics is the use of symbols as tools to make the observable more focused. The symbols, in turn become the medium for communicating mathematical perspectives of reality. The invention of such systems in a problem setting is mathematical modeling. If the modeling process is made more creative through increased freedom and flexibility within planned activities, mathematics becomes more meaningful and personal to students (Niss, 1989). Modeling naturally uses mathematics as a tool and language for solving problems. These two perspectives create the framework for discussing mathematical modeling. Mathematical modeling instruction is discussed in three parts: theoretical, pedagogical, and assessment.

Theoretical Basis for Mathematical Modeling Instruction

Mathematical modeling is a subset of problem solving in which the task presented to the solver is nonroutine and the finding or designing of an appropriate mathematical model is part of the task (Burkhardt, 1989). Many examples of modeling have been reviewed and they fall into one of two categories. Representations of the modeling process either focused on the student's knowledge or the student's processes and products. An explanation of these two modeling types is reviewed and becomes the foundation for discussing modeling instruction.

Models of Mathematical Modeling. The first modeling type focuses on the knowledge that the student brings to the modeling situation. Skovsmose (1989) broke mathematical modeling (Figure 1) into three different cognitive domains: (a) mathematical knowledge, (b) modeling knowledge, and (c) reflective knowledge used to monitor the process of modeling. Reflective knowledge focuses on a conceptual or metaknowledge used by the student to synthesize their knowledge of mathematical concepts and modeling theories.

Figure 1. Skovsmose (1989) Model of Mathematical Modeling



Many mathematics education reform programs that emphasis mathematical modeling built their curriculum using this perspective of modeling. An example of such a program is the Computer-Intensive Algebra Program (Heid & Zbiek, 1995). CIA focused on mathematical modeling instead of classic word problems; it centers on algebraic concepts, modeling strategies, and use of multiple representations (Heid & Zbiek, 1995). The three main curriculum objectives correspond directly to the three areas of knowledge identified by Skovsmose.

The second modeling type focuses on the processes and products of the student during the modeling process. In the *Standards* (NCTM, 1989), the process of mathematical modeling is illustrated by the diagram shown in Figure 2. First , students must formulate the problem and decide what is important from the given information.