Integrated calculus
by Michel Helfgott

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Education
Montana State University

Abstract:
This research addressed the question of whether there were differences in achievement between students that followed an integrated approach to calculus that integrated mathematics and physics, compared to students that followed a non-integrated approach.

The subjects in the study were the 151 students that completed Calculus II (second semester calculus intended mainly for engineering students) at Montana State University-Bozeman during the fall of 1996. There were a total of five sections with five different instructors. All the sections used the Harvard Calculus book and took common exams. Three sections were assigned to the experimental group, which followed the integrated method, while two sections acted as the control group.

Both groups covered the main topics of chapters 6 -10 of the Harvard Calculus book. The instructors in the experimental group stressed problems about applications to physics, as well as the conceptual and computational aspects of calculus. In addition, students in this group received enrichment notes that supplemented the textbook. The instructors in the control group also stressed the conceptual and computational aspects of calculus as well as applications to physics. However, the control group did not delve as deeply into these applications and did not have the support of the enrichment notes.

Analysis of Covariance (ANCOVA), with Calculus I scores and SAT - math scores acting as covariates, was the technique of choice to compare methods with regard to Calculus II and Physics I scores. Physics I is the first semester calculus-based physics course. ANCOVAS were also used with gender as a factor, and when students take Physics I as a factor (not yet, concurrently with Calculus II, or before Calculus II). For interaction analyses, two-way analyses of variance were employed once students were categorized into three groups according to their scores in Calculus I, SAT- math, and Calculus II.

Students in the integrated group did significantly better in Calculus II. Interaction was found when Physics I scores were analyzed, with method and SAT-math groups as factors. Students with high mathematical aptitude in the integrated group scored significantly better than students with high mathematical aptitude in the non-integrated group, when Physics I scores were analyzed. No other interactions were detected. Furthermore, there were no differences in Calculus II achievement according to when students took Physics I. No differences in achievement according to gender were found either.

On the basis of the findings of this study, an integrated approach to the teaching of second semester calculus is recommended.

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of

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APPROVAL

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Michel Helfgott

This thesis has been read by each member of the graduate committee and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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ABSTRACT

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Both groups covered the main topics of chapters 6 - 10 of the Harvard Calculus book. The instructors in the experimental group stressed problems about applications to physics, as well as the conceptual and computational aspects of calculus. In addition, students in this group received enrichment notes that supplemented the textbook. The instructors in the control group also stressed the conceptual and computational aspects of calculus as well as applications to physics. However, the control group did not delve as deeply into these applications and did not have the support of the enrichment notes.

Analysis of Covariance (ANCOVA), with Calculus I scores and SAT - math scores acting as covariates, was the technique of choice to compare methods with regard to Calculus II and Physics I scores. Physics I is the first semester calculus-based physics course. ANCOVAS were also used with gender as a factor, and when students take Physics I as a factor (not yet, concurrently with Calculus II, or before Calculus II). For interaction analyses, two-way analyses of variance were employed once students were categorized into three groups according to their scores in Calculus I, SAT- math, and Calculus II.

Students in the integrated group did significantly better in Calculus II. Interaction was found when Physics I scores were analyzed, with method and SAT-math groups as factors. Students with high mathematical aptitude in the integrated group scored significantly better than students with high mathematical aptitude in the non-integrated group, when Physics I scores were analyzed. No other interactions were detected. Furthermore, there were no differences in Calculus II achievement according to when students took Physics I. No differences in achievement according to gender were found either.

On the basis of the findings of this study, an integrated approach to the teaching of second semester calculus is recommended.

CHAPTER 1

PROBLEM STATEMENT AND REVIEW OF LITERATURE

Introduction

Since calculus was created in the late seventeenth century, the teaching of it has been a matter of concern to mathematicians and educators. Probably, in no other basic mathematical subject can one find so many pedagogical difficulties. This should not be a surprise; for two hundred years (1670 - 1870) the best mathematicians of each generation struggled trying to understand the underlying structure that held calculus together as a coherent whole.

Since Newton's time, calculus has played a central role in mathematics. In its beginnings, it was intimately linked to physics, and it remained so for a long time. Problems in physics led to new mathematical theories that originated from calculus, for instance, differential equations or the calculus of variations. These mathematical theories helped to explain innumerable aspects of physics. Well into the 19th century there was almost no gap between mathematicians and physicists; quite often, their concerns were similar.

Euler's "Introductio in Analysin Infinitorum" (1748) can be considered to be the first textbook on calculus in a modern sense. One way or another, every book on this branch of mathematics published since then stems from Euler's work. The books by Leonhard Euler epitomize a whole epoch concerned with the accelerated development of the subject and its multiple applications.

At the end of the 18th century and the beginning of the 19th century, mathematicians started to analyze with care the foundations of calculus. Joseph Lagrange, and especially Louis Cauchy in the first decades of the past century, started a movement towards rigorization that culminated around 1870 with the works of Karl Weierstrass. Calculus was built around the real line, and depended solely on it.

It took a long time to put calculus on a solid foundation. No wonder that the solution to the problem of rigorization happened to be a sophisticated structure that was out of reach of most beginning college students. While students in Europe benefited from the famous collection of "Cours' d' Analyse" by Cauchy, Picard, Jordan, and other great 19th century mathematicians, teaching of calculus in America lagged far behind. With few exceptions, well into the 1950's, most calculus textbooks lacked rigor and motivation. Applications were few and scattered across the textbooks; little or no connection between mathematics and the real world could be found. Calculus was presented as a series of clever tricks and procedures.

Starting in 1957, with the launching of the Sputnik, mathematics and the natural sciences received great impetus in the United States of America. In the next two decades, new books on the calculus were published, stressing the theoretical aspects of the subject but paying little attention to the links between calculus and physics or chemistry. The pendulum swung completely in the other direction, from a lack of rigor to excessive rigor, somehow blurring the distinction between calculus and real analysis. Quite often during their first year in college, students had to deal with techniques that baffled them. Moreover, applications were relegated to the end of each chapter, many times as optional material. The net result of this way of presenting calculus was a high degree of student failure. Voices of discontent were raised among mathematicians, natural scientists, educators, and the community in general.

A conference was convened at Tulane University in 1986 to address the problem of calculus teaching. This conference is considered to be the beginning of what is now called "Reform Calculus." Radical steps were taken towards promoting a "lean and lively calculus," wherein a balance could be reached between conceptual developments and applications, and where modern technologies would be used in a pervasive way.

Several projects came to light under the inspiration of the Tulane conference. The best known is the one started by a Consortium based at Harvard University. Montana State University-Bozeman has adopted the Harvard Consortium textbook (Hughes-Hallet et al. 1994) for its two semester first year calculus sequence, intended mostly for engineering majors. This book is a radical departure from the traditional presentation since it focuses on enhancing student's understanding, and applications are more numerous than in the past.

It is an open question whether or not the introduction of substantial applications from the natural sciences, within the calculus course, has a pedagogical impact (Ferrini-Mundy and Geuther Graham 1991, p. 633). The purpose of this work is to investigate whether there is a difference in achievement between students that follow the Harvard Calculus textbook, and those students who, besides using the same textbook, receive supplemental materials and study several mathematical aspects of physics related to the course they are taking concurrently in physics.

Statement of the Problem

Is there a difference in achievement between students that follow an integrated Harvard Calculus approach, integrating mathematics and physics, compared to students that follow a Harvard Calculus non-integrated approach?

The Importance of the Study

There is ample evidence that an integrated approach to mathematics teaching, integrating the natural sciences and mathematics, is an advisable path to follow at all levels. However, most of this evidence is anecdotal. There is a need for carefully conducted research to address the question of whether or not integration of academic areas (e.g., mathematics and physics) benefits students as their proponents assert. Some research on the subject has been done at the middle school level, but little at the high school level and almost none at the college level. In particular, calculus is a subject taken by more than half a million college students every year in the USA. Thus, it is important to carefully determine whether steps toward integrating calculus with the natural sciences is a sound option or not. This study purports to give an answer to this question.

Definitions of Terms

For the purpose of this study, the researcher used the following definitions:

Calculus II: Second semester calculus, using the Harvard Consortium book. This course is intended primarily for engineering, mathematics, and the natural sciences students.

Achievement in Calculus II: Achievement in Calculus II was measured by three one-hour examinations and a comprehensive final examination.

Physics I: First semester of a three semester sequence, primarily for engineering and physical sciences students. Covers topics in mechanics.

Achievement in Physics I: Achievement in Physics I was measured by a midterm examination and a comprehensive final examination.

Calculus I: First semester calculus, using the Harvard Consortium book. Intended primarily for engineering, mathematics, and the natural sciences students.

Previous knowledge of Calculus: Previous knowledge of calculus was measured by three one-hour examinations and the final comprehensive examination in Calculus I taken at Montana State University-Bozeman the previous semester.

Harvard Calculus: A reform calculus approach developed by a consortium based at Harvard University.

Integrated Harvard Calculus Approach: This is the approach that follows the content of the Harvard Calculus textbook, and supplements it with enrichment notes that cover several mathematical aspects of mechanics and chemical kinetics that go beyond the textbook. Additionally, application problems from the book are discussed thoroughly.

Non-integrated Harvard Calculus Approach: This is the approach that follows the content of the Harvard Calculus textbook, without the enrichment notes. The non-integrated approach does not delve as deeply into the applications.

Scholastic Aptitude in Mathematics: Scholastic aptitude was measured by the mathematics portion of the SAT or ACT tests.

Interaction: "An interaction between two factors is said to exist if the mean differences among levels of factor A are not constant across levels (categories) of factor B" (Glass and Hopkins, 1996, p. 483).

Questions to be Answered

This study has attempted to answer the following questions:

1. Is there a difference in the adjusted Calculus II achievement means, between the integrated and non-integrated groups, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics?

2. In the analysis of Calculus II achievement, does method of teaching (integrated or non-integrated) interact with previous knowledge of calculus?

3. In the analysis of Calculus II achievement, does method of teaching (integrated or non-integrated) interact with scholastic aptitude in mathematics?

4. Is there a difference in the adjusted Calculus II achievement means, between students that have not yet taken Physics I, are taking Physics I concurrently with Calculus II, and took Physics I before Calculus II, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics?

5. In the analysis of Calculus II achievement, does method of teaching (integrated or non-integrated) interact with the time when students take Physics I (not yet, concurrently with calculus II, or before Calculus II)?

6. Is there a difference in the adjusted Physics I achievement means, between the integrated and non-integrated groups, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics?

7. In the analysis of Physics I achievement, does method of teaching (integrated or non-integrated) interact with previous knowledge of calculus?

8. In the analysis of Physics I achievement, does method of teaching (integrated or non-integrated) interact with scholastic aptitude in mathematics?

9. In the analysis of Physics I achievement, does method of teaching (integrated or non-integrated) interact with achievement in Calculus II?

10. Is there a difference in the adjusted Calculus II achievement means between female and male students, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics?

11. Is there a difference in the adjusted Physics I achievement means between female and male students, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics?

12. In the analysis of Calculus II achievement, does method of teaching (integrated or non-integrated) interact with gender?

13. In the analysis of Physics I achievement, does method of teaching (integrated or non-integrated) interact with gender?

Conceptual Framework

Mathematics has been used in physics for a long time, especially since Galileo Galilei established the basis of the scientific method at the beginning of the 17th century. Mathematics became "the language of science," and its use, both in science research and science instruction, steadily won ground. A natural science was considered to have achieved full maturity once it lost a merely descriptive stage, and a mathematical approach had been established. Chemistry followed the path of physics, and in our century biology started to use a mathematical approach to some degree too.

In contrast, mathematics research and instruction has drifted away, little by little, from the natural sciences. Despite the fact that physics played a very important role in the development of mathematics (calculus is a well-known example of this assertion), contemporary mathematicians - with few and scattered exceptions - have become isolated in their discipline, and have ceased to use physics in their research and instruction almost completely. It is common knowledge that the scientific method, with its stress on exploration, gathering of data, and prediction, is a tool that can be used with great profit in the learning of mathematics.

When we talk about integrating mathematics and the natural sciences in the classroom, we do not mean integration of content but of methodologies. Lynn Steen expresses this idea very clearly, when he analyzes one of the possibilities for integrating mathematics and science (Steen 1992, p. 8):

> [one such possibility is to employ] mathematical methods thoroughly in science, and scientific methods thoroughly in mathematics, coordinating both subjects sufficiently to make this feasible. This is, I submit, an ideal situation. Each discipline, science and mathematics, would accrue benefits from an infusion of methods of the other, but neither would lose its identity or distinguishing features in an artificial effort at union. There are, after all, important differences between science and mathematics, both philosophical, methodological, and historical. These should not be lost in a misguided effort at homogenization.

Further on (p. 12) the same author writes:

> Similarly, the compelling logic of inference and deduction can help the students experience the special power of science. Absent the rigorous logic of inference that is typical of mathematics, science instruction can easily degenerate into description, demonstration, and memorization. Without the intrinsic authority of inference, the authority found in science becomes extrinsic, hence heretical: students believe what teachers tell them, not what they have logically demonstrated from evidence.

The theoretical basis which underlies our study stems from Steen's thoughts, more than from any other thinker's.

Teachers of mechanics use calculus pervasively. It is time that teachers of calculus use physics and chemistry as pedagogical tools in the classroom, drawing illuminating examples from both sciences to impress upon the students the proper idea of the close links between mathematics and the natural sciences. There seems to be much to gain and nothing to lose.

In consonance with the aforementioned concepts, NCTM (1991, p. 70) recommends "connecting mathematics to other subjects and to the world outside the classroom," while AMATYC (1995, p. 16) stresses the fact that " students must have the opportunity to observe the inter-relatedness of scientific and mathematical investigations."

Review of Literature

The Precursors of Integrated Calculus

Kline (1967) was among the first in modern times to warn of the dangers of excessive rigor in the presentation of the calculus. He advocated an intuitive approach. Besides, he saw the need to use physics extensively in the calculus classes:

> The second essential respect in which this book differs from current ones is that the relationship of mathematics to science is taken seriously. The present trend to separate mathematics from science is tragic. There are chapters of mathematics that have value in and for themselves. However, the calculus divorced from applications is meaningless. We should also keep in mind that most of the students taking calculus will be scientists and engineers and these students must learn how to use mathematics. But the step from mathematics to its applications is not simple and straightforward and it creates difficulties for the student from the time he is called upon to solve verbal problems in algebra. The mathematics courses fail to teach students how to formulate physical problems mathematically. The science and engineering courses, on the other hand, assume that students know how to translate physical problems into mathematical language and how to make satisfactory idealizations. The gap between mathematics and science instruction must be filled, and we can do so to our own advantage because thereby we give meaning and motivation to the calculus (Kline 1967, p. vii).

The same author (Kline 1970) quite forcefully advocates an intuitive approach to calculus teaching, with physical arguments playing an important role. This proposal is a continuation of the criticism voiced three years earlier.

Boas (1971) called attention to the advisability of using an approach to calculus teaching similar to the one used by scientists when they teach science. Whether in physics, chemistry, or biology, teachers of science base many of their lectures on well-established experiments carried out in the past. Often these experiments are difficult to replicate or time-consuming, so scientists take them for granted in the classroom setting and continue ahead. Boas maintains that proofs are to mathematics what experiments are to the natural sciences, an analogy that can be used with great profit by teachers of mathematics:

> ... I claim that the teacher of calculus would do well to follow the lead of the experimental scientist. Let him give proofs when they are easy and justify unexpected things; let him omit tedious or difficult proofs, especially those of plausible things. Let him give easy proofs under simplified assumptions rather than complicated proofs under general hypotheses. Let him by all means give correct statements, but not necessarily the most general ones that he knows (Boas 1971, p. 666).

Despite the warnings of noted teachers as Kline and Boas, the same inclination toward rigor at the expense of understanding was followed in the seventies with regard to calculus teaching. In fact, two trends coexisted during this decade: excessive rigor and "watered-down" versions with no rigor at all. The latter trend had dominated calculus teaching for most part of the first half of this century, in concordance with the belief that the theory behind the calculus could only be understood in advanced courses on real analysis. The pendulum started to swing to the other extreme after the launching of the Sputnik when great changes in education were started. Both trends lacked links to physics or to any other natural science.

In the sixties and seventies, students were failing in calculus at an alarming rate, so several solutions to the problem were proposed. Self-pacing, discovery, collaborative learning, computers in calculus instruction, and programmed learning were among them (Teles 1992). However, none of them offered conclusive evidence as to their merits. In some cases there was a small, but not significant difference when compared to traditional calculus courses taught through the lecture format. In others it was difficult to replicate the experiments, leaving the impression that a superb teacher could do marvels with whatever approach he/she adopted.

The Calculus Reform Movement

The situation came to a crisis around 1984 when Ralston (1984) presented the idea to put calculus on a coequal role with discrete mathematics in the first two years of college. This would amount to downsizing calculus, a subject that for a long time had been the center of first and second year college mathematics. The idea was hotly disputed by many mathematicians, Daniel Kleitman and Peter D. Lax among them. Kleitman and Lax contended that calculus should continue to be the core of first year mathematics, since its methods and ideas are at the origin of some of the most impressive developments in mathematics. Lax wrote: "As to calculus, mathematicians need not less, but more of it. The real crisis is that it is badly taught, the syllabus has remained stationary, and modern points of view, especially those having to do with the role of applications and computing are poorly represented" (Ralston 1984, p. 380).

At the 1986 Tulane University conference there was a consensus with regard to the need of using modern technology in the classroom, and of striking a balance between the conceptual and computational aspects of calculus, together with significant applications. The Mathematical Association of America published the proceedings of the conference (Douglass 1986) and a continuation of the proceedings (Steen 1987), both of which have become very influential.

Several reform projects were developed after the Tulane conference. The best known, and the one that has had the greatest impact on calculus teaching, is the project started by a consortium based at Harvard University (Hughes-Hallet et al. 1994). Two basic principles guided their efforts:

1. Every topic should be presented geometrically, numerically, and algebraically.
2. Formal definitions and procedures evolve from the investigation of practical problems.

Research on the Impact of Reform Calculus

Does research support the claims of the Harvard Consortium when compared to traditional approaches? Ratay (1993) used a preliminary edition of the Harvard Consortium textbook with the class of '95 at the United States Merchant Marine Academy, while the class of '94 had used a traditional textbook. The grades of the classes of '94 and '95 were compared for each of the three quarters of their freshman year. Students were grouped according to their mathematics scholastic aptitude test (SAT-math) scores into four categories (500 - 540, 550 - 590, 600 - 640, 650 - 800 ranges), and their average grades were plotted both for the '94 and '95 classes. Results show that the class of '95 consistently outperforms the class of '94, especially for the first quarter among those in the 500 - 540 range. The mean difference is almost a full letter grade for those in the lowest aptitude group. A similar result was obtained when, instead of the SAT-math scores, the researcher used the CPT scores (CPT is an algebra examination administered to all entering freshman at the Academy). The students were divided into four groups according to their CPT scores (20 - 49, 50 - 69, 70 - 89, 90- 120 ranges) and their average grade was calculated. In summary, Ratay found that students earned higher calculus grades using the Harvard consortium book and the benefit was larger among those students with less preparation and aptitude in mathematics. It is to be noted that the work under consideration is of a preliminary nature. The fact that the groups that were compared took different examinations at different times, does diminish the validity of its conclusions. Besides, no significance tests were conducted; the results could be due to chance. Certain trends can be noticed from the graphs of Ratay's paper, but no conclusive statement can be done since no statistical tests were performed.

At Brigham Young University, three calculus programs are taught simultaneously as part of the regular curriculum (Armstrong, Garner and Wynn 1994). Two of them are reform calculus (Harvard Calculus and CUM, calculus using Mathematica), and the third is a traditional approach. The reform calculus programs are characterized as programs where essential use of technology is made, with students learning pertinent applications in an environment where teaching is innovative (group work, interactive teaching methods, et cetera). Two early evaluations compared grades of students in several courses with calculus as a prerequisite. Grades of former calculus students in courses in nine areas, namely biology, chemical engineering, chemistry, civil engineering, electrical engineering, electrical engineering technology, mathematics, physics, and statistics were surveyed.

The first study concluded that Harvard Calculus students did better in six of the nine areas, CUM students did better in mathematics, while traditional calculus students obtained the highest grades in electrical engineering and in statistics courses. However, only the better achievement of the CUM students in subsequent mathematics courses was statistically significant (0.05 level). The second study comprised more students (the authors do not specify the number of students involved in each study). Again, Harvard Calculus students did best in the same six areas, CUM students did best in mathematics and electrical engineering courses, while traditional calculus students did best in statistics. None of the differences were statistically significant.

A third study was also conducted at Brigham Young University, considering calculus grades and ACT (American College Testing) scores. Neither student selection strategies nor instructor differences were taken into consideration. Besides, the CUM group was very small compared to those in the other two programs. No statistical differences in grade point averages in subsequent courses (linear algebra, multidimensional calculus, engineering mathematics, mathematical statistics, and two principles of physics courses) were found.

> While these statistics were not definitive, we were happy to find that reformed calculus students did not do worse statistically than traditional students in any of the courses surveyed. This is so despite the fact that subsequent courses depend upon traditional calculus information. Also, such surveys do not account for other factors making reformed calculus more advantageous, such as more positive student and instructor attitudes, better mastery of concepts and applications, benefits obtained through group study, and advantages to students due to their increased technological expertise (Armstrong, Garner and Wynn 1994, p. 309).

A secondary finding was reported: Students who took second semester traditional calculus after having taken Harvard Calculus the first semester, suffered a significant drop in grades (0.05 alpha level).

Kerry Johnson conducted a four-semester study at Oklahoma State University, comparing Harvard calculus with traditional Calculus (Johnson 1995). Answers were sought to the following questions:

1. Do Harvard students get better grades in calculus than traditional calculus students?
2. Are Harvard students more likely to enroll in subsequent mathematics courses?
3. Do Harvard students perform better in subsequent mathematics courses than other students?
4. How do students that go from Harvard Calculus 1 into Traditional Calculus 2 perform?

With regard to the first question, the answer found is that a higher percentage of the Harvard Calculus students pass the course and make a C or better in the course than traditional calculus students (67% vs. 62% in Calculus 1, 80% vs. 71% in Calculus 2.) There was a varied response to the second question, depending on the course. For example, among students who got a D or better in Calculus 1, 63% of Harvard Calculus students took Calculus 2 compared to 56% of traditional calculus students that took

Calculus 2. Enrollments in Differential Equations were 36% Harvard, 33% traditional, while in Linear Algebra it was Harvard 20%, traditional 27%. With regard to the third question, Johnson found that the answer is no. For example, in Differential Equations, 50% of the D or better Calculus 2 students maintained or improved their grades, while 58% of the traditional calculus students maintained or improved their grades. In Linear Algebra, the difference was Harvard 60% vs. Traditional 69%. Both Differential Equations and Linear Algebra are traditional courses. Finally, as one might expect, it is not advisable to go from Calculus 1 - Harvard Calculus into Calculus 2 - Traditional Calculus. Only 55.3% of these students made a C or better in Calculus 2 compared to more than 80% in the other three possible combinations (Calculus 1 - traditional into calculus 2 - traditional, Calculus 1 - traditional into Calculus 2 - Harvard, and Calculus 1 - Harvard into Calculus 2 - Harvard.) The greater emphasis on algebraic skills in traditional calculus may explain these percentages. No statistical analysis of any kind (besides simple percentages) is reported in the paper under consideration.

The three papers mentioned above have shortcomings which are diverse. For instance, there are no common measures of achievement or carefully set conditions with an experimental and a control group. Some of these difficulties are recognized by the authors, when they write about the "preliminary nature" of their research. There has been little published research concerning reform calculus initiatives (Becker and Pence 1994, p.6), so the research done at the USA Merchant Marine Academy, Brigham Young University, and Oklahoma State University, are a first step forward.

Physical Applications within the Calculus Courses

Joan Ferrini-Mundy and Karen Geuther Graham (1991) put in the forefront of future research the idea tò determine whether or not examples drawn from physics can help in the learning of mathematical concepts related to calculus:

A number of mathematics education research questions arise in conjuction with the calculus effort. Several questions relate to the scope and sequence of the mathematics content. Examples include: How does one decide which parts of the traditional curriculum can best be omitted? Is it more helpful to students to introduce the idea of limit before the idea of derivative? What are the effects of introducing substantial physical applications within the calculus course? Many questions arise that relate directly to student learning and background matters. Examples of relatively broad questions include: Does lack of algebraic facility truly hinder calculus learning? How does the experience of secondary school calculus relate to the experience of college calculus? How does student "intuition" develop? Do physical examples help in the learning of concepts?" (Ferrini-Mundy and Geuther Graham 1991, p. 633).

These research questions, and others mentioned by both authors in the same paper, could have very important consequences since, annually, 600,000 students enroll in some type of calculus course in four-year colleges and universities in the United States of America. Almost half of these students are in mainstream "engineering" calculus, and only 46% finish the year with a grade of D or higher (Ferrini-Mundy and Geuther Graham 1991, p. 627.)

A pilot project was conducted at Dutchess Community College (Poughkeepsie, New York) by Wesley Ostertag, a mathematician, and Tony Zito, a physicist (Ostertag and Zito 1995), fully integrating first year Harvard Calculus with first year physics. Both of them teach this rather unique course, which blends all the topics in first year calculus and physics. Students at Dutchess Community College can enroll in the integrated course or enroll separately in a two semester Harvard Calculus and in a two semester "traditional" physics course. They found that 65% of those in the integrated one-year course obtained a grade of C or better in both semesters, while only 50% of those enrolled in the non-integrated sequence accomplished this goal. The authors do not mention whether or not the objectives and tests were the same. Moreover, a standardized test designed to measure student's understanding of basic kinematics was given as a pre- and post-test to students in both integrated and non-integrated sections. The mean post-test score for the integrated section was 69% (an improvement of 22% over the pre-test mean score), whereas the mean score for the non-integrated section was 61% (an improvement of 15% over the pre-test mean score). The authors do not report whether these results are significant or not.

This type of fully integrated first year calculus-physics course might be of crucial importance for community colleges, since they offer two-year degrees, and thus cannot afford to have first-semester calculus as a prerequisite for first-semester physics (as often happens in four-year colleges and universities).

Integrating College Mathematics with the Natural Sciences

There has been a limited number of efforts toward integrating mathematics teaching and the natural sciences at the college level. Among these we can mention Helfgott (1990), Jean and Iglesias (1990), and Helfgott (1995). The first one describes an integrated approach to differential equations, used in the classroom setting by the author, blending mathematics and several aspects of chemistry and physics:

> Student proficiency and surveys conducted among former pupils show that an integrated approach to several aspects of the natural sciences together with differential equations is highly recommended. Students who followed the integrated approach, instead of the classical differential equations course with few examples of applications outside mathematics, found it less difficult to do the work in later courses in control theory, heat transfer, transport phenomena and chemical kinetics (Helfgott 1990, p. 1014).

The paper by Jean and Iglesias is based on a course developed by the authors, wherein biology and mathematics are blended in a coherent whole. This is a radical departure from traditional courses in mathematics for biology majors. The third paper describes a first-year calculus course where history and the natural sciences (physics and chemistry) are used extensively:

> In teaching the calculus, it is useful to supply many examples of applications from the natural sciences. We go further than usual, developing classical examples from mechanics and chemical kinetics. The latter are particularly helpful because they are simple, require few prerequisites, and use a significant amount of readily available data. We stress the meaning of the scientific method in its different stages of building models, obtaining consequences, and contrasting them with data. Applications appear everywhere, not necessarily at the end of a section (Helfgott 1995, p. 136).

K-12 Integration of Mathematics and Science

The problem of integrating mathematics and science in the K-12 curriculum in the United States has a long history that goes back to E.H. Moore at the turn of the century. Moore, Professor of Mathematics at the University of Chicago, advocated teaching mathematics in close relationship to problems in physics, chemistry, and engineering. His idea found a great deal of support among teachers of high school mathematics and also by teachers of the High School sciences, and eventually led to the formation of the Central Association of Science and Mathematics Teachers, and its influential publication "School Science and Mathematics." Breslich (1936, p. 58) wrote about the Association in the following terms:

> One of the major purposes of the association was to find and establish legitimate contacts between the mathematical subjects and the sciences. It was hoped that the constant training which the pupil derives from applying mathematics to problems in science would increase his mathematical power and that his interest in mathematics would grow with the opportunities of using it in other school subjects. Indeed, some of the leaders of the movement were advocating that algebra, geometry, and physics be organized into a coherent course. If possible, this course was to be taught by the same teacher or at least by two teachers who were in sympathy with the ideas of correlation.

What happened? The efforts toward integration have not been successful, especially at the high school level. The trend toward specialization and the lack of proper training among teachers have conspired against integration. All too often mathematics teachers had little knowledge about science, and teachers of science had little knowledge about mathematics.

Few steps were taken with regard to how mathematics teaching could be improved by using examples drawn from the natural sciences. Fortunately, in the last decade there has been a renewed interest in the subject. Systemic initiatives have started to foster an interdisciplinary approach to mathematics, among these the Systemic Initiative in Montana Mathematics and Science (SIMMS) project in Montana. Its first objective (SIMMS 1993) is to redesign the 9-12 mathematics curriculum using an integrated approach for all students. The project is in its fifth year, and has had a marked impact on high school education in Montana, through its workshops, the publication of high-quality, fully integrated modules, technological support to schools, and the like. Besides, high school teachers all over the nation are developing ways to integrate mathematics and the natural sciences, trying to close the gap between them (Abad 1994, Longhart and Hughes 1995).

At the elementary and middle school level some research has been done about the effect of science and mathematics integration (Friend 1985, Kren and Huntsberger 1977). Friend's main purpose was to determine how integrating science and mathematics in a seventh grade physics unit affected achievement in science. Divided into 4 classes, 108 seventh graders were involved in the study. Two classes consisted of students with standardized reading and mathematics scores at least two years above grade level (AGL), while two classes consisted of students with standardized reading and mathematics scores on grade level (GL). One AGL class and one GL class followed the science and mathematics integration approach.

The investigation lasted 10 weeks, and students were assessed on a common test of physics. An analysis of variance showed that only AGL students taught by the integrated approach, demonstrated significantly greater achievement (0.01 level) than AGL students taught by the non-integrated approach. For GL students there was no significant difference between those in the integrated and non-integrated classes. Moreover, analysis of variance showed that AGL students that followed the non-integrated approach scored significantly higher (0.01 level) than GL students that followed the integrated approach. Friend recommends, on the basis of his findings, that science and mathematics should be integrated for AGL students.

Kren and Huntsberger investigated the effect of integrating science and mathematics instruction on fourth and fifth-grade student achievement in two mathematical skills (measuring and constructing angles, and interpreting and constructing graphs). A total of 161 children from eight classrooms participated in the study. The authors report (Kren and Huntsberger 1977, p. 558) that the treatments of the study were:

1. To present the concept in mathematics first so that the child may be able to apply it in science at a later date.

2. To present the concept in science and mathematics concurrently so that the two disciplines will enhance each other.

3. To present the mathematical concept in science, and follow the presentation with a similar one in mathematics.

An analysis of variance showed that there was no significant difference among the groups. Thus, the aforementioned mathematical skills could be taught with equal effectiveness under any of the three approaches. The authors recommend further investigations of ways science can be used to enhance the teaching of mathematics. The issue of integration is actively discussed nowadays at the middle school level, both by its advocates and by those that have some concerns about its implementation (George 1996, Beane 1996).

The Critics

Reform calculus has provoked a backlash in the academic community. Some critics adopt an emotional stance without justifying their claims that the use of applications in calculus courses may harm the integrity of the subject (Kleinfeld 1996, p. 230) while others raise important questions with regard to calculus teaching, which have to be addressed and discussed objectively. Hu (1996, p. 1538) quite appropriately singularizes four main problems that seem to affect several reform calculus books:

1. Confusion between heuristics and mathematical proof.
2. Less emphasis on symbolic manipulation.
3. Use of computers as a replacement of mathematical thinking.
4. Lack of mathematical closure in the discussion of applications.

There is an ongoing discussion, a healthy development in the mathematical community, which paid little attention to pedagogical issues in the past (Cipra 1996, Wilson 1997). Proponents and detractors of reform calculus, and, in particular, of the utilization of applications as a learning device in the classroom, have laid out their arguments. Everyone hopes that the ongoing debate will remain civil and fruitful.

Final Remarks

Should mathematics and science lose their identity in the quest of integration? Lynn A. Steen, former President of the Mathematical Association of America, gives a negative answer (Steen, 1992). He would rather recommend teachers to employ mathematical methods thoroughly in science, and scientific methods thoroughly in mathematics. That is to say, rather than blending content, Steen advocates blending methods. He sees mathematics and the sciences as different enterprises, one revealing order and pattern, the other seeking to understand nature. Nonetheless, they can contribute a lot to each other through cross-fertilization.

CHAPTER 2

METHODOLOGY

Sample and Population Description

The sample comprised all the students that completed second semester reform calculus (Math 182) at Montana State University-Bozeman, during the fall semester of 1996 (Math 182 is a four credit course that meets five days per week.) Five sections with five different instructors were scheduled at different hours. Three sections were set aside for the experimental treatment, while two sections acted as the control group. Students chose the sections according to their timetables and none of them knew who his or her instructor was going to be until the first day of classes. The students did not know which sections were in the experimental group and which sections were in the control group. In other words, the experimental group did not know that they were being "experimented upon." The population under consideration is intended to simulate the students that enroll and complete second-semester calculus in land-grant institutions of the United States.

Statistical Hypotheses

The questions to be answered by this study, stated in hypothesis form, are the following:

1. There is no difference between the adjusted Calculus II achievement means, of the integrated and the non-integrated groups, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics.

2. Method of instruction (integrated or non-integrated) and previous knowledge of calculus do not interact on Calculus II achievement.

3. Method of instruction (integrated or non-integrated) and scholastic aptitude in mathematics do not interact on Calculus II achievement.

4. When statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics, there is no difference in the adjusted Calculus II achievement means between those students that have not yet taken Physics I, those students that are concurrently taking Physics I and Calculus II, and those students that have taken Physics I before Calculus II.

5. Method of instruction (integrated or non-integrated) and when students take Physics I (not yet, concurrently with Calculus II, or before Calculus II) do not interact on Calculus II achievement.

6. There is no difference between the adjusted Physics I achievement means, of the integrated and the non-integrated groups, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics.

7. Method of instruction (integrated or non-integrated) and previous knowledge of calculus do not interact on Physics I achievement.

8. Method of instruction (integrated or non-integrated) and scholastic aptitude in mathematics do not interact on Physics I achievement.

9. Method of instruction (integrated or non-integrated) and achievement in Calculus II do not interact on Physics I achievement.

10. There is no difference in the adjusted Calculus II achievement means between female and male students, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics.

11. There is no difference in the adjusted Physics I achievement means between female and male students, when statistically equated with respect to previous knowledge of calculus and scholastic aptitude in mathematics.

12. Method of instruction (integrated or non-integrated) and gender do not interact on Calculus II achievement.

13. Method of instruction (integrated or non-integrated) and gender do not interact on Physics I achievement.

Explanation of Experimental Treatments

The main objective of this study was to determine whether or not the integrated method of instruction, which interrelates calculus and mechanics, affected performance in calculus and physics. The three experimental sections covered the same basic materials as the two control sections, namely chapters 6 through 10 of the Harvard Calculus textbook. However, the experimental sections discussed in class many examples from physics. These examples go beyond those presented in the textbook, have strong mathematical components, and are related to topics covered in Physics I. In addition, students in the experimental sections received enrichment notes that supplemented the textbook, a sample of which can be found in appendix E.

The original sample of students that completed Calculus II was statistically analyzed in order to answer hypothesis 5. This sample was smaller whenever Calculus I achievement or SAT scores intervened as factors or covariables, because not every student had taken Calculus I the previous semester and not all the mathematics scholastic aptitude scores were available. Similar considerations had to be applied when Physics I scores were analyzed, corresponding to students that took this course concurrently with

Calculus II. The need to obtain an unbiased score, limited the sample to those students that took Calculus I the previous semester. In other words, the need of common examinations to judge achievement in first semester calculus determined a smaller sample. The latter was analyzed in order to answer several questions of this research, namely questions 1, 2, 6, 7, and 10.

It should be emphasized that students that dropped out from Calculus II were not considered in the study. They had to take all the exams, including the two-hour final, to remain in the sample . The same applies to Physics I students wherever their scores were analyzed. Quizzes, homework, or group work were not analyzed because of the impossibility of obtaining a common measure acceptable to all instructors. These three activities together determined only one sixth of the total score in Calculus I or Calculus II. Neither are physics labs considered.

The Maxmincon Principle

The difference between the integrated and non-integrated groups was made as large as possible, so as to maximize the systematic variance. There were constraints, due to the fact that all sections of Calculus II had to cover the same core materials. Nonetheless, the experimental group dealt with enrichment notes, mainly related to several mathematical aspects of physics that are not normally covered in calculus classes. Besides, the physics problems in the textbook were heavily stressed in the experimental group.

There were two contaminating variables that had to be controlled: previous knowledge of calculus and scholastic aptitude in the area of mathematics. Both were controlled by means of an analysis of covariance. The best way to control all extraneous variables would be through randomization (Kerlinger and Pedhazur 1973, p. 82).

However, in the setting of our study, random assignment to treatments could not be accomplished since students chose the section to be in, according to their class schedules in many different courses. It is to be noted that they did not know who their instructor was until the first day of classes, when groups were already immutable. Thus, because of these conditions, we can assume that there was not any systematic selection bias.

The teacher variable, always present in this type of research, was controlled by having five different instructors; three in the experimental group, one of them the researcher, and two in the control group. The instructors were all graduate teaching assistants with similar backgrounds.

Minimization of error variance was accomplished through reduction of errors of measurement. Students in all five sections of Calculus II took common examinations simultaneously, at a certain common time in the evening especially set aside for this purpose (outside the regular class hours). Moreover, the reliability of the graders of the essay type questions in Calculus II was controlled carefully by establishing a common rubric for partial credit. With regard to the physics examinations, there is confidence that there were not any errors of measurement in grading beyond the normal standard errors since all the exam questions were of the multiple-choice type.

The Hawthorne effect

The Hawthorne effect holds that the researcher's impact on his or her subjects may actually affect the research results. This effect did not affect the results since students in Calculus II did not know that a research project was under way. If students in the experimental and control groups compared their class notes or hand-outs, they may have noticed that there was a difference on how some topics are developed, with applications being stressed, but they could not have noticed anything else.

Enrichment Notes

A sample of the type of additional materials -- to be called "enrichment notes"-- that students in the integrated group used, can be found in appendix E. These notes constituted a distinctive difference between the experimental and control groups. The topics covered there went beyond the common textbook and the common syllabus used by the five sections (experimental and control), and stressed the close links between calculus and physics. In these notes, several models of physics problems were built from first principles; not as accepted differential equations whose origin was unknown to the student. Also, some basic aspects of chemical kinetics were covered in class due to their strong mathematical content. Mathematical closure was sought in the experimental group, in the sense that the enrichment notes served to highlight important aspects of calculus. These notes did not discuss isolated topics, but were fully integrated with the textbook .

Besides, the enrichment notes dealt with some proofs and techniques that cannot be found in the Harvard Calculus book. For instance, the enrichment notes dealt with the fundamental theorem of calculus, the criteria for comparison of improper integrals, and the usual tests to determine the convergence of series. Even though students were not tested on the theoretical aspects of calculus, the instructors in the experimental group considered that it was advisable to provide proofs of some very important propositions.

The Instructors in the Experimental Group

Two instructors were invited by the researcher to participate in the experimental group. They willingly accepted, even though they knew that the integrated method required greater effort on the part of the teacher. A long session took place before classes started, wherein the researcher explained the purpose and methodology of the study.

Thereafter, each week the three instructors met on a regular basis to discuss the enrichment notes, practice exams, quizzes and the like. These meetings went beyond the regular meetings of the five instructors with the course supervisor -- the latter a regular member of the faculty in the department of mathematical sciences. A careful record of the experimental group meetings was kept, and is shown in Appendix C.

Methods of Data Collection

Collection of Quantitative Data

Achievement in Calculus II was measured by three one-hour exams and a comprehensive final, comprising a total of 500 points, 100 for each term exam and 200 for the final. All were common examinations, taken outside the regular hours. A first draft of each of them was made by the course supervisor, an experienced faculty member that was not in charge of teaching a section. This draft was carefully analyzed by each instructor, whose collective responsibility was to check for its content validity. All the instructors got then together and discussed the final version.

With regard to grading, each instructor was assigned the job of grading one question of all the tests. Since the questions were open-ended, of the essay type, the grader had to adopt a consistent policy, especially concerning partial credit. This was achieved by adopting a common rubric, which was strictly followed.

The conditions under which the study was carried out, did not allow a reliability analysis of the test itself (test-retest technique, for example), but we may assume that these tests are reliable due to the extensive body of knowledge accumulated on calculus testing since reform calculus was adopted at MSU-Bozeman for the 181-182 series.

Physics I was taught by one member of the physics department faculty, in a large lecture setting. Achievement in Physics I was measured by the midterm exam and a comprehensive final, both common examinations of the multiple-choice type.

This study deals only with raw scores. Moreover, as was mentioned before, 100 points allotted in Calculus II to group work and quizzes was not considered. These were given at the discretion of each instructor, who could adopt the policy that he/she found best suited for the group. The very nature of the group work and quizzes did not allow a common standard of measurement, thus determining an insurmountable barrier for statistical analysis. For a similar reason, laboratory work in physics or chemistry was not taken into account. The Calculus II exams taken by the students during the fall of 96 are included in appendix A.

The SAT math scores were provided by the Office of Admissions at MSU-Bozeman. Since some students had taken the ACT math but not the SAT math test, a concordance table between SAT and ACT -- supplied by the ACT company and shown in appendix D-- was used. The other covariate scores (Calculus I) were provided by the instructors of first semester calculus, on the basis of three one-hour exams and a comprehensive two-hour final (a total of 500 points).

<u>Instructor Feedback</u>

The researcher interviewed the two instructors that accompanied him in the experimental group. These interviews were conducted at the end of the semester, with a pre-established questionnaire geared toward their experiences in teaching reform calculus with a strong applied component. The almost verbatim transcriptions of the interviews are to be found in the next chapter.

Analytical Techniques and Research Design

Definition of Variables

The scores obtained in the one-hour exams and the final by all students in Math 182 (Fall 96) were added, constituting a number called CalcII. Similarly, their scores obtained in their previous semester calculus (Math 181) -- both in the one-hour exams and the final -- were added; this value was called CalcI. Under the symbol Phy we have scores for those students enrolled in Math 182 that were taking Physics I (Physics 211) concurrently. These scores were obtained in the same way as in Math 181 and Math 182, by adding the scores corresponding to the midterm exam and the final. Each student's scholastic aptitude test in mathematics can be found under a column called SAT. Furthermore, those students that belonged to the experimental group (three sections of Math 182) were coded 1 with regard to Method, while those that belonged to the control group (two sections of Math 182) were coded 2 with regard to Method. The variable WhenPh had three levels (1= not yet taken Physics 211, 2= taking Physics 211 concurrently with Math 182, 3= taken Physics 211 before), while female students were coded 0 and male students were coded 1.

Categorization of Some Variables and Display of Data

In order to study possible interactions, some of the variables were categorized. With this purpose in mind, the lower and upper quartiles (Q1 and Q3, respectively) of CalcI, CalcII, and SAT data were calculated to divide the students in three groups (low, medium, and high) for each of the aforementioned variables. Thus, three new columns were added: CalcIG, CalcIIG, and SATG. The following ten entries were the basis for all subsequent analyses:

Method CalcI CalcII Phy SAT CalcIG CalcIIG SATG WhenPh Gender

Methods of Analysis

Two-way analysis of variance (ANOVA) was the statistical technique used in order to answer questions 2, 3, 5, 7, 8, 9, 12, and 13, while analysis of covariance (ANCOVA) with two covariates, CalcI and SAT, was the chosen statistical tool to answer questions 1, 4, 6, 10, and 11.

Two-way ANOVAS , on eight different instances, were calculated:

- ANOVA for CalcII, with Method and CalcIG as factors
- ANOVA for CalcII, with Method and SATG as factors
- ANOVA for CalcII, with Method and WhenPh as factors
- ANOVA for Phy, with Method and CalcIG as factors
- ANOVA for Phy, with Method and SAT as factors
- ANOVA for Phy, with Method and CalcIIG as factors
- ANOVA for CalcII, with Method and Gender as factors
- ANOVA for Phy, with Method and Gender as factors

Five different ANCOVAS (each of them with two covariates: SAT and CalcI) were calculated:

- ANCOVA for CalcII, with Method as factor
- ANCOVA for CalcII, with WhenPhy as factor
- ANCOVA for Phy, with Method as factor
- ANCOVA for CalcII, with Gender as factor
- ANCOVA for Phy, with Gender as factor

Alpha Level

An alpha level of 0.05 was set before the collection of data took place. This level was adopted instead of a more conservative 0.01 because it is hard to imagine any harm that could be done to the students by advocating an integrated approach to calculus even if no measurable advantages exist. That is to say, a type I error (rejecting the null even though it is true) could not possibly have serious negative effects. This researcher was quite concerned about the possibility of making a type II error (failing to reject the null even though it is false).

Limitations and Delimitations

Limitations

1. One limitation to the study is that students of Calculus II were not randomly assigned to the experimental and control groups. They chose the section to be in well before the beginning of the semester and according to their timetables. Calculus II was taught in five sections, at five different times and by five different instructors. Students did not know that there were going to be experimental and control groups. Thus, despite lack of random assignment, one might expect that chance was not absent from the process of selection of sections. In other words, there is no apparent selection bias.

2. Another limitation of the study was the fact that some students enrolled in Calculus II did not concurrently take Physics I. Only those students that concurrently took Calculus II and Physics I were included as part of the sample used in order to answer questions 6, 7, 8, 9, 11, and 13. A smaller sample does create problems in the realm of statistical analysis. However, the data collected from all students enrolled in Calculus II was used to answer questions 1, 2, 3, 4,

and 10 (provided the student had taken, at MSU-Bozeman, Calculus I the previous semester, and the student's scholastic aptitude test in mathematics was available). Questions 5 and 12 had no restrictions, in the sense that the scores of all students that completed Calculus II could be analyzed.

3. For one reason or another, some students dropped from courses during the semester, reducing the size of the sample. Furthermore, some students had not taken Calculus I at MSU-Bozeman, or their scholastic aptitude test was not kept at the Admissions Office. These two factors also reduced the size of the sample.

4. The background and experience of the teachers in charge of Calculus II were not exactly the same. This fact introduces the well-known "teacher effect" phenomenon, maybe unavoidable in educational research of this type, but nonetheless a limitation. In order to minimize the teacher effect, three sections followed the integrated approach: one taught by the researcher, two others by instructors willing to participate in the experiment. The five sections used the same textbook and followed the same syllabus, a fact that lessened the influence of teaching styles.

5. Once classes start, the groups are immutable. Nonetheless, it can happen that a student may want to move from an integrated group to a non-integrated group or vice versa, for some compelling reason.

6. The course load taken by each student is an uncontrolled variable that may affect student's achievement in Calculus II or Physics I. However, this researcher trusts that the variability due to course load is reflected in Calculus I achievement. A similar consideration can be applied to uncontrolled variables such as number of hours spent on homework, or on part-time jobs. The grades in Calculus I reflect not only mathematical aptitude -- the correlation with SAT scores is rather low as

can be seen in the next chapter -- but also the life style of the student in the sense of how much time he/she devotes to study.

7. Some sections, following the integrated or non-integrated approaches, did group work once per week. In one of the integrated sections, students were encouraged to meet on weekends in order to complete the assigned group problems. Thus, academic activities outside the classroom were an uncontrolled variable.

Delimitations

1. The study was conducted during the 1996 fall semester, at MSU-Bozeman. It can be assumed that this study could be replicated with a sample of students of similar background, in land-grant institutions of the United States.

2. Another delimitation of the study is related to the fact that the study was carried out with students enrolled in a reform calculus course. Reform calculus welcomes applications, thus creating an atmosphere where the integrated approach can be tried. Traditional calculus courses, offered at many institutions (Math 170, 175, and 176 at MSU-Bozeman are an example), may not be as receptive as reform calculus courses with regard to an integrated approach.

CHAPTER 3

RESULTS

The Pilot Study

A pilot study was conducted by the researcher during the spring of 96. All the students that completed Math 182 (CalcII), second semester reform calculus, participated in the study. There were six sections, in charge of six different instructors. The experimental group, taught by the researcher, met daily (Monday through Friday) for 50 minutes, while the other five sections (constituting the control group) also met five times per week for 50 minutes.

There were 32 students in the experimental group (Method 1) and 178 students in the control group (Method 2). The next table shows the number of participants in the study, as a function of the four main variables. MINITAB was the statistical software used.

Table 1. Number of students by method and the four main variables

Row: Variable	Column: Method 1	2	total
CalcII	32	178	210
Phy	18	52	70
CalcI	20	136	156
SAT	28	150	178

In other words, among the 210 students, 156 took Math 181 (CalcI) the previous semester, while 70 students took Physics 211 (first semester physics) during the spring of 96. Through the MSU-Bozeman Office of Admissions, it was possible to obtain the scholastic aptitude scores in mathematics with regard to 178 of the total number of 210 students enrolled in Calculus II.

Some students had taken the ACT, others -- a majority -- the SAT; an official concordance table allowed us to assign a single SAT score to each one of the 178 students.

It is to be noted that all the scores of Calculus I - Fall 95 (three one-hour exams and the final), Calculus II - Spring 96 (three one-hour exams and the final), and Physics 211 (one midterm exam and the final), were obtained directly from the instructors. All the tests were common examinations.

The quartiles (Q1 and Q3) corresponding to Calculus I, Calculus II, and SAT were obtained in order to categorize students in three groups (low, medium, and high). The maximum score for both Calculus I and Calculus II was 500 points, while students could achieve a maximum of 150 points in physics.

Table 2. Means, Medians, and Quartiles Corresponding to the Main Variables

Variable	N	Mean	Median	Q1	Q3
CalcI	156	406.1	409	383.25	441.75
CalcII	210	386.66	391.5	347.00	440.50
SAT	178	613.71	620.00	570.00	662.50
Phy	70	113.74	115.50	106.75	127.00

Table 3. Ranks According to Quartiles

	CalcI	CalcII	SAT
Low	1 - 383	1 - 347	1 - 570
Medium	384 - 441	348 - 441	571 - 662
High	442 - 500	442 - 500	663 - 800

How strong is the linear association between the four main variables? The following table gives an answer to this question.

Table 4. Pearson Correlations Among the Four Main Variables

	CalcI	CalcII	SAT
CalcII	0.504		
SAT	0.366	0.337	
Phy	0.414	0.805	0.289

From this table it can be concluded that there is a strong linear association (0.805) between CalcII and Phy, a moderate linear association (0.504) between CalcI and CalcII, and a weak linear association (0.289) between SAT and Phy. Next, it is important to look at a table of descriptive statistics comparing methods.

Table 5. Descriptive Statistics Comparing Methods

Variable	Method	N	Mean	Median	StDev
CalcII	1	32	386.8	378.0	62.9
	2	178	386.63	393.50	67.16
Phy	1	18	114.17	114.00	20.00
	2	52	113.60	118.00	22.95

There is a striking similarity between the means in CalcII, when method 1 (integrated approach) and method 2 (non-integrated approach) are compared: 386.8 and 386.63, respectively. The means in physics are also very close between both methods (114.17 and 113.60). Their medians do not differ much either. In addition, students in the experimental and control groups were compared with regard to their SAT and CalcI scores.

Table 6. SAT and CalcI Scores in the Experimental and Control Groups

Variable	Method	N	Mean	Median	StDev
SAT	1	28	620.4	650	71.8
	2	150	612.47	620	74.23
CalcI	1	20	391.1	391	46.8
	2	136	408.19	409.50	47.68

Five different two-way ANOVAS, with their respective interaction plots, were calculated so as to reject or retain the null hypothesis related to questions 2, 3, 7, 8, and 9. During the pilot project the variables Gender and WhenPhy were not considered.

Figure 1. Interaction Plot for CalcII Grades with Method and CalcIG as Factors