Thermal properties of extreme black hole spacetimes
by Daniel J Loranz

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy
Physics in
Montana State University

Abstract:
Constraints axe placed on the thermal properties of extreme black hole spacetimes by examining the expectation value of the stress-energy tensor for a free conformally invariant scalar field in such spacetimes.

Working in a general two dimensional static black hole spacetime, the stress-energy of the quantized field in thermal equilibrium with a black hole is found to diverge strongly on the event horizon of the black hole unless the field is in a thermal state with a temperature defined by the surface gravity of the event horizon. This divergence occurs for both extreme and nonextreme black holes in precisely the same manner. Thus, extreme black hole spacetimes cannot be assigned an arbitrary temperature without serious consequences.

Studying the stress-energy for a quantized scalar field in the reduced two dimensional charged dilatonic black hole spacetime of Garfinkle, Horowitz, and Strominger further confirms that extreme black hole spacetimes have well defined thermal properties. Despite the lack of horizons in the string description of the extreme dilatonic black hole, a unique temperature is obtained for the extreme black hole by extrapolating the temperature of the nonextreme black hole to the extreme state. Any other temperature is shown to result in a divergent stress-energy in the conformally related physical description of the extreme dilatonic black hole.

Finally, the expectation value of the stress-energy tensor for a massless conformally invariant scalar field is numerically calculated for the full four dimensional extreme dilatonic black hole of Garfinkle, Horowitz, and Strominger. Working in the string metric, the components of this stress-energy are found to be finite everywhere in the extreme dilatonic black hole spacetime. These results are the first values calculated for the stress-energy of a quantized field in a superstring black hole spacetime.

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APPROVAL

of a thesis submitted by

Daniel J. Loranz

This thesis has been read by each member of the thesis committee, and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Date 6-10-97

The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful.

-Henri Poincaré

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CONVENTIONS

Throughout this dissertation I use the conventions and notation of Misner, Thorne, and Wheeler [6]. In addition I use natural units ($c = G = \hbar = k_B = 1$).

ABSTRACT

Constraints are placed on the thermal properties of extreme black hole spacetimes by examining the expectation value of the stress-energy tensor for a free conformally invariant scalar field in such spacetimes.

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Finally, the expectation value of the stress-energy tensor for a massless conformally invariant scalar field is numerically calculated for the full four dimensional extreme dilatonic black hole of Garfinkle, Horowitz, and Strominger. Working in the string metric, the components of this stress-energy are found to be finite everywhere in the extreme dilatonic black hole spacetime. These results are the first values calculated for the stress-energy of a quantized field in a superstring black hole spacetime.

CHAPTER 1

Introduction

This dissertation examines the temperature of extreme black holes and the expectation values of the stress-energy for quantized fields in general static black hole spacetimes. Black hole radiance and stable extreme black hole states play an important role in many contemporary studies exploring the quantum nature of gravity. That black holes have temperatures and can radiate may seem an oxymoron to the reader not familiar with quantum gravity. Even the purely classical notion of an extreme black hole may be new for the lay person. This chapter introduces necessary definitions and sets the context for the work that follows.

Black Hole Structure

Most people have a basic understanding that a black hole is an object with a strong gravitational attraction from which nothing, not even light, can escape. One does not need general relativity to imagine such objects. In the late 1700's Michell [1] and Laplace [2] independently theorized such invisible stars using Newtonian gravity and the corpuscular theory of light. Their calculations showed that a star with Earth-like density and a radius two hundred and fifty times larger than the sun would trap its own radiation and allow no light to escape.

This intuitive idea of a black hole as a region of "no escape" is formalized by examining the causal structure of a black hole spacetime. Because of the maximum velocity defined by special relativity, not all points in a spacetime are causally connected to a given event. The boundaries of a causally connected region are defined by the trajectories followed by massless particles, such as photons. These trajectories, called null rays, mark the past and future light cones of an event. Points lying within the light cone are causally connected to the event, while points outside the light cone are not. Normally, all points in a spacetime lie in the causal past (i.e. within the past light cone) of future infinity and in the causal future (future light cone) of past infinity. However, a black hole spacetime has a region that is not causally connected to future infinity but is causally connected to past infinity. This region of points represents the black hole, and the boundary between this region and the region that is causally connected to future infinity is called the event horizon (see figures (1) and (2)). Thus, a black hole is formally defined as a region of spacetime that is not causally connected to future infinity.

The simplest black hole described by general relativity is given by the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \qquad (1)$$

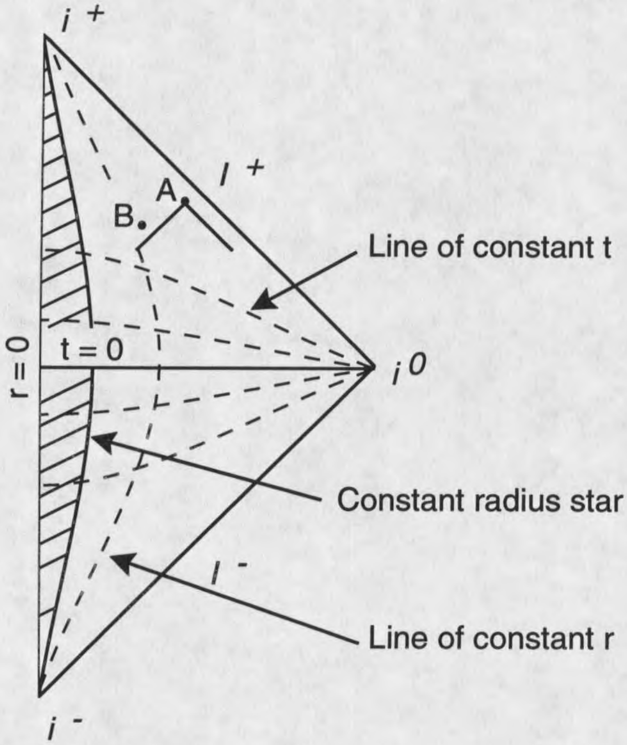

Figure 1: This figure show a conformal diagram of a constant radius star. As for all conformal diagrams, the null rays in this figure travel along lines tilted at 45° and infinity has been mapped to a finite boundary located at i and I. Note that events A and B are causally disconnected since event B lies outside A's light cone. In contrast, all events lie in the causal past of future infinity, i^+, since all events lie within the past light cone of i^+ defined by the surface I^+. For reference, a few lines of constant r and constant t are shown. These will be suppressed in succeeding diagrams.

where M is the mass of the black hole, $d\Omega^2$ is the metric of the two sphere, $d\theta^2 + \sin^2\theta d\phi^2$, and the metric components $g_{\alpha\beta}$ are read from the interval ds by $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ in the usual manner. The Schwarzschild metric describes a static, spherically symmetric, vacuum spacetime with an event horizon at $r = 2M$ and a curvature singularity at $r = 0$ (see figure (3)). One might suspect this curvature singularity results from the spherical symmetry of the Schwarzschild geometry. However a set of proofs by

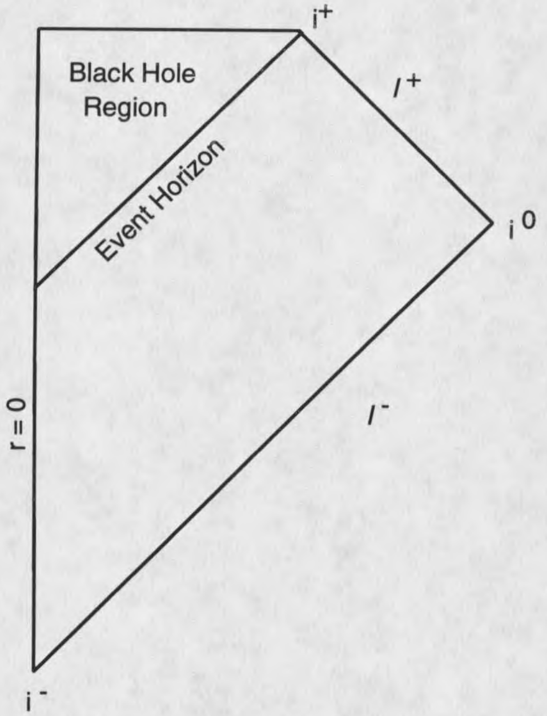

Figure 2: In this diagram, not all events lie in the causal past of future infinity, i^+. Those points of the spacetime causally disconnected from i^+ represent the black hole. Note that timelike paths exist which enter the black hole and become trapped.

Hawking and Penrose show that singularities are a generic feature of black holes [3, 4]. These proofs require only that general relativity correctly describes gravity, that local energy density be positive, and that an event horizon forms during the gravitational collapse. Thus, the general relativistic picture of a black hole includes a singularity enclosed by an event horizon.

The Schwarzschild metric is the unique time independent, spherically symmetric, vacuum solution to the Einstein equations [5]. General relativity permits only two other stationary electrovacuum black hole solutions [6]. One solution generalizes

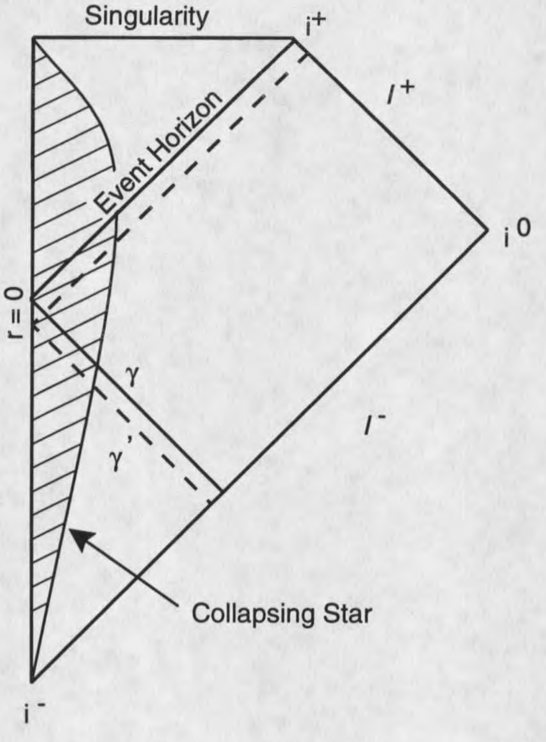

Figure 3: This conformal diagram depicts a spherically symmetric star collapsing to form a Schwarzschild black hole. The null ray γ' just manages to escape to infinity after reflecting from $r = 0$. The very next null ray, γ, marks the event horizon of the black hole. Any null ray reflecting from $r = 0$ after γ enters the black hole and becomes trapped. Any observer traveling along a timelike geodesic that crosses the event horizon also becomes trapped within the black hole.

Schwarzschild by including an electromagnetic field and is given by the Reissner-Nordström metric

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{2}$$

where Q is the charge on the black hole. The other solution generalizes Reissner-Nordström by including angular momentum and is given by the Kerr-Newman metric

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2 + a^2)d\phi - adt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2, \tag{3}$$

where $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$, $\rho^2 \equiv r^2 + a^2\cos^2\theta$, and a is the angular momentum of the black hole per unit mass, $a = J/M$. For $Q = 0$, the metric of Eq. (3) reduces to the Kerr metric. A general analysis of gravitational collapse remains unsolved. However, the gravitational collapse of a star with more than twice the mass of the sun is expected to form a black hole which will settle down to a final state described by the Kerr-Newman metric [3, 6, 7]. Thus, general relativity suggests a picture of complete gravitational collapse that ends in a black hole parameterized only by the black hole's mass, charge, and angular momentum. This result is often stated colloquially by saying, "black holes have no hair."

The simple picture of a black hole as a singularity enclosed by an event horizon still holds true for these more general black holes. Still, charge and angular momentum will change the structure of a black hole in some interesting ways. For the work that follows, the most important change is the creation of an inner horizon. Unlike the Schwarzschild case, not all timelike geodesics that cross the event horizon of a Reissner-Nordström or Kerr-Newman black hole terminate on the singularity within. In fact, an observer who crosses the event horizon of a Reisnner-Nordström or Kerr-Newman black hole, while forever trapped inside, is free to travel to regions within the black hole that are far from the singularity and have small curvature. However, during these maneuvers the observer eventually enters a region causally connected to the singularity. The boundary of such a region is a Cauchy horizon, and for the Reissner-Nordström black hole, this boundary marks the inner horizon (see figure (4)).

Generally, determining the horizon structure of a particular black hole geometry requires global knowledge of the entire spacetime. However, for the special case of a static spacetime, the horizons are located where $g_{tt} = 0$. For a Schwarzschild black hole $g_{tt} = -(1 - 2M/r)$, and one thus finds a single horizon at $r_0 = 2M$. For a Reissner-Nordström black hole, one finds inner and outer horizons (r_- and r_+ respectively) at $r_\pm = M \pm \sqrt{M^2 - Q^2}$. Note that as the charge, Q, goes to zero, the inner horizon vanishes and the outer horizon moves outward to $r_+ = 2M$, the Schwarzschild value. On the other hand, when $Q \to M$, the horizons become degenerate ($r_- \to r_+ \to M$), and the event horizon becomes a Cauchy horizon. In fact, if Q is greater than M, the metric component g_{tt} has no roots, no horizons exist in the spacetime, and instead of describing a black hole, Eq. (2) describes a "naked" singularity. This continuum from Schwarzschild-like black hole to naked singularity

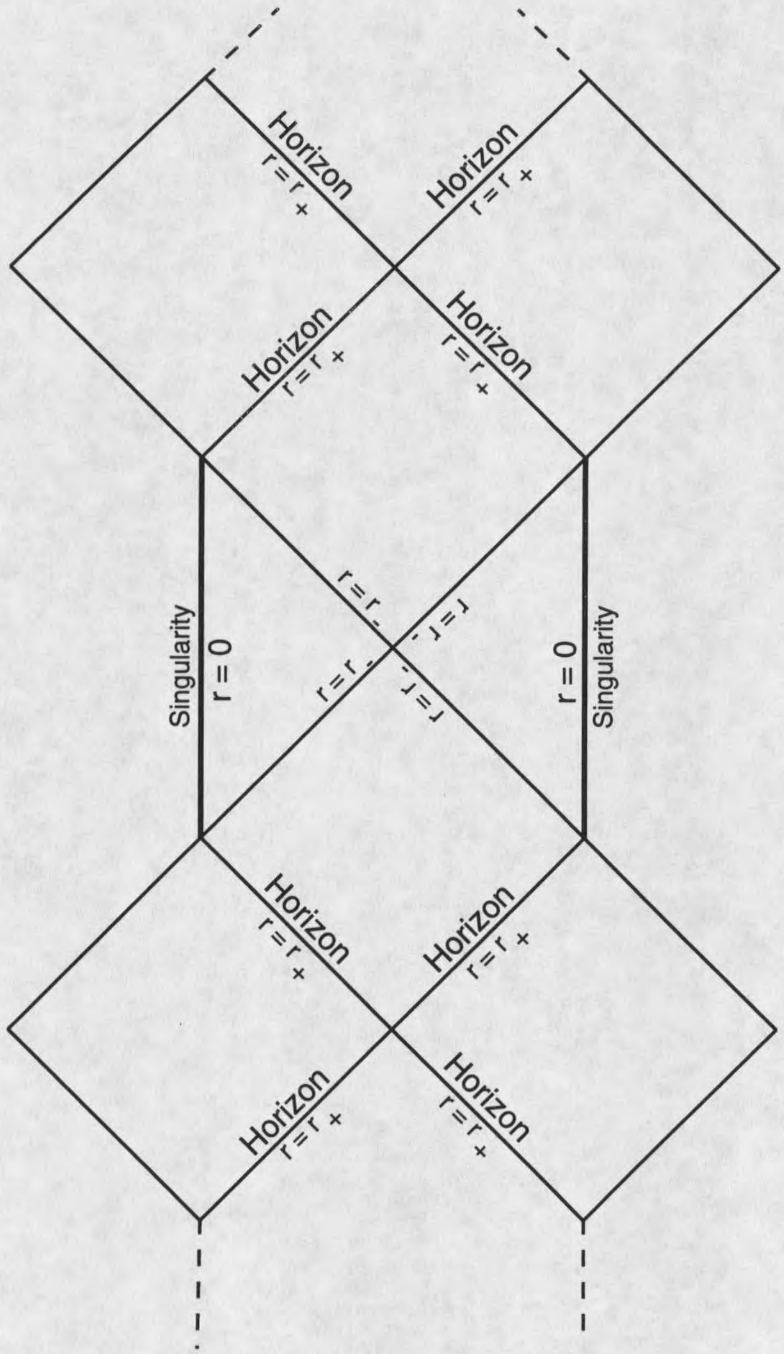

Figure 4: The Reissner-Nordström black hole has multiple horizons. An observer who crosses the inner horizon, r_-, enters a region causally connected to a singularity. The symmetry of the Reissner-Nordström solution implies this geometry continues, resulting in an infinite set of black holes within black holes.

is a general description of the possible configurations of black holes with multiple horizons. A black hole with degenerate horizons is called an extreme black hole.

A property of black holes important to this dissertation is the surface gravity, κ. As in Newtonian theory, surface gravity measures the force required to keep a unit test mass at rest just above the surface of a gravitating body. The general metric for a static, spherically symmetric black hole is

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2. \tag{4}$$

For Eq. (4) one finds the surface gravity to be

$$\kappa = \frac{1}{2} \frac{f'}{\sqrt{fh}}\bigg|_{r \to r_0} , \tag{5}$$

where r_0 is the location of the event horizon, and a prime denotes differentiation with respect to r. Using Eq. (5) one finds the surface gravity for a Schwarzschild black hole to be $\kappa = 1/(4M)$. For a Reissner-Nordström black hole the surface gravity is $\kappa = \sqrt{M^2 - Q^2}/{r_+}^2$, where r_+, recall, is the radius of the outer horizon. The surface gravity of a Reissner-Nordström black hole thus depends on the charge as well as the mass of the black hole. As the charge increases from zero to M, the surface gravity decreases from $1/(4M)$, the Schwarzschild value, to zero. Hence, an extreme Reissner-Nordström black hole has zero surface gravity.

This section defined both the black hole in general and the extreme black hole configuration. It also highlighted some of the physical details of black holes as presented by general relativity. As interesting as this picture may be, the general relativistic description of black holes is incomplete. One must still consider the interaction of matter fields with the black hole spacetime. The quantum characteristic of matter is well established. Accounting for this quantum aspect, one finds that black holes radiate a thermal spectrum of particles.

Black Hole Radiation

The singularity lurking at the heart of a black hole is a disturbing presence. Singularities are locations where the description of spacetime as a manifold with pseudo-Reimannian geometry breaks down. Since all known physical laws depend on this description of spacetime, all of physics breaks down at the singularity. In the words of Stephen Hawking, "This is a great crisis for physics because it means that one cannot predict the future: One does not know what will come out of the singularity." [8] In this sense, general relativity predicts its own failure.

One might argue that the event horizon enclosing the singularity makes the breakdown of predictability a nonissue. Observers outside a black hole are causally disconnected from the singularity. They cannot see phenomena occurring within the event horizon. However, can one assume that a newly formed singularity is always safely enclosed by an event horizon? The assumption that singularities are always enclosed by an event horizon is known as the weak cosmic censorship hypothesis, and while