Quantum fluctuations and thermodynamic processes in the presence of closed timelike curves
by Tsunefumi Tanaka

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics
Montana State University

Abstract:
A closed timelike curve (CTC) is a closed loop in spacetime whose tangent vector is everywhere timelike. A spacetime which contains CTC's will allow time travel. One of these spacetimes is Grant space. It can be constructed from Minkowski space by imposing periodic boundary conditions in spatial directions and making the boundaries move toward each other. If Hawking's chronology protection conjecture is correct, there must be a physical mechanism preventing the formation of CTC's. Currently the most promising candidate for the chronology protection mechanism is the back reaction of the metric to quantum vacuum fluctuations. In this thesis the quantum fluctuations for a massive scalar field, a self-interacting field, and for a field at nonzero temperature are calculated in Grant space. The stress-energy tensor is found to remain finite everywhere in Grant space for the massive scalar field with sufficiently large field mass. Otherwise it diverges on chronology horizons like the stress-energy tensor for a massless scalar field.

If CTC's exist they will have profound effects on physical processes. Causality can be protected even in the presence of CTC's if the self-consistency condition is imposed on all processes. Simple classical thermodynamic processes of a box filled with ideal gas in the presence of CTC's are studied. If a system of boxes is closed, its state does not change as it travels through a region of spacetime with CTC's. But if the system is open, the final state will depend on the interaction with the environment. The second law of thermodynamics is shown to hold for both closed and open systems. A similar problem is investigated at a statistical level for a gas consisting of multiple selves of a single particle in a spacetime with CTC's.

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APPROVAL

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Tsunefumi Tanaka

This thesis has been read by each member of the thesis committee, and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Date 5/21/97

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I wish to express my gratitude and appreciation to my grandparents who had set the goal of my life for which I will be always striving.

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Throughout our calculations natural units in which $c = G = \hbar = 1$ are used and the metric signature is $+2$.

ABSTRACT

A closed timelike curve (CTC) is a closed loop in spacetime whose tangent vector is everywhere timelike. A spacetime which contains CTC's will allow time travel. One of these spacetimes is Grant space. It can be constructed from Minkowski space by imposing periodic boundary conditions in spatial directions and making the boundaries move toward each other. If Hawking's chronology protection conjecture is correct, there must be a physical mechanism preventing the formation of CTC's. Currently the most promising candidate for the chronology protection mechanism is the back reaction of the metric to quantum vacuum fluctuations. In this thesis the quantum fluctuations for a massive scalar field, a self-interacting field, and for a field at nonzero temperature are calculated in Grant space. The stress-energy tensor is found to remain finite everywhere in Grant space for the massive scalar field with sufficiently large field mass. Otherwise it diverges on chronology horizons like the stress-energy tensor for a massless scalar field.

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CHAPTER 1

Introduction

In recent years the physics of time travel has been hotly debated. The study of time travel falls into two categories: the (im)possibility argument on time travel and the exploration of physical effects due to time travel if it is possible. The first part of this thesis deals with a physical process, the growth of vacuum fluctuations of quantized fields, which might be able to prevent time travel. The quantized fields are analyzed in a particular model spacetime, called Grant space. It will be shown that the vacuum fluctuations do not always diverge. In the second half simple thermodynamic processes and statistical mechanics of particles in a spacetime allowing time travel are discussed.

Closed Timelike Curves

The concept of time travel in general suggests "going back in time." However, this statement is too ambiguous. A spacetime in which time travel is allowed is one with closed timelike curves. A closed timelike curve (CTC) is defined as *a world line which is a closed loop whose tangent vector is everywhere timelike.* According to a clock carried by an observer on a CTC, time always moves forward. But since his world line is closed, he comes back to the same point in spacetime. To a second observer who is not on a CTC, the first observer appears to be traveling from the future to the past. On a CTC the choice of an event divides other events on the curve into future events and past events only locally. If the observer follows a CTC in the future direction based on his proper time starting from an event X, he will eventually reaches the same event again. This implies that events to the future of X can influence the outcome of an observation at X.

Chronology Horizons

A region of spacetime without any CTC's is called a chronal region; a region with CTC's is called a nonchronal region. At the boundary between chronal and nonchronal regions there exists a chronology horizon. The nonchronal region is bounded to the past by a future chronology horizon and to the future by the past chronology horizon (see Fig. 1). The future chronology horizon is a special type of future Cauchy horizon. It is generated by null geodesics that have no past end points but can leave the horizon when followed into the future [1]. A past chronology horizon is generated by null geodesics that have no future endpoints but can leave the horizon when followed into the past. These null geodesics, called generators, appear to originate from a smoothly closed null geodesic, called the fountain. There must be something deflecting null geodesics around the fountain in order for the generators

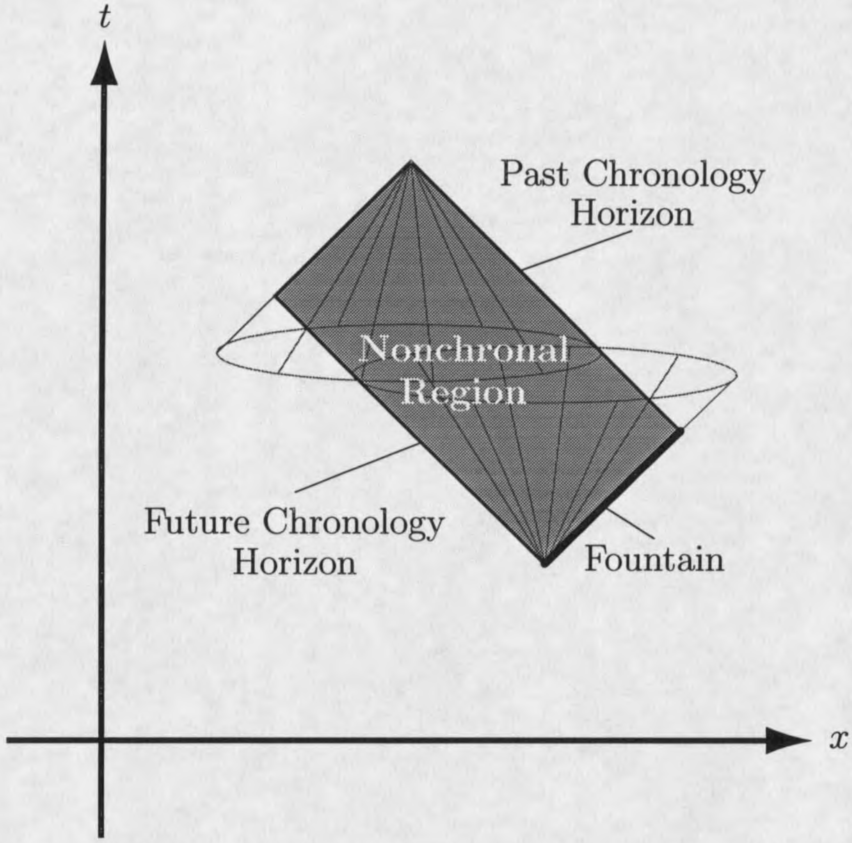

Figure 1: A spacetime with a compact nonchronal region.

to emerge from the fountain [1]. The total energy density of all matter fields around the fountain need to be negative so that a bundle of null geodesics spreads out as it travels along the fountain.

Spacetimes with CTC's

Closed timelike curves appear in some solutions of Einstein field equation, such as Van Stockum space and the Gödel universe, and also in spacetimes with nontrivial topology, for example, Gott space, Misner space, Grant space, and wormhole spacetimes. In the cases of Van Stockum space and the Gödel universe, light cones are tilted in the spatial direction due to the gravitational field. In other cases the spacetime manifold, or at least a part of it, becomes periodic in the time direction. General relativity does not impose any restrictions on the topology of spacetime. Therefore, the topology is a mathematical choice rather than a physical requirement. Misner space, Grant space, and wormhole spacetimes have a non-Hausdorff topology. A brief description of each of these spacetimes follows.

Van Stockum Space

In 1937 Van Stockum discovered a solution to Einstein field equations consisting of an infinitely long cylinder made of rigidly and rapidly rotating dust [2, 3]. The dust particles are held in position by gravitational attractions between them and the centrifugal force due to rotation. Near the surface of the cylinder inertial frames are dragged by rotation so strongly that light cones tilt over in the circumferential direction (See Fig. 2). Frame dragging tilts the light cone so strongly that a velocity vector of a timelike worldline inside the light cone can have a negative time component as seen by an observer far away from the cylinder. A particle following this trajectory can travel backward to an arbitrary point in the past by circling around the cylinder a sufficient number of times. By moving away from the cylinder the particle can start moving forward in time again and reach a point where it started originally. In Van Stockum space CTC's pass through every point in the spacetime, even through the

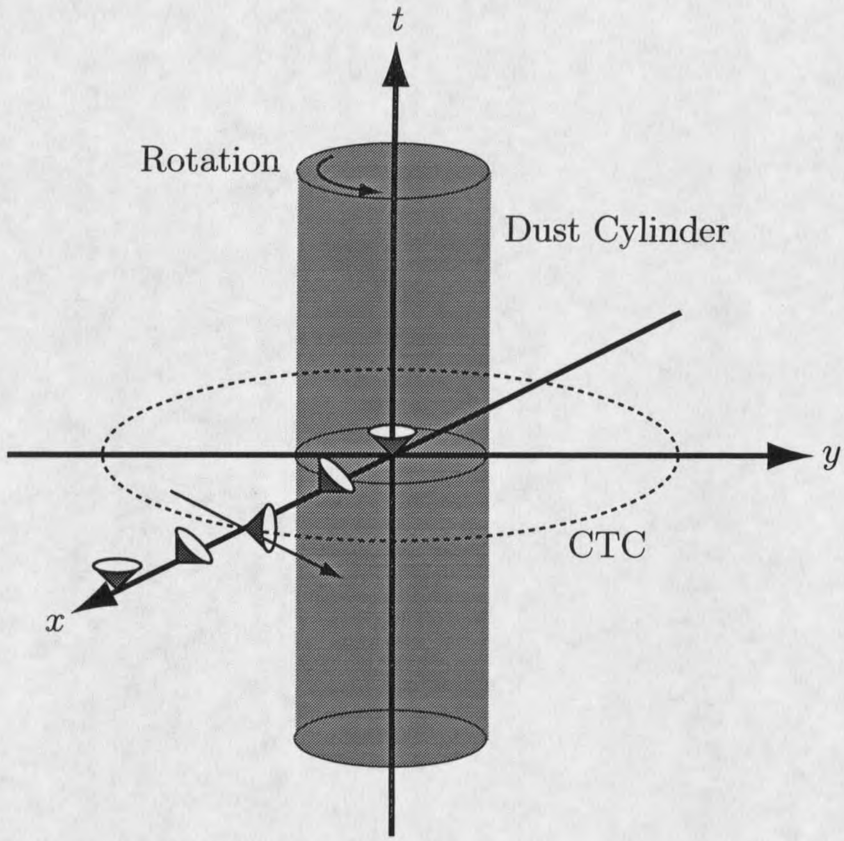

Figure 2: Light cones are tilted in the spatial direction near the surface of the cylinder in Van Stockum space.

center of the cylinder where the light cone is not tilted.

The Gödel Universe

Another solution of Einstein field equation with CTC's is the Gödel universe [4]. It is a stationary, homogeneous cosmological model with nonzero cosmological constant. The universe is filled with rotating, homogeneously distributed dust. The spacetime is rotationally symmetric about any points. Like Van Stockum space CTC's are formed by the tilting of light cones due to inertial frame dragging. On any rotational symmetry axis the light cone is not tilted; it is in the $\frac{\partial}{\partial t}$ direction. As the radial distance from the axis increases, the light cone starts to tilt in the $\frac{\partial}{\partial \phi}$ direction. For radial distances greater than a particular value, $\frac{\partial}{\partial \phi}$ becomes a timelike vector, and a circle of a constant r becomes a closed timelike geodesic. Because the spacetime is homogeneous and stationary, all points in the spacetime are equivalent and CTC's pass through every point [4].

Gott Space

In Gott space two infinitely long, parallel cosmic strings move past each other at high speed without intersecting [5]. Spacetime is flat except on the cosmic string where a conical singularity exists. A circle around the string has a circumference less than $2\pi r$. Gott space can be constructed by cutting out two wedges of a deficit angle $8\pi\mu$ from Minkowski space, where μ is the mass per unit length of the cosmic strings in Planck units, then identifying two edges of each wedge. The apexes of these wedges moves on parallel lines in opposite directions at a high speed. In the center of momentum frame of the strings a point on one side of the wedge and its identified point on the other side of the wedge do not have the same time coordinate due to the motion of the string. Therefore, a path entering the wedge from the leading side in the future exits from the trailing side in the past. By using two cosmic strings a closed timelike path can be formed (Fig. 3).

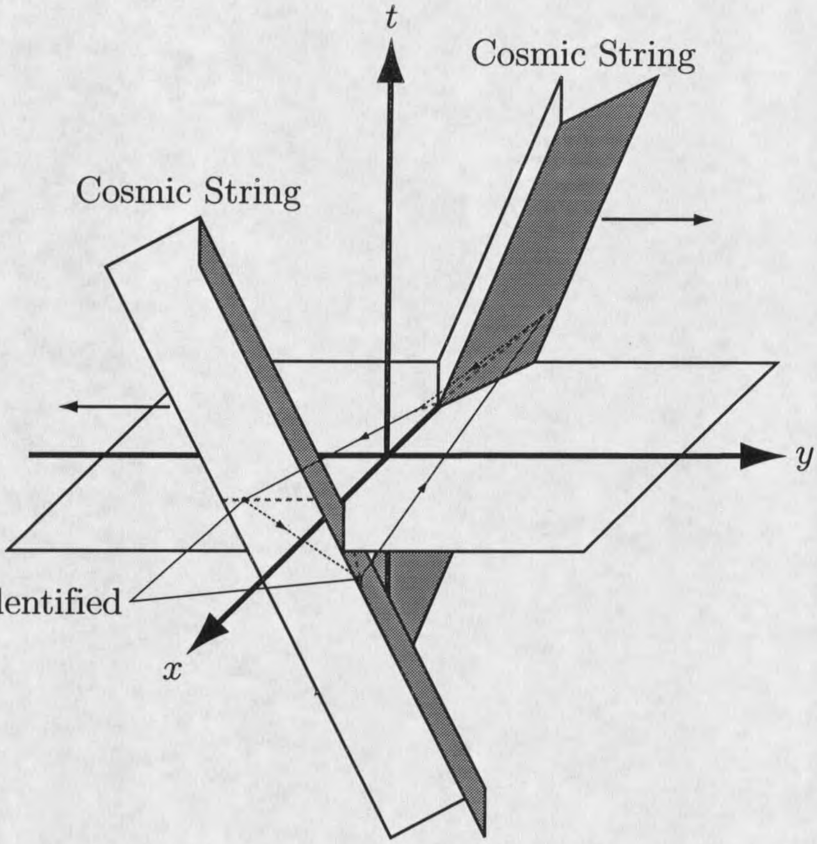

Figure 3: Closed timelike curves are formed around two cosmic strings in Gott space.

Misner Space

Misner space can be constructed from Minkowski space by imposing periodic boundary conditions in a spatial direction [4, 6]. A time shift is then introduced between the proper times of the boundary walls by moving them toward each other at a constant speed [1]. As the walls get closer the time shift becomes equal to the spatial separation between the wall. First a closed null geodesic (fountain) then CTC's are formed as shown in Fig. 4.