Numerical analysis of two-phase fluid flow and heat transfer in porous media
by Matthew William Waite

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Mechanical Engineering
Montana State University
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Abstract:
Heat transfer and fluid flow in porous media have several important engineering applications. Among
these applications are geothermal reservoirs, the dryout of moist organic soils due to forest or grass
fires, and agricultural soil considerations. This study investigated transient and steady state heat and
mass transfer of water with phase change in a homogeneous and isotropic Darcian porous medium
heated from the side. The top surface was isothermal at atmospheric temperature, the lower surface was
isothermal at an elevated temperature, while the left wall was adiabatic. The right side was subjected to
a constant, uniform heat flux. The two vertical walls were impermeable, and the upper surface was
maintained at a constant pressure. Fluid crossed the lower surface at a constant, uniform velocity.

The mathematical equations that govern two-phase fluid flow in porous media are highly nonlinear. In
this study, these coupled nonlinear governing equations were solved using numerical methods. The
energy equation was discretized and solved by using the control-volume based finite difference
method. The continuity and momentum equations were combined, discretized, and solved using the
error vector propagation method.

The resulting solutions were expressed in plots of constant temperature, streamlines, and velocity
vectors. The parameters studied include the porous material, expressed as the Rayleigh number (Ra),
the imposed side heat flux, the bottom surface temperature, and the bottom surface permeability. The
resulting isotherm, streamline, and velocity vector plots were studied, and conclusions were drawn
regarding each parameter. Additionally, transient plots were run for several cases. These transient
solutions provided additional insight into the current problem.

The results obtained show that the Rayleigh number is a significant parameter. As Ra was decreased,
the porous material was less permeable, and there was a shift in heat transfer from convection to
conduction. The result was more vapor formation and an overall hotter enclosure. As the imposed side
heat flux was increased, there was more vapor formation, an enclosure with more fluid at or near the
saturation temperature, and faster fluid motion. As the bottom surface temperature was increased, there
was more vapor formation and more fluid was at or near the saturation temperature. A permeable
bottom surface enhanced heat transport from the bottom surface upward through the enclosure.
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of

Master of Science

in

Mechanical Engineering

MONTANA STATE UNIVERSITY - BOZEMAN
Bozeman, Montana

April 1997
APPROVAL

of a thesis submitted by

Matthew William Waite

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Approved for the College of Graduate Studies

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(Date)
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Signature ________________________________
Date ________ , ________
I would like to thank Dr. Ruhul Amin for his guidance in my thesis work. I would also like to thank Dr. Thomas Reihman and Dr. Alan George for their support as committee members. I would also like to express my appreciation to Dr. C. Y. Wang for the initial version of the computer code used in this study.

My appreciation goes to the Department of Mechanical and Industrial Engineering for providing me with financial assistance. I would also like to thank my fellow graduate students and the staff of the Department of Mechanical and Industrial Engineering, whose support made this project successful, enjoyable, and rewarding.
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<tr>
<td>A</td>
<td>aspect ratio of porous enclosure (=L_y/L_x)</td>
<td>-----</td>
</tr>
<tr>
<td>(C_p)</td>
<td>specific heat</td>
<td>(J/kg\cdot K)</td>
</tr>
<tr>
<td>(D(s))</td>
<td>capillary diffusion coefficient, Eq. (25)</td>
<td>(m^2/s)</td>
</tr>
<tr>
<td>(f(s))</td>
<td>hindrance function, Eq. (22)</td>
<td>-----</td>
</tr>
<tr>
<td>(g)</td>
<td>gravitational acceleration</td>
<td>(m/s^2)</td>
</tr>
<tr>
<td>(g)</td>
<td>gravitational acceleration vector</td>
<td>(m/s^2)</td>
</tr>
<tr>
<td>(h)</td>
<td>enthalpy</td>
<td>(J/kg)</td>
</tr>
<tr>
<td>(h_{fg})</td>
<td>latent heat of liquid-vapor phase change</td>
<td>(J/kg)</td>
</tr>
<tr>
<td>(H)</td>
<td>volumetric enthalpy</td>
<td>(J/m^3)</td>
</tr>
<tr>
<td>(j)</td>
<td>diffusive mass flux, Eq. (26)</td>
<td>(kg/m^2\cdot s)</td>
</tr>
<tr>
<td>(J(s))</td>
<td>J-function for capillary pressure, Eq. (24)</td>
<td>-----</td>
</tr>
<tr>
<td>(k_r)</td>
<td>relative permeability</td>
<td>-----</td>
</tr>
<tr>
<td>(k_{eff})</td>
<td>effective thermal conductivity, Eq. (15)</td>
<td>(W/m\cdot K)</td>
</tr>
<tr>
<td>(K)</td>
<td>absolute permeability</td>
<td>(m^2)</td>
</tr>
<tr>
<td>(L_x)</td>
<td>length in x (horizontal) direction</td>
<td>(m)</td>
</tr>
<tr>
<td>(L_y)</td>
<td>length in y (vertical) direction</td>
<td>(m)</td>
</tr>
<tr>
<td>(Le_c)</td>
<td>Lewis number of capillary diffusion [=(\varepsilon K)^{1/2} \alpha C_{pi}/\nu k_{eff}]</td>
<td>-----</td>
</tr>
<tr>
<td>(p)</td>
<td>pressure</td>
<td>(Pa)</td>
</tr>
<tr>
<td>(p_c)</td>
<td>capillary pressure, Eq. (23)</td>
<td>(Pa)</td>
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<tr>
<td>q</td>
<td>heat flux</td>
<td>W/m²</td>
</tr>
<tr>
<td>Q</td>
<td>dimensionless heat flux (= qL_j/[k_{eff}(T_{sat}-T_0)])</td>
<td>-----</td>
</tr>
<tr>
<td>Ra</td>
<td>single-phase Rayleigh number (= KL_g \rho_f \beta (T_{sat}-T_0) C_{pl}/(\nu k_{eff}))</td>
<td>-----</td>
</tr>
<tr>
<td>Ra_{2ph}</td>
<td>two-phase Rayleigh number (= KL_g \rho_f C_{pl}/(\nu k_{eff}))</td>
<td>-----</td>
</tr>
<tr>
<td>s</td>
<td>liquid saturation</td>
<td>-----</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
<td>s</td>
</tr>
<tr>
<td>t^*</td>
<td>dimensionless time (= t/(L_y^2/\alpha_{eff}))</td>
<td>-----</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td>°C</td>
</tr>
<tr>
<td>u</td>
<td>velocity vector</td>
<td>m/s</td>
</tr>
<tr>
<td>u_x</td>
<td>velocity in x (horizontal) direction</td>
<td>m/s</td>
</tr>
<tr>
<td>u_x^*</td>
<td>dimensionless velocity in x direction (= u_x/L_x/\alpha_{eff})</td>
<td>-----</td>
</tr>
<tr>
<td>u_y</td>
<td>velocity in y (vertical) direction</td>
<td>m/s</td>
</tr>
<tr>
<td>u_y^*</td>
<td>dimensionless velocity in y direction (= u_y/L_y/\alpha_{eff})</td>
<td>-----</td>
</tr>
<tr>
<td>x</td>
<td>coordinate in horizontal direction</td>
<td>m</td>
</tr>
<tr>
<td>X</td>
<td>dimensionless x coordinate (= x/L_x)</td>
<td>-----</td>
</tr>
<tr>
<td>y</td>
<td>coordinate in vertical direction</td>
<td>m</td>
</tr>
<tr>
<td>Y</td>
<td>dimensionless y coordinate (= y/L_y)</td>
<td>-----</td>
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**Greek Symbols**

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_{eff})</td>
<td>effective thermal diffusivity (= k_{eff}/(\rho C_{pl}))</td>
<td>m²/s</td>
</tr>
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</table>
### NOMENCLATURE - continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>$\alpha_s$</td>
<td>heat capacitance ratio (solid-liquid) [= \rho_s C_{ps} / (\rho_f C_{pf})]</td>
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</tr>
<tr>
<td>$\alpha_v$</td>
<td>heat capacitance ratio (vapor-liquid) [= \rho_v C_{pv} / (\rho_v C_{pV})]</td>
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</tr>
<tr>
<td>$\beta$</td>
<td>thermal expansion coefficient</td>
<td>K(^{-1})</td>
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<tr>
<td>$\gamma_h$</td>
<td>two phase advection correction coefficient, Eq. (12)</td>
<td>-----</td>
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<tr>
<td>$\Gamma_h$</td>
<td>effective diffusion coefficient, Eq. (14)</td>
<td>kg/m·s</td>
</tr>
<tr>
<td>$\Delta\rho$</td>
<td>density difference [= \rho_l - \rho_s]</td>
<td>kg/m(^3)</td>
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<tr>
<td>$\epsilon$</td>
<td>porosity</td>
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<tr>
<td>$\epsilon_v$</td>
<td>volume fraction of vapor phase, Eq. (47)</td>
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<tr>
<td>$\theta$</td>
<td>dimensionless temperature [= (T-T_0)/(T_{sar}-T_0)]</td>
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<tr>
<td>$\lambda$</td>
<td>relative mobility</td>
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<tr>
<td>$\mu$</td>
<td>dynamic viscosity [= \rho \nu]</td>
<td>Pa·s</td>
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<tr>
<td>$\nu$</td>
<td>kinematic viscosity</td>
<td>m(^2)/s</td>
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<tr>
<td>$\rho$</td>
<td>density</td>
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<tr>
<td>$\sigma$</td>
<td>surface tension</td>
<td>N/m</td>
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<tr>
<td>$\psi$</td>
<td>stream function</td>
<td>m(^2)/s</td>
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<tr>
<td>$\psi^*$</td>
<td>nondimensional stream function [= \psi / \alpha_{eff}]</td>
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</tr>
<tr>
<td>$\Omega$</td>
<td>effective heat capacitance ratio, Eq. (13)</td>
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**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
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NOMENCLATURE - continued

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>c</td>
<td>capillary</td>
<td>N/A</td>
</tr>
<tr>
<td>l</td>
<td>liquid phase</td>
<td>N/A</td>
</tr>
<tr>
<td>o</td>
<td>initial</td>
<td>N/A</td>
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<tr>
<td>sat</td>
<td>saturated state</td>
<td>N/A</td>
</tr>
<tr>
<td>T</td>
<td>top</td>
<td>N/A</td>
</tr>
<tr>
<td>v</td>
<td>vapor phase</td>
<td>N/A</td>
</tr>
<tr>
<td>W</td>
<td>wall</td>
<td>N/A</td>
</tr>
<tr>
<td>x</td>
<td>x (horizontal) direction</td>
<td>N/A</td>
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<tr>
<td>y</td>
<td>y (vertical) direction</td>
<td>N/A</td>
</tr>
<tr>
<td>\kappa</td>
<td>&quot;kinetic&quot; property</td>
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</table>
Heat transfer and fluid flow in porous media have several important engineering applications. Among these applications are geothermal reservoirs, the dryout of moist organic soils due to forest or grass fires, and agricultural soil considerations. This study investigated transient and steady state heat and mass transfer of water with phase change in a homogeneous and isotropic Darcian porous medium heated from the side. The top surface was isothermal at atmospheric temperature, the lower surface was isothermal at an elevated temperature, while the left wall was adiabatic. The right side was subjected to a constant, uniform heat flux. The two vertical walls were impermeable, and the upper surface was maintained at a constant pressure. Fluid crossed the lower surface at a constant, uniform velocity.

The mathematical equations that govern two-phase fluid flow in porous media are highly nonlinear. In this study, these coupled nonlinear governing equations were solved using numerical methods. The energy equation was discretized and solved by using the control-volume based finite difference method. The continuity and momentum equations were combined, discretized, and solved using the error vector propagation method.

The resulting solutions were expressed in plots of constant temperature, streamlines, and velocity vectors. The parameters studied include the porous material, expressed as the Rayleigh number (Ra), the imposed side heat flux, the bottom surface temperature, and the bottom surface permeability. The resulting isotherm, streamline, and velocity vector plots were studied, and conclusions were drawn regarding each parameter. Additionally, transient plots were run for several cases. These transient solutions provided additional insight into the current problem.

The results obtained show that the Rayleigh number is a significant parameter. As Ra was decreased, the porous material was less permeable, and there was a shift in heat transfer from convection to conduction. The result was more vapor formation and an overall hotter enclosure. As the imposed side heat flux was increased, there was more vapor formation, an enclosure with more fluid at or near the saturation temperature, and faster fluid motion. As the bottom surface temperature was increased, there was more vapor formation and more fluid was at or near the saturation temperature. A permeable bottom surface enhanced heat transport from the bottom surface upward through the enclosure.
INTRODUCTION

A study of heat transfer and fluid flow with liquid-vapor phase change in a porous medium heated from the side has several important engineering applications. Among these applications are geothermal systems, oil reservoir engineering, thermal energy storage, heat pipes, and post-accident analysis of nuclear reactors. Additional applications include the dryout of moist organic soil due to a forest or grass fire [Hungerford et al. (1995)] and agricultural considerations concerning soil [Hungerford et al. (1991)]. Recently, scientists and engineers have been investigating how fluid flow in porous media relates to thermal energy storage. As the world’s supplies of fossil fuels dwindle, engineers have shown an increased interest in alternative means of energy storage and recovery.

Fluid flow in a porous medium is due to either forced or natural convection. Forced convection is caused by fluid moving due to an externally applied pressure difference. Natural, or free, convection is caused by buoyancy effects. Buoyancy is due to the combination of a fluid density gradient and a body force, usually gravity, that proportional to a fluid density gradient. The necessary density gradient can result from a temperature gradient in the fluid. The result of the buoyancy effect is fluid motion in which warm, light fluid rises and cool, heavy fluid falls.

In general, a porous medium is a material that consists of a solid matrix with an interconnected void [Nield and Bejan (1992)]. Usually the solid matrix is assumed to be
rigid, or undergoes only small deformations. The void spaces, or pores, allow one or more fluids to flow through the material. In two-phase flow, the void space can be shared by gases, liquids, or a combination of gas and liquid. A typical porous medium filled with a two-phase fluid is shown in Figure 1. The amount of void space relative to the volume of a porous enclosure can vary significantly from one material to another, and thus fluid flow and heat transfer characteristics change significantly from one material to another.

Figure 1. Typical Porous Medium With Two-Phase Fluid
Porous materials are in general defined by two quantities. These quantities, porosity and permeability, characterize the nature of fluid flow through porous media. Porosity is the fraction of total volume of the medium occupied by void space. Permeability is an empirical property that describes the ability of the porous medium to transmit fluid through it. In mathematical terms, permeability is the constant of proportionality between velocity (or mass flux) and the pressure gradient present in the porous enclosure. A high permeability porous medium allows easy, fast fluid motion, while a low permeability medium greatly restricts the motion of the fluid.

The mathematically expressed conservation equations that govern two-phase fluid flow in porous media are highly nonlinear. The equations are coupled and must be solved simultaneously. Therefore, analytical results are mostly limited to one-dimensional problems with several simplifying assumptions. For multi-dimensional problems governed by the complete conservation equations, such as most interesting engineering applications, numerical methods are the most common way to obtain solutions. The validity and accuracy of numerical methods are established by comparing numerical results with experimental results for the same case. After successful validation, the numerical methods can be satisfactorily extended to other situations.
Motivation for Present Research

The majority of previous research in porous media has been limited to single phase systems. Single phase studies limit the applicability of the research to situations in which the temperature is either significantly above or below the saturation temperature of the working fluid. For situations like geothermal reservoirs, temperature varies in such a range that studying the two-phase flow of the fluid is necessary. A typical geothermal reservoir consists of soil saturated with water at or near the boiling point.

Another interesting application of two-phase flow in porous media is that of oil reservoir engineering. In many underground oil reservoirs, the liquid crude oil may be mixed with another fluid such as natural gas. In such situations, the study of two-phase flow is valuable to determine the most effective method for crude oil extraction.

A porous enclosure heated from the side simulates many naturally occurring phenomena. Besides geothermal wells and oil reservoirs, such a study simulates the dryout of moist organic soil as a forest or grass fire passes through the region. The related application is the effect of heat and mass transfer on agricultural soils. If soil is heated above a certain temperature due to a forest fire, natural regrowth of the trees in that soil is not possible. So, the study of heat and mass transport of fluid in such media is essential.
Geometry of the problem analyzed is shown in Figure 2. The problem considered in this study consisted of a square, two-dimensional porous enclosure. The porous medium was assumed to be of uniform porosity and permeability. Initially, the enclosure was at ambient temperature and pressure and saturated with liquid water.

The thermal boundary conditions considered were as follows. The top surface was isothermal, at the ambient temperature. The bottom surface was isothermal, with various temperatures studied to cover a broad range of water temperatures in geothermal situations. The left wall was adiabatic and the right wall was subjected to a constant, uniform heat flux.

The hydrodynamic boundary conditions for the current problem were as follows. The top surface was maintained at atmospheric pressure. This allowed fluid to flow into or out of the enclosure through the top surface. The left and right walls were impermeable, not allowing fluid to pass through them. The flow conditions at the bottom surface studied included both zero and nonzero normal velocity components. In this way, the effect of permeability of the bottom surface was analyzed. A more rigorous description of both thermal and hydrodynamic boundary conditions is presented in a later section.

As the porous enclosure heated up, buoyancy caused the fluid next to the heated wall to move upward. Cooler fluid then replaced the warm fluid as it moved upward after
heating and left the enclosure. Cold fluid was then drawn downward from the top permeable wall to maintain conservation of mass. Thus began a pattern of fluid motion through the enclosure. Since fluid could cross the upper permeable surface in either direction, the warm fluid adjacent to the heated wall escaped through the upper surface while cool fluid was drawn into the enclosure through the other side of the upper surface. Eventually, the fluid adjacent to the heated wall boiled and a two-phase mixture was observed.

**Figure 2.** Geometry of Porous Enclosure Analyzed
Of interest in this study was the effect of different porous materials, imposed side wall heat flux, bottom surface temperature, and flow conditions at the bottom surface. The imposed heating conditions were applied to a porous enclosure initially at ambient temperature and pressure and saturated with liquid water. The effect of varying the above parameters was seen on the steady state temperature distribution, flow patterns, and vapor formation. Several cases were run as transient problems to determine fluid flow and heat transfer behavior as a function of time.
Literature Survey

General References

The textbook by Incropera and DeWitt (1990) provided information about the mechanisms of boiling and natural convection. This text was also important as a reference for thermophysical properties of fluids and solids.

The textbook by Roberson and Crowe (1993) was a general reference for fluid mechanics. This text was also a source for understanding the nature of fluid flow and interpreting stream functions and streamline plots. The Kays and Crawford (1993) textbook supplied information about natural and forced convection heat transfer.

Patankar (1980) was an excellent reference for numerical methods in heat transfer and fluid flow. The author discussed the derivation and application of the control-volume based finite difference method for heat conduction and fluid flow. The text by Anderson et al. (1984) provided additional information about finite difference methods for fluid mechanics with heat transfer.

The references by Roache (1971, 1972) gave information about the error vector propagation (EVP) method for solving Poisson-type equations. In the current study using the two-phase mixture model, the pressure field was solved for via the EVP method.
Bear (1972) provided an excellent general reference for fluid flow in a porous medium. Among the relevant topics covered were appropriate boundary conditions, two-phase flow, and Darcy fluid flow. A Darcy fluid obeys Darcy's law, a relation between fluid velocity (mass flux) and pressure gradient for porous media. Bear also presented a rigorous development of the traditional method of solving for two-phase flows in porous media, the separate flow model (SFM).

Scheidegger (1957) presented a good introduction to the topic of fluid flow through porous media. The author covered the basics of porous flow and applications. Scheidegger also provided examples of experiments to measure geometrical properties such as porosity and permeability. This text was also a good source of porosity and permeability data for various materials.

Nield and Bejan (1992) discussed the topic of convection in porous media. Several different heating conditions were covered, with particular attention to the onset of convective flows in each case. Also addressed were appropriate forms of hydrodynamic boundary conditions for Darcy flow and the mechanisms of boiling in porous media. This text was an excellent reference for values of porosity and permeability of different porous media.

Cheng (1978) discussed how fluid flow and heat transfer in a porous medium apply to geothermal situations. He also presented an explanation of Darcy's law, the equations that govern flow in porous media, and appropriate thermal and hydrodynamic boundary
conditions. This reference was essentially a compilation of porous media literature to date, and had examples of specific geothermal systems.

Ribando and Torrance (1976) studied natural convection in a porous medium heated from below. The porous enclosure studied was heated from below and cooled from above. This paper presented an excellent discussion of appropriate thermal and hydrodynamic boundary conditions for heat transfer and fluid flow in porous media. The authors discussed critical Rayleigh numbers for the onset of convection. An important result was that for a permeable upper surface, the fluid streamlines crossed the upper boundary, while for an impermeable surface, the fluid was prohibited from crossing the upper boundary, and turned away. They observed circulating convection cells for each case.

Cao and Faghri (1994) presented an analytical solution for a capillary pumped loop (CPL) with a single phase fluid. In this study, a two dimensional porous enclosure was heated on part of its upper surface and had a fluid of constant velocity coming in the bottom permeable and isothermal surface. The vertical walls were impermeable and insulated. The portion of the top surface over which the heat flux occurred was impermeable, while the rest of the top surface was permeable. They found that the fluid flowed vertically into the porous structure, remained approximately one dimensional until the middle section, and then turned toward the permeable part of the upper surface. Since the upper surface was not permeable across the entire width, the fluid sped up as it turned. The authors also presented a relationship between the heat flux at the top surface and the mass flux of fluid coming out of the top.
Two-phase Flow in Porous Media

Ramesh and Torrance (1990a) presented a numerical algorithm for boiling problems in porous media. The problem analyzed was a porous enclosure heated from below and cooled from above. Fluid was allowed to cross the upper boundary, but not the sides or the bottom. The vertical walls were well insulated. They used the traditional SFM and tracked the moving phase change boundary. A liquid region overlying a two-phase mixture was observed. They found that the phase change interface was strongly influenced by liquid phase natural convection, and that the liquid flowed both directions across the phase change interface. The authors verified the method with experimental results and discussed convergence criteria for numerical methods.

Ramesh and Torrance (1990b) analyzed the onset and stability of thermal convection in a porous medium that contained both one- and two-phase regions. This was a semi-analytical solution for the same problem as the previously mentioned paper by the same authors (1990a). They found that in a liquid phase dominated system, convective instability was mainly driven by buoyancy in the liquid region. For vapor phase dominated systems, the instability was driven by a density difference between the liquid and two-phase regions.

Parmentier (1979) presented an analytical study for two-phase flow in a porous medium adjacent to a vertical heated plate. As an early attempt at studying phase change in porous media, this solution was based on a similarity type of approximation for the vapor layer. Since an assumption in this method was that the fluid transforms directly
from liquid to vapor with no mixed phase, the solution is not applicable to many geothermal problems.

Vafai and Tien (1989) examined the phase change effects of fluids in porous media. Aimed at porous thermal insulation applications, this study was mostly concerned with condensation. The authors found that most of the liquid condensate accumulated in the hot and humid regions of the porous enclosure.

Forsyth and Simpson (1991) presented a two-phase, two-component model for natural convection in a porous medium heated from below. Among their results was the observation that a constant heat flux boundary condition was more suited to studying vapor formation than an isothermal condition.

Tung and Dhir (1990) obtained a finite element solution for phase change problems with two-dimensional fluid flow in porous media. The authors used a blocked porous channel in which the permeability was not constant. The partition in the channel did not cover the entire width, and it was observed that the fluid sped up as it went through the opening and slowed again on the other side. The results they obtained agreed with experiments.

Udell (1983) studied a porous enclosure heated from above and cooled from below. Udell found that such an enclosure had three distinct regions at steady state. Near the top was a conduction dominated region with stationary superheated vapor. The middle region was two-phase, approximately isothermal, and convection dominated. Capillary forces pushed liquid components upward, while a pressure gradient forced vapor components downward. The bottom region was also conduction dominated and filled
with compressed liquid. Udell also observed that the length of the two-phase region increased as the prescribed boundary heat flux was decreased.

Udell (1985) presented a study on the effect of capillarity and how it relates to countercurrent flow. Among his results was that the effect of capillarity on the two-phase flow in a porous medium was a resulting countercurrent flow. Several important results were observed. First, a dry vapor zone did not form below a certain critical, or dryout, heat flux. Second, the flow in both the liquid and vapor regions was dominated by viscous forces, and therefore Darcy’s law was valid. Last, the heat transfer was several orders of magnitude higher than that of a purely conductive case, due to the effect of the evaporation, convection, and condensation process.

Torrance (1983) studied a porous enclosure heated from below and cooled from above to determine the interaction of boiling and convection. Torrance found that vapor formed near the heated surface and observed the critical Rayleigh numbers for the onset of convection, mode switching, and oscillatory convection. He also observed that in low permeability media, a non-convecting liquid layer overlay the two-phase region, while in higher permeability media, convection initiated in the liquid region and penetrated into the two-phase zone.

Schubert and Strauss (1977) investigated the stability of two-phase convection in porous media. The authors asserted that convection was driven by a phase change instability mechanism, and that the buoyancy force played little or no role in determining the onset of stability. They also stated that if steam and water were in thermodynamic equilibrium, temperature perturbations were responsible for pressure variations that
moved the fluid against the frictional resistance of the medium.

Sheu et al. (1979) presented a numerical solution for a geothermal reservoir above a cooling igneous intrusion. They simulated boiling due to a pressure drop. A liquid water region above and below a two-phase zone was observed.

Chung and Catton (1988) analyzed heat transfer in a porous enclosure due to a heat source. This study was applicable to post-accident analysis of nuclear reactors. The authors found that the saturation distribution and two-phase flow are mainly controlled by vapor escape from the top surface, rather than liquid supply in the bottom surface.

*Two-phase Flow Using the Two-Phase Mixture Model*

Wang and Beckermann (1993a) derived a two-phase mixture model for boiling in capillary porous media. A capillary porous medium is one in which the capillary force is significant. The new model was based on the idea that the two phases are constituents of a binary mixture. The two-phase mixture model was derived from the conventional separate flow model (SFM) with no additional assumptions.

Wang and Beckermann (1993b) used the two-phase mixture model to analyze forced convection adjacent to a vertical heated plate. For this study, a constant liquid saturation was assumed at the vertical wall, while a constant uniform upward velocity prevailed away from the plate. The authors found that a thin capillary layer existed over the solid heated surface at high Peclet numbers, and that the two-phase flow was confined to this layer. They also confirmed the numerical results with a similarity approximation.
analogous to a boundary layer solution.

Wang et al. (1994) presented a numerical methodology for phase change problems in porous media using the two-phase mixture model. The authors proposed that the energy equation be solved via the control-volume based finite difference method of Patankar (1980), and that the continuity and momentum equations be combined and solved using the error vector propagation (EVP) method of Roache (1972). The authors proved the accuracy of the new model by applying the mixture model to boiling in a porous enclosure heated from below and cooled from above. The vertical walls were insulated and all surfaces except the top were impermeable. They found that for low permeability media, there was conduction both before and after boiling began. In medium permeability media, there was conduction before boiling and convection after boiling began. In high permeability media, heat transfer was convection dominated both before and after the onset of boiling.

Experimental Studies in Porous Media Fluid Flow

Sondergeld and Turcotte (1977) carried out two-phase convection experiments in a sandbox. The porous enclosure was heated from below and the vertical walls were insulated. They observed that a stable form of two-phase convection can exist in a porous medium, and that the two-phase region had a definite nonhorizontal water-mixture interface. The authors also obtained valuable plots of the temperature distribution in the cross-section of the enclosure and measured vapor formation due to the imposed heat flux.
Another paper by the same authors (1978) presented a flow visualization study of two-phase thermal convection. In this second paper, Sondergeld and Turcotte presented plots of fluid streamlines and measured fluid velocity in the porous enclosure.

Rahli et al. (1996) studied heat transfer and phase change in a forced convection porous medium. The porous enclosure analyzed was heated from the sides and the fluid had a constant upward velocity away from the heated surface. This experimental analysis found three distinct zones of fluid: all liquid near the bottom, a liquid-vapor mixture in the middle, and all vapor near the top.

Bau and Torrance (1982) studied boiling in low permeability porous media. They determined that near the onset of boiling in an enclosure heated from below, the two-phase zone was approximately isothermal with the fluid at its saturation temperature. They also observed that this two-phase region was overlain by a liquid layer with an approximately vertical temperature gradient. In an isothermal two-phase zone, heat transfer can occur only by convection. The authors used a low permeability medium so that the heat flow was approximately vertical, with negligible free convection circulations.

Moseley and Dhir (1992) examined capillarity in porous media. Capillary pressure is a pressure discontinuity at the interface of two fluids. They concluded that the commonly used Leverett capillary pressure function that describes a relationship between capillarity and liquid saturation works very well.

Manteufel et al. (1992) investigated heat and mass transfer in two-phase, two-component flow in a porous medium. They concluded that the dominant mechanisms for mass transfer were vaporization, condensation, and liquid advection.
PROBLEM FORMULATION

Introduction

The problem analyzed in this study was a two dimensional square porous enclosure. The porous material was assumed to be homogeneous, isotropic, and initially saturated with liquid water. A homogeneous medium is one in which the physical properties, namely porosity, permeability, and effective (fluid-solid) thermal conductivity, do not vary with position in the enclosure, and in an isotropic porous medium, properties are independent of direction [Bear (1972)].

There were also assumptions made about the fluid inside the porous medium. It was assumed that the fluid obeyed Darcy's law for the conservation of momentum. By making this assumption, inertial, drag, and acceleration forces were neglected, with the flow dominated by viscous forces. The Boussinesq approximation was also applied to the governing equations. The Boussinesq approximation states that fluid is incompressible except for variable density in the buoyancy term. The fluid was also assumed to have constant physical properties, such as density and viscosity, in each phase. The properties of the two-phase mixture vary with the relative amounts of each phase. Mixture enthalpy was assumed to be related to temperature in the liquid region and to liquid saturation in the two-phase region. It was assumed that the two-phase fluid was in thermodynamic equilibrium with the solid matrix and was isothermal at the boiling temperature.
Traditionally, two-phase flow in a porous medium has been modeled using the so-called separate flow model (SFM). In the SFM, the gas and liquid phases are considered two distinct fluids with individual thermodynamic properties and different flow velocities [Bear (1972)]. The principles of conservation for mass, momentum, and energy are then applied to each fluid, with appropriate interfacial conditions. The result is a large set of nonlinear equations, and the problem is compounded greatly in two and three-dimensional problems. The SFM equations can usually be simplified and solved numerically by neglecting the capillary force. The capillary force results from a pressure difference, due to surface tension, between the liquid and vapor phases. This assumption, however, restricts the usefulness of the SFM in low permeability and heterogeneous media, where the capillary force is significant [Wang et al. (1994)]. Another significant problem with the SFM is that the phase change interface line is not known a priori, and must therefore be tracked through the problem calculations. This results in the need for a moving coordinate system or numerically recalculating the mesh point locations with each iteration.

Recently, Wang and Beckermann (1993) developed a new model in which the conservation equations are formulated for a two-phase mixture. This new model, called the two-phase mixture model by the authors, has several advantages over the SFM. The conservation equations are valid in all regions throughout the domain. This means fewer

Governing Equations

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governing equations, and there is no need to track the phase change transition line. The new method also has no additional approximations from the traditional SFM. Since the governing equations are valid throughout the calculation domain, numerical implementation of this method is much easier than the SFM.

With the previously mentioned assumptions and neglecting volumetric heat generation, the conservation equations for mass (continuity), momentum (Darcy's law), and energy that govern two-phase flow in porous media using the two-phase mixture model are given below. A rigorous derivation of the conservation equations for the two-phase mixture model is given in two papers by the originators of the method, Wang and Beckermann (1993a) and Wang et al. (1994). Only the final form of the equations, along with some explanatory notes, is presented here.

The equation for conservation of mass was obtained by combining the mass conservation equations for each phase as given in the SFM and observing that the mass flow rate of vapor must come at the expense of the liquid phase. With that in mind, the continuity equation for the two-phase mixture model is:

\[ \nabla \cdot (\rho \mathbf{u}) = 0 \]  

Conservation of Momentum (Darcy's law):

\[ \mathbf{u} = -\frac{K}{\mu(s)} \left[ \nabla p - \rho_\ell(s)g \right] \]  

Here, it was necessary for later calculations to substitute the equation for
conservation of momentum into the equation for conservation of mass. This results in a single, Poisson-type equation for pressure, given below. This resulting equation was solved using the error vector propagation (EVP) method of Roache (1971, 1972). This solution method will be discussed later.

\[
\nabla^2 p = \frac{\nabla \cdot \rho \frac{\partial p}{\partial t}}{K} + \nabla \cdot \left( \frac{K}{\rho} \frac{\rho_v \mathbf{g}}{g} \right)
\]

Conservation of Energy:

\[
\Omega \frac{\partial (\rho h)}{\partial t} + \nabla \cdot (\gamma_h \rho \mathbf{u} h) = \nabla \cdot (\nabla_h \nabla h) + \nabla \cdot \left[ (h - h_{\text{sat}}) \nabla \ln(\rho) \right] + \nabla \cdot \left[ f(s) \frac{K \Delta \rho h_{\text{fg}} \mathbf{g}}{\rho_v} \right]
\]

Eq. (4) is given in terms of mixture enthalpy, \( h \). The conservation equations, Eqs. (1), (2), and (4), are valid throughout the calculation domain, in both the single and two-phase regions. It is valuable to note that the energy equation represents a temperature equation in the single-phase region, and reduces to an equation for liquid saturation in the isothermal two-phase region [Wang et al. (1994)]. The liquid saturation, \( s \), is the volumetric fraction of void space occupied by the liquid phase.

It is convenient to introduce volumetric enthalpy and rewrite the energy equation.

\[
H = \rho(h - h_{\text{sat}})
\]

\[
\Omega \frac{\partial H}{\partial t} + \nabla \cdot (\gamma_h \mathbf{u} H) = \nabla \cdot \left( \frac{\nabla_h \nabla H}{\rho} \right) + \nabla \cdot \left[ f(s) \frac{K \Delta \rho h_{\text{fg}} \mathbf{g}}{\rho_v} \right]
\]

\[
(5)
\]

\[
(6)
\]
It is important to note that the volumetric enthalpy, $H$, is related to mixture enthalpy, $h$, by Eq. (5). It was assumed that $h=C_p T$ in the single phase regions and $h=h_{sat}+sh_{fg}$ in the isothermal two-phase mixture region. With the mixture enthalpy defined in this manner, the energy equation, Eq. (6), reduces to an expression for temperature in the single phase regions and for liquid saturation in the two-phase region. The assumption that $h=C_p T$ is accurate for the liquid phase, but is not a very good approximation for the vapor phase. However, this study was concerned primarily with the onset of boiling, and a dryout (all vapor) condition was never observed. By limiting the study to regions of all liquid and liquid-vapor mixture, the inaccuracy associated by assuming $h=C_p T$ in the all-vapor region was avoided.

Several mixture properties and variables need to be defined here. A property without a subscript denotes a mixture property, while the subscripts $l$ and $v$ indicate liquid and vapor properties, respectively.

Density:

$$\rho = \rho_l s + \rho_v (1-s)$$

(7)

Kinetic density, which accounts for the relative mobilities of individual phases:

$$\rho_k = \rho_l [1 - \beta_l (T - T_0)] \lambda_l (s) + \rho_v [1 - \beta_v (T - T_{sat})] \lambda_v (s)$$

(8)
Viscosity:

\[ \mu(s) = \frac{\rho_s s + \rho_v (1-s)}{(k_r/v_r) + (k_r/v_v)} \]  

(9)

Velocity:

\[ \rho u = \rho \mu_s + \rho \mu_v \]  

(10)

Enthalpy:

\[ \rho h = \rho_s h_s + \rho_v (1-s) h_v \]  

(11)

The additional terms seen in the energy equation, namely the two-phase advection correction coefficient \( \gamma_h \), the effective heat capacitance ratio \( \Omega \), and the effective diffusion coefficient \( \Gamma_h \), are defined below.

Two-phase advection correction coefficient:

\[ \gamma_h = \frac{s + \frac{\rho_v (1-s)}{\rho_s}}{s} \frac{\lambda_f(s)}{\lambda_f(s)} \]  

(12)

Effective heat capacitance ratio:

\[ \Omega = \varepsilon + \rho_s C_p (1-\varepsilon) \frac{dT}{dH} \]  

(13)
Effective diffusion coefficient:

\[ \Gamma_h = \rho [D(s) + k_{\text{eff}} \frac{dT}{dH}] \]  

(14)

The effective thermal conductivity, \( k_{\text{eff}} \), in Eq. (14) reflects the thermal conductivities of the solid, liquid, and vapor components. It is defined as:

\[ k_{\text{eff}} = k_s (1 - \varepsilon) + k_f \varepsilon s + k_v \varepsilon (1 - s) \]  

(15)

The temperature and liquid saturation can be recovered from the volumetric enthalpy by the following relations. Also included is the derivative of temperature with respect to volumetric enthalpy, as used in the equations for effective heat capacitance ratio and diffusion coefficient, Eqs. (13) and (14).

For \( H \leq -\rho h_{fg} \):

\[ T = \frac{H + \rho h_{\text{sat}}}{\rho_f C_{pl}} \quad ; \quad s = 1 \quad ; \quad \frac{dT}{dH} = 0 \]  

(16)

For \( -\rho h_{fg} < H \leq 0 \):

\[ T = T_{\text{sat}} \quad ; \quad s = \frac{H}{-\rho h_{fg}} \quad ; \quad \frac{dT}{dH} = 0 \]  

(17)

For \( 0 < H \):

\[ T = T_{\text{sat}} + \frac{H}{\rho_v C_{pv}} \quad ; \quad s = 0 \quad ; \quad \frac{dT}{dH} = \frac{1}{\rho_v C_{pv}} \]  

(18)
There are several supplementary relations needed to complete the set of equations necessary for solution of the governing equations. These relations include the relative permeability and mobility of each phase, the hindrance function $f(s)$, the capillary pressure function $p_c(s)$, and the capillary diffusion coefficient $D(s)$.

The relative permeabilities were chosen as linear functions, commonly used in geothermal systems.

$$k_{r_l}(s) = s \quad k_{r_v}(s) = 1 - s$$ \hfill (19)

Consequently, the relative mobilities for phase migration are:

$$\lambda_l(s) = \frac{s/v_l}{(s/v_l) + [(1-s)/v_v]}$$ \hfill (20)

$$\lambda_v(s) = \frac{(1-s)/v_v}{(s/v_l) + [(1-s)/v_v]}$$ \hfill (21)

The hindrance function for phase migration and eventual separation, $f(s)$:

$$f(s) = k_{r_v}(s)\lambda_l(s) = \frac{s(1-s)/v_l}{(s/v_l) + [(1-s)/v_v]}$$ \hfill (22)

The Leverett function is used for the capillary pressure function, $p_c(s)$. Moseley and Dhir (1992) verified that the Leverett function is an accurate relation for capillary pressure.

$$p_c(s) = \sigma J(s) \left( \frac{\kappa}{K} \right)$$ \hfill (23)
The form of $J(s)$ recommended by Udell (1983) is used, i.e.,

$$J(s) = 1.417(1 - s) - 2.120(1 - s)^2 + 1.263(1 - s)^3$$  \((24)\)

The capillary diffusion coefficient $D(s)$ can be written as:

$$D(s) = \frac{\sqrt{\epsilon K \sigma}}{\mu_t} \frac{s(1 - s)}{[(\nu_f/\nu_s)s + (1 - s)][1.417 - 4.240(1 - s) + 3.789(1 - s)^2]}$$  \((25)\)

To recover the velocities of each phase from the overall fluid velocity, several additional relations are required. Notably, the definition of a diffusive mass flux is helpful in obtaining these velocities. This diffusive mass flux is defined as:

$$j = -\rho_j D(s) \nabla s f(s) \frac{K \Delta \rho}{\nu_v} g$$  \((26)\)

The velocity of each phase can be obtained from this mass flux via:

$$\rho_j \mu_j = \lambda_j \rho u + j$$  \((27)\)

$$\rho_v \mu_v = \lambda_v \rho u - j$$  \((28)\)
Initial and Boundary Conditions

Appropriate initial conditions for the problem studied are given below. Initially, the fluid in the medium was assumed to be at rest and at ambient temperature and pressure. If we note that $h = C_p T$ in the single phase (liquid) region, the initial and boundary conditions for volumetric enthalpy can be expressed in terms of temperature. The initial conditions are written mathematically below.

At $t=0$:

$$H = H_o = \rho \left( C_p T_o - h_{vad} \right)$$  \hspace{1cm} (29)

$$p = 0$$  \hspace{1cm} (30)

For $t>0$, the top surface was isothermal, maintained at the ambient temperature. The left wall was adiabatic, and the right wall was subjected to a constant, uniform heat flux. The bottom temperature was also isothermal.

At $x=0$:

$$\frac{\partial H}{\partial x} = 0$$  \hspace{1cm} (31)
At \( x= L_x \):

\[
- \frac{\Gamma_h}{\rho} \frac{\partial H}{\partial x} = q_w
\]  \hspace{1cm} (32)

At \( y=0 \):

\[
H = \rho \left( C_{p} T_B - h_{vat} \right)
\]  \hspace{1cm} (33)

At \( y= L_y \):

\[
H = \rho \left( C_{p} T_0 - h_{vat} \right)
\]  \hspace{1cm} (34)

The hydrodynamic boundary conditions deserve some discussion here. As can be seen in Eq. (2), the momentum equation is only first order in the spatial derivative. Therefore, only one condition at each boundary can be specified [Nield and Bejan (1992)]. Consequently, the presence of a slip condition is possible at the boundaries if Darcy's law is assumed to be valid. According to Cheng (1978), Darcy's law is valid so long as the Reynold's number based on pore size (permeability) is less than one. In the current notation, this Reynold's number is related as:

\[
Re_K = \frac{u_{\text{max}} \sqrt{K}}{v}
\]  \hspace{1cm} (35)

If this number does not exceed one, Darcy's law can be applied. As will be shown in a later section, this was the case for the problem studied.
If specifying a no-slip condition at the boundaries is necessary, the use of a modified form of Darcy's law is required. There are several such forms, the most commonly used of which are Forchheimer's equation, which accounts for quadratic drag due to higher fluid velocities, and Brinkman's extension of Darcy's law, which contains an additional Laplacian term similar to the Navier-Stokes relations to describe viscous effects.

A permeable surface can be defined mathematically in two ways. The first way is to specify a constant pressure at the impermeable surface. The second way is to specify a fluid velocity crossing the surface. The first method is more useful if the velocity is not known, such as an upper surface exposed to the atmosphere or a fluid reservoir. The second method is useful if fluid of a known velocity is crossing the permeable surface. An impermeable surface is usually defined by setting the normal component of velocity equal to zero at the surface. The hydrodynamic boundary conditions for the problem studied are given below.

At $x=0$ and $x=L_x$:

$$\frac{\partial p}{\partial x} = 0$$  \hspace{1cm} (36)

At $y=0$:

$$\frac{\partial p}{\partial y} = -\left[ \rho \frac{g}{K} + u_y \frac{\mu}{K} \right]$$  \hspace{1cm} (37)

At $y=L_y$:

$$p = 0$$  \hspace{1cm} (38)
Thus, the vertical boundaries were impermeable, while at the top surface, fluid could pass either into or out of the enclosure, depending on the pressure gradient. A fluid velocity was specified at the bottom surface, which was permeable if $u_{y,B} \neq 0$. When the velocity $u_{y,B}$ was set to zero, Eq. (37) reduced to that for an impermeable surface. If a nonzero velocity was specified for the bottom surface, the fluid properties of the incoming fluid were taken as liquid values.

The water in the porous enclosure was assumed to have constant properties in each phase, while the two-phase mixture varied with the composition of the mixture.

Water properties used in this study are given in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Liquid Phase</th>
<th>Vapor Phase</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, $\rho$</td>
<td>957.85</td>
<td>0.5956</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Specific heat, $C_p$</td>
<td>4217</td>
<td>2029</td>
<td>J/kg·K</td>
</tr>
<tr>
<td>Kinematic viscosity, $v$</td>
<td>$4.670 \times 10^{-7}$</td>
<td>$2.018 \times 10^{-5}$</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>Thermal conductivity, $k$</td>
<td>0.654</td>
<td>0.025</td>
<td>W/m·K</td>
</tr>
<tr>
<td>Saturation enthalpy, $h_{sat}$</td>
<td>$4.178 \times 10^5$</td>
<td>$2.675 \times 10^6$</td>
<td>J/kg</td>
</tr>
<tr>
<td>Surface tension, $\sigma$</td>
<td>0.0589</td>
<td>-----</td>
<td>N/m</td>
</tr>
<tr>
<td>Thermal expansion, $\beta$</td>
<td>$5.229 \times 10^{-4}$</td>
<td>-----</td>
<td>K$^{-1}$</td>
</tr>
<tr>
<td>Latent heat, $h_{fg}$</td>
<td>$2.257 \times 10^6$</td>
<td>-----</td>
<td>J/kg</td>
</tr>
</tbody>
</table>
Three porous materials were used in this study. These were sand, soil, and glass beads. Sand represented a high permeability medium, while soil was considered low permeability. The glass bead system was considered a medium permeability porous material. Properties for each material are listed in Table 2.

Table 2. Thermophysical properties for porous media.

<table>
<thead>
<tr>
<th>Property</th>
<th>Soil</th>
<th>Glass Beads</th>
<th>Sand</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability, K</td>
<td>$8.25 \times 10^{-12}$</td>
<td>$7.00 \times 10^{-11}$</td>
<td>$9.88 \times 10^{-11}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Porosity, $\epsilon$</td>
<td>0.50</td>
<td>0.35</td>
<td>0.40</td>
<td>-----</td>
</tr>
<tr>
<td>Density, $\rho$</td>
<td>2050</td>
<td>2700</td>
<td>1515</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$k_{eff}$</td>
<td>0.587</td>
<td>0.85</td>
<td>0.423</td>
<td>W/m-K</td>
</tr>
<tr>
<td>$\alpha_{eff}$</td>
<td>$1.453 \times 10^{-7}$</td>
<td>$2.104 \times 10^{-7}$</td>
<td>$1.047 \times 10^{-7}$</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>Specific heat, $C_p$</td>
<td>1840</td>
<td>870</td>
<td>800</td>
<td>J/kg-K</td>
</tr>
<tr>
<td>$\alpha_s = \rho_s C_p / \rho_l C_{pl}$</td>
<td>0.934</td>
<td>0.582</td>
<td>0.300</td>
<td>-----</td>
</tr>
</tbody>
</table>

The values of thermal conductivity presented in Table 2 reflect the values without any vapor formation. Since this study was concerned primarily with the onset of vapor formation and due to the relatively small value for thermal conductivity of the vapor phase, the contribution of the vapor conductivity to the effective conductivity was small. Therefore, for this study, the effective thermal conductivity was assumed constant with very little loss in accuracy [Wang et al. (1994)].
Normalizing the Governing Equations

As stated by Wang et al. (1994), rewriting the governing equations, Eqs. (1), (2), (6), and the initial and boundary conditions Eqs. (29-34, 36-38) in nondimensional form is difficult. Due to the varying size of the single-phase region and flow cross-sectional area, suitable length and velocity scales are impossible to establish. According to the authors, there are ten important nondimensional parameters. Four of these are related to the physical properties of the fluid. These are $\rho^*$ (density ratio), $\nu^*$ (viscosity ratio), $\alpha_s$ (solid-liquid heat capacitance ratio), and $\alpha_v$ (vapor-liquid heat capacitance ratio). The other six parameters depend on enclosure geometry, fluid properties, and properties of the solid matrix. These are $A$ (aspect ratio), $\lambda$ (latent heat parameter), $Le_s$ (Lewis Number, ratio of capillary diffusion coefficient and thermal diffusivity), $Ra$ and $Ra_{2\phi}$ (Rayleigh numbers in the single and two-phase regions, respectively), and $Q_w$ (nondimensional imposed wall heat flux). Each of these properties is defined below or in the nomenclature section at the beginning of the paper.

Additionally, there is a new nondimensional parameter, the nondimensional velocity of the fluid at the bottom surface, $u_{y,B}^*$. This number reflects the hydrodynamic boundary condition at the bottom surface in a dimensionless form. The parameter $u_{y,B}^*$ is defined in the nomenclature. A value of zero for $u_{y,B}^*$ reflects an impermeable bottom surface, while a nonzero value reflects the specified velocity of fluid coming through the bottom surface.
This concentrated on the effect of different porous materials (Ra), imposed wall heat flux (Qw), bottom surface temperature, (θb), and bottom imposed fluid velocity (u_y,B). A list of the range of these parameters studied is given in Table 3 in the next section. A broad range of these nondimensional parameters was studied. Each of these parameters is defined and explained below.

A change in the Rayleigh number, Ra, was equivalent to a change in the porous material studied. The Rayleigh number for the single-phase region is defined as:

$$Ra = \frac{KL_s g \rho \beta (T_{sat} - T_o) C_p l}{v l k_{eff}}$$

Thus, a change in the permeability, K, was reflected in the Rayleigh number. For the current study, soil (Ra=10), glass beads (Ra=56), and sand (Ra=160) are used. A higher Rayleigh number reflected a more permeable porous material.

A change in the imposed wall heat flux was seen in a change in the dimensionless wall heat flux, Qw:

$$Q_w = \frac{q_w L_y}{k_{eff} (T_{sat} - T_o)}$$

As the value of the imposed heat flux was increased, so was the value of the dimensionless heat flux. Note that a change in the porous material was also seen in Qw, through the effective thermal conductivity. Thus, the same value of imposed heat flux did not produce the same value of Qw for different materials.
The dimensionless form of bottom surface temperature has the same form as the dimensionless temperature as defined in the nomenclature:

\[
\theta_B = \frac{T_B - T_0}{T_{sat} - T_0}
\]  (41)

This form of dimensionless temperature was particularly useful, since fluid at its initial temperature, \( T_0 \), had \( \theta = 0 \), while fluid at the saturation temperature had \( \theta = 1 \). For the current study, bottom surface temperatures of 75 and 100°C were used (\( \theta_B = 0.7 \) and 1.0 respectively).

A dimensionless form of the bottom surface fluid velocity is given as:

\[
u_{y,B}^* = \frac{u_{y,B} L_y}{\alpha_{eff}}
\]  (42)

An impermeable bottom surface had \( u_{y,B}^* = 0 \), while permeable bottom surfaces had a non-zero values. In addition to the impermeable bottom surface, two specified velocities were studied. The values of incoming fluid velocity were chosen to cover a broad range of suitable velocities for geothermal situations. Velocities of \( 1.5 \times 10^{-6} \) and \( 3.0 \times 10^{-6} \) m/s were chosen for study. The incoming fluid was specified as a liquid, and appropriate thermophysical properties were used.

A complete list of the parameters covered in the current study is given in the computational matrix, Table 3.

The above nondimensional parameters permit generalization of results, but should
not be used for direct physical interpretation. As can be seen from the definition of these properties, varying one physical property will change more than one dimensionless parameter. For example, varying the permeability, $K$, will affect the two Rayleigh numbers and the Lewis Number. A variation in porosity will be reflected in the Lewis number, both Rayleigh numbers, and the dimensionless wall heat flux (through effective thermal conductivity).
NUMERICAL METHOD

Introduction

Analytical solutions to fluid flow problems in porous media are mostly limited to one dimension and usually some simplifying assumptions are made. Some authors have obtained similarity solutions in which the vapor layer is treated like a boundary layer in viscous fluid flow [Parmentier (1979), Wang and Beckermann (1993b), and Lai and Kulacki (1991a) and (1991b)]. Although this presents some interesting insights into the formation of the vapor zone, the applicability of the problem is limited to flow near a vertical heated wall with the fluid moving at constant velocity away from the wall.

For a two-dimensional problem with arbitrary boundary conditions, the usual solution procedure is numerical methods. Numerical methods can be split into two groups: finite element and finite difference. In a finite element method (FEM), the calculation domain is divided into many small two-dimensional elements with simple geometries. The governing equations are approximated, and usually integrated by the Galerkin method of weighted residuals. The solution is then obtained for values at the corners of these elements. This method is excellent for problems with irregular geometries. However, if the problem domain is limited to circular or rectangular geometries, finite difference methods provide a much simpler and faster way to solve problems.
In a finite difference method, the calculation domain is split into discrete nodal points. The partial derivatives in the governing equations are expanded in a Taylor series and expressed as difference quotients [Anderson et al. (1984)]. For example, a second order derivative is expressed as:

\[
\frac{\partial^2 U}{\partial x^2} = \frac{U(x-\Delta x,t) - 2U(x,t) + U(x+\Delta x,t)}{\left(\Delta x\right)^2} \tag{43}
\]

The resulting equations are then solved for the values at the node points. There are several ways to approach and solve finite difference approximations. One of the most commonly used methods in heat transfer problems with fluid flow is the control-volume based finite difference method of Patankar (1980).

Using the control-volume based finite difference method, the temperature at a certain node is expressed in terms of properties of the surrounding nodes and flow conditions at the control volume faces. The control volume is taken to be the region surrounding the center node P. For a general temperature field \( T \), the discretized equation has the form:

\[
\left[ a_E + a_w + a_N + a_S + \frac{\rho \Delta x \Delta y}{\Delta t} \right] T_p = a_E T_E + a_w T_w + a_N T_N + a_S T_S + S_p \Delta x \Delta y + \frac{\rho \Delta x \Delta y}{\Delta t} T_p^o \tag{44}
\]

Where the superscript \(^o\) indicates old values for \( \rho \) and \( T \) from the previous time step at the center point P. The terms \( \Delta x \), \( \Delta y \), and \( \Delta t \) refer to the dimensions of the computational cell (control volume) and the time step, respectively. The \( a \)'s are the combined diffusion/convection coefficients and the \( T \)'s are the values of the temperature field at the
nodes neighboring the central point \( P \). In the control-volume based finite difference method, the transient terms are treated full implicitly, guaranteeing solution independence of time step. The power law scheme is used for the combined convection/conduction terms, and the source term is linearized.

In the current problem, the energy equation, Eq. (6) was discretized using the control-volume based finite difference method of Patankar (1980), and the last term on the right-hand side of Eq. (6) was treated as a source term. The combined conductive and convective terms were discretized using the power law scheme. An arithmetic mean approximation was assumed for the interface diffusion coefficient. The energy equation was solved for the volumetric enthalpy, \( H \). From the volumetric enthalpy, the temperature and liquid saturation were recovered via Eqs. (16-18).

The governing partial differential equations were discretized as they appear in Eqs. (1), (2), and (6). By choosing a very large time step, the discretized equations reduced to those for the corresponding steady state situation [Patankar (1980)]. In the case of the steady state problem, the first terms in the continuity and energy equations, Eqs. (1) and (6), dropped out. By choosing a large time step, the terms in Eq. (44) with \( \Delta t \) in the denominator disappear. The transient solutions for the current study were obtained by using a sufficiently small time step that the transient terms in Eqs. (1) and (6) were significant.

To obtain the flow field, the combined continuity-momentum equation, Eq. (3) was discretized with a centered difference approximation similar to Eq. (43) and solved by the error vector propagation (EVP) method of Roache (1971, 1972). The resulting
pressure field was then substituted into the momentum equation, Eq. (2), to obtain the fluid velocity at each node point. From there, the individual phase fluid velocities were recovered.

The EVP method is a direct way to solve for the pressure field, and can be summarized as follows. An arbitrary value for pressure is picked for the first interior (non-boundary) row of nodes in the calculation domain. This arbitrary value is in error from the true value by a small amount. Similarly, an arbitrary value is picked for each row up to and including the upper boundary. Since the pressure conditions at the upper boundary are specified, the error between the arbitrary value and the true value is zero. With this knowledge, there is now a recursion relation for the error vector. Another sweep through the domain corrects the arbitrary values to the true pressure value.

The equations governing conservation of mass, momentum, and energy were discretized and solved using a finite difference approximation. The numerical investigations were carried out on a DEC-VAX model 6000-520.
Computational Grid

The calculation domain was split into a uniform and fixed grid of $42 \times 42$ nodes. As shown by Wang et al. (1994), a $42 \times 42$ grid was sufficient for a geometry of the same size and type used in this study, and finer grids led to no significant improvement in results. The notion of a staggered grid, as recommended by Patankar (1980), is used. In this staggered grid method, there are two nodal grids that overlie each other. The first is the grid for calculating temperature, liquid saturation, and enthalpy values, and the second is for calculating fluid velocities and stream functions. The nodes for the velocity grid lie on the control volume faces, while those for the temperature grid lie halfway between control volume faces. The governing equations were solved simultaneously, and steady state convergence was achieved when the absolute value of the relative error in enthalpy and velocity between two successive iterations was less than $10^{-5}$ (J/m$^3$ or m/s). The entire calculation grid was checked, and the summation of errors for all nodes was used to check convergence. The relative error is:

$$
e_{H} = \frac{\sum_{i,j=1}^{42} \left| H_{\text{old}}(i, j) - H(i, j) \right|}{\sum_{i,j=1}^{42} \left| H(i, j) \right|} \quad \text{and} \quad 
e_{u} = \frac{\sum_{i,j=1}^{42} \left| u_{\text{old}}(i, j) - u(i, j) \right|}{\sum_{i,j=1}^{42} \left| u(i, j) \right|}$$  \hspace{1cm} (45)

Figures 3 and 4 show the temperature and velocity grids, respectively.
Figure 3. Main Calculation Grid for Temperature, Enthalpy, and Liquid Saturation
Figure 4. Staggered Calculation Grid for Velocity and Stream Function
Several cases were run to determine the effects of various porous materials (Ra), imposed side wall heat flux \( (Q_w) \), bottom surface temperature \( (\theta_B) \), and bottom surface permeability conditions \( (u_{y,B}^*) \) on heat transfer and flow mechanisms in the enclosure. In the numerical calculations, a change in material was reflected through a change in the Rayleigh number and a change in side wall heat flux through the nondimensional side wall heat flux \( (Q_w) \). A wide range of all the parameters was used to cover a broad range of applications. The computational matrix that shows the ranges of the parameters used in the current study is shown in Table 3. A total of 155 cases were run for the current study.
Table 3. Computational matrix for the current study.

<table>
<thead>
<tr>
<th>Media</th>
<th>Ra</th>
<th>$u_{y,E}$</th>
<th>$\theta_B$</th>
<th>$Q_w$</th>
<th>Number Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>10</td>
<td>0.0</td>
<td>0.7</td>
<td>1.0, 5.0, 10.0, ..., 25.0, 30.0</td>
<td>7</td>
</tr>
<tr>
<td>Soil</td>
<td>10</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0, 5.0, 10.0, ..., 25.0, 30.0</td>
<td>7</td>
</tr>
<tr>
<td>Soil</td>
<td>10</td>
<td>2.06</td>
<td>0.7</td>
<td>1.0, 5.0, 10.0, ..., 25.0, 30.0</td>
<td>7</td>
</tr>
<tr>
<td>Soil</td>
<td>10</td>
<td>2.06</td>
<td>1.0</td>
<td>1.0, 5.0, 10.0, ..., 25.0, 30.0</td>
<td>7</td>
</tr>
<tr>
<td>Soil</td>
<td>10</td>
<td>4.13</td>
<td>0.7</td>
<td>1.0, 5.0, 10.0, ..., 25.0, 30.0</td>
<td>7</td>
</tr>
<tr>
<td>Soil</td>
<td>10</td>
<td>4.13</td>
<td>1.0</td>
<td>1.0, 5.0, 10.0, ..., 25.0, 30.0</td>
<td>7</td>
</tr>
<tr>
<td>Glass</td>
<td>56</td>
<td>0.0</td>
<td>0.7</td>
<td>1.0, 5.0, 10.0, ..., 35.0, 40.0</td>
<td>9</td>
</tr>
<tr>
<td>Glass</td>
<td>56</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0, 5.0, 10.0, ..., 35.0, 40.0</td>
<td>9</td>
</tr>
<tr>
<td>Glass</td>
<td>56</td>
<td>1.43</td>
<td>0.7</td>
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<td>9</td>
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Validation of the Numerical Method

To accurately study two-phase flow in porous media using the two-phase mixture model, the results obtained by this method must be verified with other published results to establish the accuracy of the current method. Once the accuracy of the numerical predictions has been established, the method can be extended to situations for which there have been no previous studies.

The validation for the current numerical method was obtained by comparing the results obtained by the present method with the corresponding experimental results. For this purpose, the experimental works by Sondergeld and Turcotte (1977, 1978) were chosen. In these studies the porous enclosure was heated from below. The first study (1977) was an experiment in which the authors obtained temperature profiles for the enclosure and measured the amount of vapor formed for different values of the bottom heat flux. The second study (1978) provided plots of fluid streamlines and velocity vectors for the same heating condition.

The first experiment provided data for a new variable, \( \epsilon_v \), the volumetric fraction of vapor present in the entire porous enclosure. This vapor fraction is defined as:

\[
\epsilon_v = A \int_0^1 \int_0^1 (1 - s) \, dX \, dY
\]  

\( (46) \)
Sondergeld and Turcotte carried out their experiments for two different heights of porous enclosures, 0.1048 m and 0.1984 m. The numerical simulations run for similar geometries with identical heat flux values provided excellent matches, both qualitatively and quantitatively. Both temperature values and fluid flow patterns corresponded well with the experimental results and examples are shown in Wang et al. (1994). In addition, numerical calculation of the volumetric vapor fraction, $\epsilon_v$, provided an average deviation of about 20-25% from the experimentally measured values, excellent matches considering the relative difficulty in measuring vapor fraction experimentally. Figure 5 shows a plot of the numerically calculated vapor fraction and the measured values obtained by Sondergeld and Turcotte (1977) for a range of bottom heat fluxes. The two spurious data points seen in Figure 5(b) are attributed to fluidization of the solid matrix due to violent boiling at high heat flux values [Wang et al. (1994)].

It should be noted here that the same method of validation was used by the originators of the two-phase mixture model in verifying their code [Wang et al. (1994)]. The authors also compared their work to previously published numerical results, notably Ramesh and Torrance (1990) and Ribando and Torrance (1976). These studies also analyzed a porous enclosure heated from below. Wang et al. (1994) found that results given by the current method matched nearly identically with other results that used the SFM. For example, the authors found that the maximum value of stream function calculated by the two-phase mixture model differed from the value calculated by the SFM by less than one percent for the cases mentioned.
Figure 5. Experimental (dots) and Numerical (solid line) Results for Volumetric Vapor Fraction, $\varepsilon_v$. 

(a) Enclosure with H=0.1048 m. 

(b) Enclosure with H=0.1984 m.
To establish the validity of a 42×42 node grid as used in this study, a number of cases were also run with a finer grid to determine the effect on solution accuracy. For the finer grid, 75 nodes were used in each direction and the results were compared with those of a 42×42 node grid. An average improvement in deviation from experimental values of about 2-3% was seen in most cases by using the 75×75 grid. Figure 6 shows a plot of vapor fraction versus bottom heat flux for the 0.1984 m high enclosure of Sondergeld and Turcotte (1977) using a 75×75 node grid and a 42×42 node grid. It can be seen from Figure 6 that increasing the number of nodes provided no significant increase in accuracy. Additionally, computation time was severely increased by using the 75×75 grid. Therefore, for the present study the use of a 42×42 node grid provided results of sufficient accuracy.
Figure 6. Results of Grid Independence Test
RESULTS AND DISCUSSION

Introduction

The porous geometry examined in the current study was a square enclosure of dimensions $L_x=0.2$ m and $L_y=0.2$ m. The upper surface temperature was chosen to be at atmospheric conditions, with $T_o=20^\circ$C. Two bottom surface temperatures were studied, $T_B=75$ and $100^\circ$C ($\theta_B=0.7$ and 1.0). Three porous materials were studied, and a broad range of imposed wall heat flux values was analyzed. The computational matrix, shown in Table 3, gives details of the range of the current study.

As discussed in an earlier section, the objective of the current study was to investigate the heat transfer and motion of a two-phase fluid in a porous medium subjected to a constant heat flux from one side. Of primary interest were the steady state temperature distribution, fluid stream paths, heat fluxes at each surface, and vapor formation under different thermal and hydrodynamic boundary conditions. In this study, the dimensionless side heat flux ($Q_w$), bottom surface temperature ($\theta_B$), bottom surface incoming fluid velocity ($u_{y,B}$), and Rayleigh number (Ra) were varied to understand their effects on the items of interest.

The numerical solutions of the discretized equations provided data for the fluid temperature, pressure, stream function, liquid saturation, and the velocity of each phase. Additionally, average values of the heat fluxes at the top and bottom surfaces of the
enclosure were calculated by integrating the local heat flux values using Simpson's Rule. Also, the volumetric vapor fraction, Eq. (46) was calculated for all the cases studied.

The isotherm plots presented in this section represent the nondimensional temperature field, θ. The value of θ at the upper boundary is equal to zero, while a value of θ=1 represents fluid at the saturation temperature. Six values of θ are shown, with the lines nearest the top surface being the coolest at θ=0.15 and the lines nearest the heated right wall being the warmest at θ=0.90. Isotherm spacing is a uniform increment of Δθ=0.15 in all cases.

Temperature distribution plots were particularly useful in the analysis of the current problem. They provided insight into temperature gradients in the domain, the size and geometry of the two-phase region, and the nature of the fluid motion. In a region of closely packed isotherms, a steep temperature gradient was present, resulting in greater conductive and convective heat transfer.

Plots of constant values of the stream function, ψ, were also very useful. Such plots were exact "maps" of the motion of the fluid in the enclosure. The stream function was defined in the usual way as:

\[
\begin{align*}
u &= \frac{\partial \psi}{\partial y} \\
u &= -\frac{\partial \psi}{\partial x}
\end{align*}
\]

(47)

To calculate the stream function, a value of ψ=0 was arbitrarily chosen at the coordinate system origin. The values of stream function next to the origin were calculated by numerically integrating the velocity values at neighboring points. In this way, the values of the stream functions were calculated for the entire domain.
In interpreting the streamline plots, several things must be kept in mind. First, a streamline cannot cross an impermeable boundary since fluid motion across an impermeable surface is not possible. Secondly, the numerical difference between any two streamlines represents the volumetric flow rate of fluid between the two streamlines. Consequently, the broader the range of streamlines in the domain, the higher the volumetric flow rate. Lastly, the more closely packed the streamlines are, the faster the fluid is moving [Roberson and Crowe (1993)]. This is also related to the notion of a mass or volumetric flow rate. For a constant density, if the area between two streamlines decreases, the velocity must increase to maintain the same mass flow rate between the streamlines. The fact that fluid velocity increases in regions of closely packed streamlines can also be seen in the definition of the stream function, Eq.(47). As the spacing between streamlines decreases, the stream function gradient and therefore, fluid velocity, increases.

The stream function $\psi$ was normalized with the effective thermal diffusivity of the porous enclosure, $\alpha_{\text{eff}}$. A broad range of values for nondimensional stream function $\psi^*$ is shown in each plot. The maximum stream function value is shown on the streamline plots.

Another useful tool in the analysis of the fluid motion were plots of the velocity vectors for each phase. These plots helped to verify the direction of the fluid motion, and provided insight into the different motion of each phase. The lengths of these vectors were calculated relative to the maximum fluid velocity in the enclosure. By calculating the lengths in this way, the velocity vector plots were produced so that the region of fastest fluid motion was shown by the vectors with the longest arrows. However, care should be exercised in studying the velocity vector plots. To obtain a clear picture of fluid motion in
a relatively small plot area, it was necessary to plot only the velocity vectors at every other node. Therefore, while it might appear that a velocity vector crosses an impermeable surface, a study of the streamline plot will show this to be untrue, and merely a shortcoming of the plotting mechanism.

Additionally, lines of constant liquid saturation were superimposed on the isotherm plots. These isosaturation lines gave insight into the nature and location of vapor formation, and the size of the two-phase region.

To calculate the average heat flux values at the upper and lower surfaces, the local values of heat flux were numerically integrated using Simpson's Rule. The local heat flux was calculated based on the total volumetric enthalpy using the equation:

\[ q_y = -\frac{\Gamma_h \frac{\partial H}{\partial y}}{\rho \frac{\partial y}{\partial y}} \]  

Eq. (48) reduces to an equation for heat flux in the single phase region, while the flux should be thought of as an energy gradient in the two-phase region. More on this is discussed later in this section. For convention, heat flux was considered positive in the positive axis direction.

To make a clear presentation of the nature of two-phase flow under the given heating conditions, the results obtained are summarized by explaining the effect of varying each parameter. In such a comparison, only one parameter at a time was changed, to make proper conclusions about the contribution of that parameter. In the following sections, a summary of the influence due to each parameter is presented.
It was expected that varying the imposed side heat flux would have a significant impact on the steady state temperature distribution and fluid motion. As the nondimensional side heat flux $Q_w$ is increased, more energy is being transferred into the enclosure. Therefore, intuition suggests that as $Q_w$ is increased, the enclosure heats up more and begins to form more vapor. The numerical calculations verified that this is the case. The effect of varying $Q_w$ is summarized below. With increasing $Q_w$ there was:

1. An increase in the total stored energy in the porous enclosure. In other words, there was more fluid at or near the saturation temperature.

2. An increase in vapor formation. This was reflected in the increased size of the two-phase fluid region and in an increase in the volumetric vapor fraction, $\epsilon_v$.

3. An increase in the speed of fluid motion. This was verified by observing that the range of stream function values increased with $Q_w$.

4. An increase in the nondimensional heat flux at the upper surface, $Q_T$.

These observations were noticed regardless of the type of porous material present in the enclosure. However, these results were dependent on Rayleigh number. Each of the four results above was noticed for each porous medium, but the degree to which they occurred varied with $Ra$. Therefore, looking at the results for each material individually is useful, to learn the degree to which the phenomena occur. The effect of Rayleigh number is summarized later in the section.
At low values of the imposed heat flux, the temperature distribution was uniform throughout the enclosure. As the heat flux was increased, vapor began to form, and the two-phase region increased in size. As this happened, the isotherms began to move closer together, and eventually became concentrated in a small region. This resulted in a large temperature gradient.

Besides the observations noted earlier, an interesting phenomenon occurred in a high permeability porous medium such as sand. The isotherm and streamline plots, as shown in Figure 7, are very similar to porous enclosures heated from below as reported in Wang et al. (1994). In the bottom heated solutions presented in that paper, most of the steady state solutions had two convection cells, and temperature distribution plots that were symmetric about the vertical center of the enclosure. In the current study, very similar fluid motion and temperature distribution were observed in high permeability media at relatively low values of imposed heat flux.

Figures 7(a) and 7(b) show isotherm and streamline plots for the case $Ra=160$, $\theta_b=0.7$, $u_y^*=-0.0$, and $Q_w=1.0$. As one can see from the streamline plot, Figure 7(b), the steady state solution has two convection cells. The right-hand cell rotates counterclockwise, and the left-hand cell rotates clockwise. The imposed heat flux at the right wall heats the liquid adjacent to the wall, and due to this increase in temperature, buoyancy forces cause the fluid to rise. This begins a pattern of circulation in which hot fluid next to the heated wall rises and escapes through the permeable upper surface. To
Figure 7. Isotherm and Streamline Plots for Ra=160, θ_B=0.7, U_y Q_1=0.0, and Q_w=1.0
maintain conservation of mass, cool fluid is drawn downward into the enclosure through part of the upper surface remote from the heated wall. The fluid motion is fastest adjacent to the heated wall, indicated by the closely packed streamlines. The hot bottom surface added a significant amount of heat to the enclosure, and this is particularly noticeable in the lower left corner of the enclosure. In this region, the fluid is moving slowly, and the heat from the bottom surface results in buoyancy forces that begin a second convection cell.

The fluid motion of the two convection cells is reflected in the isotherm plot, Figure 7(a). As one can see in this plot, the counterclockwise convection cell brings hot fluid from the bottom of the enclosure upward along the right wall. Similarly, the clockwise rotating cell brings hot fluid from the bottom upward along the left wall. The two cells meet at the center of the upper surface, and the downward flow of both cells draws cool fluid from the upper surface into the center of the enclosure.

In Figure 7(a), the temperature distribution is nearly symmetric about a line at X=0.5, and is similar to the temperature distribution for a porous enclosure heated from below. However, by examining the left and right edges, it can be seen that the isotherms have a nonzero slope at the heated right wall. This is due to the added energy from the imposed wall heat flux. The slope of the isotherms at the left wall is zero, consistent with an adiabatic surface. The shape of the temperature distribution suggested that the enclosure got a significant amount of heat from the hot bottom surface. In this case, the heating conditions were not sufficient to start any vapor formation, and therefore \( e_v = 0.0 \).

In Figure 7(b), it can be observed that the nature of the fluid motion is consistent
with that of a bottom heated enclosure, with fluid in the right-hand convection cell moving counterclockwise, and fluid in the left-hand cell moving clockwise. This type of counterflow motion drew hot fluid from the lower surface upward along the vertical walls and out the permeable upper surface, while drawing cool fluid from the upper surface downward into the center of the enclosure. This fluid motion is reflected in the plot of temperature distribution, Figure 7(a).

Figures 8(a) and 8(b) are isotherm and streamline plots, respectively, for the case $Ra=160$, $\theta_b=0.7$, $u_y=b=0.0$, and $Q_w=20.0$. As can be seen from Figure 8(b), the fluid motion has switched to one convection cell, moving counterclockwise. In this case, the effect of the imposed wall heat flux has become more significant. The result is a single convection cell driven by phase change instability. The temperature distribution, Figure 8(a), is consistent with the nature of the fluid motion. As the fluid next to the heated wall heats, buoyancy forces will cause the fluid to rise. This begins a cycle of counterclockwise motion. As the hot fluid moves upward along the right vertical wall, it leaves the enclosure upon reaching the top permeable surface. To make up the lost mass, cold fluid enters through the region of the top permeable wall remote from the hot right wall. This relatively cooler fluid moves downward. This pattern is clearly visible from the streamline and velocity vector plots. A detailed discussion of the fluid flow mechanisms that cause the shift from two to one convection cell is presented in the discussion of transient studies at the end of this section. In summary, the enclosure starts with a single convection cell and shifts to two cells as time progresses. This shift is due to slow-moving fluid in the lower left corner of the enclosure and heat added from the hot bottom wall. The slow-
moving fluid allows the hot bottom wall to create enough buoyancy to begin a second convection cell. As $Q_w$ is increased and the fluid speeds up, the buoyant forces are no longer sufficient to start a second convection cell.

Figures 8(c) and 8(d) are vapor and liquid velocity vectors plots, respectively, for the same case. The vapor components of the fluid move away from the heated wall and turn upward near the single to two-phase transition line. The motion of the liquid phase is nearly the same as that of the bulk fluid movement. The region of fastest fluid motion is between the center of the convection cell and the right heated wall.

Figures 9(a) and 9(b) are isotherm and streamline plots for the case $Ra=160$, $\theta_b=0.7$, $u_{yB}^*=0.0$, and $Q_w=35.0$. The nature of fluid motion and temperature distribution are nearly the same as Figures 8(a) and 8(b). In Figure 9(b), it can be observed that the values of fluid stream functions are larger than Figure 8(b), indicating faster fluid motion. Evidence can also be seen of faster fluid motion in the temperature distribution plots. In Figure 9(a), the isotherm lines are nearer to the bottom surface than in Figure 8(a), indicating that the hot fluid is becoming more concentrated near the bottom of the enclosure. The temperature distribution suggests that the cool fluid being drawn into the enclosure is penetrating farther as $Q_w$ is increased. The conclusion drawn is that the cool fluid has more momentum and therefore higher velocity as $Q_w$ is increased.

Figures 9(c) and 9(d) are vapor and liquid velocity vector plots for the same case. The fluid motion is nearly the same as in Figures 8(c) and 8(d), with the vapor components moving away from the heated wall and turning upward. Again, the region of fastest fluid motion is adjacent to the heated right wall.
Figure 8. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=20.0$
Figure 8 (continued). Vector Plots for $Ra=160$, $\theta_B=0.7$, $u_{y,B}=0.0$, and $Q_w=20.0$

(c) Vapor Velocity Vectors

(d) Liquid Velocity Vectors
Figure 9. Isotherm and Streamline Plots for Ra=160, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=35.0$
Figure 9 (continued). Vector Plots for $Ra=160$, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=35.0$
Another interesting thing to note as $Q_w$ is increased is the increasing size of the two-phase region. The size of the two-phase region is indicated by the isosaturation (dashed) lines in the isotherm plots. A value of $s=1.0$ would show the one to two-phase transition, but the transition line is difficult to plot since the entire single phase region has $s=1.0$. As the imposed heat flux is increased, more vapor forms and an isosaturation line of a given value moves farther from the heated wall. This can be seen by comparing Figures 8(a) and 9(a). For the case with $Q_w=35.0$ (Figure 9) the $s=0.95$ line is much farther from the heated wall than the case with $Q_w=20.0$ (Figure 8). The conclusion drawn is that the size of the two-phase region increases with increasing $Q_w$.

Figures 10(a) and 10(b) are isotherm and streamline plots for the case $Ra=160$, $\theta_b=1.0$, $u_{y,*}=0.0$, and $Q_w=1.0$. Similar to the case shown in Figure 7(b), the steady state solution to this case has two convection cells. In Figure 10(a), it is observed that the temperature distribution is again nearly symmetric about $X=0.5$, with the right wall being slightly hotter than the left. By comparing Figure 10(a) with Figure 7(a), it can be seen that the enclosure is hotter for $\theta_b=1.0$ than for $\theta_b=0.7$ for the same value of $Q_w$. This is indicated by the maximum value of isotherm line plotted in each case ($\theta=0.60$ for $\theta_b=0.7$ and $\theta=0.90$ for $\theta_b=1.0$). More about the effect of bottom surface temperature will be discussed later.

Figures 11(a) and 11(b) are isotherm and streamline plots for the case $Ra=160$, $\theta_b=1.0$, $u_{y,*}=0.0$, and $Q_w=25.0$. Apparent in Figure 11(b) is the fact that the steady state solution still has two convection cells. However, the range of stream function values suggests that the fluid in the right-hand cell is moving much faster than the fluid in the left-
Figure 10. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=1.0$, $u_{y,0}^{*}=0.0$, and $Q_w=1.0$
Figure 11. Isotherm and Streamline Plots for Ra=160, θ_B=1.0, u_y,B*=0.0, and Q_w=25.0
hand cell. The fluid motion is also reflected in the isotherm plot, Figure 11(a). The cool fluid drawn into the enclosure from the upper surface is being pulled into the right half of the enclosure, due to the greater momentum of the right-hand cell. The result is a loss of symmetry about the vertical center as seen in Figures 7(a) and 10(a).

The next increment of $Q_w$ has eliminated the second, clockwise rotating convection cell. Figures 12(a) and 12(b) are isotherm and streamline plots for the case $Ra=160$, $\theta_B=1.0$, $u_{y,B}^*=0.0$, and $Q_w=30.0$. From Figure 12(b), it can be seen that the solution has only one convection cell. The temperature distribution plot, Figure 12(a), is also indicative of a single convection cell, as described for Figures 8(a) and 9(a). In the single cell solutions, most of the fluid in the right half of the enclosure is hot while the hot fluid in the left half is concentrated near the bottom surface. As mentioned earlier, an increase in $Q_w$ leads to an increase in fluid speed. Due to this higher speed of fluid motion, the buoyant forces produced by the hot bottom surface are not sufficient to reverse the direction of fluid motion and begin a second convection cell.

Figures 13(a) and 13(b) are isotherm and streamline plots for the case $Ra=160$, $\theta_B=1.0$, $u_{y,B}^*=0.0$, and $Q_w=35.0$. By comparing Figures 12 and 13, it can be observed that the fluid motion speeds up with increasing $Q_w$. This is verified by the higher values of stream function and by the cool fluid being drawn farther down the left wall. The increased momentum of fluid in the convection cell causes the cool fluid to penetrate farther into the enclosure. The result is that the hot fluid is becoming more concentrated near the bottom of the enclosure as $Q_w$ is increased. Evidence of this can be seen in the location of isotherm lines in Figures 12(a) and 13(a).
Figure 12. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=1.0$, $u_{y,B}^*=0.0$, and $Q_w=30.0$
Figure 13. Isotherm and Streamline Plots for Ra=160, $\theta_B=1.0$, $u_{yB}^*=0.0$, and $Q_w=35.0$
Another observation noted with increasing $Q_w$ is that there is a definite increase in vapor formation. By comparing the isosaturation line $s=0.95$ in Figures 12(a) and 13(a), it can be seen that the size of the two-phase region is larger for the higher value of $Q_w$. The increased size of the two-phase region is evident from the fact that the $s=0.95$ line is farther from the heated wall for the higher $Q_w$, indicating that the two-phase transition line has penetrated farther into the enclosure. The increase in vapor formation has been verified by an increase in the calculated volumetric vapor fraction, $\epsilon_v$ (not shown here).

Figures 7 through 13 clearly demonstrate the first three observations noted in the list describing the effect of increasing $Q_w$. The fourth observation, that the heat flux at the top surface increases with $Q_w$, is most easily seen by plotting $Q_T$ versus $Q_w$. Figure 14 shows plots of $Q_T$ versus $Q_w$ for two situations with $Ra=160$. The solid line in Figure 14 is a plot of $Q_T$ versus $Q_w$ for $\theta_B=1.0$ and $u_{y,B}^*=0.0$ and the dashed line is a similar plot for $\theta_B=0.7$ and $u_{y,B}^*=0.0$. As can be seen in Figure 14, there is a distinct increase in the heat flux at the upper surface, $Q_T$, as the imposed wall heat flux $Q_w$ is increased. A change in the slope of the $Q_T$ vs. $Q_w$ line is noticed in the range of $Q_w$ where the shift from a two to one cell convection pattern occurs. It is also clear from Figure 14 that the higher bottom surface temperature produces a higher value of top surface heat flux for a given imposed heat flux. More about this observation will be discussed later in the section.
Figure 14. Top Heat Flux vs. Imposed Heat Flux (Ra=160)
Glass Beads - Medium Permeability Porous Media (Ra=56)

Many of the same observations noted for the case of Ra=160 were also noted in medium permeability media. Namely, as the imposed side heat flux $Q_w$ is increased, the enclosure heats up, more vapor is formed, the speed of fluid motion is increased, and the heat flux at the upper surface is increased.

Figures 15(a) and 15(b) are isotherm and streamline plots for the case Ra=56, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=1.0$. From the evenly spaced isotherm lines in Figure 15(a), it can be seen that the temperature distribution is uniform throughout the porous enclosure. The streamline plot, Figure 15(b), shows that the fluid circulation has only one convection cell. This is different from the corresponding case with Ra=160, shown in Figure 7(b). The conclusion drawn is that fluid motion is more difficult in medium permeability media, and a second convection cell is not established. The temperature distribution reflects that the hot fluid adjacent to the heated wall rises, and cool fluid is brought down from the permeable upper surface along the left wall. The pattern of heat distribution is consistent with single convection cell solutions, as described earlier for Figures 8(a) and 9(a).

Figures 16(a) and 16(b) are isotherm and streamline plots for Ra=56, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=20.0$. Evident from the values of stream function in the streamline plot, Figure 16(b), the fluid motion has increased in speed with the increase in $Q_w$. The region of fastest fluid motion is adjacent to the heated wall, but the convection cell center is moving to the left. The reason for this is revealed in the isotherm plot, Figure 16(a). Due to the increase in $Q_w$, there is more vapor formation, and the isothermal two-phase region
Figure 15. Isotherm and Streamline Plots for $Ra=56, \theta_B=0.7, u_{x,B,*}=0.0$, and $Q_w=1.0$
Figure 16. Isotherm and Streamline Plots for $Ra=56$, $\theta_B=0.7$, $u_{y,b}=0.0$, and $Q_w=20.0$
has increased in size. The result is that the region of temperature gradient that drives the convection cell is pushed to the left by new vapor formation.

Figures 17(a) and 17(b) are isotherm and streamline plots for the case $Ra=56$, $\theta_B=0.7$, $u_{y,B}=0.0$, and $Q_w=35.0$. From the values of stream function indicated in Figure 17(b), it can be seen that the fluid circulation speed has not significantly increased from the corresponding case with $Q_w=20.0$ (Figure 16). However, now the center of circulation has moved into the left half of the enclosure. As a result, fluid is channeled between the left wall and the convection cell center, and the fastest fluid motion is near the left edge of the enclosure. The velocity vector plots for this case (not shown) are consistent with the streamline plots and verify the region of fastest fluid motion. Comparing isosaturation lines in Figures 17(a) and 16(a), a significant increase in the amount of vapor formation and the size of the two-phase region can be observed.

Figures 18(a) and 18(b) are plots for the case $Ra=56$, $\theta_B=1.0$, $u_{y,B}=0.0$, and $Q_w=1.0$. At this low value of $Q_w$, the isotherm lines are evenly distributed throughout the enclosure. The even distribution of temperature is clear from the approximately equal spacing between isotherms in Figure 18(a). By comparing Figure 18(a) with Figure 15(a), it can be seen that the enclosure is hotter with $\theta_B=1.0$ than with $\theta_B=0.7$, indicated by the maximum value of isotherm line plotted (0.60 for $\theta_B=0.7$ and 0.90 for $\theta_B=1.0$).

Figures 19 and 20 show isotherm and streamline plots for the case $Ra=56$, $\theta_B=1.0$, $u_{y,B}=0.0$, and $Q_w=20.0$ and $Q_w=35.0$, respectively. The same trends noticed for $\theta_B=0.7$ are noted here as well. Namely, the fluid speeds up with increasing $Q_w$, and due to the increasing size of the two-phase region, the center of circulation moves into the left half of
Figure 17. Isotherm and Streamline Plots for $Ra=56$, $\theta_B=0.7$, $u_x^*=0.0$, and $Q_w=35.0$
Figure 18. Isotherm and Streamline Plots for $Ra=56$, $\theta_B=1.0$, $u_{y,B}^*=0.0$, and $Q_w=1.0$
Figure 19. Isotherm and Streamline Plots for $Ra=56$, $\theta_B=1.0$, $u_{x,B}^*=0.0$, and $Q_w=20.0$
Figure 20. Isotherm and Streamline Plots for $Ra=56$, $\theta_b=1.0$, $u_{y,b}=0.0$, and $Q_w=35.0$
the enclosure. By comparing the isosaturation lines in Figure 20(a) and Figure 17(a) it is observed that there is more vapor formation with $\theta_B = 1.0$ than with $\theta_B = 0.7$.

In the glass bead porous system, two convection cells never form, even at very low values of the imposed side heat flux. Due to the lower permeability of the glass bead system, fluid motion is more difficult, and the bottom heating is not sufficiently high to establish two convection cells. The result of single convection cell motion is that the hot fluid concentrating in the upper right corner of the enclosure and along the right wall, with cool fluid being drawn downward along the left edge.

As more liquid boils to vapor at the heated right edge, the increased size of the two-phase region moves the center of the convection cell to the left. When the two-phase region has pushed the center of the cell across the horizontal center of the enclosure, the fluid is channeled between the cell center and the left wall. The result is that the region of fastest fluid motion moves from near the right wall to near the left wall as $Q_w$ is increased.

Since the permeability of the glass bead system is lower than sand, the resulting fluid motion is much slower for glass beads than for sand. Due to this slower motion, heat transfer across the enclosure is shifting slightly from convection to conduction. Convection is still the dominant mode of heat transport, but the influence of conduction is greater for the glass bead system than for sand. Because of less convective cooling, there is more vapor formation for a given value of $Q_w$ as $Ra$ decreases. This is evident from the increased size of the two-phase region, indicated by the isosaturation lines, in Figures 13(a) and 20(a). In high permeability media (Figure 13), the two-phase region is much smaller than the corresponding case for medium permeability material (Figure 20).
The velocity vector plots (not shown) indicate that in the two-phase region, the liquid components flow upward while the vapor components flow to the left, away from the heated wall, and then turn upward near the one and two-phase transition line. Therefore, in the two-phase region, the liquid and vapor components are flowing in different directions. As the two-phase region increases in size with $Q_w$, the corresponding increase in vapor saturation decreases the bulk fluid speed. Since the region of fastest fluid motion is usually the same location as the two-phase region, the conclusion is that bulk fluid speed increases up to a certain value of $Q_w$ and then levels off. Evidence of this phenomenon is seen in the streamline plots for the case $Ra=56$, $\theta_B=1.0$, $u_{yB^*}=0.0$, and $Q_w=20.0$ and 35.0 (Figures 19(b) and 20(b), respectively). Indicated by the values of the stream functions, the fluid has slowed for the case with the higher $Q_w$.

Figure 21 is a plot of $\epsilon_v$ versus $Q_w$ for medium permeability media. The solid line in Figure 21 is $\epsilon_v$ vs. $Q_w$ for $\theta_B=1.0$ and $u_{yB^*}=1.43$ and the dashed line is a similar plot for $\theta_B=0.7$ and $u_{yB^*}=1.43$. As seen in this plot, there is a distinct rise in the volumetric vapor fraction with increasing $Q_w$. Another interesting item to note is that for a given value of $Q_w$, there is a larger value of $\epsilon_v$ for $\theta_B=1.0$ than for $\theta_B=0.7$. This indicates that there is more vapor formation with the higher bottom surface temperature.

Figure 22 is a similar plot for $Q_T$ versus $Q_w$. In this plot, there is a linear increase in heat flux at the upper surface as the imposed wall heat flux is increased. Also, by comparing the solid line for $\theta_B=1.0$ and the dashed line for $\theta_B=0.7$, it can be observed that there is more heat flux at the upper surface for the higher bottom surface temperature.
Figure 21. Volumetric Vapor Fraction vs. Imposed Heat Flux (Ra=56)
Figure 22. Top Heat Flux vs. Imposed Heat Flux (Ra=56)
Soil - Low Permeability Porous Media (Ra=10)

The same trends noted for Ra=160 and Ra=56 were also noted in low permeability media (Ra=10) as well. As Q_w increases, the enclosure heated up, more vapor was formed, fluid circulation sped up, and the heat flux at the upper surface increased.

Figures 23(a) and 23(b) are isotherm and streamline plots for the case Ra=10, $\theta_b=0.7$, $u_{y,B}^*=0.0$, and Q_w=1.0. As in the corresponding cases with Ra=56 and Ra=160, the temperature distribution is uniformly spread throughout the enclosure (see Figure 15 for Ra=56 and Figure 7 for Ra=160). However, for low permeability media the enclosure is hotter than for corresponding cases with Ra=56 or Ra=160, indicated by the maximum value of $\theta$ shown in the isotherm plots. This is due to a greater amount of conductive heat transfer and less convective cooling due to slower fluid motion. As with medium permeability, there is only one convection cell at even the lowest value of Q_w.

Figures 24(a) and 24(b) are isotherm and streamline plots for Ra=10, $\theta_b=0.7$, $u_{y,B}^*=0.0$, and Q_w=10.0. By comparing these plots with Figures 23(a) and (b), it can be seen that there is a significant increase in vapor formation and that the fluid has increased in velocity. The increase in vapor formation is seen by comparing the location of the isosaturation lines and fluid speed is indicated by the range of stream function values. The convection cell center has moved to the left with increasing Q_w as observed for Ra=56.

Figures 25(a) and 25(b) are isotherm and streamline plots for Ra=10, $\theta_b=0.7$, $u_{y,B}^*=0.0$, and Q_w=20.0. As Q_w is increased, there is more vapor formation, indicated by the increased size of the two-phase region.
Figure 23. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{yB}^*=0.0$, and $Q_w=1.0$
Figure 24. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=10.0$
Figure 25. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{yB}=0.0$, and $Q_w=20.0$
Figures 26(a) and 26(b) are isotherm and streamline plots for $Ra=10$, $\theta_b=1.0$, $u_{yB}^*=0.0$, and $Q_w=1.0$. In this case, the temperature distribution is again uniform throughout the enclosure. There is also a much larger two-phase region than was observed for either $Ra=56$ or $Ra=160$ with $\theta_b=1.0$ and $Q_w=1.0$ (see Figure 10 for $Ra=160$ and Figure 18 for $Ra=56$). This suggests that as the Rayleigh number of a fluid decreases and fluid motion becomes more difficult, there is an increase in conduction heat transfer in the enclosure, resulting in more vaporization. One convection cell is seen in Figure 26(b).

Figures 27 and 28 show isotherm and streamline plots for $Ra=10$, $\theta_b=1.0$, $u_{yB}^*=0.0$, and $Q_w=10.0$ and 20.0, respectively. It can be seen by the isosaturation lines in the temperature distribution plots for each case that there is an increase in vapor formation corresponding to the increase in $Q_w$. This is quite expected because of the increased heating rate.

In low permeability media ($Ra=10$), fluid speed is nearly an order of magnitude lower than for the corresponding cases with $Ra=160$ or $Ra=56$. This verifies the intuitive notion that fluid motion is much more difficult to achieve in low permeability porous media. Resulting from this slower motion, there is a shift in the dominant mode of heat transfer from convection to conduction. The shift of heat transfer mode from convection to conduction can be seen in the nature of the isotherm lines. For $Ra=10$ (Figure 26) the isotherm lines are nearly straight indicating a significant amount of conduction. As the Rayleigh number of the material increases, the isotherm lines for corresponding cases are more curved, indicating significant convection. Cases corresponding to Figure 26 are shown in Figure 18 for $Ra=56$ and Figure 10 for $Ra=160$. Since there is less convective
Figure 26. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=1.0$, $u_{y,B}^*=0.0$, and $Q_w=1.0$
Figure 27. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=1.0$, $u_{y_B}=0.0$, and $Q_w=10.0$
Figure 28. Isotherm and Streamline Plots for $Ra=10, \theta_B=1.0, u_{y,*}=0.0,$ and $Q_w=20.0$
cooling due to this lower velocity, more vapor is allowed to form adjacent to the heated wall. The degree of vapor formation is indicated by the isosaturation curves in the isotherm plots.

Similar to medium permeability porous material ($Ra=56$), the circulation is always limited to one counterclockwise rotating convection cell. There is more vapor formation for $Ra=10$ than for either $Ra=56$ or $Ra=160$, and at high values of $Q_w$, the two-phase region covers nearly the entire enclosure. The amount of vapor formation can be measured by the size of the two-phase region and by the calculated value of volumetric vapor fraction, $\epsilon_v$.

Figure 29 shows a plot for $\epsilon_v$ versus $Q_w$ for $Ra=10$. The solid line in Figure 29 is a plot of $\epsilon_v$ vs. $Q_w$ for $\theta_B=1.0$ and $u_{y,B}^*=4.13$ and the dashed line in Figure 29 is a similar plot for $\theta_B=0.7$ and $u_{y,B}^*=4.13$. As noted before with medium permeability porous media, there is a distinct rise in vapor formation with increasing imposed wall heat flux. By comparing the solid and dashed lines in Figure 29, it can be observed that there is much more vapor formation for the cases with $\theta_B=1.0$ than for $\theta_B=0.7$.

Figure 30 is a plot for top surface heat flux, $Q_T$, versus imposed wall heat flux $Q_w$ for $Ra=10$. The solid line in Figure 30 is for $\theta_B=1.0$ and $u_{y,B}^*=4.13$ and the dashed line is for $\theta_B=0.7$ and $u_{y,B}^*=4.13$. As noted for the other porous materials, there is a nearly linear increase in $Q_T$ with increasing $Q_w$. Also, there is a greater value of heat flux at the upper surface for a given $Q_w$ with $\theta_B=1.0$ than with $\theta_B=0.7$. 
Figure 29. Volumetric Vapor Fraction vs. Imposed Heat Flux (Ra=10)
Figure 30. Top Heat Flux vs. Imposed Heat Flux (Ra=10)
Effect of Bottom Surface Temperature ($\theta_B$)

The next parameter studied was the specified bottom surface temperature boundary condition. It was expected that with a higher bottom temperature, the enclosure should heat up more and form more vapor. This is seen in the following results. The effect of changing the bottom surface temperature on the heating of the porous enclosure is summarized as follows:

1. There was more vapor formation at a fixed value of $Q_w$ for $\theta_B=1.0$ than for $\theta_B=0.7$.
2. Circulation speed was greater for $\theta_B=1.0$ than for $\theta_B=0.7$.
3. The heat flux across the upper surface was higher for $\theta_B=1.0$ than for $\theta_B=1.0$.
4. At the lowest value of $Q_w$ (1.0), there was a small amount of vapor formation for $\theta_B=1.0$, but never for $\theta_B=0.7$.
5. The lower right-hand corner of the enclosure was hotter when $\theta_B=1.0$ than when $\theta_B=0.7$. This is reflected in the temperature distribution plots and in the amount of vapor formation in the corner. An example of this phenomena is seen by comparing Figures 35 and 36.

To show the above observations, specific examples are shown for each of the three porous materials studied.
Particular to high permeability media was the ability of the fluid to form a second convection cell, as described earlier. Figures 31 and 32 are isotherm and streamline plots for $Ra=160$, $Q_w=5.0$, $u_y=0.0$, and $\theta_b=0.7$ and $\theta_b=1.0$, respectively. As can be seen from these plots, the same value of $Q_w$ will produce a single convection cell solution with $\theta_b=0.7$, while a two-cell solution exists for $\theta_b=1.0$. This suggests that the higher bottom surface temperature produces a stronger two-cell circulation pattern than the lower bottom surface temperature. This is due to the presence of a larger temperature gradient at the bottom surface for $\theta_b=1.0$ than for $\theta_b=0.7$. Therefore, the higher bottom surface temperature requires a larger imposed side heat flux to overcome the bottom heating dominance and establish a single convection cell solution.

Another interesting observation in Figures 31(a) and 32(a) is the location of the two-phase region, indicated by the isosaturation lines superimposed on the isotherm plot. For the lower bottom surface temperature case ($\theta_b=0.7$), water has not yet boiled to produce vapor and indeed has not yet reached the saturation temperature. However, for the higher bottom temperature case ($\theta_b=1.0$), water has begun to form vapor in two locations - near the lower right corner of the enclosure and at $X=1.0$ and $Y=0.65$. This seems to indicate that at low values of $Q_w$, the lower right-hand corner is the hottest part of the enclosure for $\theta_b=1.0$. For the case of $\theta_b=1.0$, boiling begins in the lower right corner for low values of $Q_w$, then shifts upward along the right edge of the enclosure as $Q_w$ is increased.
Figure 31. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=0.7$, $u_{y_B}^*=0.0$, and $Q_w=1.0$
Figure 32. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=1.0$, $u_{y,B}=0.0$, and $Q_w=1.0$
Verified by checking the values of volumetric vapor fraction, $\epsilon_v$, (not shown here), there is a small amount of vapor formation for $\theta_B=1.0$ even at the lowest value of $Q_w$. This is to be expected since the bottom surface temperature is specified at the saturation temperature of the working fluid. However, this fraction is very small and not enough vapor is produced to show up on an isosaturation plot for $Q_w=1.0$.

Figures 33 and 34 are isotherm and streamline plots for $Ra=160$, $Q_w=25.0$, $u_{y,B}=0.0$, and $\theta_B=0.7$ and $\theta_B=1.0$, respectively. By comparing Figure 33(a) and 34(a), it is evident that the higher bottom surface temperature aids in the formation of vapor, indicated by the larger size of the two-phase region. For a given $Q_w$, the $\theta_B=1.0$ cases consistently produced more vapor than the $\theta_B=0.7$ counterpart. By comparing these two figures, it can be observed that the solution for $\theta_B=1.0$ has two convection cells, while $\theta_B=0.7$ has only one cell. This further indicates that the higher bottom surface temperature aids in heat transfer from the bottom surface upward into the porous enclosure.
Figure 33. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=25.0$
Figure 34. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=1.0$, $u_{y,B}^*=0.0$, and $Q_w=25.0$
Glass Beads - Medium Permeability Porous Media (Ra=56)

As observed for Ra=160, a higher bottom surface temperature created more vapor for a given value of $Q_w$, increased the heat flux at the upper surface, increased the speed of fluid motion, and had some vapor formation at $Q_w=1.0$. Also observed in both Ra=160 and Ra=56 was a change in location of new vapor formation at low values of the imposed heat flux. For $\theta_b=0.7$, the new vapor forms at $X=1.0$ and $Y=0.65$. For $\theta_B=1.0$, new vapor formed near the lower right corner of the enclosure.

Figures 35 and 36 are isotherm and streamline plots for Ra=56, $Q_w=5.0$, $u_{y,B}^*=0.0$, and $\theta_B=0.7$ and $\theta_B=1.0$, respectively. By comparing these figures, it can be observed that the two-phase region is larger for $\theta_B=1.0$ than for $\theta_B=0.7$. It is also interesting to note the shape of the two-phase region for each case. For $\theta_B=1.0$ (Figure 36), the $s=0.99$ isosaturation line penetrates more into the enclosure and the two-phase region is approximately rectangular. For $\theta_B=0.7$ (Figure 35), the two-phase region is nearly triangular, with the phase transition line tapering toward the right vertical wall at $Y=0.15$. This indicates that the lower right corner of the enclosure is hotter for the higher bottom surface temperature. Indicated by the stream function values in Figures 35(b) and 36(b), it can be seen that the fluid motion is faster for the higher bottom surface temperature.

Figures 37 and 38 are isotherm and streamline plots for Ra=56, $Q_w=25.0$, $u_{y,B}^*=0.0$, and $\theta_B=0.7$ and $\theta_B=1.0$, respectively. The same observations noted above for $Q_w=5.0$ are apparent here as well. The two-phase region is larger, the enclosure is hotter, and the fluid is moving faster for the case of $\theta_B=1.0$ than for $\theta_B=0.7$. 
Figure 35. Isotherm and Streamline Plots for $Ra=56$, $\theta_B=0.7$, $u_{y*B}=0.0$, and $Q_w=5.0$
Figure 36. Isotherm and Streamline Plots for Ra=56, \( \theta_B=1.0 \), \( u_{y,B}^*=0.0 \), and \( Q_w=5.0 \)
Figure 37. Isotherm and Streamline Plots for \( Ra=56 \), \( \theta_B=0.7 \), \( u_{y,B}^* = 0.0 \), and \( Q_w=25.0 \)
Figure 38. Isotherm and Streamline Plots for $Ra=56$, $\theta_B=1.0$, $u_{yB}=0.0$, and $Q_w=25.0$
Soil - Low Permeability Porous Media (Ra=10)

As for medium and high permeability media, a study of low permeability media indicated that an increase in bottom surface temperature had significant effects on the heating of the porous enclosure. The observations indicated that with increasing bottom surface temperature there was an increase in: (i) vapor formation, (ii) heat flux at the upper surface, and (iii) speed of fluid motion.

Figures 39 and 40 are isotherm and streamline plots for Ra=10, Q_w=5.0, u_yB*=0.0, and θ_B=0.7 and θ_B=1.0, respectively. In comparing the temperature distribution plots, Figures 39(a) and 40(a), it can be seen that the size of the two-phase region increases with an increase in θ_B. This indicates that the energy transport in the enclosure is increased as the bottom surface temperature is increased.

Figures 41 and 42 are isotherm and streamline plots for the cases Ra=10, Q_w=15.0, u_yB*=0.0, and θ_B=0.7 and θ_B=1.0, respectively. The same observations noted above for Q_w=5.0 are seen here as well. That is, the size of the two-phase region, and therefore the energy transferred to the enclosure, increases with an increase in the bottom surface temperature.
Figure 39. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=5.0$
Figure 40. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=1.0$, $u_{x,B}^*=0.0$, and $Q_w=5.0$
Figure 41. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=15.0$
Figure 42. Isotherm and Streamline Plots for $Ra=10, \theta_B=1.0, u_{yB}^*=0.0$, and $Q_w=15.0$
Effect of Incoming Fluid Velocity at the Bottom Surface ($u_{y,B}^*$)

This section describes the effects on boiling and heat transfer when the bottom of the porous enclosure is made permeable. This type of phenomenon occurs in geothermal activities, and hence, is of interest. When heat is applied to a porous medium saturated with liquid water, the water evaporates. The water vapor thus produced moves upward due to the buoyancy effect. Hence, for different heating rates, different rates of vapor velocity will occur. When vapor moves upward, liquid water will be drawn from the permeable bottom surface to maintain conservation of mass. In this section, the effects of various values of fluid velocity ($u_{y,B}^*$) at the bottom wall are described. To determine an appropriate range for $u_{y,B}^*$, incoming fluid velocity was calculated using an energy balance equation from Cao and Faghri (1994):

$$-(\rho u_y)_{out} \frac{h_{fg}}{h_{fg}} = -q_{out}$$

(49)

By calculating the velocity from this equation, a suitable range of incoming fluid velocities was established.

The effect of varying the incoming fluid velocity from the bottom is summarized below. As $u_{y,B}^*$ was increased from zero (impermeable surface):

1. The heat flux at the upper surface, $Q_T$, increased.
2. The volumetric vapor fraction increased.
3. The heat (energy) flux at the bottom surface, $Q_B$, decreased.
The above results suggested that a nonzero incoming velocity will aid in transporting the heat from the bottom surface, upward through the enclosure, and out through the upper permeable surface. Again, specific examples from each porous material studied are presented next to show the observations noted above.

For all of the porous materials studied, a permeable bottom surface with a nonzero specified incoming fluid velocity aided in heat transport. This was evident in the amount of vapor formation and in the heat fluxes at the top and bottom surface. By observing that the vapor formation and $Q_T$ increased with $u_{y,B}^*$, and that $Q_B$ decreased with $u_{y,B}^*$, it can be concluded that a nonzero $u_{y,B}^*$ effectively carried the hot fluid from the bottom surface upward into the enclosure.
Figures 43 and 44 are isotherm and streamline plots for the case $Ra=160$, $\theta_B=0.7$, $Q_w=10.0$, and $u_{y,B}^*=0.0$ and 2.86, respectively. By comparing streamline plots, Figures 43(b) and 44(b), it can be seen that for the given heating conditions an impermeable bottom surface, Figure 43(b), will result in a single convection cell. However, a similar case with a permeable bottom surface, Figure 44(b), will result in a double convection cell steady state solution. Since the formation of the left-hand cell in a two-cell solution is due to heat transfer from the bottom surface, the conclusion drawn is that a permeable bottom surface enhances heat transport upward from the hot bottom wall. This is expected since an incoming fluid stream will help to move the fluid heated by the bottom wall upward into the enclosure.

It is also interesting to note that with an increase in $u_{y,B}^*$ comes an increase in the size of the two-phase region. The size of the two-phase region is indicated by the isosaturation line in Figures 43(a) and 44(a). By carefully examining these two plots, it can be seen that the $s=0.99$ line is farther from the heated right wall for the case with $u_{y,B}^*=2.86$ than for $u_{y,B}^*=0.0$. This suggests that the two-phase region penetrates farther into the enclosure for a permeable bottom wall.

Figures 45 and 46 are isotherm and streamline plots for the case $Ra=160$, $\theta_B=1.0$, $Q_w=40.0$, and $u_{y,B}^*=2.86$ and 5.73, respectively. As can be seen in the streamline plots, Figures 45(b) and 46(b), there is a double convection cell solution for $u_{y,B}^*=5.73$ and single convection cell solution for $u_{y,B}^*=2.86$. As discussed above, this is due to the
Figure 43. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=0.7$, $u_{_B^*}=0.0$, and $Q_w=10.0$
Figure 44. Isotherm and Streamline Plots for Ra=160, $\theta_B=0.7$, $u_{y,*}=2.86$, and $Q_w=10.0$
Figure 45. Isotherm and Streamline Plots for $Ra=160$, $\theta_b=1.0$, $u_{y,B}^*=2.86$, and $Q_w=40.0$
Figure 46. Isotherm and Streamline Plots for $Ra=160$, $\theta_B=1.0$, $u_{y_B}^*=5.73$, and $Q_w=40.0$
enhancement of heat transfer from the bottom surface brought by the increased velocity of the incoming fluid stream.

An observation to note in the isotherm plots, Figures 45(a) and 46(a), is that the $s=0.95$ and $s=0.99$ isosaturation lines are farther from the heated right wall for the larger value of $u_{x,B}^*$. As discussed above, this suggests that an increase in the velocity of the incoming fluid stream will increase the size of the two-phase region. The increase in the amount of vapor formation is due to the decrease in convective cooling that comes with the formation of the second cell. As the secondary cell forms, less cool fluid from the upper surface will be drawn into the hot right half of the enclosure. The result is less convective cooling in the region of the heated right wall and an increase in vapor formation.
An interesting observation for medium permeability porous media was that the specified incoming fluid velocity was never sufficient to begin a two convection cell solution. In other noticeable ways, a nonzero \( u_{y,B}^* \) facilitated heat transfer. By examining the calculated values for \( Q_T \) and \( Q_B \) (not shown), it was observed that an increase in \( Q_T \) and a decrease in \( Q_B \) came with an increase in \( u_{y,B}^* \). The increase in heat flux at the upper surface was due to a larger temperature gradient at the upper wall. An increase in temperature gradient suggested that more hot fluid was near the cool upper surface for increasing \( u_{y,B}^* \). Likewise, a decrease in \( Q_B \) indicated a decrease in temperature gradient at the bottom wall with increasing \( u_{y,B}^* \). This decreased gradient was a result of the incoming fluid stream moving the fluid heated by the bottom wall upward into the enclosure and closer to the upper surface. The conclusion drawn from these observations was that an increase in \( u_{y,B}^* \) enhanced heat transfer from the bottom surface.

Figures 47 and 48 are isotherm and streamline plots for the cases \( Ra=56, \theta_B=1.0, Q_w=15, \) and \( u_{y,B}^*=0.0 \) and 2.85, respectively. These two plots are representative results for varying \( u_{y,B}^* \) in a medium permeability porous material. By comparing the isosaturation lines in the temperature distribution plots, Figures 47(a) and 48(a), it can be seen that the size of the two-phase region is larger for a permeable bottom surface. The increase in size of the two-phase region is indicated by the \( s=0.95 \) and \( s=0.90 \) isosaturation lines in the two plots. For the permeable bottom surface (Figure 48), these isosaturation lines are farther away from the heated wall.
Figure 47. Isotherm and Streamline Plots for $Ra=56$, $\Theta_b=1.0$, $u_{y,b}^*=0.0$, and $Q_w=15.0$
Figure 48. Isotherm and Streamline Plots for $Ra=56$, $\theta_B=1.0$, $u_{y,B}^*=2.85$, and $Q_w=15.0$
Due to the difficulty with which fluid moved through a low permeability media such as soil, a specified velocity at the bottom surface had a greater effect on the fluid motion in the enclosure. The fluid velocity for Ra=10 was about an order of magnitude lower than for Ra=56 or Ra=160. Therefore, the same specified fluid velocity had a more significant impact on fluid motion in a low permeability medium.

Figures 49, 50, and 51 are isotherm and streamline plots for Ra=10, θ_b=1.0, Q_w=10, and u_y,B* =0.0, 2.06, and 4.13, respectively. In Figure 49(b), it can be seen that the enclosure had the typical counterclockwise circulation that arises from the phase change and buoyancy forces described earlier. However, as u_y,B* was increased, the streamline plots indicated that the fluid motion was almost straight through the porous enclosure. In Figure 50(b) it can be seen that most of the enclosure was subjected to forced convection. The upper right corner, remote from the heated surface, still had a small natural convection cell. Figure 51(b) shows an enclosure without any natural convection. The conclusion drawn was that the incoming fluid velocity had such an effect on the fluid motion that the result was nearly total forced convection.
Figure 49. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{y,B}^*=0.0$, and $Q_w=10.0$
Figure 50. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{y, B}^*=2.06$, and $Q_w=10.0$
Figure 51. Isotherm and Streamline Plots for $Ra=10$, $\theta_B=0.7$, $u_{y,B}^*=4.13$, and $Q_w=10.0$
Effect of the Porous Material (Ra)

The effect of different porous materials has been described earlier in this chapter, and is only summarized here. Increasing the Rayleigh number of the material increases its permeability, and hence the ease with which fluid moves through a material. Therefore, with decreasing Ra:

1. There was a shift from a two convection cell solution with significant bottom heating to solutions with only one counterclockwise rotating convection cell.

2. There was much more vapor formation due to the slower fluid motion and hence less convective cooling. The conclusion drawn was that with decreasing Rayleigh number, there was a shift from convection to conduction dominated heat transfer. This is intuitively obvious since a material with Ra=0 is a solid, and hence heat transport is entirely by conduction.

3. The fluid in the porous enclosure was more sensitive to an incoming fluid stream with a given velocity. Due to the low permeability, fluid motion driven by natural convection was much slower as Ra decreases.
Transient Solutions

After obtaining steady state solutions for the entire range of parameters investigated in this study, a select number of transient cases were run to gain additional insight into the nature of the current problem. Running transient cases for the entire range of parameters would have been a useful study, but due to lack of computer resources, such a study must be deferred until later.

As mentioned earlier, to obtain solutions for the steady state solutions, an extremely large time step was used in the discretized governing equations, similar to Eq. (44). This large time step effectively eliminated the transient part of the continuity and energy equations, Eqs. (1) and (6), from consideration. The transient solutions were achieved by using a small enough time step that the transient contributions to these equations were still significant.

Transient studies were done for five cases. These cases were chosen based upon results obtained from steady state solutions to gain further insight about heat and mass transfer in porous media. They covered a broad range of parameters and hence a broad range of applications. Transient solutions were obtained for:

1. $Ra=160$, $\theta_B=0.7$, $u_{y,B}^*=2.86$, $Q_w=10.0$
2. $Ra=160$, $\theta_B=1.0$, $u_{y,B}^*=2.86$, $Q_w=10.0$
3. $Ra=160$, $\theta_B=1.0$, $u_{y,B}^*=2.86$, $Q_w=20.0$
4. $Ra=10$, $\theta_B=0.7$, $u_{y,B}^*=2.06$, $Q_w=10.0$
To make meaningful conclusions regarding the transient analyses, three parameters were plotted as a function of time. These parameters were: (i) the heat flux at the upper surface $Q_T$, (ii) the heat flux at the bottom surface $Q_B$, and (iii) the volumetric vapor fraction $\epsilon_v$.

A short explanation of the method by which heat flux was calculated is discussed first. The local energy flux values at the top and bottom surfaces were calculated using Eq. (48). From Eq. (5), volumetric enthalpy is expressed as $H = \rho(h-h_{sat})$. As mentioned earlier, the assumption was made that $h = C_p T$ in the single phase (nonisothermal) regions. In the isothermal two-phase region, enthalpy is expressed in terms of liquid saturation as $h = h_{sat} - s h_f$. From these definitions for enthalpy, it can be seen that volumetric enthalpy $H$ reduces to an expression for temperature in the single phase region and for liquid saturation in the two-phase region. Therefore, Eq. (48) reduces to a heat flux expression in the single phase region and an expression for a liquid saturation gradient in the two-phase region. Having a nonzero $Q_B$ in the isothermal two-phase region was possible, since the enthalpy (liquid saturation) gradient was not necessarily zero.

The transient plots for the cases studied are shown in Figure 52 through Figure 56. In these plots, $Q_T$, $Q_B$, and $\epsilon_v$ were plotted versus time on the same nondimensional time axis. The left-hand vertical axis corresponds to the dashed line representing $\epsilon_v$, and the right-hand axis corresponds to the solid lines representing $Q_T$ and $Q_B$. It is important to note the scale of the left-hand axis when comparing $\epsilon_v$ in the transient plots. The enthalpy gradient at the upper surface was equivalent to heat flux since fluid near the top surface is
single phase.

Referring to Figure 52, the top wall heat flux, $Q_T$, started at zero since the upper surface and the fluid immediately under it were at the same temperature at the beginning. As the enclosure heated up, the fluid below the top surface got hot, while the top surface itself remained at the boundary temperature. Therefore, there was an increase in $Q_T$ with time. When the system reached steady state conditions, the temperature gradient and hence the heat flux reached a constant value.

Due to a finite temperature difference between the bottom surface and the fluid immediately above it at the beginning of the calculation, there was a positive, finite value of $Q_B$ at $t^*=0$. Since the region near the bottom surface in this case was still single phase, $Q_B$ represents heat flux. As the enclosure heated up, the fluid above the bottom surface heated up and approached the temperature of the bottom boundary, decreasing the temperature gradient and heat flux. If the temperature above the bottom row of nodes became greater than the bottom boundary, $Q_B$ became negative, indicating a downward directed gradient.

Figures 52, 53, and 54 show transient plots for high permeability medium ($Ra=160$). Figure 52 is for $\theta_B=0.7$, $u_{yB^*}=2.86$, and $Q_w=10.0$. Figure 53 is for the same case, but with $\theta_B=1.0$. Figure 54 is the same as Figure 52 with $Q_w=20.0$.

The steady state solutions for the cases presented in Figures 52 and 53 had two cell convection patterns (see Figure 44 for the steady state solution to the case in Figure 52). Common to this type of solution was a sudden increase in each of the values in the transient plots. Investigation showed that this increase in values of $Q_T$, $Q_B$, and $\epsilon_v$. 

Figure 52. Plot of $Q_T$, $Q_B$, and $\epsilon_v$ vs. Time for $Ra=160$, $\theta_B=0.7$, $u_y^* B=2.86$, $Q_w=10.0$
Figure 53. Plot of $Q_T$, $Q_B$, and $\epsilon_v$ vs. Time for $Ra=160$, $\theta_B=1.0$, $u_{y,B}*=2.86$, $Q_w=10.0$
Figure 54. Plot of $Q_T$, $Q_B$, and $\epsilon_v$ vs. Time for $Ra=160$, $\theta_B=0.7$, $u_{y,B}^*=2.86$, $Q_w=20.0$
occurred as the convection pattern shifted from one to two convection cells. Therefore, the enclosure started with a single counterclockwise rotating convection cell established by the imposed side heat flux, and then switched to a two cell counter flow convection pattern. This was due to the significant contribution of heating from the bottom surface. As the enclosure heated up, the additional heat from the bottom surface began a second convection cell. Fluid in the lower left corner of the enclosure was slow moving relative to the rest of the enclosure, as verified by the velocity vector plots in each case. Heat transfer from the hot bottom surface created buoyancy in the fluid near the bottom. As time progressed, the buoyancy forces became great enough to penetrate the existing convection cell and change the direction of fluid motion. With the single convection cell in the enclosure, fluid in the lower left corner was moving downward. The buoyancy forces created by the hot bottom surface reversed the direction of this fluid motion. The result was that hot fluid rose along the left wall, and established a second convection cell. In cases with a single convection cell at steady state, the buoyancy forces were not sufficient to reverse the downward direction fluid motion.

Figure 54 shows transient plots for a steady state solution with a single convection cell. In this case, the high value of the imposed side wall heat flux created faster fluid circulation, and the second cell was never allowed to start in the lower left corner. This single cell solution had smooth transient plots, without a sudden increase in the value for $Q_T$, $Q_B$, and $\epsilon_v$.

The sudden increase in value of $Q_T$, $Q_B$, and $\epsilon_v$ in Figures 52 and 53 was caused by the fluid motion that arose from the onset of the second convection cell. As the second
convection cell formed, there was a decrease in the length of the upper surface through which cold fluid was drawn in. The result was less cool fluid drawn in with the onset of the secondary convection cell, and less convective cooling. Due to the decrease in convective cooling, the enclosure became much hotter with the formation of the secondary cell, and resulted in increased values of heat flux at the upper and lower surfaces and of volumetric vapor fraction.

As described above, the heat added from the hot bottom wall created fluid buoyancy that reversed the direction of fluid motion and began a second convection cell. The fluid motion due to the second convection cell brought hot fluid from the lower left corner of the enclosure upward along the left edge. The result was that hot fluid became more concentrated near the upper surface in the cases with two convection cells. The result of this concentration of hot fluid near the cool upper surface was a steeper temperature gradient and therefore higher heat flux. The increase in $Q_t$ resulting from the formation of the second convection cell is indicated in the transient plots by the sudden increase at $t^* \approx 0.12$ in Figure 52 and at $t^* \approx 0.06$ in Figure 53.

The sudden increase in the value of $\epsilon_v$ at approximately the same point is also due to the onset of the secondary convection cell. With the onset of the secondary convection cell, less cool fluid was drawn into the enclosure from the upper surface. The result was less convective cooling and an increase in the amount of liquid that boiled to vapor.

Figures 55 and 56 show plots of $Q_t$, $Q_b$, and $\epsilon_v$ versus time for low permeability media ($Ra=10$), with $u_{y,B}*=2.06$, $Q_w=10.0$, and $\theta_B=0.7$ and 1.0, respectively. In low permeability media, a two cell steady state convection pattern was never observed, and
Figure 55. Plot of $Q_T$, $Q_B$, and $\epsilon_v$ vs. Time for $Ra=10$, $\theta_B=0.7$, $u_{y,B}^{-}=2.06$, $Q_w=10.0$
Figure 56. Plot of $Q_T$, $Q_B$, and $\epsilon_v$ vs. Time for $Ra=10$, $\theta_B=1.0$, $u_{y,B}^*=2.06$, $Q_w=10.0$
hence the smooth transient plots.

An interesting behavior to note about the low permeability transient plots, Figures 55 and 56, was that the enthalpy flux at the bottom surface becomes negative. This indicated that the enthalpy gradient was directed in the negative y-axis direction. In Figure 55 (steady state solution in Figure 50) $Q_b$ became negative. This was because the two-phase region had increased in size enough that fluid near the boiling temperature overlay the $\theta_B=0.7$ bottom surface. The result was a temperature gradient directed outward through the bottom surface. In the steady state solution to the case shown in Figure 56 (not shown), the fluid at the bottom surface was at the boiling temperature, as was the fluid above the bottom. However, the fluid above the bottom had a higher fraction of vapor formation than the bottom surface. The result was an enthalpy gradient directed outward through the bottom surface, and $Q_b$ became negative.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>Effect on Solution</th>
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<tbody>
<tr>
<td>$Q_w$</td>
<td>As $Q_w$ was increased</td>
<td>Enclosure heated up</td>
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<tr>
<td></td>
<td></td>
<td>More liquid was boiled to vapor</td>
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<tr>
<td></td>
<td></td>
<td>Speed of circulation increased</td>
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<tr>
<td></td>
<td></td>
<td>Heat flux at upper surface increased</td>
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<tr>
<td>$\theta_B$</td>
<td>As $\theta_B$ was increased</td>
<td>Shift in location of new vapor formation</td>
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<tr>
<td></td>
<td></td>
<td>Vapor formation at lowest imposed heat flux</td>
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<tr>
<td></td>
<td></td>
<td>Heat flux across upper surface increased</td>
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<tr>
<td></td>
<td></td>
<td>More vapor formation for given $Q_w$</td>
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<tr>
<td></td>
<td></td>
<td>Speed of circulation increased</td>
</tr>
<tr>
<td>$u_{y,B}^*$</td>
<td>As $u_{y,B}^*$ was increased</td>
<td>Heat flux at upper surface increased</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Heat flux at lower surface decreased</td>
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<tr>
<td></td>
<td></td>
<td>More vapor formation</td>
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<tr>
<td>$Ra$</td>
<td>As $Ra$ was increased</td>
<td>Shift from one to two convection cells</td>
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<tr>
<td></td>
<td></td>
<td>Significantly less vapor formation</td>
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<td>Decrease in sensitivity to an incoming stream</td>
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The results obtained from the transient study provided additional information about the steady state conditions. The transient study helped to verify some of the results in Table 4, and provided some new information as well. The results from the transient study are summarized as:

1. Two cell steady state solutions began with one convection cell, then formed a second cell as time progresses. This second cell arose from significant buoyant forces due to the hot bottom surface.

2. There was a sudden increase in the plots of $Q_T$, $Q_B$, and $\epsilon_v$ versus time for the two cell convection cases. This sudden rise in all three values was due to the redistribution of hot fluid by the second cell, and the decrease in cool fluid drawn into the enclosure from the copper surface.

3. The enthalpy gradient at the bottom surface became negative for certain cases in low permeability media. This was due to the absence of convective cooling and the formation of more vapor. As more heat was added to the enclosure, the fluid overlying the bottom surface became hotter and had a greater volumetric enthalpy than the fluid at the bottom.
CONCLUSIONS AND RECOMMENDATIONS

Based on the numerical calculations conducted in this study, several conclusions and recommendations were made. The recommendations presented below are intended for future researchers in the study of two-phase fluid flow in porous media.

Conclusions

Conclusions regarding heat transfer and fluid flow in a porous enclosure heated from one side were as follows:

1. The Rayleigh number had a significant effect on the heating of the porous enclosure. With increased media permeability, the Rayleigh number increased. Therefore, a high permeability media, and hence a high Ra, allowed fluid to move relatively easily through it. The results observed in this study indicated that as Ra decreased, much more vapor formed for a given heating situation. This was due to slower fluid motion, and the resulting decrease in convective cooling of the enclosure. As Ra increased, fluid motion was less difficult, and the convective circulation in the enclosure helped to cool the hotter regions.

2. Heat flux imposed on the right vertical wall also had a significant effect on the heat transfer of the enclosure. As $Q_w$ was increased, the added energy aided the
buoyant effect, and therefore increased the speed of fluid circulation. The result was more vapor formation along the heated wall, while more convective cooling in certain regions of the enclosure.

3. The imposed fluid velocity at the bottom surface contributed to the heat transfer and fluid motion in the enclosure. By specifying a fluid of certain velocity crossing the lower boundary, the study simulated geothermal beds that draw fluid in the bottom to replace the fluid that has boiled away. Such a nonzero normal fluid velocity at the bottom surface aided heat transport. The fluid effectively carried the heat from the bottom surface and upward through the enclosure.

4. The bottom surface fluid temperature played an important role in the heat transfer in the enclosure. The higher value of bottom wall temperature added heat to the enclosure, resulting in increased vapor formation and increased speed of fluid circulation. Additionally, for high permeability porous media the higher bottom surface temperature aided in the formation of two convection cells.

5. The applicability of Darcy’s law in the present problem was verified. To verify that viscous forces dominate in the current problem, the maximum fluid velocity in the enclosure was calculated. From Eq. (35) the maximum allowable fluid velocity can be calculated as:

\[ u_{\text{max}} = \frac{Re_K v}{\sqrt{K}} = \frac{v}{\sqrt{K}} \]

As reported earlier, Cheng [1978] stated that for Darcy’s law to be valid \( Re_K \leq 1.0 \).
Therefore, so long as the maximum fluid velocity did not exceed the ratio of kinematic viscosity to the square root of absolute permeability, Darcy's law was valid. In each of the cases run for this study, this maximum velocity was checked, and never exceeded the above ratio. Therefore, Darcy's law was a valid form of the momentum equation for this study.

**Recommendations**

Several recommendations for further study are made. These recommendations came from certain situations encountered in the current study. These recommendations for future researchers include:

1. A study involving a wider range of porous materials would be very useful. From such a study, one could learn more about the manner in which two convection cells form. Establishing the minimum Rayleigh number for such solutions would be an interesting goal.

2. A more extensive transient analysis would be useful. A very careful analysis that studied closely the nature of fluid motion would provide a great deal of knowledge about water-filled porous systems.

3. A study in which $k_{eff}$ is varied with the liquid saturation would be useful if one is particularly interested in a significant amount of vapor formation. The current study loses a certain amount of accuracy for high values of $\varepsilon_r$. Hence, a study with variable $k_{eff}$ would provide more accurate results.
REFERENCES


REFERENCES-continued


REFERENCES—continued


REFERENCES-continued


